

# Users' manual for the **Sollya** tool 4.0

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## 1 Compilation and installation of **Sollya**

**Sollya** comes in two flavors:

- Either as an interactive tool. This is achieved by running the **Sollya** executable file.
- Or as a C library that provides all the features of the tool within the C programming language.

The installation of the tool and the library follow the same steps, described below. The present documentation focuses more on the interactive tool. As a matter of fact, the library works exactly the same way as the tool, so it is necessary to know a little about the tool in order to correctly use the library. The reader who is only interested in the library should at least read the following Sections 2, 3 and 5. A documentation specifically describing the library usage is available in Appendix 10 at the end of the present documentation.

### 1.1 Compilation dependencies

The **Sollya** distribution can be compiled and installed using the usual `./configure`, `make`, `make install` procedure. Besides a C and a C++ compiler, **Sollya** needs the following software libraries and tools to be installed. The `./configure` script checks for the installation of the libraries. However **Sollya** will build without error if some of its external tools are not installed. In this case an error will be displayed at runtime.

- GMP
- MPFR
- MPFI
- `fp111`
- `libxml2`
- `gnuplot` (external tool)

The use of the external tool `rlwrap` is highly recommended but not required to use the **Sollya** interactive tool. Use the `-A` option of `rlwrap` for correctly displayed ANSI X3.64/ ISO/IEC 6429 colored prompts (see below).

### 1.2 **Sollya** command line options

**Sollya** can read input on standard input or in a file whose name is given as an argument when **Sollya** is invoked. The tool will always produce its output on standard output, unless specifically instructed by a particular **Sollya** command that writes to a file. The following lines are valid invocations of **Sollya**, assuming that `bash` is used as a shell:

```
~/ % sollya
...
~/ % sollya myfile.sollya
...
~/ % sollya < myfile.sollya
```

If a file given as an input does not exist, an error message is displayed.

All configurations of the internal state of the tool are done by commands given on the **Sollya** prompt or in **Sollya** scripts. Nevertheless, some command line options are supported; they work at a very basic I/O-level and can therefore not be implemented as commands.

The following options are supported when calling **Sollya**:

- **--donotmodifystacksize**: When invoked, **Sollya** tries to increase the stack size that is available to a user process to the maximum size supported by the kernel. On some systems, the correspondent **ioctl** does not work properly. Use the option to prevent **Sollya** from changing the stack size.
- **--flush**: When this option is given, **Sollya** will flush all its input and output buffers after parsing and executing each command resp. sequence of commands. This option is needed when pipes are used to communicate with **Sollya** from another program.
- **--help**: Prints help on the usage of the tool and quits.
- **--nocolor**: **Sollya** supports coloring of the output using ANSI X3.64/ ISO/IEC 6429 escape sequences. Coloring is deactivated when **Sollya** is connected on standard input to a file that is not a terminal. This option forces the deactivation of ANSI coloring. This might be necessary on very old grey-scale terminals or when encountering problems with old versions of **rlwrap**.
- **--noprompt**: **Sollya** prints a prompt symbol when connected on standard input to a pseudo-file that is a terminal. The option deactivates the prompt.
- **--oldautoprint**: The behaviour of an undocumented feature for displaying values has changed in **Sollya** from version 1.1 to version 2.0. The old feature is deprecated. If you wish to use it nevertheless, use this deprecated option.
- **--oldrlwrapcompatible**: This option is deprecated. It makes **Sollya** emit a non ANSI X3.64 compliant coloring escape sequence for making it compatible with versions of **rlwrap** that do not support the **-A** option. The option is considered a hack since it is known to garble the output of the tool under some particular circumstances.
- **--warninfile[append] <file>**: Normally, **Sollya** emits warning and information messages together with all other displayed information on either standard output or standard error. This option allows all warning and information messages to get redirected to a file. The filename to be used must be given after the option. When **--warninfile** is used, the existing content (if any) of the file is first removed before writing to the file. With **--warninfileappend**, the messages are appended to an existing file. Even if coloring is used for the displaying all other **Sollya** output, no coloring sequences are ever written to the file. Let us emphasize on the fact that any file of a unixoid system can be used for output, for instance also a named pipe. This allows for error messaging to be performed on a separate terminal. The use of this option is mutually exclusive with the **--warnonstderr** option.
- **--warnonstderr**: Normally, **Sollya** prints warning and information messages on standard output, using a warning color when coloring is activated. When this option is given, **Sollya** will output all warning and information messages on standard error. Coloring will be used even on standard error, when activated. The use of this option is mutually exclusive with the **--warninfile[append]** option.

## 2 Introduction

Sollya is an interactive tool for handling numerical functions and working with arbitrary precision. It can evaluate functions accurately, compute polynomial approximations of functions, automatically implement polynomials for use in math libraries, plot functions, compute infinity norms, etc. Sollya is also a full-featured script programming language with support for procedures etc.

Let us begin this manual with an example. Sollya does not allow command line edition; since this may quickly become uncomfortable, we highly suggest to use the `rlwrap` tool with Sollya:

```
~/ % rlwrap -A sollya
>
```

Sollya manipulates only functions in one variable. The first time that an unbound variable is used, this name is fixed. It will be used to refer to the free variable. For instance, try

```
> f = sin(x)/x;
> g = cos(y)-1;
Warning: the identifier "y" is neither assigned to, nor bound to a library function nor external procedure, nor equal to the current free variable.
Will interpret "y" as "x".
> g;
cos(x) - 1
```

Now, the name  $x$  can only be used to refer to the free variable:

```
> x = 3;
Warning: the identifier "x" is already bound to the free variable, to a library function, library constant or to an external procedure.
The command will have no effect.
Warning: the last assignment will have no effect.
```

If you really want to unbind  $x$ , you can use the `rename` command and change the name of the free variable:

```
> rename(x,y);
Information: the free variable has been renamed from "x" to "y".
> g;
cos(y) - 1
> x=3;
> x;
3
```

Sollya has a reserved keyword that can always be used to refer to the free variable. This keyword is `_x_`. This is particularly useful in contexts when the name of the variable is not known: typically when referring to the free variable in a pattern matching or inside a procedure.

```
> f == sin(_x_)/_x_;
true
```

As you have seen, you can name functions and easily work with them. The basic thing to do with a function is to evaluate it at some point:

```
> f(-2);
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
0.45464871341284084769800993295587242135112748572394
> evaluate(f,-2);
0.45464871341284084769800993295587242135112748572394
```



The printed value is generally a faithful rounding of the exact value at the working precision (i.e. one of the two floating-point numbers enclosing the exact value). Internally **Sollya** represents numbers as floating-point numbers in arbitrary precision with radix 2: the fact that a faithful rounding is performed in binary does not imply much on the exactness of the digits displayed in decimal. The working precision is controlled by the global variable `prec`:

```
> prec;
165
> prec=200;
The precision has been set to 200 bits.
> prec;
200
> f(-2);
Warning: rounding has happened. The value displayed is a faithful rounding to 200 bits of the true result.
0.4546487134128408476980099329558724213511274857239451341894865
```

Sometimes a faithful rounding cannot easily be computed. In such a case, a value is printed that was obtained using floating-point approximations without control on the final accuracy:

```
> log2(5)/log2(17) - log(5)/log(17);
Warning: rounding may have happened.
If there is rounding, the displayed value is *NOT* guaranteed to be a faithful rounding of the true result.
0
```

The philosophy of **Sollya** is: *Whenever something is not exact, print a warning.* This explains the warnings in the previous examples. If the result can be shown to be exact, there is no warning:

```
> sin(0);
0
```

Let us finish this Section with a small complete example that shows a bit of what can be done with **Sollya**:

```
> restart;
The tool has been restarted.
> prec=50;
The precision has been set to 50 bits.
> f=cos(2*exp(x));
> d=[-1/8;1/8];
> p=remez(f,2,d);
> derivativeZeros = dirtyfindzeros(diff(p-f),d);
> derivativeZeros = inf(d)..derivativeZeros..sup(d);
> maximum=0;
> for t in derivativeZeros do {
  r = evaluate(abs(p-f), t);
  if r > maximum then { maximum=r; argmaximum=t; };
};
> print("The infinity norm of", p-f, "is", maximum, "and is reached at", argmaximum);
The infinity norm of -3.89710727796949e-2 * x^2 + -1.79806720921853 * x + (-0.4162655728752966) - cos(2 * exp(x)) is 8.630659444322699e-4 and is reached at 6.663550880713786e-2
```

In this example, we define a function  $f$ , an interval  $d$  and we compute the best degree-2 polynomial approximation of  $f$  on  $d$  with respect to the infinity norm. In other words,  $\max_{x \in d} \{|p(x) - f(x)|\}$  is minimal amongst polynomials with degree not greater than 2. Then, we compute the list of the zeros of the derivative of  $p - f$  and add the bounds of  $d$  to this list. Finally, we evaluate  $|p - f|$  for each point in the list and store the maximum and the point where it is reached. We conclude by printing the result in a formatted way.

Let us mention as a sidenote that you do not really need to use such a script for computing an infinity norm; as we will see, the command `dirtyinfnorm` does this for you.

### 3 General principles

The first purpose of **Sollya** is to help people using numerical functions and numerical algorithms in a safe way. It is first designed to be used interactively but it can also be used in scripts<sup>1</sup>.

One of the particularities of **Sollya** is to work with multi-precision arithmetic (it uses the **MPFR** library). For safety purposes, **Sollya** knows how to use interval arithmetic. It uses interval arithmetic to produce tight and safe results with the precision required by the user.

The general philosophy of **Sollya** is: *When you can perform a computation exactly and sufficiently quickly, do it; when you cannot, do not, unless you have been explicitly asked for.*

The precision of the tool is set by the global variable `prec`. In general, the variable `prec` determines the precision of the outputs of commands: more precisely, the command will internally determine how much precision should be used during the computations in order to ensure that the output is a faithfully rounded result with `prec` bits.

For decidability and efficiency reasons, this general principle cannot be applied every time, so be careful. Moreover certain commands are known to be unsafe: they give in general excellent results and give almost `prec` correct bits in output for everyday examples. However they are merely based on heuristics and should not be used when the result must be safe. See the documentation of each command to know precisely how confident you can be with their result.

A second principle (that comes together with the first one) is the following one: *When a computation leads to inexact results, inform the user with a warning.* This can be quite irritating in some circumstances: in particular if you are using **Sollya** within other scripts. The global variable `verbosity` lets you change the level of verbosity of **Sollya**. When the variable is set to 0, **Sollya** becomes completely silent on standard output and prints only very important messages on standard error. Increase `verbosity` if you want more information about what **Sollya** is doing. Please keep in mind that when you affect a value to a global variable, a message is always printed even if `verbosity` is set to 0. In order to silently affect a global variable, use `!`:

```
> prec=30;
The precision has been set to 30 bits.
> prec=30!;
>
```

For conviviality reasons, values are displayed in decimal by default. This lets a normal human being understand the numbers they manipulate. But since constants are internally represented in binary, this causes permanent conversions that are sources of roundings. Thus you are loosing in accuracy and **Sollya** is always complaining about inexact results. If you just want to store or communicate your results (to another tools for instance) you can use bit-exact representations available in **Sollya**. The global variable `display` defines the way constants are displayed. Here is an example of the five available modes:

<sup>1</sup>Remark: some of the behaviors of **Sollya** slightly change when it is used in scripts. For example, no prompt is printed.

```

> prec=30!;
> a = 17.25;
> display=decimal;
Display mode is decimal numbers.
> a;
1.725e1
> display=binary;
Display mode is binary numbers.
> a;
1.000101_2 * 2^(4)
> display=powers;
Display mode is dyadic numbers in integer-power-of-2 notation.
> a;
69 * 2^(-2)
> display=dyadic;
Display mode is dyadic numbers.
> a;
69b-2
> display=hexadecimal;
Display mode is hexadecimal numbers.
> a;
0x1.14p4

```

Please keep in mind that it is possible to maintain the general verbosity level at some higher setting while deactivating all warnings on roundings. This feature is controlled using the `roundingwarnings` global variable. It may be set to `on` or `off`. By default, the warnings are activated (`roundingwarnings = on`) when `Sollya` is connected on standard input to a pseudo-file that represents a terminal. They are deactivated when `Sollya` is connected on standard input to a real file. See 8.156 for further details; the behavior is illustrated with examples there.

As always, the symbol `e` means  $\times 10^\square$ . The same way the symbol `b` means  $\times 2^\square$ . The symbol `p` means  $\times 16^\square$  and is used only with the `0x` prefix. The prefix `0x` indicates that the digits of the following number until a symbol `p` or white-space are hexadecimal. The suffix `_2` indicates to `Sollya` that the previous number has been written in binary. `Sollya` can parse these notations even if you are not in the corresponding `display` mode, so you can always use them.

You can also use memory-dump hexadecimal notation frequently used to represent IEEE 754 `double` and `single` precision numbers. Since this notation does not allow for exactly representing numbers with arbitrary precision, there is no corresponding `display` mode. However, the commands `printdouble` respectively `printsingle` round the value to the nearest `double` respectively `single`. The number is then printed in hexadecimal as the integer number corresponding to the memory representation of the IEEE 754 `double` or `single` number:

```

> printdouble(a);
0x4031400000000000
> printsingle(a);
0x418a0000

```

`Sollya` can parse these memory-dump hexadecimal notation back in any `display` mode. The difference of this memory-dump notation with the hexadecimal notation (as defined above) is made by the presence or absence of a `p` indicator.

## 4 Variables

As already explained, `Sollya` can manipulate variate functional expressions in one variable. These expressions contain a unique free variable the name of which is fixed by its first usage in an expression

that is not a left-hand-side of an assignment. This global and unique free variable is a variable in the mathematical sense of the term.

**Sollya** also provides variables in the sense programming languages give to the term. These variables, which must be different in their name from the global free variable, may be global or declared and attached to a block of statements, i.e. a begin-end-block. These programming language variables may hold any object of the **Sollya** language, as for example functional expressions, strings, intervals, constant values, procedures, external functions and procedures, etc.

Global variables need not to be declared. They start existing, i.e. can be correctly used in expressions that are not left-hand-sides of assignments, when they are assigned a value in an assignment. Since they are global, this kind of variables is recommended only for small **Sollya** scripts. Larger scripts with code reuse should use declared variables in order to avoid name clashes for example in loop variables.

Declared variables are attached to a begin-end-block. The block structure builds scopes for declared variables. Declared variables in inner scopes shadow (global and declared) variables of outer scopes. The global free variable, i.e. the mathematical variable for variate functional expressions in one variable, cannot be shadowed. Variables are declared using the **var** keyword. See section 8.187 for details on its usage and semantic.

The following code examples illustrate the use of variables.

```
> f = exp(x);
> f;
exp(x)
> a = "Hello world";
> a;
Hello world
> b = 5;
> f(b);
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
1.48413159102576603421115580040552279623487667593878e2
> {var b; b = 4; f(b); };
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
5.45981500331442390781102612028608784027907370386137e1
> {var x; x = 3; };
Warning: the identifier "x" is already bound to the current free variable.
It cannot be declared as a local variable. The declaration of "x" will have no e
ffect.
Warning: the identifier "x" is already bound to the free variable, to a library
function, library constant or to an external procedure.
The command will have no effect.
Warning: the last assignment will have no effect.
> {var a, b; a=5; b=3; {var a; var b; b = true; a = 1; a; b;}; a; b; };
1
true
5
3
> a;
Hello world
```

Let us state that a variable identifier, just as every identifier in **Sollya**, contains at least one character, starts with a ASCII letter and continues with ASCII letters or numerical digits.

## 5 Data types

**Sollya** has a (very) basic system of types. If you try to perform an illicit operation (such as adding a number and a string, for instance), you will get a typing error. Let us see the available data types.

## 5.1 Booleans

There are two special values **true** and **false**. Boolean expressions can be constructed using the boolean connectors **&&** (and), **||** (or), **!** (not), and comparisons.

The comparison operators **<**, **<=**, **>** and **>=** can only be used between two numbers or constant expressions.

The comparison operators **==** and **!=** are polymorphic. You can use them to compare any two objects, like two strings, two intervals, etc. As a matter of fact, polymorphism is allowed on both sides: it is possible to compare objects of different type. Such objects of different type, as they can never be syntactically equal, will always compare unequal (see exception for **error**, section 8.53) and never equal. It is important to remember that testing the equality between two functions will return **true** if and only if the expression trees representing the two functions are exactly the same. See 8.53 for an exception concerning the special object **error**. Example:

```
> 1+x==1+x;  
true
```

## 5.2 Numbers

Sollya represents numbers as binary multi-precision floating-point values. For integer values and values in dyadic, binary, hexadecimal or memory dump notation, it automatically uses a precision needed for representing the value exactly (unless this behaviour is overridden using the syntax given below). Additionally, automatic precision adaption takes place for all integer values (even in decimal notation) written without the exponent sign **e** or with the exponent sign **e** and an exponent sufficiently small that they are less than  $10^{999}$ . Otherwise the values are represented with the current precision **prec**. When a number must be rounded, it is rounded to the precision **prec** before the expression get evaluated:

```
> prec=12!;  
> 4097.1;  
Warning: Rounding occurred when converting the constant "4097.1" to floating-point with 12 bits.  
If safe computation is needed, try to increase the precision.  
4098  
> 4098.1;  
Warning: Rounding occurred when converting the constant "4098.1" to floating-point with 12 bits.  
If safe computation is needed, try to increase the precision.  
4098  
> 4097.1+1;  
Warning: Rounding occurred when converting the constant "4097.1" to floating-point with 12 bits.  
If safe computation is needed, try to increase the precision.  
4099
```

As a matter of fact, each variable has its own precision that corresponds to its intrinsic precision or, if it cannot be represented, to the value of **prec** when the variable was set. Thus you can work with variables having a precision higher than the current precision.

The same way, if you define a function that refers to some constant, this constant is stored in the function with the current precision and will keep this value in the future, even if **prec** becomes smaller.

If you define a function that refers to some variable, the precision of the variable is kept, independently of the current precision:

```

> prec = 50!;
> a = 4097.1;
Warning: Rounding occurred when converting the constant "4097.1" to floating-point with 50 bits.
If safe computation is needed, try to increase the precision.
> prec = 12!;
> f = x + a;
> g = x + 4097.1;
Warning: Rounding occurred when converting the constant "4097.1" to floating-point with 12 bits.
If safe computation is needed, try to increase the precision.
> prec = 120;
The precision has been set to 120 bits.
> f;
4.097099999999998544808477163314819335e3 + x
> g;
4098 + x

```

In some rare cases, it is necessary to read in decimal constants with a particular precision being used in the conversion to the binary floating-point format, which **Sollya** uses. Setting **prec** to that precision may prove to be an insufficient means for doing so, for example when several different precisions have to be used in one expression. For these rare cases, **Sollya** provides the following syntax: decimal constants may be written *%precision%constant*, where *precision* is a constant integer, written in decimal, and *constant* is the decimal constant. **Sollya** will convert the constant *constant* with precision *precision*, regardless of the global variable **prec** and regardless if *constant* is an integer or would otherwise be representable.

[illegible]

Sollya is an environment that uses floating-point arithmetic. The IEEE 754-2008 standard on floating-point arithmetic does not only define floating-point numbers that represent real numbers but also floating-point data representing infinities and Not-a-Numbers (NaNs). Sollya also supports infinities and NaNs in the spirit of the IEEE 754-2008 standard without taking the standard’s choices literally.

- Signed infinities are available through the `Sollya` objects `infy`, `-infy`, `@Inf@` and `@-Inf@`.
- Not-a-Numbers are supported through the `Sollya` objects `NaN` and `@NaN@`. `Sollya` does not have support for NaN payloads, signaling or quiet NaNs or signs of NaNs. Signaling NaNs are supported on input for single and double precision memory notation (see section 3). However, they immediately get converted to plain `Sollya` NaNs.

The evaluation of an expression involving a NaN or the evaluation of a function at a point being NaN always results in a NaN.

Infinites are considered to be the limits of expressions tending to infinity. They are supported as bounds of intervals in some cases. However, particular commands might prohibit their use even though there might be a mathematical meaning attached to such expressions. For example, while `Sollya` will evaluate expressions such as  $\lim_{x \rightarrow -\infty} e^x$ , expressed e.g. through `evaluate(exp(x), [-infy; 0])`, it will

not accept to compute the (finite) value of

$$\int_{-\infty}^0 e^x dx.$$

The following examples give an idea of what can be done with Sollya infinities and NaNs. Here is what can be done with infinities:

```
> f = exp(x) + 5;
> f(-infty);
5
> evaluate(f, [-infty; infty]);
[5; @Inf@]
> f(infty);
@Inf@
> [-infty; 5] * [3; 4];
[-@Inf@; 20]
> -infty < 5;
true
> log(0);
-@Inf@
> [log(0); 17];
Warning: the given expression is not a constant but an expression to evaluate
and a faithful evaluation is not possible.
Will use a plain floating-point evaluation, which might yield a completely wrong
value.
Warning: inclusion property is satisfied but the diameter may be greater than the
least possible.
[-@Inf@; 17]
>
```

And the following example illustrates NaN behavior.



```

> 3/0;
Warning: the given expression is undefined or numerically unstable.
@NaN@
> (-3)/0;
Warning: the given expression is undefined or numerically unstable.
@NaN@
> infty/infy;
Warning: the given expression is undefined or numerically unstable.
@NaN@
> infy + infy;
@Inf@
> infy - infy;
Warning: the given expression is undefined or numerically unstable.
@NaN@
> f = exp(x) + 5;
> f(NaN);
@NaN@
> NaN == 5;
false
> NaN == NaN;
false
> NaN != NaN;
false
> X = "Vive la Republique!";
> !(X == X);
false
> X = 5;
> !(X == X);
false
> X = NaN;
> !(X == X);
true
>

```

### 5.3 Rational numbers and rational arithmetic

The **Sollya** tool is mainly based on floating-point arithmetic: wherever possible, floating-point algorithms, including algorithms using interval arithmetic, are used to produce approximate but safe results. For some particular cases, floating-point arithmetic is not sufficient: some algorithms just require natural and rational numbers to be handled exactly. More importantly, for these applications, it is required that rational numbers be displayed as such.

**Sollya** implements a particular mode that offers a lightweight support for rational arithmetic. When needed, it can be enabled by assigning **on** to the global variable **rationalmode**. It is disabled by assigning **off**; the default is **off**.

When the mode for rational arithmetic is enabled, **Sollya**'s behavior will change as follows:

- When a constant expression is given at the **Sollya** prompt, **Sollya** will first try to simplify the expression to a rational number. If such an evaluation to a rational number is possible, **Sollya** will display that number as an integer or a fraction of two integers. Only if **Sollya** is not able to simplify the constant expression to a rational number, it will launch the default behavior of evaluating constant expressions to floating-point numbers that are generally faithful roundings of the expressions.
- When the global mode **autosimplify** is **on**, which is the default, **Sollya** will additionally use rational arithmetic while trying to simplify expressions given in argument of commands.

