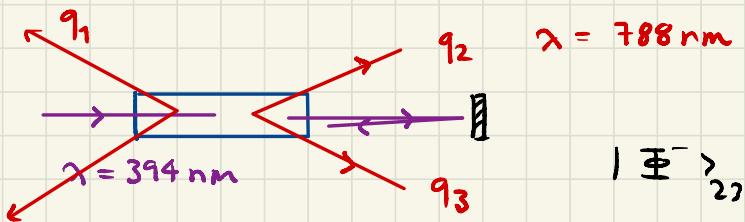


GENERAZIONE DI COPPIE DI FOTONI ENTANGLED

Parametric down-conversion



Equazioni d'onda

$$\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = \frac{1}{c^2 \epsilon_0} \frac{\partial^2}{\partial t^2} \vec{P}$$

\vec{P} polarizzabilità

$$\vec{P} = \epsilon_0 \chi \vec{E}$$

mezzi lineari

↑ suscettività

$$P_i = \epsilon_0 \chi^{(2)}_{ijk} E_j E_k$$

mezzi NON lineari

Onde piane, 3 onde, $\omega_3 = \omega_1 + \omega_2$

$$\vec{E}(\vec{r}, t) = \sum_{j=1}^3 \vec{E}_j(z) e^{i(k_j z - \omega_j t)} + \text{c.c.}$$

Separando le componenti di Fourier nell'onda

Ipotesi $A_1(z)$, $A_2(z)$ variano lentamente

$$\frac{1}{A_1} \frac{\partial}{\partial z} A_1 \ll k_1, \quad \frac{1}{A_2} \frac{\partial}{\partial z} A_2 \ll k_2,$$

$$\left\{ \begin{array}{l} \frac{d}{dz} A_1 = i \frac{\omega_1}{n_1 c} \chi^{(2)} A_2^* A_3 e^{i(\Delta k z)} \\ \frac{d}{dz} A_2 = i \frac{\omega_2}{n_2 c} \chi^{(2)} A_1^* A_3 e^{i \Delta k z} \\ \frac{d}{dz} A_3 \approx 0 \quad A_3 \text{ non varia} \end{array} \right.$$

$$\Delta k = k_3 - k_2 - k_1$$

$$\text{Nell' ipotesi } \Delta k = 0$$

$$\begin{cases} A_1(z) = A_1(0) \cosh(\alpha z) \\ A_2(z) = i \sqrt{\frac{\omega_2 n_1}{\omega_1 n_2}} \sinh(\alpha z) \end{cases}$$

Classicamente PDC non esiste per $A_1(0) = 0$

→ ottica quantistica

Hamiltoniana in rappresentazione d' interazione del campo e.m. nel cristallo

$$H_{\text{SPDC}}(t) \propto \sqrt{\hat{a}_3^+ e^{-i\omega_3 t}} \underbrace{\hat{a}_1^+ e^{+i\omega_1 t}}_{\text{distruzione di } \omega_1} \underbrace{\hat{a}_2^+ e^{+i\omega_2 t}}_{\text{creazione di } \omega_1 + \omega_2}$$

- Resonanza $\omega_3 = \omega_1 + \omega_2$

- $V \propto \int_0^L e^{i\Delta k z} dz$ L lunghezza cristallo

$$= L e^{i\Delta k \frac{L}{2}} \text{sinc}\left(\frac{\Delta k L}{2}\right) \quad \text{sinc}(x) \equiv \frac{\sin x}{x}$$

$\Delta k \ll 1/L$ "phase matching",

$$\Delta k = k_3 - k_2 - k_1 = \left[n_3 \frac{\omega_3}{c} - n_2 \frac{\omega_2}{c} - n_1 \frac{\omega_1}{c} \right]$$

n_i indice di rifrazione

$$= \frac{1}{c} \left[n_3 \omega_3 - n_2 \omega_2 - n_1 \omega_1 \right]$$

Se $\omega_3 = \omega_1 + \omega_2$

$$\Delta k = 0 \quad \text{solo se} \quad n_3 = \frac{n_2 + n_1}{2}$$

In generale, a causa dispersione cromatica, NON accade.

CRISTALLI BIRIFRANGENTI

Indice di rifrazione dipende dalla polarizzazione del campo e.m.

