Exam-2025-07-15-exam-mod2

Friday, February 6, 2026 9:53 AM

(5 points) Transform the following propositional logic formula into an equivalent formula in Disjunctive Normal Form:

$$((A \to B) \land (C \lor \neg D)) \lor ((B \land \neg C) \to (A \lor D))$$

$$((A \to B) \land (C \lor \neg D)) \lor ((B \land \neg C) \to (A \lor D))$$

$$((\neg A \lor B) \land (C \lor \neg D)) \lor (\neg(B \land \neg C) \lor (A \lor D))$$

$$((\neg A \lor B) \land C) \lor ((\neg A \lor B) \land \neg D) \lor \neg B \lor C \lor A \lor D$$

SOLUTION $[(C \land \neg A) \lor (C \land B) \lor (\neg A \land \neg D) \lor (B \land \neg D) \lor \neg B \lor C \lor A \lor D$

$$\underbrace{\underbrace{C \lor (C \land \neg A)}_{C} \lor (C \land B)}_{C} \qquad \neg B \lor (B \land \neg D) \qquad \boxed{\text{SAME FOR } A \text{ AND } (\neg A \land \neg D)}$$

$$\underbrace{(\neg B \lor B)}_{T} \land (\neg B \lor \neg D)$$

$$C \lor A \lor \neg D \lor \neg B \lor \neg D \lor D$$

BETTER SOL $[C \lor A \lor \neg B \lor \underbrace{\neg D \lor D}$ } always true tautology

(5 points) Consider the following statement:

"If the alarm is set, then either the door is locked or the security system is active. If the security system is inactive, then the room is secure if and only if the door is locked. If the door is not locked, then the alarm is not set. If the alarm is not set and the room is secure, then the security system must be active. The security system is inactive."

Formalise this statement and determine (with truth tables or otherwise) whether it is inconsistent.

A : allarm is set DL : DOOR LOCKED SA : SECURITY ACTIVE

RS : ROOM SECURE

1) $A \to (DL \lor SA)$

2) $\neg SA \to (RS \leftrightarrow DL)$

3) $\neg DL \to \neg A$

3) $\neg DL \rightarrow \neg A$

4) $(\neg A \wedge RS) \rightarrow SA$

5) $\neg SA$

5) $\neg SA = T$ WHEN $SA = \bot$

2) $\underbrace{\neg SA}_{T} \rightarrow \underbrace{(RS \leftrightarrow DL)}_{\text{WHEN IS } T}$

CASE 1 $RS = \bot$, $DL = \bot$, $SA = \bot$
CASE 2 $RS = T$, $DL = T$, $SA = \bot$

4) $\underbrace{\neg A \wedge RS}_{\text{MUST BE } \bot} \rightarrow \bot$

Case 1 $RS = \bot$ $A = ?$, $DL = \bot$, $SA = \bot$
Case 2 $RS = T$ $A = \bot$, $DL = T$, $SA = \bot$

3) case 1: $\underbrace{\neg \bot}_{T} \rightarrow \underbrace{\neg A}_{T}$ so

case 2: $\underbrace{\bot \rightarrow T}_{T}$

Case 1 $RS = \bot$ $A = \bot$, $DL = \bot$, $SA = \bot$

Case 2 $RS = T$ $A = \bot$, $DL = T$, $SA = \bot$

1) CASE 1: $\bot \rightarrow \ldots$
CASE 2: $\bot \rightarrow \ldots$

always TRUE BOTH IN CASE 1 AND 2
TWO SOLUTION $\rightarrow$ CONSISTENT

2. (5 points) Formalize in FOL the following statements:

- Every coach who trains a tennis player respects at least one other player that the player he trains dislike.
- Every scientist who wrote a paper with a co-author admires at least one person who has never collaborated with any of their co-authors.