Even when `rationalmode` is on, Sollya will not be able to exhibit integer ratios between transcendental quantities. For example, Sollya will not display  $\frac{1}{6}$  for  $\arcsin(\frac{1}{2})/\pi$  but 0.16666.... Sollya's evaluator for rational arithmetic is only able to simplify rational expressions based on addition, subtraction, multiplication, division, negation, perfect squares (for square root) and integer powers.

The following example illustrates what can and what cannot be done with Sollya's mode for rational arithmetic:

```
> 1/3 - 1/7;
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
0.19047619047619047619047619047619047619047619
> rationalmode = on;
Rational mode has been activated.
> 1/3 - 1/7;
4 / 21
> (2 + 1/7)^2 + (6/7)^2 + 2 * (2 + 1/7) * 6/7;
9
> rationalmode = off;
Rational mode has been deactivated.
> (2 + 1/7)^2 + (6/7)^2 + 2 * (2 + 1/7) * 6/7;
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
9
> rationalmode = on;
Rational mode has been activated.
> asin(1)/pi;
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
0.5
> sin(1/6 * pi);
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
0.5
> exp(1/7 - 3/21) / 7;
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
0.142857142857142857142857142857142857142857145
> rationalmode = off;
Rational mode has been deactivated.
> exp(1/7 - 3/21) / 7;
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
0.142857142857142857142857142857142857142857145
> print(1/7 - 3/21);
1 / 7 - 3 / 21
> rationalmode = on;
Rational mode has been activated.
> print(1/7 - 3/21);
0
```

## 5.4 Intervals and interval arithmetic

Sollya can manipulate intervals that are closed subsets of the real numbers. Several ways of defining intervals exist in Sollya. There is the most common way where intervals are composed of two numbers or constant expressions representing the lower and the upper bound. These values are separated either by commas or semi-colons. Interval bound evaluation is performed in a way that ensures the inclusion

property: all points in the original, unevaluated interval will be contained in the interval with its bounds evaluated to floating-point numbers.

```
> d=[1;2];
> d2=[1,1+1];
> d==d2;
true
> prec=12!;
> 8095.1;
Warning: Rounding occurred when converting the constant "8095.1" to floating-point with 12 bits.
If safe computation is needed, try to increase the precision.
8096
> [8095.1; 8096.1];
Warning: Rounding occurred when converting the constant "8095.1" to floating-point with 12 bits.
The inclusion property is nevertheless satisfied.
Warning: Rounding occurred when converting the constant "8096.1" to floating-point with 12 bits.
The inclusion property is nevertheless satisfied.
[8094;8098]
```

Sollya has a mode for printing intervals that are that thin that their bounds have a number of decimal digits in common when printed. That mode is called `midpointmode`; see below for an introduction and section 8.104 for details. As Sollya must be able to parse back its own output, a syntax is provided to input intervals in midpoint mode. However, please pay attention to the fact that the notation used in midpoint mode generally increases the width of intervals: hence when an interval is displayed in midpoint mode and read again, the resulting interval may be wider than the original interval.

```
> midpointmode = on!;
> [1.725e4;1.75e4];
0.17~2/5~e5
> 0.17~2/5~e5;
0.17~2/5~e5
> midpointmode = off!;
> 0.17~2/5~e5;
[17200;17500]
```

In some cases, intervals become infinitely thin in theory, in which case one tends to think of point intervals even if their floating-point representation is not infinitely thin. Sollya provides a very convenient way for input of such point intervals. Instead of writing `[a;a]`, it is possible to just write `[a]`. Sollya will expand the notation while making sure that the inclusion property is satisfied:

```
> [3];
[3;3]
> [1/7];
Warning: the given expression is not a constant but an expression to evaluate. A faithful evaluation to 29 bits will be used.
[0.14285713;0.142857149]
> [exp(8)];
Warning: the given expression is not a constant but an expression to evaluate. A faithful evaluation to 29 bits will be used.
[2.9809577e3;2.980958e3]
```

When the mode `midpointmode` is set to on (see 8.104), Sollya will display intervals that are provably reduced to one point in this extended interval syntax. It will use `midpointmode` syntax for intervals that are sufficiently thin but not reduced to one point (see section 8.104 for details):

```

> midpointmode = off;
Midpoint mode has been deactivated.
> [17;17];
[17;17]
> [exp(pi);exp(pi)];
Warning: the given expression is not a constant but an expression to evaluate. A
faithful evaluation to 170 bits will be used.
[2.31406926327792690057290863679485473802661062426e1;2.3140692632779269005729086
3679485473802661062426008e1]
> midpointmode = on;
Midpoint mode has been activated.
> [17;17];
[17]
> [exp(pi);exp(pi)];
Warning: the given expression is not a constant but an expression to evaluate. A
faithful evaluation to 170 bits will be used.
0.231406926327792690057290863679485473802661062426~0/1~e2
>

```

Sollya intervals are internally represented with floating-point numbers as bounds; rational numbers are not supported here. If bounds are defined by constant expressions, these are evaluated to floating-point numbers using the current precision. Numbers or variables containing numbers keep their precision for the interval bounds.

Constant expressions get evaluated to floating-point values immediately; this includes  $\pi$  and rational numbers, even when `rationalmode` is on (see section 5.3 for this mode).

```

> prec = 300!;
> a = 4097.1;
Warning: Rounding occurred when converting the constant "4097.1" to floating-poi
nt with 300 bits.
If safe computation is needed, try to increase the precision.
> prec = 12!;
> d = [4097.1; a];
Warning: Rounding occurred when converting the constant "4097.1" to floating-poi
nt with 12 bits.
The inclusion property is nevertheless satisfied.
> prec = 300!;
> d;
[4096;4.0971e3]
> prec = 30!;
> [-pi;pi];
Warning: the given expression is not a constant but an expression to evaluate. A
faithful evaluation to 35 bits will be used.
Warning: the given expression is not a constant but an expression to evaluate. A
faithful evaluation to 35 bits will be used.
[-3.141592655;3.141592655]

```

You can get the upper-bound (respectively the lower-bound) of an interval with the command `sup` (respectively `inf`). The middle of the interval can be computed with the command `mid`. Let us also mention that these commands can also be used on numbers (in that case, the number is interpreted as an interval containing only one single point. In that case the commands `inf`, `mid` and `sup` are just the identity):

```

> d=[1;3];
> inf(d);
1
> mid(d);
2
> sup(4);
4

```

Let us mention that the `mid` operator never provokes a rounding. It is rewritten as an unevaluated expression in terms of `inf` and `sup`.

`Sollya` permits intervals to also have non-real bounds, such as infinities or NaNs. When evaluating certain expressions, in particular given as interval bounds, `Sollya` will itself generate intervals containing infinities or NaNs. When evaluation yields an interval with a NaN bound, the given expression is most likely undefined or numerically unstable. Such results should not be trusted; a warning is displayed.

While computations on intervals with bounds being NaN will always fail, `Sollya` will try to interpret infinities in the common way as limits. However, this is not guaranteed to work, even if it is guaranteed that no unsafe results will be produced. See also section 5.2 for more detail on infinities in `Sollya`. The behavior of interval arithmetic on intervals containing infinities or NaNs is subject to debate; moreover, there is no complete consensus on what should be the result of the evaluation of a function  $f$  over an interval  $I$  containing points where  $f$  is not defined. `Sollya` has its own philosophy regarding these questions. This philosophy is explained in Appendix 9 at the end of this document.

```

> evaluate(exp(x),[-infy;0]);
[0;1]
> dirtyinfnorm(exp(x),[-infy;0]);
Warning: a bound of the interval is infinite or NaN.
This command cannot handle such intervals.
@NaN@
>
> f = log(x);
> [f(0); f(1)];
Warning: the given expression is not a constant but an expression to evaluate
and a faithful evaluation is not possible.
Will use a plain floating-point evaluation, which might yield a completely wrong
value.
Warning: inclusion property is satisfied but the diameter may be greater than th
e least possible.
[-@Inf@;0]
>

```

`Sollya` internally uses interval arithmetic extensively to provide safe answers. In order to provide for algorithms written in the `Sollya` language being able to use interval arithmetic, `Sollya` offers native support of interval arithmetic. Intervals can be added, subtracted, multiplied, divided, raised to powers, for short, given in argument to any `Sollya` function. The tool will apply the rules of interval arithmetic in order to compute output intervals that safely encompass the hull of the image of the function on the given interval:

```

> [1;2] + [3;4];
[4;6]
> [1;2] * [3;4];
[3;8]
> sqrt([9;25]);
[3;5]
> exp(sin([10;100]));
[0.36787942;2.7182819]

```

When such expressions involving intervals are given, **Sollya** will follow the rules of interval arithmetic in precision **prec** for immediately evaluating them to interval enclosures. While **Sollya**'s evaluator always guarantees the inclusion property, it also applies some optimisations in some cases in order to make the image interval as thin as possible. For example, **Sollya** will use a Taylor expansion based evaluation if a composed function, call it  $f$ , is applied to an interval. In other words, in this case **Sollya** will behave as if the **evaluate** command (see section 8.54) were implicitly used. In most cases, the result will be different from the one obtained by replacing all occurrences of the free variable of a function by the interval the function is to be evaluated on:

```
> f = x - sin(x);
> [-1b-10;1b-10] - sin([-1b-10;1b-10]);
[-1.95312484477957829894e-3;1.95312484477957829894e-3]
> f([-1b-10;1b-10]);
[-1.55220421701117626897e-10;1.55220421701117626897e-10]
> evaluate(f, [-1b-10;1b-10]);
[-1.55220421701117626897e-10;1.55220421701117626897e-10]
```

## 5.5 Functions

**Sollya** knows only about functions with one single variable. The first time in a session that an unbound name is used (without being assigned) it determines the name used to refer to the free variable.

The basic functions available in **Sollya** are the following:

- $+$ ,  $-$ ,  $*$ ,  $/$ ,  $^$
- **sqrt**
- **abs**
- **sin**, **cos**, **tan**, **sinh**, **cosh**, **tanh**
- **asin**, **acos**, **atan**, **asinh**, **acosh**, **atanh**
- **exp**, **expm1** (defined as  $\text{expm1}(x) = \exp(x) - 1$ )
- **log** (natural logarithm), **log2** (binary logarithm), **log10** (decimal logarithm), **log1p** (defined as  $\text{log1p}(x) = \log(1 + x)$ )
- **erf**, **erfc**
- **halfprecision**, **single**, **double**, **doubleextended**, **doubledouble**, **quad**, **tripledouble** (see sections 8.75, 8.165, 8.46, 8.48, 8.47, 8.138 and 8.184)
- **HP**, **SG**, **D**, **DE**, **DD**, **QD**, **TD** (see sections 8.75, 8.165, 8.46, 8.48, 8.47, 8.138 and 8.184)
- **floor**, **ceil**, **nearestint**.

The constant  $\pi$  is available through the keyword **pi** as a 0-ary function:

```
> display=binary!;
> prec=12!;
> a=pi;
> a;
Warning: rounding has happened. The value displayed is a faithful rounding to 12
bits of the true result.
1.10010010001_2 * 2^(1)
> prec=30!;
> a;
Warning: rounding has happened. The value displayed is a faithful rounding to 30
bits of the true result.
1.10010010000111111011010101001_2 * 2^(1)
```

The reader may wish to see Sections 8.93 and 8.70 for ways of dynamically adding other base functions to Sollya.

## 5.6 Strings

Anything written between quotes is interpreted as a string. The infix operator `@` concatenates two strings. To get the length of a string, use the `length` function. You can access the  $i$ -th character of a string using brackets (see the example below). There is no character type in Sollya: the  $i$ -th character of a string is returned as a string itself.

```
> s1 = "Hello "; s2 = "World!";
> s = s1@s2;
> length(s);
12
> s[0];
H
> s[11];
!
```

Strings may contain the following escape sequences: `\\`, `\`, `\?`, `\``, `\n`, `\t`, `\a`, `\b`, `\f`, `\r`, `\v`, `\x`[hexadecimal number] and `\`[octal number]. Refer to the C99 standard for their meaning.

## 5.7 Particular values

Sollya knows about some particular values. These values do not really have a type. They can be stored in variables and in lists. A (possibly not exhaustive) list of such values is the following one:

- `on`, `off` (see sections 8.115 and 8.114)
- `dyadic`, `powers`, `binary`, `decimal`, `hexadecimal` (see sections 8.49, 8.126, 8.18, 8.34 and 8.77)
- `file`, `postscript`, `postscriptfile` (see sections 8.63, 8.123 and 8.124)
- `RU`, `RD`, `RN`, `RZ` (see sections 8.157, 8.143, 8.152 and 8.158)
- `absolute`, `relative` (see sections 8.2 and 8.146)
- `floating`, `fixed` (see sections 8.66 and 8.65)
- `halfprecision`, `single`, `double`, `doubleextended`, `doubledouble`, `quad`, `tripledouble` (see sections 8.75, 8.165, 8.46, 8.48, 8.47, 8.138 and 8.184)
- `HP`, `SG`, `D`, `DE`, `DD`, `QD`, `TD` (see sections 8.75, 8.165, 8.46, 8.48, 8.47, 8.138 and 8.184)
- `perturb` (see section 8.118)
- `honorcoeffprec` (see section 8.78)
- `default` (see section 8.35)
- `error` (see section 8.53)
- `void` (see section 8.189)

## 5.8 Lists

Objects can be grouped into lists. A list can contain elements with different types. As for strings, you can concatenate two lists with `@`. The function `length` also gives the length of a list.

You can prepend an element to a list using `.:` and you can append an element to a list using `:`. The following example illustrates some features:

```
> L = [| "foo" |];
> L = L.:1;
> L = "bar".:L;
> L;
[|"bar", "foo", 1|]
> L[1];
foo
> L@L;
[|"bar", "foo", 1, "bar", "foo", 1|]
```

Lists can be considered arrays and elements of lists can be referenced using brackets. Possible indices start at 0. The following example illustrates this point:

```
> L = [|1,2,3,4,5|];
> L;
[|1, 2, 3, 4, 5|]
> L[3];
4
```

Lists may contain ellipses indicated by `,...`, between elements that are constant and evaluate to integers that are incrementally ordered. `Sollya` translates such ellipses to the full list upon evaluation. The use of ellipses between elements that are not constants is not allowed. This feature is provided for ease of programming; remark that the complexity for expanding such lists is high. For illustration, see the following example:

```
> [|1,...,5|];
[|1, 2, 3, 4, 5|]
> [| -5,...,5|];
[|-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5|]
> [|3,...,1|];
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
error
> [|true,...,false|];
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
error
```

Lists may be continued to infinity by means of the `...` indicator after the last element given. At least one element must explicitly be given. If the last element given is a constant expression that evaluates to an integer, the list is considered as continued to infinity by all integers greater than that last element. If the last element is another object, the list is considered as continued to infinity by re-duplicating this last element. Let us remark that bracket notation is supported for such end-elliptic lists even for implicitly given elements. However, evaluation complexity is high. Combinations of ellipses inside a list and in its end are possible. The usage of lists described here is best illustrated by the following examples:



```

> L = [|1,2,true,3...|];
> L;
[|1, 2, true, 3...|]
> L[2];
true
> L[3];
3
> L[4];
4
> L[1200];
1200
> L = [|1,...,5,true...|];
> L;
[|1, 2, 3, 4, 5, true...|]
> L[1200];
true

```

## 5.9 Structures

In a similar way as in lists, Sollya allows data to be grouped in – untyped – structures. A structure forms an object to which other objects can be added as elements and identified by their names. The elements of a structure can be retrieved under their name and used as usual. The following sequence shows that point:

```

> s.a = 17;
> s.b = exp(x);
> s.a;
17
> s.b;
exp(x)
> s.b(1);
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
2.71828182845904523536028747135266249775724709369998
> s.d.a = [-1;1];
> s.d.b = sin(x);
> inf(s.d.a);
-1
> diff(s.d.b);
cos(x)

```

Structures can also be defined literally using the syntax illustrated in the next example. They will also be printed in that syntax.

```

> a = { .f = exp(x), .dom = [-1;1] };
> a;
{ .f = exp(x), .dom = [-1;1] }
> a.f;
exp(x)
> a.dom;
[-1;1]
> b.f = sin(x);
> b.dom = [-1b-5;1b-5];
> b;
{ .dom = [-3.125e-2;3.125e-2], .f = sin(x) }
> { .f = asin(x), .dom = [-1;1] }.f(1);
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
1.57079632679489661923132169163975144209858469968754

```

If the variable `a` is bound to an existing structure, it is possible to use the “dot notation” `a.b` to assign the value of the field `b` of the structure `a`. This works even if `b` is not yet a field of `a`: in this case a new field is created inside the structure `a`.

Besides, the dot notation can be used even when `a` is unassigned. In this case a new structure is created with a field `b`, and this structure is bound to `a`. However, the dot notation cannot be used if `a` is already bound to something that is not a structure.

These principles apply recursively: for instance, if `a` is a structure that contains only one field `d`, the command `a.b.c = 3` creates a new field named `b` inside the structure `a`; this field itself is a structure containing the field `c`. The command `a.d.c = 3` is allowed if `a.d` is already a structure, but forbidden otherwise (e.g. if `a.d` was equal to `sin(x)`). This is summed up in the following example.

```

> restart;
The tool has been restarted.
> a.f = exp(x);
> a.dom = [-1;1];
> a.info.text = "My akrnoximation problem";
> a;
{ .info = { .text = "My akrnoximation problem" }, .dom = [-1;1], .f = exp(x) }
>
> a.info.text = "My approximation problem";
> a;
{ .info = { .text = "My approximation problem" }, .dom = [-1;1], .f = exp(x) }
>
> b = exp(x);
> b.a = 5;
Warning: cannot modify an element of something that is not a structure.
Warning: the last assignment will have no effect.
> b;
exp(x)
>
> a.dom.a = -1;
Warning: cannot modify an element of something that is not a structure.
Warning: the last assignment will have no effect.
> a;
{ .info = { .text = "My approximation problem" }, .dom = [-1;1], .f = exp(x) }

```

When printed, the elements of a structure are not sorted in any manner. They get printed in an arbitrary order that just maintains the order given in the definition of literate structures. That said, when compared, two structures compare equal iff they contain the same number of identifiers, with the

same names and iff the elements of corresponding names all compare equal. This means the order does not matter in comparisons and otherwise does only for printing.

The following example illustrates this matter:

```
> a = { .f = exp(x), .a = -1, .b = 1 };
> a;
{ .f = exp(x), .a = -1, .b = 1 }
> a.info = "My function";
> a;
{ .info = "My function", .f = exp(x), .a = -1, .b = 1 }
>
> b = { .a = -1, .f = exp(x), .info = "My function", .b = 1 };
> b;
{ .a = -1, .f = exp(x), .info = "My function", .b = 1 }
>
> a == b;
true
>
> b.info = "My other function";
> a == b;
false
>
> b.info = "My function";
> a == b;
true
> b.something = true;
> a == b;
false
```

## 6 Iterative language elements: assignments, conditional statements and loops

### 6.1 Blocks

Statements in Sollya can be grouped in blocks, so-called begin-end-blocks. This can be done using the key tokens `{` and `}`. Blocks declared this way are considered to be one single statement. As already explained in section 4, using begin-end-blocks also opens the possibility of declaring variables through the keyword `var`.

### 6.2 Assignments

Sollya has two different assignment operators, `=` and `:=`. The assignment operator `=` assigns its right-hand-object “as is”, i.e. without evaluating functional expressions. For instance, `i = i + 1;` will dereferenciate the identifier `i` with some content, notate it  $y$ , build up the expression (function)  $y + 1$  and assign this expression back to `i`. In the example, if `i` stood for the value 1000, the statement `i = i + 1;` would assign “1000 + 1” – and not “1001” – to `i`. The assignment operator `:=` evaluates constant functional expressions before assigning them. On other expressions it behaves like `=`. Still in the example, the statement `i := i + 1;` really assigns 1001 to `i`.

Both Sollya assignment operators support indexing of lists or strings elements using brackets on the left-hand-side of the assignment operator. The indexed element of the list or string gets replaced by the right-hand-side of the assignment operator. When indexing strings this way, that right-hand side must evaluate to a string of length 1. End-elliptic lists are supported with their usual semantic for this kind of assignment. When referencing and assigning a value in the implicit part of the end-elliptic list, the list gets expanded to the corresponding length.

The following examples well illustrate the behavior of assignment statements:

```

> autosimplify = off;
Automatic pure tree simplification has been deactivated.
> i = 1000;
> i = i + 1;
> print(i);
1000 + 1
> i := i + 1;
> print(i);
1002
> L = [|1,...,5|];
> print(L);
[|1, 2, 3, 4, 5|]
> L[3] = L[3] + 1;
> L[4] := L[4] + 1;
> print(L);
[|1, 2, 3, 4 + 1, 6|]
> L[5] = true;
> L;
[|1, 2, 3, 5, 6, true|]
> s = "Hello world";
> s;
Hello world
> s[1] = "a";
> s;
Hallo world
> s[2] = "foo";
Warning: the string to be assigned is not of length 1.
This command will have no effect.
> L = [|true,1,...,5,9...|];
> L;
[|true, 1, 2, 3, 4, 5, 9...|]
> L[13] = "Hello";
> L;
[|true, 1, 2, 3, 4, 5, 9, 10, 11, 12, 13, 14, 15, "Hello", 17...|]

```

The indexing of lists on left-hand sides of assignments is reduced to the first order. Multiple indexing of lists of lists on assignment is not supported for complexity reasons. Multiple indexing is possible in right-hand sides.

```

> L = [| 1, 2, [|"a", "b", [|true, false] |] |];
> L[2][2][1];
false
> L[2][2][1] = true;
Warning: the first element of the left-hand side is not an identifier.
This command will have no effect.
> L[2][2] = "c";
Warning: the first element of the left-hand side is not an identifier.
This command will have no effect.
> L[2] = 3;
> L;
[|1, 2, 3|]

```

### 6.3 Conditional statements

Sollya supports conditional statements expressed with the keywords **if**, **then** and optionally **else**. Let us mention that only conditional statements are supported and not conditional expressions.

