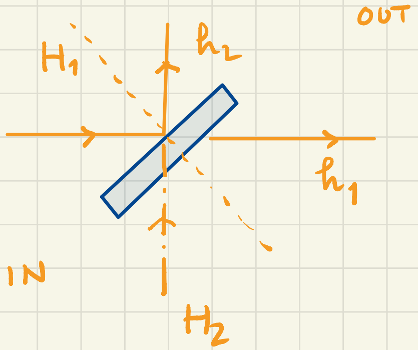


$$\begin{cases} \hat{b}_1 = \overbrace{\cos\theta}^{t}\ \hat{a}_1 + \overbrace{\sin\theta\, e^{i\phi}}^{r'}\ \hat{a}_2 \\ \hat{b}_2 = \underbrace{-\sin\theta\, e^{-i\phi}}_{r}\ \hat{a}_1 + \underbrace{\cos\theta}_{t'}\ \hat{a}_2 \end{cases}$$

Beam-splitter 50/50

$$\frac{1}{\sqrt{2}} = \cos\theta = \sin\theta \Rightarrow \quad \theta = \pi/4$$

$$\begin{cases} \hat{b}_1 = \frac{1}{\sqrt{2}} \left(\hat{a}_1 + e^{i\phi} \hat{a}_2 \right) \\ \hat{b}_2 = \frac{1}{\sqrt{2}} \left(- \hat{a}_1 e^{-i\phi} + \hat{a}_2 \right) \end{cases}$$

$$\begin{cases} \hat{a}_1 = \frac{1}{\sqrt{2}} \left(\hat{b}_1 - \hat{b}_2 e^{i\phi} \right) \\ \hat{a}_2 = \frac{1}{\sqrt{2}} \left(\hat{b}_1 e^{-i\phi} + \hat{b}_2 \right) \end{cases}$$

Un fotone per ciascun modo ingresso

$$|\alpha\rangle = \hat{a}_1^\dagger \hat{a}_2^\dagger |0\rangle$$
$$= \frac{1}{2} (\hat{b}_1^\dagger - \hat{b}_2^\dagger e^{-i\phi})(\hat{b}_1^\dagger e^{+i\phi} + \hat{b}_2^\dagger)|0\rangle$$
$$= \frac{1}{2} (e^{i\phi} \hat{b}_1^\dagger \hat{b}_1^\dagger + \hat{b}_1^\dagger \hat{b}_2^\dagger - \hat{b}_2^\dagger \hat{b}_1^\dagger - e^{-i\phi} \hat{b}_2^\dagger \hat{b}_2^\dagger)|0\rangle$$
$$= \frac{1}{2} (e^{i\phi} \hat{b}_1^\dagger \hat{b}_1^\dagger - e^{-i\phi} \hat{b}_2^\dagger \hat{b}_2^\dagger)|0\rangle$$

Introduciamo le polarizzazioni

- stato $|\alpha\rangle = \hat{H}_1^\dagger \hat{H}_2^\dagger |0\rangle$
$$= \frac{1}{2} (e^{i\phi} \hat{h}_1^\dagger \hat{h}_1^\dagger - e^{-i\phi} \hat{h}_2^\dagger \hat{h}_2^\dagger)|0\rangle$$
- stato $|\alpha\rangle = \hat{V}_1^\dagger \hat{V}_2^\dagger |0\rangle$
$$= \frac{1}{2} (e^{i\phi} \hat{v}_1^\dagger \hat{v}_1^\dagger - e^{-i\phi} \hat{v}_2^\dagger \hat{v}_2^\dagger)|0\rangle$$

$$|\Phi^+\rangle_{12} = \frac{1}{\sqrt{2}} (|H_1 H_2\rangle + |V_1 V_2\rangle)$$
$$|\Phi^-\rangle_{12} = \frac{1}{\sqrt{2}} (|H_1 H_2\rangle - |V_1 V_2\rangle)$$

Per questi 2 stati di Bell, sempre 2 fotoni sulla stessa porta uscita del beam-splitter

$$\bullet \quad |\alpha\rangle = \hat{H}_1^\dagger \hat{V}_2^\dagger |0\rangle$$

$$= \frac{1}{2}\left(e^{i\phi}\, \hat{h}_1^\dagger \hat{v}_1^\dagger - e^{-i\phi}\, \hat{h}_2^\dagger \hat{v}_2^\dagger - \hat{v}_1^\dagger \hat{h}_2^\dagger + \hat{h}_1^\dagger \hat{v}_2^\dagger\right)|0\rangle$$

$$\bullet \quad |\alpha\rangle = \hat{V}_1^\dagger \hat{H}_2^\dagger |0\rangle$$

$$= \frac{1}{2}\left(e^{i\phi}\, \hat{v}_1^\dagger \hat{h}_1^\dagger - e^{-i\phi}\, \hat{v}_2^\dagger \hat{h}_2^\dagger - \hat{h}_1^\dagger \hat{v}_2^\dagger + \hat{v}_1^\dagger \hat{h}_2^\dagger\right)|0\rangle$$