1) COACH(X) TRAINS(X,Y) DISLIKES(Y,Z)
PLAYER(X) RESPECT(X,Z)

$$\forall x \forall y ((\text{COACH}(x) \wedge \text{PLAYER}(y) \wedge \text{TRAINS}(x,y)) \rightarrow \exists z (\text{PLAYER}(z) \wedge (z \neq y) \wedge \text{RESPECTS}(x,z) \wedge \text{DISLIKES}(y,z)))$$

2) SCIENTIST(X)
COAUTHOR(Y,X) Y IS COAUTOR OF X
ADMIRE(X,Z)
COLLABORATED(Z,W) PERSON(Z)

COAUTHOR(Y, X)   Y is COAUTHOR OF X

ADMIRE(X, Z)

COLLABORATED(Z, W)   PERSON(Z)

$$\forall x((\text{SCIENTIST}(x) \land \exists y\, \text{COAUTHOR}(y,x)) \rightarrow$$
$$\exists z\, (\text{PERSON}(z) \land \text{ADMIRES}(x,z) \land \forall w(\text{COAUTHOR}(w,x) \rightarrow \neg\text{COLLABORATED}(z,w))))$$

(5 points) Considering the formula F resulting as the formalization of one of the two cases of previous exercise, provide, if possible, an interpretation which satisfies F and an interpretation which does not satisfies F.

NOT SATISFY F1

COACH(a).
PLAYER(b)
TRAIN(a, b)

SATISFY F1

COACH(a)
PLAYER(b)
TRAIN(a, b)
PLAYER(c)
RESPECT(a, c)
DISLIKES(b, c)

(6 points) Consider the following Prolog program:

```
q(alpha, X, Y)    :- q(beta, X, Y), r(Y).
q(beta, X, f(Z))  :- p(X),       r(Z), s(Z).
q(alpha, beta, Z) :- r(Z),       s(Z).
q(gamma, X, g(Z)) :- q(alpha, X, Z), p(Z).

p(c).
p(d).

r(f(a)).
r(g(b)).
r(h(d)).
r(f(f(a))).
r(f(h(d))).

s(g(b)).
```

What is the result of evaluating the query:

```
q(alpha, c, W).
```

Provide all possible solutions for W, if any. Motivate the answer.

The Prolog Program

Rules:

1. q(alpha, X, Y) :- q(beta, X, Y), r(Y).
2. q(beta, X, f(Z)) :- p(X), r(Z), s(Z).
3. q(alpha, beta, Z) :- r(Z), s(Z).
4. q(gamma, X, g(Z)) :- q(alpha, X, Z), p(Z).

Facts:

- p(c)., p(d).
- r(f(a))., r(g(b))., r(h(d))., r(f(f(a)))., r(f(h(d))).
- s(g(b)).

:- q(alpha, c, W)

SOLVE

MATCH RULE 1
X=c   Y=W
:- q(beta, X, Y), r(Y)

NO OTHER MATCH

X=c Y=W

:- q(beta, X, Y), r(Y)

SOLVE (q(beta, X, Y))

NO OTHER MATCH

MATCH RULE 2

X=c W = h(Z)

:- P(c), r(Z), S(Z)

P(c)

↑

IT IS NOT a VARIABLE WE CAN'T TRY OTHER MATCH

NO SOLUTIONS

SOLVE P(c)

P(c) MATCH WITH FACT

SOLVE r(Z)

r(Z) MATCH WITH Z=h(a)

SOLVE S(h(a))

S(h(a)) NO MATCH

MATCH WITH Z=g(b)

S(g(b)) MATCH

FAIL FOR Z=H(d), Z=h(h(a))
Z=(h(h,(d)))

X=c W=h(g(b))

:- q(beta, c, h(g(b))), r(h(g(b)))

SOLVE r(h(g(b)))

NO MATCH

Provide all possible solutions for W, if any. ...

✓ (6 points) Four room types-Economy, Standard, Deluxe, and Suite,Âmust be assigned nightly rates (in euros) that satisfy the following constraints:

- Economy may cost 100, 200, or 300 €, but must cost more than Deluxe.*
- Standard must cost exactly 50 €more than Deluxe,** and its rate must be an even number.
- Deluxe cannot exceed 250 €, and its rate must be a multiple of 50 €.

```
VAR {100, 200, 300} : ECONOMY
VAR INT : STANDARD
VAR INT : DELUXE
VAR INT : SUITE

CONSTRAINT   ECONOMY > DELUXE  *
CONSTRAINT   STANDARD = DELUXE + 50  **
CONSTRAINT   (STANDARD MOD 2) == 0

CONSTRAINT   DELUXE <= 250
CONSTRAINT   (DELUXE MOD 50) == 0

SOLVE   SATISFY
```

* I KNOW IS STRANGE BUT THE TEXT SAY SO