The following examples illustrate both syntax and semantic of conditional statements in Sollya. Concerning syntax, be aware that there must not be any semicolon before the `else` keyword.

```
> a = 3;
> b = 4;
> if (a == b) then print("Hello world");
> b = 3;
> if (a == b) then print("Hello world");
Hello world
> if (a == b) then print("You are telling the truth") else print("Liar!");
You are telling the truth
```

## 6.4 Loops

Sollya supports three kinds of loops. General *while-condition* loops can be expressed using the keywords `while` and `do`. One has to be aware of the fact that the condition test is executed always before the loop, there is no *do-until-condition* loop. Consider the following examples for both syntax and semantic:

```
> verbosity = 0!;
> prec = 30!;
> i = 5;
> while (expm1(i) > 0) do { expm1(i); i := i - 1; };
1.474131591e2
5.359815e1
1.908553692e1
6.3890561
1.718281827
> print(i);
0
```

The second kind of loops are loops on a variable ranging from a numerical start value and a end value. These kind of loops can be expressed using the keywords `for`, `from`, `to`, `do` and optionally `by`. The `by` statement indicates the width of the steps on the variable from the start value to the end value. Once again, syntax and semantic are best explained with an example:

```
> for i from 1 to 5 do print ("Hello world",i);
Hello world 1
Hello world 2
Hello world 3
Hello world 4
Hello world 5
> for i from 2 to 1 by -0.5 do print("Hello world",i);
Hello world 2
Hello world 1.5
Hello world 1
```

The third kind of loops are loops on a variable ranging on values contained in a list. In order to ensure the termination of the loop, that list must not be end-elliptic. The loop is expressed using the keywords `for`, `in` and `do` as in the following examples:

```

> L = [|true, false, 1,...,4, "Hello", exp(x)|];
> for i in L do i;
true
false
1
2
3
4
Hello
exp(x)

```

For both types of `for` loops, assigning the loop variable is allowed and possible. When the loop terminates, the loop variable will contain the value that made the loop condition fail. Consider the following examples:

```

> for i from 1 to 5 do { if (i == 3) then i = 4 else i; };
1
2
5
> i;
6

```

## 7 Functional language elements: procedures and pattern matching

### 7.1 Procedures

Sollya has some elements of functional languages. In order to avoid confusion with mathematical functions, the associated programming objects are called *procedures* in Sollya.

Sollya procedures are common objects that can be, for example, assigned to variables or stored in lists. Procedures are declared by the `proc` keyword; see section 8.135 for details. The returned procedure object must then be assigned to a variable. It can hence be applied to arguments with common application syntax. The `procedure` keyword provides an abbreviation for declaring and assigning a procedure; see section 8.136 for details.

Sollya procedures can return objects using the `return` keyword at the end of the begin-end-block of the procedure. Section 8.150 gives details on the usage of `return`. Procedures further can take any type of object in argument, in particular also other procedures that are then applied to arguments. Procedures can be declared inside other procedures.

Common Sollya procedures are declared with a certain number of formal parameters. When the procedure is applied to actual parameters, a check is performed if the right number of actual parameters is given. Then the actual parameters are applied to the formal parameters. In some cases, it is required that the number of parameters of a procedure be variable. Sollya provides support for the case with procedures with an arbitrary number of actual arguments. When the procedure is called, those actual arguments are gathered in a list which is applied to the only formal list parameter of a procedure with an arbitrary number of arguments. See section 8.136 for the exact syntax and details; an example is given just below.

Let us remark that declaring a procedure does not involve any evaluation or other interpretation of the procedure body. In particular, this means that constants are evaluated to floating-point values inside Sollya when the procedure is applied to actual parameters and the global precision valid at this moment.

Sollya procedures are well illustrated with the following examples:

```
> succ = proc(n) { return n + 1; };
> succ(5);
6
> 3 + succ(0);
4
> succ;
proc(n)
{
  nop;
  return (n) + (1);
}
```

```
> add = proc(m,n) { var res; res := m + n; return res; };
> add(5,6);
11
> hey = proc() { print("Hello world."); };
> hey();
Hello world.
> print(hey());
Hello world.
void
> hey;
proc()
{
  print("Hello world.");
  return void;
}
```

```
> fac = proc(n) { var res; if (n == 0) then res := 1 else res := n * fac(n - 1);
  return res; };
> fac(5);
120
> fac(11);
39916800
> fac;
proc(n)
{
  var res;
  if (n) == (0) then
    res := 1
  else
    res := (n) * (fac((n) - (1)));
  return res;
}
```

```

> sumall = proc(args = ...) { var i, acc; acc = 0; for i in args do acc = acc +
i; return acc; };
> sumall;
proc(args = ...)
{
var i, acc;
acc = 0;
for i in args do
acc = (acc) + (i);
return acc;
}
> sumall();
0
> sumall(1);
1
> sumall(1,5);
6
> sumall(1,5,9);
15
> sumall @ [|1,5,9,4,8|];
27
>

```

Let us note that, when writing a procedure, one does not know what will be the name of the free variable at run-time. This is typically the context when one should use the special keyword `_x_`:

```

> ChebPolynomials = proc(n) {
  var i, res;
  if (n<0) then res = [|]|
  else if (n==0) then res = [|1|]
  else {
    res = [|1, _x_|];
    for i from 2 to n do res[i] = horner(2*_x_*res[i-1]-res[i-2]);
  };
  return res;
};
>
> f = sin(x);
> T = ChebPolynomials(4);
> canonical = on!;
> for i from 0 to 4 do T[i];
1
x
-1 + 2 * x^2
-3 * x + 4 * x^3
1 + -8 * x^2 + 8 * x^4

```

Sollya also supports external procedures, i.e. procedures written in C (or some other language) and dynamically bound to Sollya identifiers. See 8.61 for details.

## 7.2 Pattern matching

Starting with version 3.0, Sollya supports matching expressions with expression patterns. This feature is important for an extended functional programming style. Further, and most importantly, it allows expression trees to be recursively decomposed using native constructs of the Sollya language. This means no help from external procedures or other compiled-language mechanisms is needed here anymore.



Basically, pattern matching supports relies on one **Sollya** construct:

```
match expr with
  pattern1 : (return-expr1)
  pattern2 : (return-expr2)
  ...
  patternN : (return-exprN)
```

That construct has the following semantic: try to match the expression *expr* with the patterns *pattern1* through *patternN*, proceeding in natural order. If a pattern *patternI* is found that matches, evaluate the whole **match ... with** construct to the return expression *return-exprI* associated with the matching pattern *patternI*. If no matching pattern is found, display an error warning and return **error**. Note that the parentheses around the expressions *return-exprI* are mandatory.

Matching a pattern means the following:

- If a pattern does not contain any programming-language-level variables (different from the free mathematical variable), it matches expressions that are syntactically equal to itself. For instance, the pattern `exp(sin(3 * x))` will match the expression `exp(sin(3 * x))`, but it does not match `exp(sin(x * 3))` because the expressions are not syntactically equal.
- If a pattern does contain variables, it matches an expression *expr* if these variables can be bound to subexpressions of *expr* such that once the pattern is evaluated with that variable binding, it becomes syntactically equal to the expression *expr*. For instance, the pattern `exp(sin(a * x))` will match the expression `exp(sin(3 * x))` as it is possible to bind `a` to `3` such that `exp(sin(a * x))` evaluates to `exp(sin(3 * x))`.

If a pattern *patternI* with variables is matched in a **match ... with** construct, the variables in the pattern stay bound during the evaluation of the corresponding return expression *return-exprI*. This allows subexpressions to be extracted from expressions and/or recursively handled as needed.

The following examples illustrate the basic principles of pattern matching in **Sollya**. One can remark that it is useful to use the keyword `_x_` when one wants to be sure to refer to the free variable in a pattern matching:

```

> match exp(x) with
  exp(x)      : (1)
  sin(x)      : (2)
  default     : (3);
1
>
> match sin(x) with
  exp(x)      : (1)
  sin(x)      : (2)
  default     : (3);
2
>
> match exp(sin(x)) with
  exp(x)      : ("Exponential of x")
  exp(sin(x)) : ("Exponential of sine of x")
  default     : ("Something else");
Exponential of sine of x
>
> match exp(sin(x)) with
  exp(x)      : ("Exponential of x")
  exp(a)      : ("Exponential of " @ a)
  default     : ("Something else");
Exponential of sin(x)
>
>
> procedure differentiate(f) {
  return match f with
    g + h      : (differentiate(g) + differentiate(h))
    g * h      : (differentiate(g) * h + differentiate(h) * g)
    g / h      : ((differentiate(g) * h - differentiate(h) * g) / (h^2))
    exp(_x_)   : (exp(_x_))
    sin(_x_)   : (cos(_x_))
    cos(_x_)   : (-sin(_x_))
    g(h)       : ((differentiate(g))(h) * differentiate(h))
    _x_        : (1)
    h(_x_)     : (NaN)
    c          : (0);
  };
>
> rename(x,y);
Information: the free variable has been renamed from "x" to "y".
> differentiate(exp(sin(y + y)));
exp(sin(y * 2)) * cos(y * 2) * 2
> diff(exp(sin(y + y)));
exp(sin(y * 2)) * cos(y * 2) * 2
>

```

As Sollya is not a purely functional language, the `match ... with` construct can also be used in a more imperative style, which makes it become closer to constructs like `switch` in C or Perl. In lieu of a simple return expression, a whole block of imperative statements can be given. The expression to be returned by that block is indicated in the end of the block, using the `return` keyword. That syntax is illustrated in the next example:

```

> match exp(sin(x)) with
  exp(a) : {
    write("Exponential of ", a, "\n");
    return a;
  }
  sin(x) : {
    var foo;
    foo = 17;
    write("Sine of x\n");
    return foo;
  }
  default : {
    write("Something else\n");
    bashexecute("LANG=C date");
    return true;
  };
Exponential of sin(x)
sin(x)
>
> match sin(x) with
  exp(a) : {
    write("Exponential of ", a, "\n");
    return a;
  }
  sin(x) : {
    var foo;
    foo = 17;
    write("Sine of x\n");
    return foo;
  }
  default : {
    write("Something else\n");
    bashexecute("LANG=C date");
    return true;
  };
Sine of x
17
>
> match acos(17 * pi * x) with
  exp(a) : {
    write("Exponential of ", a, "\n");
    return a;
  }
  sin(x) : {
    var foo;
    foo = 17;
    write("Sine of x\n");
    return foo;
  }
  default : {
    write("Something else\n");
    bashexecute("LANG=C date");
    return true;
  };
Something else
Fri Mar 15 10:04:02 CET 2013
true

```

In the case when no return statement is indicated for a statement-block in a `match ... with` construct, the construct evaluates to the special value `void` if that pattern matches.

In order to well understand pattern matching in **Sollya**, it is important to realize the meaning of variables in patterns. This meaning is different from the one usually found for variables. In a pattern, variables are never evaluated to whatever they might have set before the pattern is executed. In contrast, all variables in patterns are new, free variables that will freshly be bound to subexpressions of the matching expression. If a variable of the same name already exists, it will be shadowed during the evaluation of the statement block and the return expression corresponding to the matching expression. This type of semantic implies that patterns can never be computed at run-time, they must always be hard-coded beforehand. However this is necessary to make pattern matching context-free.

As a matter of course, all variables figuring in the expression *expr* to be matched are evaluated before pattern matching is attempted. In fact, *expr* is a usual **Sollya** expression, not a pattern.

In **Sollya**, the use of variables in patterns does not need to be linear. This means the same variable might appear twice or more in a pattern. Such a pattern will only match an expression if it contains the same subexpression, associated with the variable, in all places indicated by the variable in the pattern.

The following examples illustrate the use of variables in patterns in detail:

```
> a = 5;
> b = 6;
> match exp(x + 3) with
    exp(a + b) : {
        print("Exponential");
        print("a = ", a);
        print("b = ", b);
    }
    sin(x)      : {
        print("Sine of x");
    };

Exponential
a =  x
b =  3
> print("a = ", a, ", b = ", b);
a =  5 , b =  6
>
> a = 5;
> b = 6;
> match exp(x + 3) with
    exp(a + b) : {
        var a, c;
        a = 17;
        c = "Hallo";
        print("Exponential");
        print("a = ", a);
        print("b = ", b);
        print("c = ", c);
    }
    sin(x)      : {
        print("Sine of x");
    };

Exponential
a =  17
b =  3
c =  Hallo
> print("a = ", a, ", b = ", b);
a =  5 , b =  6
```

```

> match exp(sin(x)) + sin(x) with
  exp(a) + a : {
    print("Winner");
    print("a = ", a);
  }
  default    : {
    print("Loser");
  };

Winner
a = sin(x)
>
> match exp(sin(x)) + sin(3 * x) with
  exp(a) + a : {
    print("Winner");
    print("a = ", a);
  }
  default    : {
    print("Loser");
  };

Loser
>
> f = exp(x);
> match f with
  sin(x) : (1)
  cos(x) : (2)
  exp(x) : (3)
  default : (4);

3

```

Pattern matching is meant to be a means to decompose expressions structurally. For this reason and in an analogous way to variables, no evaluation is performed at all on (sub-)expressions that form constant functions. As a consequence, patterns match constant expressions only if they are structurally identical. For example  $5 + 1$  only matches  $5 + 1$  and not  $1 + 5$ ,  $3 + 3$  nor  $6$ .

This general rule on constant expressions admits one exception. Intervals in **Sollya** can be defined using constant expressions as bounds. These bounds are immediately evaluated to floating-point constants, though. In order to permit pattern matching on intervals, constant expressions given as bounds of intervals that form patterns are evaluated before pattern matching. However, in order not conflict with the rules of no evaluation of variables, these constant expressions as bounds of intervals in patterns must not contain free variables.

```

> match 5 + 1 with
  1 + 5 : ("One plus five")
  6      : ("Six")
  5 + 1 : ("Five plus one");
Five plus one
>
> match 6 with
  1 + 5 : ("One plus five")
  6      : ("Six")
  5 + 1 : ("Five plus one");
Six
>
> match 1 + 5 with
  1 + 5 : ("One plus five")
  6      : ("Six")
  5 + 1 : ("Five plus one");
One plus five
>
> match [1; 5 + 1] with
  [1; 1 + 5] : ("Interval from one to one plus five")
  [1; 6]      : ("Interval from one to six")
  [1; 5 + 1] : ("Interval from one to five plus one");
Interval from one to one plus five
>
> match [1; 6] with
  [1; 1 + 5] : ("Interval from one to one plus five")
  [1; 6]      : ("Interval from one to six")
  [1; 5 + 1] : ("Interval from one to five plus one");
Interval from one to one plus five
>

```

The `Sollya` keyword `default` has a special meaning in patterns. It acts like a wild-card, matching any (sub-)expression, as long as the whole expression stays correctly typed. Upon matching with `default`, no variable gets bound. This feature is illustrated in the next example:

```

> match exp(x) with
  sin(x)      : ("Sine of x")
  atan(x^2)   : ("Arctangent of square of x")
  default     : ("Something else")
  exp(x)      : ("Exponential of x");
Something else
>
> match atan(x^2) with
  sin(x)      : ("Sine of x")
  atan(default^2) : ("Arctangent of the square of something")
  default     : ("Something else");
Arctangent of the square of something
>
> match atan(exp(x)^2) with
  sin(x)      : ("Sine of x")
  atan(default^2) : ("Arctangent of the square of something")
  default     : ("Something else");
Arctangent of the square of something
>
> match exp("Hello world") with
  exp(default) : ("A miracle has happened")
  default      : ("Something else");
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
error

```

In Sollya, pattern matching is possible on the following Sollya types and operations defined on them:

- Expressions that define univariate functions, as explained above,
- Intervals with one, two or no bound defined in the pattern by a variable,
- Character sequences, iterate or defined using the @ operator, possibly with a variable on one of the sides of the @ operator,
- Lists, iterate, iterate with variables or defined using the .:, :. and @ operators, possibly with a variable on one of the sides of the @ operator or one or two variables for .: and :.,
- Structures, iterate or iterate with variables, and
- All other Sollya objects, matchable with themselves (DE matches DE, on matches on, perturb matches perturb etc.)

```

> procedure detector(obj) {
  match obj with
    exp(a * x)          : { "Exponential of ", a, " times x"; }
    [ a; 17 ]           : { "An interval from ", a, " to 17"; }
    [| |]               : { "Empty list"; }
    [| a, b, 2, exp(c) |] : { "A list of ", a, ", ", b, ", 2 and ",
                              "exponential of ", c; }
    a @ [| 2, 3 |]       : { "Concatenation of the list ", a, " and ",
                              "the list of 2 and 3"; }
    a .: [| 9 ... |]     : { a, " prepended to all integers >= 9"; }
    "Hello" @ w          : { "Hello concatenated with ", w; }
    { .a = sin(b);
      .b = [c;d] }       : { "A structure containing as .a the ",
                              "sine of ", b,
                              " and as .b the range from ", c,
                              " to ", d; }
    perturb              : { "The special object perturb"; }
    default               : { "Something else"; };
};

>
> detector(exp(5 * x));
Exponential of 5 times x
> detector([3.25;17]);
An interval from 3.25 to 17
> detector([| |]);
Empty list
> detector([| sin(x), nearestint(x), 2, exp(5 * atan(x)) |]);
A list of sin(x), nearestint(x), 2 and exponential of 5 * atan(x)
> detector([| sin(x), cos(5 * x), "foo", 2, 3 |]);
Concatenation of the list [|sin(x), cos(x * 5), "foo"|] and the list of 2 and 3
> detector([| DE, 9... |]);
doubleextended prepended to all integers >= 9
> detector("Hello world");
Hello concatenated with world
> detector({ .a = sin(x); .c = "Hello"; .b = [9;10] });
A structure containing as .a the sine of x and as .b the range from 9 to 10
> detector(perturb);
The special object perturb
> detector([13;19]);
Something else

```

Concerning intervals, please pay attention to the fact that expressions involving intervals are immediately evaluated and that structural pattern matching on functions on intervals is not possible. This point is illustrated in the next example:



```

> match exp([1;2]) with
  [a;b]                : {
                        a," ",",b;
                        }
  default               : {
                        "Something else";
                        };
2.71828182845904523536028747135266249775724709369989, 7.389056098930650227230427
4605750078131803155705518
>
> match exp([1;2]) with
  exp([a;b])           : {
                        a," ",", b;
                        }
  default              : {
                        "Something else";
                        };
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
error
>
> match exp([1;2]) with
  exp(a) : {
        "Exponential of ", a;
        }
  default : {
        "Something else";
        };
Something else

```

With respect to pattern matching on lists or character sequences defined using the @ operator, the following is to be mentioned:

- Patterns like `a @ b` are not allowed as they would need to perform an ambiguous cut of the list or character sequence to be matched. This restriction is maintained even if the variables (here `a` and `b`) are constrained by other occurrences in the pattern (for example in a list) which would make the cut unambiguous.
- Recursive use of the @ operator (even mixed with the operators `.:` and `:.:`) is possible under the condition that there must not exist any other parenthization of the term in concatenations (@) such that the rule of one single variable for @ above gets violated. For instance, `( [1 1] @ a) @ (b @ [1 4])` is not possible as it can be re-parenthesized `[1 1] @ (a @ b) @ [1 4]`, which exhibits the ambiguous case.

These points are illustrated in this example:

```

> match [| exp(sin(x)), sin(x), 4, DE(x), 9... |] with
      exp(a) .: (a .: (([|] :. 4) @ (b @ [| 13... |]))) :
      { "a = ", a, ", b = ", b; };
a = sin(x), b = [|doubleextended(x), 9, 10, 11, 12|]
>
> match [| 1, 2, 3, 4, D... |] with
      a @ [| 4, D...|] : (a);
[|1, 2, 3|]
>
> match [| 1, 2, 3, 4, D... |] with
      a @ [| D...|] : (a);
[|1, 2, 3, 4|]
>
> match [| 1, 2, 3, 4... |] with
      a @ [| 3...|] : (a);
[|1, 2|]
>
> match [| 1, 2, 3, 4... |] with
      a @ [| 4...|] : (a);
[|1, 2, 3|]
>
> match [| 1, 2, 3, 4... |] with
      a @ [| 17...|] : (a);
[|1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16|]
>
> match [| 1, 2, 3, 4... |] with
      a @ [| 17, 18, 19 |] : (a)
      default                : ("Something else");
Something else

```

As mentionned above, pattern matching on **Sollya** structures is possible. Patterns for such a match are given in a literately, i.e. using the syntax `{ .a = exprA, .b = exprB, ... }`. A structure pattern *sp* will be matched by a structure *s* iff that structure *s* contains at least all the elements (like *.a*, *.b* etc.) of the structure pattern *sp* and iff each of the elements of the structure *s* matches the pattern in the corresponding element of the structure pattern *sp*. The user should be aware of the fact that the structure to be matched is only supposed to have at least the elements of the pattern but that it may contain more elements is a particular **Sollya** feature. For instance with pattern matching, it is hence possible to ensure that access to particular elements will be possible in a particular code segment. The following example is meant to clarify this point:

```

> structure.f = exp(x);
> structure.dom = [1;2];
> structure.formats = [| DD, D, D, D |];
> match structure with
  { .f = sin(x);
    .dom = [a;b]
  }
    : { "Sine, ",a," ", "b; }
  { .f = exp(c);
    .dom = [a;b];
    .point = default
  }
    : { "Exponential, ",a, " ", " ", b, " ", " ", c; }
  { .f = exp(x);
    .dom = [a;b]
  }
    : { "Exponential, ",a, " ", " ", b; }
  default
    : { "Something else"; };
Exponential, 1, 2
>
> structure.f = sin(x);
> match structure with
  { .f = sin(x);
    .dom = [a;b]
  }
    : { "Sine, ",a," ", "b; }
  { .f = exp(c);
    .dom = [a;b];
    .point = default
  }
    : { "Exponential, ",a, " ", " ", b, " ", " ", c; }
  { .f = exp(x);
    .dom = [a;b]
  }
    : { "Exponential, ",a, " ", " ", b; }
  default
    : { "Something else"; };
Sine, 1, 2
>
> structure.f = exp(x + 2);
> structure.point = 23;
> match structure with
  { .f = sin(x);
    .dom = [a;b]
  }
    : { "Sine, ",a," ", "b; }
  { .f = exp(c);
    .dom = [a;b];
    .point = default
  }
    : { "Exponential, ",a, " ", " ", b, " ", " ", c; }
  { .f = exp(x);
    .dom = [a;b]
  }
    : { "Exponential, ",a, " ", " ", b; }
  default
    : { "Something else"; };
Exponential, 1, 2, 2 + x

```

## 8 Commands and functions

### 8.1 abs

Name: **abs**  
the absolute value.

Library names:

```

sollya_obj_t sollya_lib_abs(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_abs(sollya_obj_t)
#define SOLLYA_ABS(x) sollya_lib_build_function_abs(x)

```

Description:

- **abs** is the absolute value function.  $\text{abs}(x) = \begin{cases} x & x > 0 \\ -x & x \leq 0 \end{cases}$ .