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial^2}{\partial t^2} \vec{D}$$

$$\vec{D} = \underset{\uparrow}{\epsilon} \vec{E}$$

Tensore
matrice 3×3

Scegliendo "assi principali," come assi cartesiani

$$\epsilon = \epsilon_0 \begin{bmatrix} n_1^2 & & \\ & n_2^2 & 0 \\ 0 & & n_3^2 \end{bmatrix}$$

$$\vec{E} = \vec{E}_0 e^{i\vec{k}\vec{r} - i\omega t}$$

Sostituendo

$$\left\{ \begin{array}{l} (k_x^2 - k^2) E_{ox} + k_x k_y E_{oy} + k_x k_z E_{oz} = -\mu_0 \omega^2 \epsilon_0 n_1^2 E_{ox} \\ k_x k_y E_{ox} + (k_y^2 - k^2) E_{oy} + k_y k_z E_{oz} = -\mu_0 \omega^2 \epsilon_0 n_2^2 E_{oy} \\ k_x k_z E_{ox} + k_y k_z E_{oy} + (k_z^2 - k^2) E_{oz} = -\mu_0 \omega^2 \epsilon_0 n_3^2 E_{oz} \end{array} \right.$$

Determinante nullo , $n_1 = n_2 = n_0$, $n_3 = n_e$

$$\left[\frac{\omega^2}{c^2} - \left(\frac{k_x^2 + k_y^2}{n_e^2} + \frac{k_z^2}{n_0^2} \right) \right] \left[\frac{\omega^2}{c^2} - \frac{k^2}{n_0^2} \right] = 0$$

$$\frac{\omega^2}{c^2} - \frac{k^2}{n_0^2} = 0 \rightarrow k^2 = n_0^2 \frac{\omega^2}{c^2}$$

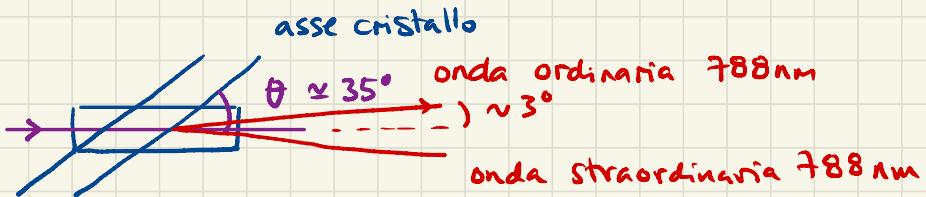
onda ordinaria
in ogni direzione

indice di rifrazione n_0

Onda straordinaria

$$\begin{aligned} \frac{\omega^2}{c^2} &= \frac{k_x^2 + k_y^2}{n_e^2} + \frac{k_z^2}{n_0^2} & k_z &= k \cos \theta \\ &= \left(\frac{\sin^2 \theta}{n_e^2} + \frac{\cos^2 \theta}{n_0^2} \right) k^2 & k_x^2 + k_y^2 &= k^2 \sin^2 \theta \\ &\quad \underbrace{\qquad\qquad\qquad}_{\perp} \qquad \qquad \qquad \qquad \end{aligned}$$

Eperimento Zellinger, cristallo uniaxiale BBO

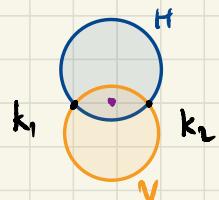


Stato dei fotoni generati a 788 nm

$$|k_1 H\rangle |k_2 V\rangle + e^{i\phi} |k_1 V\rangle |k_2 H\rangle$$

$$|H_1 V_2\rangle + e^{i\phi} |V_1 H_2\rangle$$

$$\downarrow e^{i\phi} = -1 \quad |\Psi_{12}\rangle$$



TELETRASPORTO CON (ATOMI) IONI

Science 323, 486 (2003)

Ioni Yb^+ , intrappolati $\sim 1\text{m}$

$$F = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{array}{c} |1\rangle \\ \text{---} \\ |0\rangle \end{array} - {}^2S_{1/2} \quad I = 1/2 \text{ momento angolare del nucleo}$$
$$\Delta_{HF}$$
$$F = J + I = \{0, 1\}$$