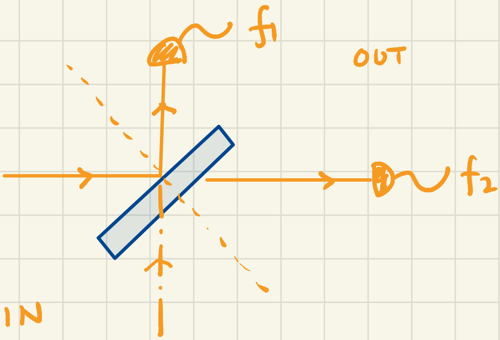
$$|\Psi_{12}^+\rangle \propto \frac{1}{2}\left(e^{i\phi}\, \hat{v}_1^\dagger \hat{h}_1^\dagger - e^{-i\phi}\, \hat{v}_2^\dagger \hat{h}_2^\dagger\right)|0\rangle$$

In uscita dal BS, due fotoni sulla stessa porta

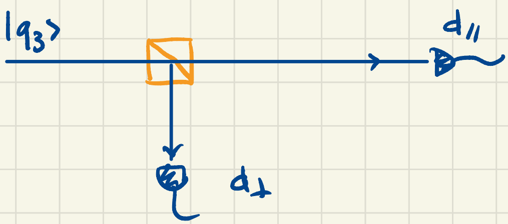
$$|\Psi_{12}^-\rangle \propto \frac{1}{2}\left(- \hat{h}_1^\dagger \hat{v}_2^\dagger + \hat{v}_1^\dagger \hat{h}_2^\dagger\right)|0\rangle$$

In uscita dal BS, un fotone per porta

Analisi dello stato di q_3



– Coincidenza nei due rivelatori
(f_1, f_2)



Osservabili, coincidenze triple

– $(f_1, f_2, d_{\parallel})$

– $(f_1, f_2, d_{\perp})$

Se il teletrasporto ha successo

$$N(f_1, f_2, d_{\parallel}) = N(f_1, f_2)$$

$$N(f_1, f_2, d_{\perp}) = 0$$

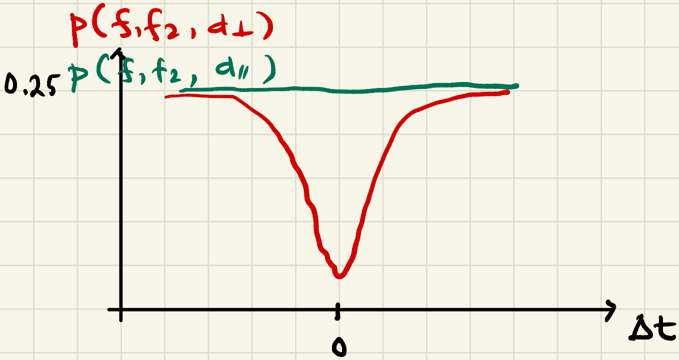
Nell' esperimento, fotoni sono pacchetti d'onda, durata temporale finita.

Se due fotoni temporalmente distinguibili

$$p(f_1, f_2) = 1/2$$

Se 2 fotoni temp. indistinguibili

$$p(f_1, f_2) = p(\Psi_{12}^-) = 1/4$$



Δt ritardo nel tempo di arrivo sul beam-splitter

— 2 fotoni distinguibili

$$p(f_1, f_2, d_\perp) = p(f_1, f_2) \cdot p(d_\perp)$$

$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$p(f_1, f_2, d_\parallel) = p(f_1, f_2) \cdot p(d_\parallel) =$$

$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

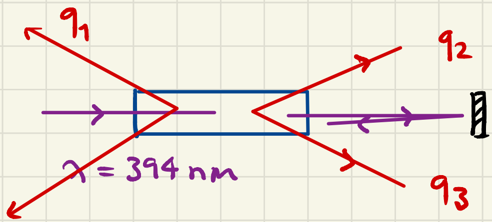
- 2 fotoni INDISTINGUIBILI

$$p(f_1, f_2, d_\perp) = 0$$

$$p(f_1, f_2, d_\parallel) = p(f_1, f_2) = \frac{1}{4}$$

GENERAZIONE DI COPPIE DI FOTONI ENTANGLED

Parametric down-conversion



$\lambda = 788$ nm

$|\Phi^-\rangle_{23}$

Equazione d'onda

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = \frac{1}{c^2 \epsilon_0}\frac{\partial^2}{\partial t^2}\vec{P}$$

$\vec{P}$ ↳ polarizzabilità

$\vec{P} = \epsilon_0 \chi \vec{E}$ mezzi lineari

χ: suscettività

$P_i = \epsilon_0 \chi_{ijk} E_j E_k$ mezzi NON lineari

Onde piane, 3 onde

$$\vec{E}(\vec{r},t) = \sum_{j=1}^{3} \vec{\epsilon}_j A_j e^{i(k_j z - \omega_j t)} + c.c.$$

Separando le componenti di Fourier nell'e. d'onda

$$\begin{cases} \frac{d}{dz} A_1 = i \frac{\omega_1}{n_1 c} \chi^{(2)} A_2^* A_3 \, e^{i(\Delta k z)} \\ \frac{d}{dz} A_2 = i \frac{\omega_2}{n_2 c} \chi^{(2)} A_1^* A_3 \, e^{i \Delta k z} \\ \frac{d}{dz} A_3 \simeq 0 \end{cases}$$