## 8.2 absolute

Name: **absolute**

indicates an absolute error for **externalplot**, **fminimax** or **supnorm**

Library names:

```

sollya_obj_t sollya_lib_absolute()
int sollya_lib_is_absolute(sollya_obj_t)

```

Usage:

**absolute** : absolute|relative

Description:

- The use of **absolute** in the command **externalplot** indicates that during plotting in **externalplot** an absolute error is to be considered.  
See **externalplot** for details.
- Used with **fminimax**, **absolute** indicates that **fminimax** must try to minimize the absolute error.  
See **fminimax** for details.
- When given in argument to **supnorm**, **absolute** indicates that an absolute error is to be considered for supremum norm computation.  
See **supnorm** for details.

Example 1:

```

> bashexecute("gcc -fPIC -c externalplotexample.c");
> bashexecute("gcc -shared -o externalplotexample externalplotexample.o -lgmp -lmpfr");
> externalplot("./externalplotexample",absolute,exp(x),[-1/2;1/2],12,perturb);

```

See also: **externalplot** (8.60), **fminimax** (8.68), **relative** (8.146), **bashexecute** (8.17), **supnorm** (8.173)

## 8.3 accurateinfnorm

Name: **accurateinfnorm**

computes a faithful rounding of the infinity norm of a function

Usage:

**accurateinfnorm**(*function,range,constant*) : (function, range, constant) → constant  
**accurateinfnorm**(*function,range,constant,exclusion range 1,...,exclusion range n*) : (function, range, constant, range, ..., range) → constant

Parameters:

- *function* represents the function whose infinity norm is to be computed
- *range* represents the infinity norm is to be considered on

- *constant* represents the number of bits in the significant of the result
- *exclusion range 1* through *exclusion range n* represent ranges to be excluded

Description:

- The command **accurateinfnorm** computes an upper bound to the infinity norm of function *function* in *range*. This upper bound is the least floating-point number greater than the value of the infinity norm that lies in the set of dyadic floating point numbers having *constant* significant mantissa bits. This means the value **accurateinfnorm** evaluates to is at the time an upper bound and a faithful rounding to *constant* bits of the infinity norm of function *function* on range *range*.

If given, the fourth and further arguments of the command **accurateinfnorm**, *exclusion range 1* through *exclusion range n* the infinity norm of the function *function* is not to be considered on.

- The command **accurateinfnorm** is now considered DEPRECATED in **Sollya**. Users should be aware about the fact that the algorithm behind **accurateinfnorm** is highly inefficient and that other, better suited algorithms, such as **supnorm**, are available inside **Sollya**. As a matter of fact, while **accurateinfnorm** is maintained for compatibility reasons with legacy **Sollya** codes, users are advised to avoid using **accurateinfnorm** in new **Sollya** scripts and to replace it, where possible, by the **supnorm** command.

Example 1:

```
> p = remez(exp(x), 5, [-1;1]);
> accurateinfnorm(p - exp(x), [-1;1], 20);
4.52055246569216251373291015625e-5
> accurateinfnorm(p - exp(x), [-1;1], 30);
4.5205513970358879305422306060791015625e-5
> accurateinfnorm(p - exp(x), [-1;1], 40);
4.520551396713923253400935209356248378753662109375e-5
```

Example 2:

```
> p = remez(exp(x), 5, [-1;1]);
> midpointmode = on!;
> infnorm(p - exp(x), [-1;1]);
0.45205~5/7~e-4
> accurateinfnorm(p - exp(x), [-1;1], 40);
4.520551396713923253400935209356248378753662109375e-5
```

See also: **infnorm** (8.86), **dirtyinfnorm** (8.42), **supnorm** (8.173), **checkinfnorm** (8.24), **remez** (8.147), **diam** (8.38)

## 8.4 acos

Name: **acos**

the arccosine function.

Library names:

```
sollya_obj_t sollya_lib_acos(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_acos(sollya_obj_t)
#define SOLLYA_ACOS(x) sollya_lib_build_function_acos(x)
```

Description:

- **acos** is the inverse of the function **cos**: **acos**(*y*) is the unique number  $x \in [0; \pi]$  such that **cos**(*x*)=*y*.
- It is defined only for  $y \in [-1; 1]$ .

See also: **cos** (8.29)

## 8.5 `acosh`

Name: **acosh**

the arg-hyperbolic cosine function.

Library names:

```
sollya_obj_t sollya_lib_acosh(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_acosh(sollya_obj_t)
#define SOLLYA_ACOSH(x) sollya_lib_build_function_acosh(x)
```

Description:

- **acosh** is the inverse of the function **cosh**: **acosh**( $y$ ) is the unique number  $x \in [0; +\infty]$  such that **cosh**( $x$ )= $y$ .
- It is defined only for  $y \in [0; +\infty]$ .

See also: **cosh** (8.30)

## 8.6 `&&`

Name: `&&`

boolean AND operator

Library name:

```
sollya_obj_t sollya_lib_and(sollya_obj_t, sollya_obj_t)
```

Usage:

$$expr1 \ \&\& \ expr2 : (\text{boolean}, \text{boolean}) \rightarrow \text{boolean}$$

Parameters:

- *expr1* and *expr2* represent boolean expressions

Description:

- `&&` evaluates to the boolean AND of the two boolean expressions *expr1* and *expr2*. `&&` evaluates to true iff both *expr1* and *expr2* evaluate to true.

Example 1:

```
> true && false;
false
```

Example 2:

```
> (1 == exp(0)) && (0 == log(1));
true
```

See also: `||` (8.116), `!` (8.111)

## 8.7 `..`

Name: `..`

add an element at the end of a list.

Library name:

```
sollya_obj_t sollya_lib_append(sollya_obj_t, sollya_obj_t)
```

Usage:

$$L..x : (\text{list}, \text{any type}) \rightarrow \text{list}$$

Parameters:

- $L$  is a list (possibly empty).
- $x$  is an object of any type.

Description:

- `::` adds the element  $x$  at the end of the list  $L$ .
- Note that since  $x$  may be of any type, it can in particular be a list.

Example 1:

```
> [|2,3,4|]::.5;  
[|2, 3, 4, 5|]
```

Example 2:

```
> [|1,2,3|]::.[|4,5,6|];  
[|1, 2, 3, [|4, 5, 6|]|]
```

Example 3:

```
> [|]|::.1;  
[|1|]
```

See also: `::` (8.129), `@` (8.27)

## 8.8 $\sim$

Name:  $\sim$

floating-point evaluation of a constant expression

Library name:

`sollya_obj_t sollya_lib_approx(sollya_obj_t)`

Usage:

$\sim$  *expression* : function  $\rightarrow$  constant  
 $\sim$  *something* : any type  $\rightarrow$  any type

Parameters:

- *expression* stands for an expression that is a constant
- *something* stands for some language element that is not a constant expression

Description:

- $\sim$  *expression* evaluates the *expression* that is a constant term to a floating-point constant. The evaluation may involve a rounding. If *expression* is not a constant, the evaluated constant is a faithful rounding of *expression* with **precision** bits, unless the *expression* is exactly 0 as a result of cancellation. In the latter case, a floating-point approximation of some (unknown) accuracy is returned.
- $\sim$  does not do anything on all language elements that are not a constant expression. In other words, it behaves like the identity function on any type that is not a constant expression. It can hence be used in any place where one wants to be sure that expressions are simplified using floating-point computations to constants of a known precision, regardless of the type of actual language elements.
- $\sim$  **error** evaluates to error and provokes a warning.

- `~` is a prefix operator not requiring parentheses. Its precedence is the same as for the unary `+` and `-` operators. It cannot be repeatedly used without brackets.

Example 1:

```
> print(exp(5));
exp(5)
> print(~ exp(5));
1.48413159102576603421115580040552279623487667593878e2
```

Example 2:

```
> autosimplify = off!;
```

Example 3:

```
> print(~sin(5 * pi));
0
```

Example 4:

```
> print(~exp(x));
exp(x)
> print(~ "Hello");
Hello
```

Example 5:

```
> print(~exp(x*5*Pi));
exp((pi) * 5 * x)
> print(exp(x* ~(5*Pi)));
exp(x * 1.57079632679489661923132169163975144209858469968757e1)
```

Example 6:

```
> print(~exp(5)*x);
1.48413159102576603421115580040552279623487667593878e2 * x
> print( (~exp(5))*x);
1.48413159102576603421115580040552279623487667593878e2 * x
> print(~(exp(5)*x));
exp(5) * x
```

See also: **evaluate** (8.54), **prec** (8.127), **error** (8.53)

## 8.9 asciiplot

Name: **asciiplot**

plots a function in a range using ASCII characters

Library name:

```
void sollya_lib_asciiplot(sollya_obj_t, sollya_obj_t)
```

Usage:

**asciiplot**(*function*, *range*) : (function, range) → void

Parameters:

- *function* represents a function to be plotted



- *range* represents a range the function is to be plotted in

Description:

- **asciipLOT** plots the function *function* in range *range* using ASCII characters. On systems that provide the necessary `TIOCGWINSZ ioctl`, **Sollya** determines the size of the terminal for the plot size if connected to a terminal. If it is not connected to a terminal or if the test is not possible, the plot is of fixed size  $77 \times 25$  characters. The function is evaluated on a number of points equal to the number of columns available. Its value is rounded to the next integer in the range of lines available. A letter **x** is written at this place. If zero is in the hull of the image domain of the function, an x-axis is displayed. If zero is in range, a y-axis is displayed. If the function is constant or if the range is reduced to one point, the function is evaluated to a constant and the constant is displayed instead of a plot.

Example 1:



Example 2:



Example 3:

```
> asciiplot(5, [-1;1]);
5
```

Example 4:

```
> asciiplot(exp(x), [1;1]);
2.71828182845904523536028747135266249775724709369998
```

See also: **plot** (8.120), **externalplot** (8.60)

## 8.10 asin

Name: **asin**  
the arcsine function.

Library names:

```
sollya_obj_t sollya_lib_asin(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_asin(sollya_obj_t)
#define SOLLYA_ASIN(x) sollya_lib_build_function_asin(x)
```

Description:

- **asin** is the inverse of the function **sin**: **asin**( $y$ ) is the unique number  $x \in [-\pi/2; \pi/2]$  such that **sin**( $x$ )= $y$ .
- It is defined only for  $y \in [-1; 1]$ .

See also: **sin** (8.164)

## 8.11 asinh

Name: **asinh**

the arg-hyperbolic sine function.

Library names:

```
sollya_obj_t sollya_lib_asinh(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_asinh(sollya_obj_t)
#define SOLLYA_ASINH(x) sollya_lib_build_function_asinh(x)
```

Description:

- **asinh** is the inverse of the function **sinh**: **asinh**( $y$ ) is the unique number  $x \in [-\infty; +\infty]$  such that **sinh**( $x$ )= $y$ .
- It is defined for every real number  $y$ .

See also: **sinh** (8.166)

## 8.12 atan

Name: **atan**

the arctangent function.

Library names:

```
sollya_obj_t sollya_lib_atan(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_atan(sollya_obj_t)
#define SOLLYA_ATAN(x) sollya_lib_build_function_atan(x)
```

Description:

- **atan** is the inverse of the function **tan**: **atan**( $y$ ) is the unique number  $x \in [-\pi/2; +\pi/2]$  such that **tan**( $x$ )= $y$ .
- It is defined for every real number  $y$ .

See also: **tan** (8.176)

## 8.13 atanh

Name: **atanh**

the hyperbolic arctangent function.

Library names:

```
sollya_obj_t sollya_lib_atanh(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_atanh(sollya_obj_t)
#define SOLLYA_ATANH(x) sollya_lib_build_function_atanh(x)
```

Description:

- **atanh** is the inverse of the function **tanh**: **atanh**( $y$ ) is the unique number  $x \in [-\infty; +\infty]$  such that **tanh**( $x$ )= $y$ .
- It is defined only for  $y \in [-1; 1]$ .

See also: **tanh** (8.177)

## 8.14 autodiff

Name: **autodiff**

Computes the first  $n$  derivatives of a function at a point or over an interval.

Library name:

```
sollya_obj_t sollya_lib_autodiff(sollya_obj_t, sollya_obj_t, sollya_obj_t)
```

Usage:

**autodiff**( $f, n, x_0$ ) : (function, integer, constant)  $\rightarrow$  list  
**autodiff**( $f, n, I$ ) : (function, integer, range)  $\rightarrow$  list

Parameters:

- $f$  is the function to be differentiated.
- $n$  is the order of differentiation.
- $x_0$  is the point at which the function is differentiated.
- $I$  is the interval over which the function is differentiated.

Description:

- **autodiff** computes the first  $n$  derivatives of  $f$  at point  $x_0$ . The computation is performed numerically, without symbolically differentiating the expression of  $f$ . Yet, the computation is safe since small interval enclosures are produced. More precisely, **autodiff** returns a list  $[f_0, \dots, f_n]$  such that, for each  $i$ ,  $f_i$  is a small interval enclosing the exact value of  $f^{(i)}(x_0)$ .
- Since it does not perform any symbolic differentiation, **autodiff** is much more efficient than **diff** and should be preferred when only numerical values are necessary.
- If an interval  $I$  is provided instead of a point  $x_0$ , the list returned by **autodiff** satisfies:  $\forall i, f^{(i)}(I) \subseteq f_i$ . A particular use is when one wants to know the successive derivatives of a function at a non representable point such as  $\pi$ . In this case, it suffices to call **autodiff** with the (almost) point interval  $I = [\mathbf{pi}]$ .
- When  $I$  is almost a point interval, the returned enclosures  $f_i$  are also almost point intervals. However, when the interval  $I$  begins to be fairly large, the enclosures can be deeply overestimated due to the dependency phenomenon present with interval arithmetic.
- As a particular case,  $f_0$  is an enclosure of the image of  $f$  over  $I$ . However, since the algorithm is not specially designed for this purpose it is not very efficient for this particular task. In particular, it is not able to return a finite enclosure for functions with removable singularities (e.g.  $\sin(x)/x$  at 0). The command **evaluate** is much more efficient for computing an accurate enclosure of the image of a function over an interval.

Example 1:

```
> L = autodiff(exp(cos(x))+sin(exp(x)), 5, 0);
> midpointmode = on!;
> for f_i in L do f_i;
0.3559752813266941742012789792982961497379810154498~2/4~e1
0.5403023058681397174009366074429766037323104206179~0/3~
-0.3019450507398802024611853185539984893647499733880~6/2~e1
-0.252441295442368951995750696489089699886768918239~6/4~e1
0.31227898756481033145214529184139729746320579069~1/3~e1
-0.16634307959006696033484053579339956883955954978~3/1~e2
```

Example 2:

```

> f = log(cos(x)+x);
> L = autodiff(log(cos(x)+x), 5, [2,4]);
> L[0];
[0;1.27643852425465597132446653114905059102580436018893]
> evaluate(f, [2,4]);
[0.45986058925497069206106494332976097408234056912429;1.207872105899641695959010
37621103012113048821362855]
> fprime = diff(f);
> L[1];
[2.53086745013099407167484456656211083053393118778677e-2;1.756802495307928251372
63909451182909413591288733649]
> evaluate(fprime, [2,4]);
[2.71048755415961996452136364304380881763456815673085e-2;1.109195306639432908373
97225788623531405558431279949]

```

Example 3:

```

> L = autodiff(sin(x)/x, 0, [-1,1]);
> L[0];
[-@Inf@;@Inf@]
> evaluate(sin(x)/x, [-1,1]);
[0.5403023058681397174009366074429766037323104206179;1]

```

See also: **diff** (8.40), **evaluate** (8.54)

## 8.15 autosimplify

Name: **autosimplify**

activates, deactivates or inspects the value of the automatic simplification state variable

Library names:

```

void sollya_lib_set_autosimplify_and_print(sollya_obj_t)
void sollya_lib_set_autosimplify(sollya_obj_t)
sollya_obj_t sollya_lib_get_autosimplify()

```

Usage:

```

autosimplify = activation value : on|off → void
autosimplify = activation value ! : on|off → void
autosimplify : on|off

```

Parameters:

- *activation value* represents **on** or **off**, i.e. activation or deactivation

Description:

- An assignment **autosimplify** = *activation value*, where *activation value* is one of **on** or **off**, activates respectively deactivates the automatic safe simplification of expressions of functions generated by the evaluation of commands or in argument of other commands.

**Sollya** commands like **remez**, **taylor** or **rationalapprox** sometimes produce expressions that can be simplified. Constant subexpressions can be evaluated to dyadic floating-point numbers, monomials with coefficients 0 can be eliminated. Further, expressions indicated by the user perform better in many commands when simplified before being passed in argument to a command. When the automatic simplification of expressions is activated, **Sollya** automatically performs a safe (not value changing) simplification process on such expressions.

The automatic generation of subexpressions can be annoying, in particular if it takes too much time for not enough benefit. Further the user might want to inspect the structure of the expression tree returned by a command. In this case, the automatic simplification should be deactivated.

If the assignment **autosimplify** = *activation value* is followed by an exclamation mark, no message indicating the new state is displayed. Otherwise the user is informed of the new state of the global mode by an indication.

Example 1:

```
> autosimplify = on !;
> print(x - x);
0
> autosimplify = off ;
Automatic pure tree simplification has been deactivated.
> print(x - x);
x - x
```

Example 2:

```
> autosimplify = on !;
> print(rationalapprox(sin(pi/5.9),7));
33 / 65
> autosimplify = off !;
> print(rationalapprox(sin(pi/5.9),7));
33 / 65
```

See also: **print** (8.130), **prec** (8.127), **points** (8.122), **diam** (8.38), **display** (8.44), **verbosity** (8.188), **canonical** (8.21), **taylorrecursions** (8.180), **timing** (8.183), **fullparentheses** (8.69), **midpointmode** (8.104), **hopitalrecursions** (8.79), **remez** (8.147), **rationalapprox** (8.141), **taylor** (8.178)

## 8.16 bashevaluate

Name: **bashevaluate**

executes a shell command and returns its output as a string

Library names:

```
sollya_obj_t sollya_lib_bashevaluate(sollya_obj_t, ...)
sollya_obj_t sollya_lib_v_bashevaluate(sollya_obj_t, va_list)
```

Usage:

```
bashevaluate(command) : string → string
bashevaluate(command,input) : (string, string) → string
```

Parameters:

- *command* is a command to be interpreted by the shell.
- *input* is an optional character sequence to be fed to the command.

Description:

- **bashevaluate**(*command*) will execute the shell command *command* in a shell. All output on the command's standard output is collected and returned as a character sequence.
- If an additional argument *input* is given in a call to **bashevaluate**(*command,input*), this character sequence is written to the standard input of the command *command* that gets executed.
- All characters output by *command* are included in the character sequence to which **bashevaluate** evaluates but two exceptions. Every NULL character ('`\0`') in the output is replaced with '?' as Sollya is unable to handle character sequences containing that character. Additionally, if the output ends in a newline character ('`\n`'), this character is stripped off. Other newline characters which are not at the end of the output are left as such.

Example 1:

```
> bashevaluate("LANG=C date");  
Thu May 23 16:40:48 CEST 2013
```

Example 2:

```
> [| bashevaluate("echo Hello") |];  
[|"Hello"|]
```

Example 3:

```
> a = bashevaluate("sed -e 's/a/e/g;', "Hallo");  
> a;  
Hello
```

See also: **bashexecute** (8.17)

## 8.17 bashexecute

Name: **bashexecute**

executes a shell command.

Library name:

```
void sollya_lib_bashexecute(sollya_obj_t)
```

Usage:

**bashexecute**(*command*) : string → void

Parameters:

- *command* is a command to be interpreted by the shell.

Description:

- **bashexecute**(*command*) lets the shell interpret *command*. It is useful to execute some external code within **Sollya**.
- **bashexecute** does not return anything. It just executes its argument. However, if *command* produces an output in a file, this result can be imported in **Sollya** with help of commands like **execute**, **readfile** and **parse**.

Example 1:

```
> bashexecute("LANG=C date");  
Thu May 23 16:40:51 CEST 2013
```

See also: **execute** (8.55), **readfile** (8.144), **parse** (8.117), **bashevaluate** (8.16)

## 8.18 binary

Name: **binary**

special value for global state **display**

Library names:

```
sollya_obj_t sollya_lib_binary()  
int sollya_lib_is_binary(sollya_obj_t)
```

Description:

- **binary** is a special value used for the global state **display**. If the global state **display** is equal to **binary**, all data will be output in binary notation.  
As any value it can be affected to a variable and stored in lists.

See also: **decimal** (8.34), **dyadic** (8.49), **powers** (8.126), **hexadecimal** (8.77), **display** (8.44)

## 8.19 bind

Name: **bind**

partially applies a procedure to an argument, returning a procedure with one argument less

Usage:

**bind**(*proc*, *ident*, *obj*) : (procedure, identifier type, any type) → procedure

Parameters:

- *proc* is a procedure to be partially applied to an argument
- *ident* is one of the formal arguments of *proc*
- *obj* is any Sollya object *ident* is to be bound to

Description:

- **bind** allows a formal parameter *ident* of a procedure *proc* to be bound to an object *obj*, hence *proc* to be partially applied. The result of this curried application, returned by **bind**, is a procedure with one argument less. This way, **bind** permits specialization of a generic procedure, parameterized e.g. by a function or range.
- In the case when *proc* does not have a formal parameter named *ident*, **bind** prints a warning and returns the procedure *proc* unmodified.
- **bind** always returns a procedure, even if *proc* only has one argument, which gets bound to *ident*. In this case, **bind** returns a procedure which does not take any argument. Hence evaluation, which might provoke side effects, is only performed once the procedure gets used.
- **bind** does not work on procedures with an arbitrary number of arguments.

Example 1:

```
> procedure add(X,Y) { return X + Y; };
> succ = bind(add,X,1);
> succ(5);
6
> succ;
proc(Y)
{
  nop;
  return (proc(X, Y)
  {
    nop;
    return (X) + (Y);
  })(1, Y);
}
```

Example 2:



```

> procedure add(X,Y) { return X + Y; };
> succ = bind(add,X,1);
> five = bind(succ,Y,4);
> five();
5
> five;
proc()
{
nop;
return (proc(Y)
{
nop;
return (proc(X, Y)
{
nop;
return (X) + (Y);
})(1, Y);
})(4);
}

```

Example 3:

```

> verbosity = 1!;
> procedure add(X,Y) { return X + Y; };
> foo = bind(add,R,1);
Warning: the given procedure has no argument named "R". The procedure is returned unchanged.
> foo;
proc(X, Y)
{
nop;
return (X) + (Y);
}

```

See also: **procedure** (8.136), **proc** (8.135), **function** (8.70), **@** (8.27)

## 8.20 boolean

Name: **boolean**

keyword representing a boolean type

Usage:

**boolean** : type type

Description:

- **boolean** represents the boolean type for declarations of external procedures by means of **externalproc**.