Base computazionale $|0\rangle = |F=0; f_z=0\rangle$

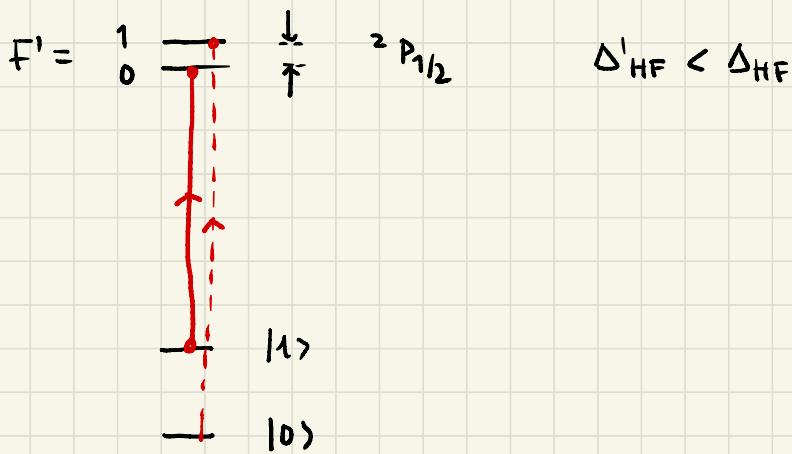
$$|1\rangle = |F=1, f_z=0\rangle$$

$$\Delta_{HF} = h \cdot 12.6 \text{ GHz}$$

$$\begin{cases} |+\rangle_A = \alpha|0\rangle + \beta|1\rangle & \text{oscillazioni di Rabi} \\ |+\rangle_B = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \end{cases}$$

Preparazione dello Stato
di ione A e B

Qubit ausiliari = fotoni



- Eccitazione ione A con impulso di luce

$$\Delta t \sim 1 \text{ ps}$$

$$\Delta f \sim \frac{1}{\Delta t} \sim 10^{12} \text{ Hz} \sim 1 \text{ THz}$$

$$|\psi\rangle_A = \alpha|0\rangle + \beta|1\rangle \rightarrow$$

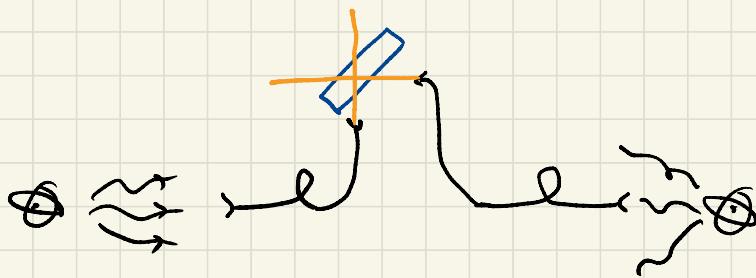
$$|\psi'\rangle_A = \alpha |F'=1, F_z=0\rangle + \beta |F'=0, F_z=0\rangle$$

- Decadimento $\tau \sim 8 \text{ ns}$

$$|\psi'_A\rangle \rightarrow |\psi''_A\rangle = \alpha |0\rangle_A |\psi_B\rangle + \beta |1\rangle_A |\psi_B\rangle$$

Stessa eccitazione + decadimento

$$|2^+\rangle_B = \frac{1}{\sqrt{2}} |0\rangle_B |b\rangle_B + \frac{1}{\sqrt{2}} |1\rangle_B |c\rangle_B$$



Stato complessivo

$$\begin{aligned}
 |4\rangle_A \otimes |4\rangle_B &= \\
 &= \frac{\alpha}{\sqrt{2}} |0\rangle_A |b\rangle_A |0\rangle_B |b\rangle_B \\
 &\quad + \frac{\beta}{\sqrt{2}} |1\rangle_A |\Gamma_A\rangle |0\rangle_B |b\rangle_B \\
 &\quad + \frac{\alpha}{\sqrt{2}} |0\rangle_A |b\rangle_A |1\rangle_B |\Gamma_B\rangle \\
 &\quad + \frac{\beta}{\sqrt{2}} |1\rangle_A |\Gamma_A\rangle |1\rangle_B |\Gamma_B\rangle
 \end{aligned}$$

$$|\Gamma_A\rangle |b\rangle_B = \frac{|\Xi^-\rangle_{AB} + |\Xi^+\rangle_{AB}}{\sqrt{2}}$$

$$|b\rangle_A |\Gamma_B\rangle = \frac{|\Xi^+\rangle_{AB} - |\Xi^-\rangle_{AB}}{\sqrt{2}}$$

→ Misura di Bell, 2 click, uno per porta del beam-splitter

Misura proiettiva su Ξ^- dei fotoni

$$- \frac{\beta}{\sqrt{2}} |1\rangle_A |0\rangle_B + \frac{\alpha}{\sqrt{2}} |0\rangle_A |1\rangle_B$$

→ Rotazione su A

$$\frac{\alpha}{\sqrt{2}} \left(\frac{|0\rangle_A + |1\rangle_A}{\sqrt{2}} \right) |1\rangle_B - \frac{\beta}{\sqrt{2}} \left(\frac{-|0\rangle_A + |1\rangle_A}{\sqrt{2}} \right) |0\rangle_B$$

- Misura proiettiva su zone A , illuminazione
risonante

$$\text{Se proietta } |1\rangle_A \rightarrow |\psi''\rangle_B = \alpha|0\rangle_B - \beta|1\rangle_B$$

$$|0\rangle_A \rightarrow |\psi''\rangle_B = \alpha|0\rangle_B + \beta|1\rangle_B$$