Remark that in contrast to other indicators, type indicators like **boolean** cannot be handled outside the **externalproc** context. In particular, they cannot be assigned to variables.

See also: **externalproc** (8.61), **constant** (8.28), **function** (8.70), **integer** (8.87), **list of** (8.95), **range** (8.140), **string** (8.169)

## 8.21 canonical

Name: **canonical**

brings all polynomial subexpressions of an expression to canonical form or activates, deactivates or

checks canonical form printing

Library names:

```
void sollya_lib_set_canonical_and_print(sollya_obj_t)
void sollya_lib_set_canonical(sollya_obj_t)
sollya_obj_t sollya_lib_canonical(sollya_obj_t)
sollya_obj_t sollya_lib_get_canonical()
```

Usage:

```
canonical(function) : function → function
canonical = activation value : on|off → void
canonical = activation value ! : on|off → void
```

Parameters:

- *function* represents the expression to be rewritten in canonical form
- *activation value* represents **on** or **off**, i.e. activation or deactivation

Description:

- The command **canonical** rewrites the expression representing the function *function* in a way such that all polynomial subexpressions (or the whole expression itself, if it is a polynomial) are written in canonical form, i.e. as a sum of monomials in the canonical base. The canonical base is the base of the integer powers of the global free variable. The command **canonical** does not endanger the safety of computations even in Sollya's floating-point environment: the function returned is mathematically equal to the function *function*.
- An assignment **canonical** = *activation value*, where *activation value* is one of **on** or **off**, activates respectively deactivates the automatic printing of polynomial expressions in canonical form, i.e. as a sum of monomials in the canonical base. If automatic printing in canonical form is deactivated, automatic printing yields to displaying polynomial subexpressions in Horner form.

If the assignment **canonical** = *activation value* is followed by an exclamation mark, no message indicating the new state is displayed. Otherwise the user is informed of the new state of the global mode by an indication.

Example 1:

```
> print(canonical(1 + x * (x + 3 * x^2)));
1 + x^2 + 3 * x^3
> print(canonical((x + 1)^7));
1 + 7 * x + 21 * x^2 + 35 * x^3 + 35 * x^4 + 21 * x^5 + 7 * x^6 + x^7
```

Example 2:

```
> print(canonical(exp((x + 1)^5) - log(asin(((x + 2) + x)^4 * (x + 1)) + x)));
exp(1 + 5 * x + 10 * x^2 + 10 * x^3 + 5 * x^4 + x^5) - log(asin(16 + 80 * x + 160 * x^2 + 160 * x^3 + 80 * x^4 + 16 * x^5) + x)
```

Example 3:

```

> canonical;
off
> (x + 2)^9;
512 + x * (2304 + x * (4608 + x * (5376 + x * (4032 + x * (2016 + x * (672 + x *
(144 + x * (18 + x))))))))))
> canonical = on;
Canonical automatic printing output has been activated.
> (x + 2)^9;
512 + 2304 * x + 4608 * x^2 + 5376 * x^3 + 4032 * x^4 + 2016 * x^5 + 672 * x^6 +
144 * x^7 + 18 * x^8 + x^9
> canonical;
on
> canonical = off!;
> (x + 2)^9;
512 + x * (2304 + x * (4608 + x * (5376 + x * (4032 + x * (2016 + x * (672 + x *
(144 + x * (18 + x))))))))))

```

See also: **horner** (8.80), **print** (8.130), **autosimplify** (8.15)

## 8.22 ceil

Name: **ceil**

the usual function ceil.

Library names:

```

sollya_obj_t sollya_lib_ceil(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_ceil(sollya_obj_t)
#define SOLLYA_CEIL(x) sollya_lib_build_function_ceil(x)

```

Description:

- **ceil** is defined as usual: **ceil**( $x$ ) is the smallest integer  $y$  such that  $y \geq x$ .
- It is defined for every real number  $x$ .

See also: **floor** (8.67), **nearestint** (8.108), **round** (8.153), **RU** (8.157)

## 8.23 chebyshevform

Name: **chebyshevform**

computes a rigorous polynomial approximation

Library name:

```

sollya_obj_t sollya_lib_chebyshevform(sollya_obj_t, sollya_obj_t,
sollya_obj_t);

```

Usage:

**chebyshevform**( $f$ ,  $n$ ,  $I$ ) : (function, integer, range)  $\rightarrow$  list

Parameters:

- $f$  is the function to be approximated.
- $n$  is the degree of the polynomial that must approximate  $f$ .
- $I$  is the interval over which the function is to be approximated. This interval cannot be a point interval, i.e. its endpoints have to be different.

Description:

- **chebyshevform** computes an approximation polynomial and an interval error bound for a given function  $f$ .  
More precisely, it returns a list  $L = [p, \text{coeffErrors}, \Delta, \text{chebCoeffs}]$  where:
  - $p$  is an approximation polynomial of degree  $n$  which is roughly speaking a numerical Chebyshev interpolation polynomial of  $f$  over  $I$  represented in monomial basis.
  - $\text{coeffsErrors}$  is a list of  $n + 1$  intervals. Each interval  $\text{coeffsErrors}[i]$  contains an enclosure of all the errors accumulated when computing the  $i$ -th coefficient of  $p$ .
  - $\Delta$  is an interval that provides a bound for the approximation error between  $p$  and  $f$ . See details below.
  - $\text{chebCoeffs}$  is a list of  $n + 1$  intervals. These are the interval coefficients obtained for the representation of interpolation polynomial considered in Chebyshev basis. See details below.
- The polynomial  $p$  and the bound  $\Delta$  are obtained using Chebyshev Models principles.
- More formally, the mathematical property ensured by the algorithm may be stated as follows:  
Using monomial basis representation given by  $p$ : there exist (small) values  $\varepsilon_i \in \text{coeffsErrors}[i]$  such that:  $\forall x \in I, \exists \delta \in \Delta, f(x) - p(x) = \sum_{i=0}^n \varepsilon_i x^i + \delta$ .  
Using Chebyshev basis representation given by  $\text{chebCoeffs}$ : there exist values  $\alpha_i \in \text{chebCoeffs}[i]$  such that:  $\forall x \in I, \exists \delta \in \Delta, f(x) - \sum_{i=0}^n \alpha_i T_i(y) = \delta$ , where  $[a, b] = I, y = (2x - b - a)/(b - a)$  and  $T_i(y)$  is the  $i$ -th Chebyshev polynomial over  $[-1, 1]$ .
- The algorithm does not guarantee that by increasing the degree of the approximation, the remainder bound will become smaller. Moreover, it may even become larger due to the dependency phenomenon present with interval arithmetic. In order to reduce this phenomenon, a possible solution is to split the definition domain  $I$  into several smaller intervals.

Example 1:

```
> TL=chebyshevform(exp(x), 10, [-1,1]);
> p=TL[0];
> Delta=TL[2];
> p; Delta;
1.000000000000000000000000000000000000000000000000000000017 + x * (1.000000000273898191
9773953471453626764777115166417 + x * (0.500000000022767543680233268825420973654
11870042311 + x * (0.166666661190450656168988993421371510533693281444003 + x * (
4.16666662114404339731887995095915661615696989046437e-2 + x * (8.333363974982552
0034240517636254576844423981913792e-3 + x * (1.388891436361480907068621055372167
77944219976325265e-3 + x * (1.98342776066472142284727329700227389403558111865906
e-4 + x * (2.47957727598277888282218067965324267955768146489014e-5 + x * (2.8254
0334409824772663163700125420195118047008360658e-6 + x * 2.8136983333718843135828
0911630401974906330455923616e-7))))))))))
[-2.71406412827174505775085010461449926572460824320373e-11;2.7140641282717450577
5085010461449926572460824320373e-11]
```

See also: **taylorform** (8.179), **diff** (8.40), **autodiff** (8.14), **taylor** (8.178), **remez** (8.147)

## 8.24 checkfnorm

Name: **checkfnorm**

checks whether the infinity norm of a function is bounded by a value

Library name:

```
sollya_obj_t sollya_lib_checkinfnorm(sollya_obj_t, sollya_obj_t,
                                     sollya_obj_t)
```

Usage:

**checkinfnorm**(*function*,*range*,*constant*) : (function, range, constant) → boolean

Parameters:

- *function* represents the function whose infinity norm is to be checked
- *range* represents the infinity norm is to be considered on
- *constant* represents the upper bound the infinity norm is to be checked to

Description:

- The command **checkinfnorm** checks whether the infinity norm of the given function *function* in the range *range* can be proven (by **Sollya**) to be less than the given bound *bound*. This means, if **checkinfnorm** evaluates to **true**, the infinity norm has been proven (by **Sollya**'s interval arithmetic) to be less than the bound. If **checkinfnorm** evaluates to **false**, there are two possibilities: either the bound is less than or equal to the infinity norm of the function or the bound is greater than the infinity norm but **Sollya** could not conclude using its internal interval arithmetic.

**checkinfnorm** is sensitive to the global variable **diam**. The smaller **diam**, the more time **Sollya** will spend on the evaluation of **checkinfnorm** in order to prove the bound before returning **false** although the infinity norm is bounded by the bound. If **diam** is equal to 0, **Sollya** will eventually spend infinite time on instances where the given bound *bound* is less or equal to the infinity norm of the function *function* in range *range*. In contrast, with **diam** being zero, **checkinfnorm** evaluates to **true** iff the infinity norm of the function in the range is bounded by the given bound.

Example 1:

```
> checkinfnorm(sin(x),[0;1.75], 1);
true
> checkinfnorm(sin(x),[0;1.75], 1/2); checkinfnorm(sin(x),[0;20/39],1/2);
false
true
```

Example 2:

```
> p = remez(exp(x), 5, [-1;1]);
> b = dirtyinfnorm(p - exp(x), [-1;1]);
> checkinfnorm(p - exp(x), [-1;1], b);
false
> b1 = round(b, 15, RU);
> checkinfnorm(p - exp(x), [-1;1], b1);
true
> b2 = round(b, 25, RU);
> checkinfnorm(p - exp(x), [-1;1], b2);
false
> diam = 1b-20!;
> checkinfnorm(p - exp(x), [-1;1], b2);
true
```

See also: **infnorm** (8.86), **dirtyinfnorm** (8.42), **supnorm** (8.173), **accurateinfnorm** (8.3), **remez** (8.147), **diam** (8.38)

## 8.25 coeff

Name: **coeff**

gives the coefficient of degree *n* of a polynomial

Library name:

**sollya\_obj\_t sollya\_lib\_coeff(sollya\_obj\_t, sollya\_obj\_t)**

Usage:

**coeff**( $f, n$ ) : (function, integer)  $\rightarrow$  constant

Parameters:

- $f$  is a function (usually a polynomial).
- $n$  is an integer

Description:

- If  $f$  is a polynomial, **coeff**( $f, n$ ) returns the coefficient of degree  $n$  in  $f$ .
- If  $f$  is a function that is not a polynomial, **coeff**( $f, n$ ) returns 0.

Example 1:

```
> coeff((1+x)^5,3);
10
```

Example 2:

```
> coeff(sin(x),0);
0
```

See also: **degree** (8.36), **roundcoefficients** (8.154), **subpoly** (8.170)

## 8.26 composepolynomials

Name: **composepolynomials**

computes an approximation to the composition of two polynomials and bounds the error

Library name:

```
sollya_obj_t sollya_lib_composepolynomials(sollya_obj_t, sollya_obj_t)
```

Usage:

**composepolynomials**( $p, q$ ) : (function, function)  $\rightarrow$  structure

Parameters:

- $p$  and  $q$  are polynomials

Description:

- Given two polynomials  $p$  and  $q$ , **composepolynomials**( $p, q$ ) computes an approximation  $r$  to the polynomial  $(p \circ q)$  and bounds the error polynomial  $r - (p \circ q)$  using interval arithmetic.
- **composepolynomials** always returns a structure containing two elements, **poly** and **radii**. The element **poly** contains the approximate composed polynomial  $r$ . The element **radii** contains a list of  $n + 1$  intervals  $a_i$  bounding the coefficients of the error polynomial, which is of the same degree  $n$  as is the composed polynomial  $(p \circ q)$ . This is, there exist  $\alpha_i \in a_i$  such that

$$\sum_{i=0}^n \alpha_i x^i = r(x) - (p \circ q)(x).$$

- In the case when either of  $p$  or  $q$  is not a polynomial, **composepolynomials** behaves like **substitute** used in a literate structure. The list of intervals bounding the coefficients of the error polynomial is returned empty.

Example 1:

```
> composepolynomials(1 + 2 * x + 3 * x^2 + 4 * x^3, 5 + 6 * x + 7 * x^2);
{ .radii = [[0;0], [0;0], [0;0], [0;0], [0;0], [0;0], [0;0]], .poly = 586 + x
* (1992 + x * (4592 + x * (6156 + x * (6111 + x * (3528 + x * 1372)))) ) }
```

Example 2:

```
> print(composepolynomials(1/5 * x + exp(17) + log(2) * x^2, x^4 + 1/3 * x^2));
{ .radii = [[-3.58732406867153170156474773322218529607747057120388e-43;3.587324
06867153170156474773322218529607747057120388e-43], [0;0], [-2.672764710092195646
14053646715148187881519688010505e-51;2.67276471009219564614053646715148187881519
688010505e-51], [0;0], [-1.06910588403687825845621458686059275152607875204202e-5
0;1.06910588403687825845621458686059275152607875204202e-50], [0;0], [-2.13821176
807375651691242917372118550305215750408404e-50;2.1382117680737565169124291737211
8550305215750408404e-50], [0;0], [-1.0691058840368782584562145868605927515260787
5204202e-50;1.06910588403687825845621458686059275152607875204202e-50]]], .poly =
2.41549527535752982147754351803858238798675673527228e7 + x^2 * (6.6666666666666
66666666666666666666666666666666666666666666666666666666666666666666666666666666
4241840897277792706698 + x^2 * (0.4620981203732968729448214143054510453836667562
4017 + x^2 * 0.693147180559945309417232121458176568075500134360259))) }
```

Example 3:

```
> composepolynomials(sin(x),x + x^2);
{ .radii = [ | ], .poly = sin(x * (1 + x)) }
```

See also: **substitute** (8.171)

## 8.27 @

Name: @

concatenates two lists or strings or applies a list as arguments to a procedure

Library name:

`sollya_obj_t sollya_lib_concat(sollya_obj_t, sollya_obj_t)`

Usage:

$$\begin{aligned} L1@L2 &: (\text{list}, \text{list}) \rightarrow \text{list} \\ string1@string2 &: (\text{string}, \text{string}) \rightarrow \text{string} \\ proc@L1 &: (\text{procedure}, \text{list}) \rightarrow \text{any type} \end{aligned}$$

Parameters:

- $L1$  and  $L2$  are two lists.
- $string1$  and  $string2$  are two strings.
- $proc$  is a procedure.

Description:

- In its first usage form, @ concatenates two lists or strings.
- In its second usage form, @ applies the elements of a list as arguments to a procedure. In the case when  $proc$  is a procedure with a fixed number of arguments, a check is done if the number of elements in the list corresponds to the number of formal parameters of the procedure. An empty list can therefore applied only to a procedure that does not take any argument. In the case of a procedure with an arbitrary number of arguments, no such check is performed.

Example 1:

```
> [[1,...,3]@[7,8,9]];
[1, 2, 3, 7, 8, 9]
```

Example 2:

```
> "Hello @"World!";  
Hello World!
```

Example 3:

```
> procedure cool(a,b,c) {  
  write(a," ", b," and ",c," are cool guys.\n");  
};  
> cool @ [| "Christoph", "Mioara", "Sylvain" |];  
Christoph, Mioara and Sylvain are cool guys.
```

Example 4:

```
> procedure sayhello() {  
  "Hello! how are you?";  
};  
> sayhello();  
Hello! how are you?  
> sayhello @ [|];  
Hello! how are you?
```

Example 5:

```
> procedure add(L = ...) {  
  var acc, i;  
  acc = 0;  
  for i in L do acc = i + acc;  
  return acc;  
};  
> add(1,2);  
3  
> add(1,2,3);  
6  
> add @ [|1, 2|];  
3  
> add @ [|1, 2, 3|];  
6  
> add @ [|];  
0
```

See also: `::` (8.129), `:::` (8.7), **procedure** (8.136), **proc** (8.135), **bind** (8.19)

## 8.28 constant

Name: **constant**

keyword representing a constant type

Usage:

**constant** : type type

Description:

- **constant** represents the **constant** type for declarations of external procedures **externalproc**.

Remark that in contrast to other indicators, type indicators like **constant** cannot be handled outside the **externalproc** context. In particular, they cannot be assigned to variables.

See also: **externalproc** (8.61), **boolean** (8.20), **function** (8.70), **integer** (8.87), **list of** (8.95), **range** (8.140), **string** (8.169)



## 8.29 cos

Name: **cos**  
the cosine function.

Library names:

```
sollya_obj_t sollya_lib_cos(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_cos(sollya_obj_t)
#define SOLLYA_COS(x) sollya_lib_build_function_cos(x)
```

Description:

- **cos** is the usual cosine function.
- It is defined for every real number  $x$ .

See also: **acos** (8.4), **sin** (8.164), **tan** (8.176)

## 8.30 cosh

Name: **cosh**  
the hyperbolic cosine function.

Library names:

```
sollya_obj_t sollya_lib_cosh(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_cosh(sollya_obj_t)
#define SOLLYA_COSH(x) sollya_lib_build_function_cosh(x)
```

Description:

- **cosh** is the usual hyperbolic function:  $\cosh(x) = \frac{e^x + e^{-x}}{2}$ .
- It is defined for every real number  $x$ .

See also: **acosh** (8.5), **sinh** (8.166), **tanh** (8.177), **exp** (8.56)

## 8.31 D

Name: **D**  
short form for **double**

See also: **double** (8.46)

## 8.32 DD

Name: **DD**  
short form for **doubledouble**

See also: **doubledouble** (8.47)

## 8.33 DE

Name: **DE**  
short form for **doubleextended**

See also: **doubleextended** (8.48)

## 8.34 decimal

Name: **decimal**  
special value for global state **display**

Library names:

```
sollya_obj_t sollya_lib_decimal()
int sollya_lib_is_decimal(sollya_obj_t)
```

Description:

- **decimal** is a special value used for the global state **display**. If the global state **display** is equal to **decimal**, all data will be output in decimal notation.  
As any value it can be affected to a variable and stored in lists.

See also: **dyadic** (8.49), **powers** (8.126), **hexadecimal** (8.77), **binary** (8.18), **display** (8.44)

### 8.35 default

Name: **default**

default value for some commands.

Library names:

```
sollya_obj_t sollya_lib_default()
int sollya_lib_is_default(sollya_obj_t)
```

Description:

- **default** is a special value and is replaced by something depending on the context where it is used. It can often be used as a joker, when you want to specify one of the optional parameters of a command and not the others: set the value of uninteresting parameters to **default**.
- Global variables can be reset by affecting them the special value **default**.

Example 1:

```
> p = remez(exp(x),5,[0;1],default,1e-5);
> q = remez(exp(x),5,[0;1],1,1e-5);
> p==q;
true
```

Example 2:

```
> prec;
165
> prec=200;
The precision has been set to 200 bits.
```

### 8.36 degree

Name: **degree**

gives the degree of a polynomial.

Library name:

```
sollya_obj_t sollya_lib_degree(sollya_obj_t)
```

Usage:

**degree( $f$ )** : function  $\rightarrow$  integer

Parameters:

- $f$  is a function (usually a polynomial).

Description:

- If  $f$  is a polynomial, **degree( $f$ )** returns the degree of  $f$ .
- Contrary to the usage, Sollya considers that the degree of the null polynomial is 0.

- If  $f$  is a function that is not a polynomial, **degree**( $f$ ) returns -1.

Example 1:

```
> degree((1+x)*(2+5*x^2));
3
> degree(0);
0
```

Example 2:

```
> degree(sin(x));
-1
```

See also: **coeff** (8.25), **subpoly** (8.170), **roundcoefficients** (8.154)

### 8.37 denominator

Name: **denominator**

gives the denominator of an expression

Library name:

```
sollya_obj_t sollya_lib_denominator(sollya_obj_t)
```

Usage:

**denominator**( $expr$ ) : function  $\rightarrow$  function

Parameters:

- $expr$  represents an expression

Description:

- If  $expr$  represents a fraction  $expr1/expr2$ , **denominator**( $expr$ ) returns the denominator of this fraction, i.e.  $expr2$ .  
If  $expr$  represents something else, **denominator**( $expr$ ) returns 1.  
Note that for all expressions  $expr$ , **numerator**( $expr$ ) / **denominator**( $expr$ ) is equal to  $expr$ .

Example 1:

```
> denominator(5/3);
3
```

Example 2:

```
> denominator(exp(x));
1
```

Example 3:

```
> a = 5/3;
> b = numerator(a)/denominator(a);
> print(a);
5 / 3
> print(b);
5 / 3
```

Example 4:

```

> a = exp(x/3);
> b = numerator(a)/denominator(a);
> print(a);
exp(x / 3)
> print(b);
exp(x / 3)

```

See also: **numerator** (8.113), **rationalmode** (8.142)

### 8.38 **diam**

Name: **diam**

parameter used in safe algorithms of Sollya and controlling the maximal length of the involved intervals.

Library names:

```

void sollya_lib_set_diam_and_print(sollya_obj_t)
void sollya_lib_set_diam(sollya_obj_t)
sollya_obj_t sollya_lib_get_diam()

```

Usage:

```

diam = width : constant → void
diam = width ! : constant → void
diam : constant

```

Parameters:

- *width* represents the maximal relative width of the intervals used

Description:

- **diam** is a global variable. Its value represents the maximal width allowed for intervals involved in safe algorithms of Sollya (namely **infnorm**, **checkinfnorm**, **accurateinfnorm**, **integral**, **findzeros**, **supnorm**).
- More precisely, **diam** is relative to the width of the input interval of the command. For instance, suppose that **diam**=1e-5: if **infnorm** is called on interval  $[0, 1]$ , the maximal width of an interval will be 1e-5. But if it is called on interval  $[0, 1e-3]$ , the maximal width will be 1e-8.

See also: **infnorm** (8.86), **checkinfnorm** (8.24), **accurateinfnorm** (8.3), **integral** (8.88), **findzeros** (8.64), **supnorm** (8.173)

### 8.39 **dieonerrormode**

Name: **dieonerrormode**

global variable controlling if Sollya is exited on an error or not.

Library names:

```

void sollya_lib_set_dieonerrormode_and_print(sollya_obj_t)
void sollya_lib_set_dieonerrormode(sollya_obj_t)
sollya_obj_t sollya_lib_get_dieonerrormode()

```

Usage:

```

dieonerrormode = activation value : on|off → void
dieonerrormode = activation value ! : on|off → void
dieonerrormode : on|off

```

Parameters:

- *activation value* controls if Sollya is exited on an error or not.

Description:

- **dieonerrormode** is a global variable. When its value is **off**, which is the default, Sollya will not exit on any syntax, typing, side-effect errors. These errors will be caught by the tool, even if a memory might be leaked at that point. On evaluation, the **error** special value will be produced.
- When the value of the **dieonerrormode** variable is **on**, Sollya will exit on any syntax, typing, side-effect errors. A warning message will be printed in these cases at appropriate **verbosity** levels.

Example 1:

```
> verbosity = 1!;
> dieonerrormode = off;
Die-on-error mode has been deactivated.
> for i from true to false do i + "Salut";
Warning: one of the arguments of the for loop does not evaluate to a constant.
The for loop will not be executed.
> exp(17);
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
2.41549527535752982147754351803858238798675673527224e7
```

Example 2:

```
> verbosity = 1!;
> dieonerrormode = off!;
> 5 */ 4;
Warning: syntax error, unexpected /.
The last symbol read has been "/".
Will skip input until next semicolon after the unexpected token. May leak memory
.
    exp(17);
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
2.41549527535752982147754351803858238798675673527224e7
```

Example 3:

```
> verbosity = 1!;
> dieonerrormode;
off
> dieonerrormode = on!;
> dieonerrormode;
on
> for i from true to false do i + "Salut";
Warning: one of the arguments of the for loop does not evaluate to a constant.
The for loop will not be executed.
Warning: some syntax, typing or side-effect error has occurred.
As the die-on-error mode is activated, the tool will be exited.
```

Example 4:

```

> verbosity = 1!;
> dieonerrormode = on!;
> 5 */ 4;
Warning: syntax error, unexpected /.
The last symbol read has been "/".
Will skip input until next semicolon after the unexpected token. May leak memory
.
Warning: some syntax, typing or side-effect error has occurred.
As the die-on-error mode is activated, the tool will be exited.

```

Example 5:

```

> verbosity = 0!;
> dieonerrormode = on!;
> 5 */ 4;

```

See also: **on** (8.115), **off** (8.114), **verbosity** (8.188), **error** (8.53)

## 8.40 diff

Name: **diff**

differentiation operator

Library name:

```
sollya_obj_t sollya_lib_diff(sollya_obj_t)
```

Usage:

**diff**(*function*) : function  $\rightarrow$  function

Parameters:

- *function* represents a function

Description:

- **diff**(*function*) returns the symbolic derivative of the function *function* by the global free variable. If *function* represents a function symbol that is externally bound to some code by **library**, the derivative is performed as a symbolic annotation to the returned expression tree.

Example 1:

```

> diff(sin(x));
cos(x)

```

Example 2:

```

> diff(x);
1

```

Example 3:

```

> diff(x^x);
x^x * (1 + log(x))

```

See also: **library** (8.93), **autodiff** (8.14), **taylor** (8.178), **taylorform** (8.179)

## 8.41 dirtyfindzeros

Name: **dirtyfindzeros**

gives a list of numerical values listing the zeros of a function on an interval.

Library name:

`sollya_obj_t sollya_lib_dirtyfindzeros(sollya_obj_t, sollya_obj_t)`

Usage:

**dirtyfindzeros**( $f, I$ ) : (function, range)  $\rightarrow$  list

Parameters:

- $f$  is a function.
- $I$  is an interval.

Description:

- **dirtyfindzeros**( $f, I$ ) returns a list containing some zeros of  $f$  in the interval  $I$ . The values in the list are numerical approximation of the exact zeros. The precision of these approximations is approximately the precision stored in **prec**. If  $f$  does not have two zeros very close to each other, it can be expected that all zeros are listed. However, some zeros may be forgotten. This command should be considered as a numerical algorithm and should not be used if safety is critical.
- More precisely, the algorithm relies on global variables **prec** and **points** and it performs the following steps: let  $n$  be the value of variable **points** and  $t$  be the value of variable **prec**.
  - Evaluate  $|f|$  at  $n$  evenly distributed points in the interval  $I$ . The working precision to be used is automatically chosen in order to ensure that the sign is correct.
  - Whenever  $f$  changes its sign for two consecutive points, find an approximation  $x$  of its zero with precision  $t$  using Newton's algorithm. The number of steps in Newton's iteration depends on  $t$ : the precision of the approximation is supposed to be doubled at each step.
  - Add this value to the list.

Example 1:

```
> dirtyfindzeros(sin(x), [-5;5]);  
[|-3.14159265358979323846264338327950288419716939937508, 0, 3.141592653589793238  
46264338327950288419716939937508|]
```

Example 2:

```
> L1=dirtyfindzeros(x^2*sin(1/x), [0;1]);  
> points=1000!;  
> L2=dirtyfindzeros(x^2*sin(1/x), [0;1]);  
> length(L1); length(L2);  
18  
25
```

See also: **prec** (8.127), **points** (8.122), **findzeros** (8.64), **dirtyinfnorm** (8.42), **numberroots** (8.112)

## 8.42 dirtyinfnorm

Name: **dirtyinfnorm**

computes a numerical approximation of the infinity norm of a function on an interval.

Library name:

`sollya_obj_t sollya_lib_dirtyinfnorm(sollya_obj_t, sollya_obj_t)`

Usage:

**dirtyinfnorm**( $f, I$ ) : (function, range)  $\rightarrow$  constant

Parameters:

- $f$  is a function.
- $I$  is an interval.

Description:

- **dirtyinfnorm**( $f, I$ ) computes an approximation of the infinity norm of the given function  $f$  on the interval  $I$ , e.g.  $\max_{x \in I} \{|f(x)|\}$ .
- The interval must be bound. If the interval contains one of -Inf or +Inf, the result of **dirtyinfnorm** is NaN.
- The result of this command depends on the global variables **prec** and **points**. Therefore, the returned result is generally a good approximation of the exact infinity norm, with precision **prec**. However, the result is generally underestimated and should not be used when safety is critical. Use **infnorm** instead.
- The following algorithm is used: let  $n$  be the value of variable **points** and  $t$  be the value of variable **prec**.
  - Evaluate  $|f|$  at  $n$  evenly distributed points in the interval  $I$ . The evaluation are faithful roundings of the exact results at precision  $t$ .
  - Whenever the derivative of  $f$  changes its sign for two consecutive points, find an approximation  $x$  of its zero with precision  $t$ . Then compute a faithful rounding of  $|f(x)|$  at precision  $t$ .
  - Return the maximum of all computed values.

Example 1:

```
> dirtyinfnorm(sin(x), [-10;10]);  
1
```

Example 2:

```
> prec=15!;  
> dirtyinfnorm(exp(cos(x))*sin(x), [0;5]);  
1.45856  
> prec=40!;  
> dirtyinfnorm(exp(cos(x))*sin(x), [0;5]);  
1.458528537135  
> prec=100!;  
> dirtyinfnorm(exp(cos(x))*sin(x), [0;5]);  
1.458528537136237644438147455024  
> prec=200!;  
> dirtyinfnorm(exp(cos(x))*sin(x), [0;5]);  
1.458528537136237644438147455023841718299214087993682374094153
```

Example 3:

```
> dirtyinfnorm(x^2, [log(0);log(1)]);  
@NaN@
```

See also: **prec** (8.127), **points** (8.122), **infnorm** (8.86), **checkinfnorm** (8.24), **supnorm** (8.173)



## 8.43 `dirtyintegral`

Name: **dirtyintegral**

computes a numerical approximation of the integral of a function on an interval.

Library name:

`sollya_obj_t sollya_lib_dirtyintegral(sollya_obj_t, sollya_obj_t)`

Usage:

**dirtyintegral**( $f, I$ ) : (function, range)  $\rightarrow$  constant

Parameters:

- $f$  is a function.
- $I$  is an interval.

Description:

- **dirtyintegral**( $f, I$ ) computes an approximation of the integral of  $f$  on  $I$ .
- The interval must be bound. If the interval contains one of `-Inf` or `+Inf`, the result of **dirtyintegral** is NaN, even if the integral has a meaning.
- The result of this command depends on the global variables **prec** and **points**. The method used is the trapezium rule applied at  $n$  evenly distributed points in the interval, where  $n$  is the value of global variable **points**.
- This command computes a numerical approximation of the exact value of the integral. It should not be used if safety is critical. In this case, use command **integral** instead.
- Warning: this command is currently known to be unsatisfactory. If you really need to compute integrals, think of using an other tool or report a feature request to [sylvain.chevillard@ens-lyon.org](mailto:sylvain.chevillard@ens-lyon.org).

Example 1:

```
> sin(10);
-0.54402111088936981340474766185137728168364301291621
> dirtyintegral(cos(x), [0;10]);
-0.54400304905152629822448058882475382036536298356281
> points=2000!;
> dirtyintegral(cos(x), [0;10]);
-0.54401997751158321972222697312583199035995837926892
```

See also: **prec** (8.127), **points** (8.122), **integral** (8.88)

## 8.44 `display`

Name: **display**

sets or inspects the global variable specifying number notation

Library names:

```
void sollya_lib_set_display_and_print(sollya_obj_t)
void sollya_lib_set_display(sollya_obj_t)
sollya_obj_t sollya_lib_get_display()
```

Usage:

**display** = *notation value* : decimal|binary|dyadic|powers|hexadecimal  $\rightarrow$  void  
**display** = *notation value* ! : decimal|binary|dyadic|powers|hexadecimal  $\rightarrow$  void  
**display** : decimal|binary|dyadic|powers|hexadecimal

Parameters:

- *notation value* represents a variable of type `decimal|binary|dyadic|powers|hexadecimal`

Description:

- An assignment `display = notation value`, where *notation value* is one of **decimal**, **dyadic**, **powers**, **binary** or **hexadecimal**, activates the corresponding notation for output of values in **print**, **write** or at the Sollya prompt.

If the global notation variable **display** is **decimal**, all numbers will be output in scientific decimal notation. If the global notation variable **display** is **dyadic**, all numbers will be output as dyadic numbers with Gappa notation. If the global notation variable **display** is **powers**, all numbers will be output as dyadic numbers with a notation compatible with Maple and PARI/GP. If the global notation variable **display** is **binary**, all numbers will be output in binary notation. If the global notation variable **display** is **hexadecimal**, all numbers will be output in C99/ IEEE754-2008 notation. All output notations can be parsed back by Sollya, inducing no error if the input and output precisions are the same (see **prec**).

If the assignment `display = notation value` is followed by an exclamation mark, no message indicating the new state is displayed. Otherwise the user is informed of the new state of the global mode by an indication.

Example 1:

```
> display = decimal;
Display mode is decimal numbers.
> a = evaluate(sin(pi * x), 0.25);
> a;
0.70710678118654752440084436210484903928483593768847
> display = binary;
Display mode is binary numbers.
> a;
1.011010100000100111100110011001111111001110111100110010010000100010110010111110
11000100110110011011101010100101010111110100111110001110101101111011000001011101
010001_2 * 2^(-1)
> display = hexadecimal;
Display mode is hexadecimal numbers.
> a;
0xb.504f333f9de6484597d89b3754abe9f1d6f60ba88p-4
> display = dyadic;
Display mode is dyadic numbers.
> a;
33070006991101558613323983488220944360067107133265b-165
> display = powers;
Display mode is dyadic numbers in integer-power-of-2 notation.
> a;
33070006991101558613323983488220944360067107133265 * 2^(-165)
```

See also: **print** (8.130), **write** (8.191), **decimal** (8.34), **dyadic** (8.49), **powers** (8.126), **binary** (8.18), **hexadecimal** (8.77), **prec** (8.127)

## 8.45 /

Name: /

division function

Library names:

```
sollya_obj_t sollya_lib_div(sollya_obj_t, sollya_obj_t)
sollya_obj_t sollya_lib_build_function_div(sollya_obj_t, sollya_obj_t)
```

Usage:

Parameters:

- Description:

- Example 1:

Example 2:

Example 3:

Example 4:

Example 5:

Example 6:

See also: + (8.121), - (8.106), \* (8.107), ^ (8.125)

## 8.46 double

Names: **double**, **D**

rounding to the nearest IEEE 754 double (binary64).

Library names:

```
sollya_obj_t sollya_lib_double(sollya_obj_t)
sollya_obj_t sollya_lib_double_obj()
int sollya_lib_is_double_obj(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_double(sollya_obj_t)
#define SOLLYA_D(x) sollya_lib_build_function_double(x)
```

Description:

- **double** is both a function and a constant.
- As a function, it rounds its argument to the nearest IEEE 754 double precision (i.e. IEEE754-2008 binary64) number. Subnormal numbers are supported as well as standard numbers: it is the real rounding described in the standard.
- As a constant, it symbolizes the double precision format. It is used in contexts when a precision format is necessary, e.g. in the commands **round**, **roundcoefficients** and **implementpoly**. See the corresponding help pages for examples.

Example 1:

```
> display=binary!;
> D(0.1);
1.10011001100110011001100110011001100110011001100110011001101_2 * 2^(-4)
> D(4.17);
1.000010101110000101000111101011100001010001111010111_2 * 2^(2)
> D(1.011_2 * 2^(-1073));
1.1_2 * 2^(-1073)
```

See also: **halfprecision** (8.75), **single** (8.165), **doubleextended** (8.48), **doubledouble** (8.47), **quad** (8.138), **tripledouble** (8.184), **roundcoefficients** (8.154), **implementpoly** (8.83), **round** (8.153), **printdouble** (8.131)

## 8.47 doubledouble

Names: **doubledouble**, **DD**

represents a number as the sum of two IEEE doubles.

Library names:

```
sollya_obj_t sollya_lib_double_double(sollya_obj_t)
sollya_obj_t sollya_lib_double_double_obj()
int sollya_lib_is_double_double_obj(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_double_double(sollya_obj_t)
#define SOLLYA_DD(x) sollya_lib_build_function_double_double(x)
```

Description:

- **doubledouble** is both a function and a constant.
- As a function, it rounds its argument to the nearest number that can be written as the sum of two double precision numbers.
- The algorithm used to compute **doubledouble**( $x$ ) is the following: let  $x_h = \mathbf{double}(x)$  and let  $x_l = \mathbf{double}(x - x_h)$ . Return the number  $x_h + x_l$ . Note that if the current precision is not sufficient to exactly represent  $x_h + x_l$ , a rounding will occur and the result of **doubledouble**( $x$ ) will be useless.

- As a constant, it symbolizes the double-double precision format. It is used in contexts when a precision format is necessary, e.g. in the commands **round**, **roundcoefficients** and **implementpoly**. See the corresponding help pages for examples.

Example 1:

```
> verbosity=1!;  
> a = 1+ 2^(-100);  
> DD(a);  
1.00000000000000000000000000000007888609052210118054  
> prec=50!;  
> DD(a);  
1.000000000000000000000000000000008
```

See also: **halfprecision** (8.75), **single** (8.165), **double** (8.46), **doubleextended** (8.48), **quad** (8.138), **tripleddouble** (8.184), **roundcoefficients** (8.154), **implementpoly** (8.83), **round** (8.153)

## 8.48 doubleextended

Names: **doubleextended, DE**

computes the nearest number with 64 bits of mantissa.

Library names:

```

sollya_obj_t sollya_lib_doubleextended(sollya_obj_t)
sollya_obj_t sollya_lib_doubleextended_obj()
int sollya_lib_is_doubleextended_obj(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_doubleextended(sollya_obj_t)
#define SOLLYA_DE(x) sollya_lib_build_function_doubleextended(x)

```

Description:

- **doubleextended** is a function that computes the nearest floating-point number with 64 bits of mantissa to a given number. Since it is a function, it can be composed with other Sollya functions such as **exp**, **sin**, etc.
- **doubleextended** now does handle subnormal numbers for a presumed exponent width of the double-extended format of 15 bits. This means, with respect to rounding, **doubleextended** behaves as a IEEE 754-2008 binary79 with a 64 bit significand (with a hidden bit normal range), one sign bit and a 15 bit exponent field would behave. This behavior may be different from the one observed on Intel-based IA32/Intel64 processors (or compatible versions from other vendors). However it is the one seen on HP/Intel Itanium when the precision specifier is double-extended and pseudo-denormals are activated.
- Since it is a function and not a command, its behavior is a bit different from the behavior of **round**(x,64,RN) even if the result is exactly the same. **round**(x,64,RN) is immediately evaluated whereas **doubleextended**(x) can be composed with other functions (and thus be plotted and so on).

Example 1:

```
> display=binary!;
> DE(0.1);
1.10011001100110011001100110011001100110011001101_2 * 2^(-4)
> round(0.1,64,RN);
1.10011001100110011001100110011001100110011001101_2 * 2^(-4)
```

Example 2:

```
> D(2^(-2000));
0
> DE(2^(-20000));
0
```

Example 3:

```
> verbosity=1!;
> f = sin(DE(x));
> f(pi);
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
-5.0165576126683320235573270803307570138315616702549e-20
> g = sin(round(x,64,RN));
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
```

See also: **roundcoefficients** (8.154), **halfprecision** (8.75), **single** (8.165), **double** (8.46), **doubledouble** (8.47), **quad** (8.138), **tripledouble** (8.184), **round** (8.153)

## 8.49 dyadic

Name: **dyadic**

special value for global state **display**

Library names:

```
sollya_obj_t sollya_lib_dyadic()
int sollya_lib_is_dyadic(sollya_obj_t)
```

Description:

- **dyadic** is a special value used for the global state **display**. If the global state **display** is equal to **dyadic**, all data will be output in dyadic notation with numbers displayed in Gappa format. As any value it can be affected to a variable and stored in lists.

See also: **decimal** (8.34), **powers** (8.126), **hexadecimal** (8.77), **binary** (8.18), **display** (8.44)

## 8.50 ==

Name: **==**

equality test operator

Library name:

```
sollya_obj_t sollya_lib_cmp_equal(sollya_obj_t, sollya_obj_t)
```

Usage:

$$expr1 == expr2 : (\text{any type}, \text{any type}) \rightarrow \text{boolean}$$

Parameters:

- *expr1* and *expr2* represent expressions

Description:

- The operator **==** evaluates to true iff its operands *expr1* and *expr2* are syntactically equal and different from **error** or constant expressions that are not constants and that evaluate to the same floating-point number with the global precision **prec**. The user should be aware of the fact that because of floating-point evaluation, the operator **==** is not exactly the same as the mathematical equality. Further remark that according to IEEE 754-2008 floating-point rules, which Sollya emulates, floating-point data which are NaN do not compare equal to any other floating-point datum, including NaN.

Example 1:

```
> "Hello" == "Hello";
true
> "Hello" == "Salut";
false
> "Hello" == 5;
false
> 5 + x == 5 + x;
true
```

Example 2:

```
> 1 == exp(0);
true
> asin(1) * 2 == pi;
true
> exp(5) == log(4);
false
```

Example 3:

```
> sin(pi/6) == 1/4;
false
```

Example 4:

```
> prec = 12;
The precision has been set to 12 bits.
> verbosity = 1!;
> 16384.1 == 16385.1;
Warning: Rounding occurred when converting the constant "16384.1" to floating-point with 12 bits.
If safe computation is needed, try to increase the precision.
Warning: Rounding occurred when converting the constant "16385.1" to floating-point with 12 bits.
If safe computation is needed, try to increase the precision.
true
```

Example 5:

```
> error == error;
false
```

Example 6:

```
> a = "Biba";
> b = NaN;
> a == a;
true
> b == b;
false
```

See also: `!=` (8.109), `>` (8.73), `>=` (8.71), `<=` (8.91), `<` (8.100), `in` (8.84), `!` (8.111), `&&` (8.6), `||` (8.116), `error` (8.53), `prec` (8.127)

## 8.51 erf

Name: **erf**

the error function.

Library names:

```
sollya_obj_t sollya_lib_erf(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_erf(sollya_obj_t)
#define SOLLYA_ERF(x) sollya_lib_build_function_erf(x)
```

Description:

- **erf** is the error function defined by:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

- It is defined for every real number  $x$ .

See also: **erfc** (8.52), **exp** (8.56)

## 8.52 erfc

Name: **erfc**

the complementary error function.

Library names:

```
sollya_obj_t sollya_lib_erfc(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_erfc(sollya_obj_t)
#define SOLLYA_ERFC(x) sollya_lib_build_function_erfc(x)
```

Description:

- **erfc** is the complementary error function defined by  $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$ .
- It is defined for every real number  $x$ .

See also: **erf** (8.51)

## 8.53 error

Name: **error**

expression representing an input that is wrongly typed or that cannot be executed

Library names:

```
sollya_obj_t sollya_lib_error()
int sollya_lib_obj_is_error(sollya_obj_t)
```

Usage:

**error** : error

Description:

- The variable **error** represents an input during the evaluation of which a type or execution error has been detected or is to be detected. Inputs that are syntactically correct but wrongly typed evaluate to **error** at some stage. Inputs that are correctly typed but containing commands that depend on side-effects that cannot be performed or inputs that are wrongly typed at meta-level (cf. **parse**), evaluate to **error**.

Remark that in contrast to all other elements of the **Sollya** language, **error** compares neither equal nor unequal to itself. This provides a means of detecting syntax errors inside the **Sollya** language itself without introducing issues of two different wrongly typed inputs being equal.



Example 1:

```
> print(5 + "foo");  
error
```

Example 2:

```
> error;  
error
```

Example 3:

```
> error == error;  
false  
> error != error;  
false
```

Example 4:

```
> correct = 5 + 6;  
> incorrect = 5 + "foo";  
> correct == correct;  
true  
> incorrect == incorrect;  
false  
> errorhappened = !(incorrect == incorrect);  
> errorhappened;  
true
```

See also: **void** (8.189), **parse** (8.117), **==** (8.50), **!=** (8.109)

## 8.54 evaluate

Name: **evaluate**

evaluates a function at a constant point or in a range

Library name:

```
sollya_obj_t sollya_lib_evaluate(sollya_obj_t, sollya_obj_t)
```

Usage:

```
evaluate(function, constant) : (function, constant) → constant | range  
evaluate(function, range) : (function, range) → range  
evaluate(function, function2) : (function, function) → function
```

Parameters:

- *function* represents a function
- *constant* represents a constant point
- *range* represents a range
- *function2* represents a function that is not constant

Description:

- If its second argument is a constant *constant*, **evaluate** evaluates its first argument *function* at the point indicated by *constant*. This evaluation is performed in a way that the result is a faithful rounding of the real value of the *function* at *constant* to the current global precision. If such a faithful rounding is not possible, **evaluate** returns a range surely encompassing the real value of the function *function* at *constant*. If even interval evaluation is not possible because the expression is undefined or numerically unstable, NaN will be produced.

- If its second argument is a range *range*, **evaluate** evaluates its first argument *function* by interval evaluation on this range *range*. This ensures that the image domain of the function *function* on the preimage domain *range* is surely enclosed in the returned range.
- In the case when the second argument is a range that is reduced to a single point (such that `[1; 1]` for instance), the evaluation is performed in the same way as when the second argument is a constant but it produces a range as a result: **evaluate** automatically adjusts the precision of the intern computations and returns a range that contains at most three floating-point consecutive numbers in precision **prec**. This corresponds to the same accuracy as a faithful rounding of the actual result. If such a faithful rounding is not possible, **evaluate** has the same behavior as in the case when the second argument is a constant.
- If its second argument is a function *function2* that is not a constant, **evaluate** replaces all occurrences of the free variable in function *function* by function *function2*.

Example 1:

```
> midpointmode=on!;
> print(evaluate(sin(pi * x), 2.25));
0.70710678118654752440084436210484903928483593768847
> print(evaluate(sin(pi * x), [2.25; 2.25]));
0.707106781186547524400844362104849039284835937688~4/5~
```

Example 2:

```
> print(evaluate(sin(pi * x), 2));
[-1.72986452514381269516508615031098129542836767991679e-12715;7.5941198201187963
145069564314525661706039084390067e-12716]
```

Example 3:

```
> print(evaluate(sin(pi * x), [2, 2.25]));
[-5.143390272677254630046998919961912407349224165421e-50;0.707106781186547524400
84436210484903928483593768866]
```

Example 4:

```
> print(evaluate(sin(pi * x), 2 + 0.25 * x));
sin((pi) * (2 + 0.25 * x))
```

Example 5:

```
> print(evaluate(sin(pi * 1/x), 0));
[@NaN@;@NaN@]
```

See also: **isevaluable** (8.90)

## 8.55 execute

Name: **execute**

executes the content of a file

Usage:

**execute**(*filename*) : string → void

Parameters:

- *filename* is a string representing a file name

Description:

- **execute** opens the file indicated by *filename*, and executes the sequence of commands it contains. This command is evaluated at execution time: this way you can modify the file *filename* (for instance using **bashexecute**) and execute it just after.
- If *filename* contains a command **execute**, it will be executed recursively.
- If *filename* contains a call to **restart**, it will be neglected.
- If *filename* contains a call to **quit**, the commands following **quit** in *filename* will be neglected.

Example 1:

```
> a=2;
> a;
2
> print("a=1;") > "example.sollya";
> execute("example.sollya");
> a;
1
```

Example 2:

```
> verbosity=1!;
> print("a=1; restart; a=2;") > "example.sollya";
> execute("example.sollya");
Warning: a restart command has been used in a file read into another.
This restart command will be neglected.
> a;
2
```

Example 3:

```
> verbosity=1!;
> print("a=1; quit; a=2;") > "example.sollya";
> execute("example.sollya");
Warning: the execution of a file read by execute demanded stopping the interpretation but it is not stopped.
> a;
1
```

See also: **parse** (8.117), **readfile** (8.144), **write** (8.191), **print** (8.130), **bashexecute** (8.17), **quit** (8.139), **restart** (8.149)

## 8.56 exp

Name: **exp**

the exponential function.

Library names:

```
sollya_obj_t sollya_lib_exp(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_exp(sollya_obj_t)
#define SOLLYA_EXP(x) sollya_lib_build_function_exp(x)
```

Description:

- **exp** is the usual exponential function defined as the solution of the ordinary differential equation  $y' = y$  with  $y(0) = 1$ .
- **exp**(*x*) is defined for every real number *x*.

See also: **exp** (8.56), **log** (8.96)

## 8.57 expand

Name: **expand**

expands polynomial subexpressions

Library name:

`sollya_obj_t sollya_lib_expand(sollya_obj_t)`

Usage:

**expand**(*function*) :  $\text{function} \rightarrow \text{function}$

Parameters:

- *function* represents a function

Description:

- **expand**(*function*) expands all polynomial subexpressions in function *function* as far as possible. Factors of sums are multiplied out, power operators with constant positive integer exponents are replaced by multiplications and divisions are multiplied out, i.e. denominators are brought at the most interior point of expressions.

Example 1:

```
> print(expand(x^3));  
x * x * x
```

Example 2:

```
> print(expand((x + 2)^3 + 2 * x));  
8 + 12 * x + 6 * x * x + x * x * x + 2 * x
```

Example 3:

```
> print(expand(exp((x + (x + 3))^5)));  
exp(243 + 405 * x + 270 * x * x + 90 * x * x * x + 15 * x * x * x * x + x * x *  
x * x * x + x * 405 + 108 * x * 5 * x + 54 * x * x * 5 * x + 12 * x * x * x * 5  
* x + x * x * x * x * 5 * x + x * x * 270 + 27 * x * x * x * 10 + 9 * x * x * x  
* x * 10 + x * x * x * x * x * 10 + x * x * x * 90 + 6 * x * x * x * x * 10 + x  
* x * x * x * x * 10 + x * x * x * x * 5 * x + 15 * x * x * x * x + x * x * x *  
x * x)
```

See also: **simplify** (8.162), **simplifysafe** (8.163), **horner** (8.80), **coeff** (8.25), **degree** (8.36)

## 8.58 expm1

Name: **expm1**

shifted exponential function.

Library names:

```
sollya_obj_t sollya_lib_expm1(sollya_obj_t)  
sollya_obj_t sollya_lib_build_function_expm1(sollya_obj_t)  
#define SOLLYA_EXPM1(x) sollya_lib_build_function_expm1(x)
```

Description:

- **expm1** is defined by  $\text{expm1}(x) = \exp(x) - 1$ .
- It is defined for every real number  $x$ .

See also: **exp** (8.56)

## 8.59 exponent

Name: **exponent**

returns the scaled binary exponent of a number.

Library name:

`sollya_obj_t sollya_lib_exponent(sollya_obj_t)`

Usage:

**exponent**( $x$ ) : constant  $\rightarrow$  integer

Parameters:

- $x$  is a dyadic number.

Description:

- **exponent**( $x$ ) is by definition 0 if  $x$  is one of 0, NaN, or Inf.
- If  $x$  is not zero, it can be uniquely written as  $x = m \cdot 2^e$  where  $m$  is an odd integer and  $e$  is an integer. **exponent**( $x$ ) returns  $e$ .

Example 1:

```
> a=round(Pi,20,RN);
> e=exponent(a);
> e;
-17
> m=mantissa(a);
> a-m*2^e;
0
```

See also: **mantissa** (8.101), **precision** (8.128)

## 8.60 externalplot

Name: **externalplot**

plots the error of an external code with regard to a function

Library names:

```
void sollya_lib_externalplot(sollya_obj_t, sollya_obj_t, sollya_obj_t,
                             sollya_obj_t, sollya_obj_t, ...)
void sollya_lib_v_externalplot(sollya_obj_t, sollya_obj_t, sollya_obj_t,
                              sollya_obj_t, sollya_obj_t, va_list)
```

Usage:

**externalplot**(*filename*, *mode*, *function*, *range*, *precision*) : (string, absolute|relative, function, range, integer)  $\rightarrow$  void

**externalplot**(*filename*, *mode*, *function*, *range*, *precision*, *perturb*) : (string, absolute|relative, function, range, integer, perturb)  $\rightarrow$  void

**externalplot**(*filename*, *mode*, *function*, *range*, *precision*, *plot mode*, *result filename*) : (string, absolute|relative, function, range, integer, file|postscript|postscriptfile, string)  $\rightarrow$  void

**externalplot**(*filename*, *mode*, *function*, *range*, *precision*, *perturb*, *plot mode*, *result filename*) : (string, absolute|relative, function, range, integer, perturb, file|postscript|postscriptfile, string)  $\rightarrow$  void

Description:

- The command **externalplot** plots the error of an external function evaluation code sequence implemented in the object file named *filename* with regard to the function *function*. If *mode* evaluates to *absolute*, the difference of both functions is considered as an error function; if *mode*

evaluates to *relative*, the difference is divided by the function *function*. The resulting error function is plotted on all floating-point numbers with *precision* significant mantissa bits in the range *range*.

If the sixth argument of the command **externalplot** is given and evaluates to **perturb**, each of the floating-point numbers the function is evaluated at gets perturbed by a random value that is uniformly distributed in  $\pm 1$  ulp around the original *precision* bit floating-point variable.

If a sixth and seventh argument, respectively a seventh and eighth argument in the presence of **perturb** as a sixth argument, are given that evaluate to a variable of type `file|postscript|postscriptfile` respectively to a character sequence of type `string`, **externalplot** will plot (additionally) to a file in the same way as the command **plot** does. See **plot** for details.

The external function evaluation code given in the object file name *filename* is supposed to define a function name *f* as follows (here in C syntax): `void f(mpfr_t rop, mpfr_t op)`. This function is supposed to evaluate *op* with an accuracy corresponding to the precision of *rop* and assign this value to *rop*.

Example 1:

```
> bashexecute("gcc -fPIC -c externalplotexample.c");
> bashexecute("gcc -shared -o externalplotexample externalplotexample.o -lgmp -lmpfr");
> externalplot("./externalplotexample",relative,exp(x),[-1/2;1/2],12,perturb);
```

See also: **plot** (8.120), **asciipLOT** (8.9), **perturb** (8.118), **absolute** (8.2), **relative** (8.146), **file** (8.63), **postscript** (8.123), **postscriptfile** (8.124), **bashexecute** (8.17), **externalproc** (8.61), **library** (8.93)

## 8.61 externalproc

Name: **externalproc**

binds an external code to a Sollya procedure

Usage:

**externalproc**(*identifier*, *filename*, *argumenttype*  $\rightarrow$  *resulttype*) : (identifier type, string, type type, type type)  $\rightarrow$  void

Parameters:

- *identifier* represents the identifier the code is to be bound to
- *filename* of type `string` represents the name of the object file where the code of procedure can be found
- *argumenttype* represents a definition of the types of the arguments of the Sollya procedure and the external code
- *resulttype* represents a definition of the result type of the external code

Description:

- **externalproc** allows for binding the Sollya identifier *identifier* to an external code. After this binding, when Sollya encounters *identifier* applied to a list of actual parameters, it will evaluate these parameters and call the external code with these parameters. If the external code indicated success, it will receive the result produced by the external code, transform it to Sollya's internal representation and return it.

In order to allow correct evaluation and typing of the data in parameter and in result to be passed to and received from the external code, **externalproc** has a third parameter *argumenttype*  $\rightarrow$  *resulttype*. Both *argumenttype* and *resulttype* are one of **void**, **constant**, **function**, **range**, **integer**, **string**, **boolean**, **list of constant**, **list of function**, **list of range**, **list of integer**, **list of string**, **list of boolean**.

If upon a usage of a procedure bound to an external procedure the type of the actual parameters given or its number is not correct, **Sollya** produces a type error. An external function not applied to arguments represents itself and prints out with its argument and result types.

The external function is supposed to return an integer indicating success. It returns its result depending on its **Sollya** result type as follows. Here, the external procedure is assumed to be implemented as a C function.

If the **Sollya** result type is **void**, the C function has no pointer argument for the result. If the **Sollya** result type is **constant**, the first argument of the C function is of C type `mpfr_t *`, the result is returned by affecting the MPFR variable. If the **Sollya** result type is **function**, the first argument of the C function is of C type `node **`, the result is returned by the `node *` pointed with a new `node *`. If the **Sollya** result type is **range**, the first argument of the C function is of C type `mpfi_t *`, the result is returned by affecting the interval-arithmetic variable. If the **Sollya** result type is **integer**, the first argument of the C function is of C type `int *`, the result is returned by affecting the int variable. If the **Sollya** result type is **string**, the first argument of the C function is of C type `char **`, the result is returned by the `char *` pointed with a new `char *`. If the **Sollya** result type is **boolean**, the first argument of the C function is of C type `int *`, the result is returned by affecting the int variable with a boolean value. If the **Sollya** result type is **list of** type, the first argument of the C function is of C type `chain **`, the result is returned by the `chain *` pointed with a new `chain *`. This chain contains for **Sollya** type **constant** pointers `mpfr_t *` to new MPFR variables, for **Sollya** type **function** pointers `node *` to new nodes, for **Sollya** type **range** pointers `mpfi_t *` to new interval-arithmetic variables, for **Sollya** type **integer** pointers `int *` to new int variables for **Sollya** type **string** pointers `char *` to new `char *` variables and for **Sollya** type **boolean** pointers `int *` to new int variables representing boolean values.

The external procedure affects its possible pointer argument if and only if it succeeds. This means, if the function returns an integer indicating failure, it does not leak any memory to the encompassing environment.

The external procedure receives its arguments as follows: If the **Sollya** argument type is **void**, no argument array is given. Otherwise the C function receives a C `void **` argument representing an array of size equal to the arity of the function where each entry (of C type `void *`) represents a value with a C type depending on the corresponding **Sollya** type. If the **Sollya** type is **constant**, the C type the `void *` is to be casted to is `mpfr_t *`. If the **Sollya** type is **function**, the C type the `void *` is to be casted to is `node *`. If the **Sollya** type is **range**, the C type the `void *` is to be casted to is `mpfi_t *`. If the **Sollya** type is **integer**, the C type the `void *` is to be casted to is `int *`. If the **Sollya** type is **string**, the C type the `void *` is to be casted to is `char *`. If the **Sollya** type is **boolean**, the C type the `void *` is to be casted to is `int *`. If the **Sollya** type is **list of** type, the C type the `void *` is to be casted to is `chain *`. Here depending on type, the values in the chain are to be casted to `mpfr_t *` for **Sollya** type **constant**, `node *` for **Sollya** type **function**, `mpfi_t *` for **Sollya** type **range**, `int *` for **Sollya** type **integer**, `char *` for **Sollya** type **string** and `int *` for **Sollya** type **boolean**.

The external procedure is not supposed to alter the memory pointed by its array argument `void **`.

In both directions (argument and result values), empty lists are represented by `chain * NULL` pointers.

In contrast to internal procedures, externally bounded procedures can be considered to be objects inside **Sollya** that can be assigned to other variables, stored in list etc.

Example 1:

```

> bashexecute("gcc -fPIC -Wall -c externalprocexample.c");
> bashexecute("gcc -fPIC -shared -o externalprocexample externalprocexample.o");

> externalproc(foo, "./externalprocexample", (integer, integer) -> integer);
> foo;
foo(integer, integer) -> integer
> foo(5, 6);
11
> verbosity = 1!;
> foo();
Warning: at least one of the given expressions or a subexpression is not correctly typed
or its evaluation has failed because of some error on a side-effect.
error
> a = foo;
> a(5,6);
11

```

See also: **library** (8.93), **libraryconstant** (8.94), **externalplot** (8.60), **bashexecute** (8.17), **void** (8.189), **constant** (8.28), **function** (8.70), **range** (8.140), **integer** (8.87), **string** (8.169), **boolean** (8.20), **list of** (8.95)

## 8.62 false

Name: **false**

the boolean value representing the false.

Library names:

```

sollya_obj_t sollya_lib_false()
int sollya_lib_is_false(sollya_obj_t)

```

Description:

- **false** is the usual boolean value.

Example 1:

```

> true && false;
false
> 2<1;
false

```

See also: **true** (8.185), **&&** (8.6), **||** (8.116)

## 8.63 file

Name: **file**

special value for commands **plot** and **externalplot**

Library names:

```

sollya_obj_t sollya_lib_file()
int sollya_lib_is_file(sollya_obj_t)

```

Description:

- **file** is a special value used in commands **plot** and **externalplot** to save the result of the command in a data file.
- As any value it can be affected to a variable and stored in lists.



Example 1:

```
> savemode=file;
> name="plotSinCos";
> plot(sin(x),0,cos(x),[-Pi,Pi],savemode, name);
```

See also: **externalplot** (8.60), **plot** (8.120), **postscript** (8.123), **postscriptfile** (8.124)

## 8.64 findzeros

Name: **findzeros**

gives a list of intervals containing all zeros of a function on an interval.

Library name:

```
sollya_obj_t sollya_lib_findzeros(sollya_obj_t, sollya_obj_t)
```

Usage:

**findzeros**( $f, I$ ) : (function, range)  $\rightarrow$  list

Parameters:

- $f$  is a function.
- $I$  is an interval.

Description:

- **findzeros**( $f, I$ ) returns a list of intervals  $I_1, \dots, I_n$  such that, for every zero  $z$  of  $f$ , there exists some  $k$  such that  $z \in I_k$ .
- The list may contain intervals  $I_k$  that do not contain any zero of  $f$ . An interval  $I_k$  may contain many zeros of  $f$ .
- This command is meant for cases when safety is critical. If you want to be sure not to forget any zero, use **findzeros**. However, if you just want to know numerical values for the zeros of  $f$ , **dirtyfindzeros** should be quite satisfactory and a lot faster.
- If  $\delta$  denotes the value of global variable **diam**, the algorithm ensures that for each  $k$ ,  $|I_k| \leq \delta \cdot |I|$ .
- The algorithm used is basically a bisection algorithm. It is the same algorithm that the one used for **infnorm**. See the help page of this command for more details. In short, the behavior of the algorithm depends on global variables **prec**, **diam**, **taylorrecursions** and **hopitalrecursions**.

Example 1:

```
> findzeros(sin(x),[-5;5]);
[[[-3.14208984375;-3.140869140625], [-1.220703125e-3;1.220703125e-3], [3.1408691
40625;3.14208984375]]]
> diam=1e-10!;
> findzeros(sin(x),[-5;5]);
[[[-3.14159265370108187198638916015625;-3.141592652536928653717041015625], [-1.1
6415321826934814453125e-9;1.16415321826934814453125e-9], [3.14159265253692865371
7041015625;3.14159265370108187198638916015625]]]
```

See also: **dirtyfindzeros** (8.41), **infnorm** (8.86), **prec** (8.127), **diam** (8.38), **taylorrecursions** (8.180), **hopitalrecursions** (8.79), **numberroots** (8.112)

## 8.65 fixed

Name: **fixed**

indicates that fixed-point formats should be used for **fpminimax**

Library names:

```
sollya_obj_t sollya_lib_fixed()
int sollya_lib_is_fixed(sollya_obj_t)
```

Usage:

**fixed** : fixed|floating

Description:

- The use of **fixed** in the command **fpminimax** indicates that the list of formats given as argument is to be considered to be a list of fixed-point formats. See **fpminimax** for details.

Example 1:

```
> fpminimax(cos(x),6,[|32,32,32,32,32,32,32|],[-1;1],fixed);
0.9999997480772435665130615234375 + x^2 * (-0.4999928693287074565887451171875 +
x^2 * (4.163351492024958133697509765625e-2 + x^2 * (-1.3382239267230033874511718
75e-3)))
```

See also: **fpminimax** (8.68), **floating** (8.66)

## 8.66 floating

Name: **floating**

indicates that floating-point formats should be used for **fpminimax**

Library names:

```
sollya_obj_t sollya_lib_floating()
int sollya_lib_is_floating(sollya_obj_t)
```

Usage:

**floating** : fixed|floating

Description:

- The use of **floating** in the command **fpminimax** indicates that the list of formats given as argument is to be considered to be a list of floating-point formats. See **fpminimax** for details.

Example 1:

```
> fpminimax(cos(x),6,[|D...|],[-1;1],floating);
0.99999974816012215939053930924274027347564697265625 + x * (-2.79593179695850233
444023069510765565920208989246498e-15 + x * (-0.49999286980201401719980935922649
223357439041137695 + x * (4.0484539189054105169841244454207387920433372507922e-1
4 + x * (4.16335155289191682914662351322476752102375030517578e-2 + x * (-4.01585
8818743733758578949218474363725507386355118e-14 + x * (-1.3382240885483781024645
2001194938929984346032142639e-3))))))
```

See also: **fpminimax** (8.68), **fixed** (8.65)

## 8.67 floor

Name: **floor**

the usual function floor.

Library names:

```

sollya_obj_t sollya_lib_floor(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_floor(sollya_obj_t)
#define SOLLYA_FLOOR(x) sollya_lib_build_function_floor(x)

```

Description:

- **floor** is defined as usual: **floor**( $x$ ) is the greatest integer  $y$  such that  $y \leq x$ .
- It is defined for every real number  $x$ .

See also: **ceil** (8.22), **nearestint** (8.108), **round** (8.153), **RD** (8.143)

## 8.68 fpminimax

Name: **fpminimax**

computes a good polynomial approximation with fixed-point or floating-point coefficients

Library names:

```

sollya_obj_t sollya_lib_fpminimax(sollya_obj_t, sollya_obj_t, sollya_obj_t,
                                   sollya_obj_t, ...)
sollya_obj_t sollya_lib_v_fpminimax(sollya_obj_t, sollya_obj_t,
                                   sollya_obj_t, sollya_obj_t, va_list)

```

Usage:

```

fpminimax( $f$ ,  $n$ ,  $formats$ ,  $range$ ,  $indic1$ ,  $indic2$ ,  $indic3$ ,  $P$ ) : (function, integer, list, range,
absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, absolute|relative |
fixed|floating | function, function)  $\rightarrow$  function
fpminimax( $f$ ,  $monomials$ ,  $formats$ ,  $range$ ,  $indic1$ ,  $indic2$ ,  $indic3$ ,  $P$ ) : (function, list, list, range,
absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, absolute|relative |
fixed|floating | function, function)  $\rightarrow$  function
fpminimax( $f$ ,  $n$ ,  $formats$ ,  $L$ ,  $indic1$ ,  $indic2$ ,  $indic3$ ,  $P$ ) : (function, integer, list, list, absolute|relative |
fixed|floating | function, absolute|relative | fixed|floating | function, absolute|relative | fixed|floating |
function, function)  $\rightarrow$  function
fpminimax( $f$ ,  $monomials$ ,  $formats$ ,  $L$ ,  $indic1$ ,  $indic2$ ,  $indic3$ ,  $P$ ) : (function, list, list, list,
absolute|relative | fixed|floating | function, absolute|relative | fixed|floating | function, absolute|relative |
fixed|floating | function, function)  $\rightarrow$  function

```

Parameters:

- $f$  is the function to be approximated
- $n$  is the degree of the polynomial that must approximate  $f$
- $monomials$  is a list of integers or a list of function. It indicates the basis for the approximation of  $f$
- $formats$  is a list indicating the formats that the coefficients of the polynomial must have
- $range$  is the interval where the function must be approximated
- $L$  is a list of interpolation points used by the method
- $indic1$  (optional) is one of the optional indication parameters. See the detailed description below.
- $indic2$  (optional) is one of the optional indication parameters. See the detailed description below.
- $indic3$  (optional) is one of the optional indication parameters. See the detailed description below.
- $P$  (optional) is the minimax polynomial to be considered for solving the problem.

Description:

- **fpminimax** uses a heuristic (but practically efficient) method to find a good polynomial approximation of a function  $f$  on an interval  $range$ . It implements the method published in the article:  
Efficient polynomial  $L^\infty$ -approximations  
Nicolas Brisebarre and Sylvain Chevillard  
Proceedings of the 18th IEEE Symposium on Computer Arithmetic (ARITH 18)  
pp. 169-176
- The basic usage of this command is **fpminimax**( $f, n, formats, range$ ). It computes a polynomial approximation of  $f$  with degree at most  $n$  on the interval  $range$ .  $formats$  is a list of integers or format types (such as **double**, **doubledouble**, etc.). The polynomial returned by the command has its coefficients that fit the formats indications. For instance, if `formats[0]` is 35, the coefficient of degree 0 of the polynomial will fit a floating-point format of 35 bits. If `formats[1]` is D, the coefficient of degree 1 will be representable by a floating-point number with a precision of 53 bits (which is not necessarily an IEEE 754 double precision number. See the remark below), etc.
- The second argument may be either an integer, a list of integers or a list of functions. An integer indicates the degree of the desired polynomial approximation. A list of integers indicates the list of desired monomials. For instance, the list `[[0, 2, 4, 6]]` indicates that the polynomial must be even and of degree at most 6. Giving an integer  $n$  as second argument is equivalent as giving `[[0, ..., n]]`. Finally, a list of function  $g_k$  indicates that the desired approximation must be a linear combination of the  $g_k$ .  
The list of formats is interpreted with respect to the list of monomials. For instance, if the list of monomials is `[[0, 2, 4, 6]]` and the list of formats is `[[161, 107, 53, 24]]`, the coefficients of degree 0 is searched as a floating-point number with precision 161, the coefficient of degree 2 is searched as a number of precision 107, and so on.
- The list of formats may contain either integers or format types (**halfprecision**, **single**, **double**, **doubledouble**, **tripledouble**, **doubleextended** and **quad**). The list may be too large or even infinite. Only the first indications will be considered. For instance, for a degree  $n$  polynomial, `formats[n + 1]` and above will be discarded. This lets one use elliptical indications for the last coefficients.
- The floating-point coefficients considered by **fpminimax** do not have an exponent range. In particular, in the format list, **double** is an exact synonym for 53. Currently, **fpminimax** only ensures that the corresponding coefficient has at most 53 bits of mantissa. It does not imply that it is an IEEE-754 double.
- By default, the list of formats is interpreted as a list of floating-point formats. This may be changed by passing **fixed** as an optional argument (see below). Let us take an example: **fpminimax**( $f, 2, [107, DD, 53], [0;1]$ ). Here the optional argument is missing (we could have set it to **floating**). Thus, **fpminimax** will search for a polynomial of degree 2 with a constant coefficient that is a 107 bits floating-point number, etc.  
Currently, **doubledouble** is just a synonym for 107 and **tripledouble** a synonym for 161. This behavior may change in the future (taking into account the fact that some double-doubles are not representable with 107 bits).  
Second example: **fpminimax**( $f, 2, [25, 18, 30], [0;1], \text{fixed}$ ). In this case, **fpminimax** will search for a polynomial of degree 2 with a constant coefficient of the form  $m/2^{25}$  where  $m$  is an integer. In other words, it is a fixed-point number with 25 bits after the point. Note that even with argument **fixed**, the formats list is allowed to contain **halfprecision**, **single**, **double**, **doubleextended**, **doubledouble**, **quad** or **tripledouble**. In this this case, it is just a synonym for 11, 24, 53, 64, 107, 113 or 161. This is deprecated and may change in the future.
- The fourth argument may be a range or a list. Lists are for advanced users that know what they are doing. The core of the method is a kind of approximated interpolation. The list given here is a list of points that must be considered for the interpolation. It must contain at least as many points as unknown coefficients. If you give a list, it is also recommended that you provide the minimax polynomial as last argument. If you give a range, the list of points will be automatically computed.

- The fifth, sixth and seventh arguments are optional. By default, **fpminimax** will approximate  $f$  while optimizing the relative error, and interpreting the list of formats as a list of floating-point formats.

This default behavior may be changed with these optional arguments. You may provide zero, one, two or three of the arguments in any order. This lets the user indicate only the non-default arguments.

The three possible arguments are:

- **relative** or **absolute**: the error to be optimized;
- **floating** or **fixed**: formats of the coefficients;
- a constrained part  $q$ .

The constrained part lets the user assign in advance some of the coefficients. For instance, for approximating  $\exp(x)$ , it may be interesting to search for a polynomial  $p$  of the form

$$p = 1 + x + \frac{x^2}{2} + a_3x^3 + a_4x^4.$$

Thus, there is a constrained part  $q = 1 + x + x^2/2$  and the unknown polynomial should be considered in the monomial basis  $[[3, 4]]$ . Calling **fpminimax** with monomial basis  $[[3, 4]]$  and constrained part  $q$ , will return a polynomial with the right form.

- The last argument is for advanced users. It is the minimax polynomial that approximates the function  $f$  in the given basis. If it is not given this polynomial will be automatically computed by **fpminimax**.

This minimax polynomial is used to compute the list of interpolation points required by the method. It is also used, when floating-point coefficients are desired, to give an initial assumption for the exponents of the coefficients. In general, you do not have to provide this argument. But if you want to obtain several polynomials of the same degree that approximate the same function on the same range, just changing the formats, you should probably consider computing only once the minimax polynomial and the list of points instead of letting **fpminimax** recompute them each time.

Note that in the case when a constrained part is given, the minimax polynomial must take that into account. For instance, in the previous example, the minimax would be obtained by the following command:

```
P = remez(1-(1+x+x^2/2)/exp(x), [[3,4]], range, 1/exp(x));
```

Note that the constrained part is not to be added to  $P$ .

In the case when the second argument is an integer or a list of integers, there is no restriction for  $P$ , as long as it is a polynomial. However, when the second argument is a list of functions, and even if these functions are all polynomials,  $P$  must be expanded in the given basis. For instance, if the second argument is 2 or  $[[0, 1, 2]]$ ,  $P$  can be given in Horner form. However, if the second argument is  $[[1, x, x^2]]$ ,  $P$  must be written as a linear combination of 1,  $x$  and  $x^2$ , otherwise, the algorithm will fail to recover the coefficients of  $P$  and will fail with an error message.

Please also note that recovering the coefficients of  $P$  in an arbitrary basis is performed heuristically and no verification is performed to check that  $P$  does not contain other functions than the functions of the basis.

- Note that **fpminimax** internally computes a minimax polynomial (using the same algorithm as **remez** command). Thus **fpminimax** may encounter the same problems as **remez**. In particular, it may be very slow when Haar condition is not fulfilled. Another consequence is that currently **fpminimax** has to be run with a sufficiently high working precision.

Example 1:

```
> P = fpminimax(cos(x),6,[|DD, DD, D...|],[-1b-5;1b-5]);
> printexpansion(P);
(0x3ff0000000000000 + 0xbc09fda15e029b00) + x * ((0x3af9eb57163024a8 + 0x37942c2
f3f3e3839) + x * (0xbfdffffffffffff98 + x * (0xbbd1693f9c028849 + x * (0x3fa55555
55145337 + x * (0x3c7a25f610ad9ebc + x * 0xbf56c138142da5b0))))))
```

Example 2:

```
> P = fpminimax(sin(x),6,[|32...|],[-1b-5;1b-5], fixed, absolute);
> display = powers!;
> P;
x * (1 + x^2 * (-357913941 * 2^(-31) + x^2 * (35789873 * 2^(-32))))
```

Example 3:

```
> P = fpminimax(exp(x), [|3,4|], [|D,24|], [-1/256; 1/246], 1+x+x^2/2);
> display = powers!;
> P;
1 + x * (1 + x * (1 * 2^(-1) + x * (375300225001191 * 2^(-51) + x * (5592621 * 2^(-27)))))
```

Example 4:

```
> f = cos(exp(x));
> pstar = remez(f, 5, [-1b-7;1b-7]);
> listpoints = dirtyfindzeros(f-pstar, [-1b-7; 1b-7]);
> P1 = fpminimax(f, 5, [|DD...|], listpoints, absolute, default, default, pstar);
> P2 = fpminimax(f, 5, [|D...|], listpoints, absolute, default, default, pstar);

> P3 = fpminimax(f, 5, [|D, D, D, 24...|], listpoints, absolute, default, default, pstar);
> print("Error of pstar: ", dirtyinfnorm(f-pstar, [-1b-7; 1b-7]));
Error of pstar: 7.9048441259903026332577436001060063099817726177425e-16
> print("Error of P1: ", dirtyinfnorm(f-P1, [-1b-7; 1b-7]));
Error of P1: 7.9048441259903026580081299123420463921479618202064e-16
> print("Error of P2: ", dirtyinfnorm(f-P2, [-1b-7; 1b-7]));
Error of P2: 8.2477144579950871737109573536791331686347620955984e-16
> print("Error of P3: ", dirtyinfnorm(f-P3, [-1b-7; 1b-7]));
Error of P3: 1.08454277156993282593701156841863009789063333951055e-15
```

Example 5:

```
> L = [|exp(x), sin(x), cos(x)-1, sin(x^3)|];
> g = (2^x-1)/x;
> p = fpminimax(g, L, [|D...|], [-1/16;1/16],absolute);
> display = powers!;
> p;
-3267884797436153 * 2^(-54) * sin(x^3) + 5247089102535885 * 2^(-53) * (cos(x) - 1) + -8159095033730771 * 2^(-54) * sin(x) + 6243315658446641 * 2^(-53) * exp(x)
```

Example 6:

```

> n = 9;
> T = [|1, x|];
> for i from 2 to n do T[i] = canonical(2*x*T[i-1]-T[i-2]);
> g = (2^x-1)/x;
> PCheb = fpminimax(g, T, [|DD,DE...|], [-1/16;1/16],absolute);
> display = dyadic!;
> print(PCheb);
8733930098894247371b-98 * (9 * x + -120 * x^3 + 432 * x^5 + -576 * x^7 + 256 * x
^9) + 15750497046710770365b-94 * (1 + -32 * x^2 + 160 * x^4 + -256 * x^6 + 128 *
x^8) + 6467380330985872933b-88 * (-7 * x + 56 * x^3 + -112 * x^5 + 64 * x^7) +
9342762606926218927b-84 * (-1 + 18 * x^2 + -48 * x^4 + 32 * x^6) + 1181452136745
6461131b-80 * (5 * x + -20 * x^3 + 16 * x^5) + 6405479474328570593b-75 * (1 + -8
* x^2 + 8 * x^4) + 11584457324781949889b-72 * (-3 * x + 4 * x^3) + 167797053124
47201161b-69 * (-1 + 2 * x^2) + 18265014280997359319b-66 * x + 11705449744817514
3902009975397253b-107

```

See also: **remez** (8.147), **dirtyfindzeros** (8.41), **absolute** (8.2), **relative** (8.146), **fixed** (8.65), **floating** (8.66), **default** (8.35), **halfprecision** (8.75), **single** (8.165), **double** (8.46), **doubleextended** (8.48), **doubledouble** (8.47), **quad** (8.138), **tripledouble** (8.184), **implementpoly** (8.83), **coeff** (8.25), **degree** (8.36), **roundcoefficients** (8.154), **guessdegree** (8.74)

## 8.69 fullparentheses

Name: **fullparentheses**

activates, deactivates or inspects the state variable controlling output with full parenthesising

Library names:

```

void sollya_lib_set_fullparentheses_and_print(sollya_obj_t);
void sollya_lib_set_fullparentheses(sollya_obj_t);
sollya_obj_t sollya_lib_get_fullparentheses();

```

Usage:

```

fullparentheses = activation value : on|off → void
fullparentheses = activation value ! : on|off → void

```

Parameters:

- *activation value* represents **on** or **off**, i.e. activation or deactivation

Description:

- An assignment **fullparentheses** = *activation value*, where *activation value* is one of **on** or **off**, activates respectively deactivates the output of expressions with full parenthesising. In full parenthesising mode, **Sollya** commands like **print**, **write** and the implicit command when an expression is given at the prompt will output expressions with parenthesising at all places where it is necessary for expressions containing infix operators to be parsed back with the same result. Otherwise parentheses around associative operators are omitted.

If the assignment **fullparentheses** = *activation value* is followed by an exclamation mark, no message indicating the new state is displayed. Otherwise the user is informed of the new state of the global mode by an indication.

Example 1:

```

> autosimplify = off!;
> fullparentheses = off;
Full parentheses mode has been deactivated.
> print(1 + 2 + 3);
1 + 2 + 3
> fullparentheses = on;
Full parentheses mode has been activated.
> print(1 + 2 + 3);
(1 + 2) + 3

```

See also: **print** (8.130), **write** (8.191), **autosimplify** (8.15)

## 8.70 function

Name: **function**

keyword for declaring a procedure-based function or a keyword representing a function type

Library names:

```

sollya_obj_t sollya_lib_procedurefunction(sollya_obj_t, sollya_obj_t)
sollya_obj_t sollya_lib_build_function_procedurefunction(sollya_obj_t,
                                                         sollya_obj_t)

```

Usage:

**function**(*procedure*) : procedure → function  
**function** : type type

Parameters:

- *procedure* is a procedure of type (range, integer, integer) → range

Description:

- For the sake of safety and mathematical consistency, **Sollya** distinguishes clearly between functions, seen in the mathematical sense of the term, i.e. mappings, and procedures, seen in the sense Computer Science gives to functions, i.e. pieces of code that compute results for arguments following an algorithm. In some cases however, it is interesting to use such Computer Science procedures as realisations of mathematical functions, e.g. in order to plot them or even to perform polynomial approximation on them. The **function** keyword allows for such a transformation of a **Sollya** procedure into a **Sollya** function.
- The procedure to be used as a function through **function**(*procedure*) must be of type (range, integer, integer) → range. This means it must take in argument an interval  $X$ , a degree of differentiation  $n$  and a working precision  $p$ . It must return in result an interval  $Y$  encompassing the image  $f^{(n)}(X)$  of the  $n$ -th derivative of the implemented function  $f$ , i.e.  $f^{(n)}(X) \subseteq Y$ . In order to allow **Sollya**'s algorithms to work properly, the procedure must ensure that, whenever  $(p, \text{diam}(X))$  tends to  $(+\infty, 0)$ , the computed over-estimated bounding  $Y$  tends to the actual image  $f^{(n)}(X)$ .
- The user must be aware that they are responsible of the correctness of the procedure. If, for some  $n$  and  $X$ , *procedure* returns an interval  $Y$  such that  $f^{(n)}(X) \not\subseteq Y$ , **function** will successfully return a function without any complain, but this function might behave inconsistently in further computations.
- For cases when the procedure does not have the correct signature or does not return a finite interval as a result **function**(*procedure*) evaluates to Not-A-Number (resp. to an interval of Not-A-Numbers for interval evaluation).
- **function** also represents the function type for declarations of external procedures by means of **externalproc**.

Remark that in contrast to other indicators, type indicators like **function** cannot be handled outside the **externalproc** context. In particular, they cannot be assigned to variables.



Example 1:

```

> procedure EXP(X,n,p) {
    var res, oldPrec;
    oldPrec = prec;
    prec = p!;

    res = exp(X);

    prec = oldPrec!;
    return res;
};
> f = function(EXP);
> f(1);
2.71828182845904523536028747135266249775724709369998
> exp(1);
2.71828182845904523536028747135266249775724709369998
> f(x + 3);
(function(proc(X, n, p)
{
var res, oldPrec;
oldPrec = prec;
prec = p!;
res = exp(X);
prec = oldPrec!;
return res;
}))(3 + x)
> diff(f);
diff(function(proc(X, n, p)
{
var res, oldPrec;
oldPrec = prec;
prec = p!;
res = exp(X);
prec = oldPrec!;
return res;
})))
> (diff(f))(0);
1
> g = f(sin(x));
> g(17);
0.382358169993866834026905546416556413595734583420876
> diff(g);
(diff(function(proc(X, n, p)
{
var res, oldPrec;
oldPrec = prec;
prec = p!;
res = exp(X);
prec = oldPrec!;
return res;
}))) (sin(x)) * cos(x)
> (diff(g))(1);
1.25338076749344683697237458088447611474812675164344
> p = remez(f,3,[-1/2;1/2]);
> p;
0.99967120901420646830315493949039176881764871951832 + x * (0.999737029835711401
34762682913614052309208076875596 + x * (0.51049729360282624921622721654643510358
3073053437 + x * 0.169814324607133287588897694747370380479108785868016))

```

See also: **proc** (8.135), **library** (8.93), **procedure** (8.136), **externalproc** (8.61), **boolean** (8.20), **constant** (8.28), **integer** (8.87), **list of** (8.95), **range** (8.140), **string** (8.169)

## 8.71 >=

Name: >=  
greater-than-or-equal-to operator

Library name:  
`sollya_obj_t sollya_lib_cmp_greater_equal(sollya_obj_t, sollya_obj_t)`

Usage:

$$expr1 \geq expr2 : (\text{constant}, \text{constant}) \rightarrow \text{boolean}$$

Parameters:

- *expr1* and *expr2* represent constant expressions

Description:

- The operator >= evaluates to true iff its operands *expr1* and *expr2* evaluate to two floating-point numbers  $a_1$  respectively  $a_2$  with the global precision **prec** and  $a_1$  is greater than or equal to  $a_2$ . The user should be aware of the fact that because of floating-point evaluation, the operator >= is not exactly the same as the mathematical operation *greater-than-or-equal-to*.

Example 1:

```
> 5 >= 4;
true
> 5 >= 5;
true
> 5 >= 6;
false
> exp(2) >= exp(1);
true
> log(1) >= exp(2);
false
```

Example 2:

```
> prec = 12;
The precision has been set to 12 bits.
> 16384.1 >= 16385.1;
true
```

See also: == (8.50), != (8.109), > (8.73), <= (8.91), < (8.100), in (8.84), ! (8.111), && (8.6), || (8.116), **prec** (8.127), **max** (8.102), **min** (8.105)

## 8.72 getsuppressedmessages

Name: **getsuppressedmessages**  
returns a list of numbers of messages that have been suppressed from message output

Library name:  
`sollya_obj_t sollya_lib_getsuppressedmessages();`

Usage:

$$\text{getsuppressedmessages}() : \text{void} \rightarrow \text{list}$$

Description:

- The **getsuppressedmessages** command allows the user to inspect the state of warning and information message suppression. When called, **getsuppressedmessages** returns a list of integers numbers that stand for the warning and information messages that have been suppressed. If no message is suppressed, **getsuppressedmessages** returns an empty list.
- Every Sollya warning or information message (that is not fatal to the tool's execution) has a message number. By default, these numbers are not displayed when a message is output. When message number displaying is activated using **showmessagenumbers**, the message numbers are displayed together with the message. This allows the user to match the numbers returned in a list by **getsuppressedmessages** with the actual warning and information messages.
- The list of message numbers returned by **getsuppressedmessages** is suitable to be fed into the **unsuppressmessage** command. This way, the user may unsuppress all warning and information messages that have been suppressed.

Example 1:

```
> verbosity = 1;
The verbosity level has been set to 1.
> 0.1;
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 165 bits.
If safe computation is needed, try to increase the precision.
0.1
> suppressmessage(174);
> 0.1;
0.1
> suppressmessage(407);
> 0.1;
0.1
> getsuppressedmessages();
[174, 407]
> suppressmessage(207, 196);
> getsuppressedmessages();
[174, 196, 207, 407]
```

Example 2:

```
> suppressmessage(174, 209, 13, 24, 196);
> suppressmessage([16, 17]);
> suppressmessage(19);
> unsuppressmessage([13, 17]);
> getsuppressedmessages();
[16, 19, 24, 174, 196, 209]
> unsuppressmessage(getsuppressedmessages());
> getsuppressedmessages();
[]
```

Example 3:

```

> verbosity = 12;
The verbosity level has been set to 12.
> suppressmessage(174);
> exp(x * 0.1);
Information: no Horner simplification will be performed because the given tree i
s already in Horner form.
exp(x * 0.1)
> getsuppressedmessages();
[|174|]
> verbosity = 0;
The verbosity level has been set to 0.
> exp(x * 0.1);
exp(x * 0.1)
> getsuppressedmessages();
[|174|]

```

See also: `getsuppressedmessages` (8.72), `suppressmessage` (8.174), `unsuppressmessage` (8.186), `verbosity` (8.188), `roundingwarnings` (8.156)

### 8.73 >

Name: >

greater-than operator

Library name:

`sollya_obj_t sollya_lib_cmp_greater(sollya_obj_t, sollya_obj_t)`

Usage:

$expr1 > expr2 : (\text{constant}, \text{constant}) \rightarrow \text{boolean}$

Parameters:

- *expr1* and *expr2* represent constant expressions

Description:

- The operator `>` evaluates to true iff its operands *expr1* and *expr2* evaluate to two floating-point numbers  $a_1$  respectively  $a_2$  with the global precision **prec** and  $a_1$  is greater than  $a_2$ . The user should be aware of the fact that because of floating-point evaluation, the operator `>` is not exactly the same as the mathematical operation *greater-than*.

Example 1:

```

> 5 > 4;
true
> 5 > 5;
false
> 5 > 6;
false
> exp(2) > exp(1);
true
> log(1) > exp(2);
false

```

Example 2:

```

> prec = 12;
The precision has been set to 12 bits.
> 16385.1 > 16384.1;
false

```

See also: `==` (8.50), `!=` (8.109), `>=` (8.71), `<=` (8.91), `<` (8.100), `in` (8.84), `!` (8.111), `&&` (8.6), `||` (8.116), `prec` (8.127), `max` (8.102), `min` (8.105)

## 8.74 guessdegree

Name: **guessdegree**

returns the minimal degree needed for a polynomial to approximate a function with a certain error on an interval.

Library names:

```
sollya_obj_t sollya_lib_guessdegree(sollya_obj_t, sollya_obj_t,
                                     sollya_obj_t, ...)
sollya_obj_t sollya_lib_v_guessdegree(sollya_obj_t, sollya_obj_t,
                                       sollya_obj_t, va_list)
```

Usage:

**guessdegree**( $f, I, \epsilon, w, bound$ ) : (function, range, constant, function, constant)  $\rightarrow$  range

Parameters:

- $f$  is the function to be approximated.
- $I$  is the interval where the function must be approximated.
- $\epsilon$  is the maximal acceptable error.
- $w$  (optional) is a weight function. Default is 1.
- $bound$  (optional) is a bound on the degree. Default is currently 128.

Description:

- **guessdegree** tries to find the minimal degree needed to approximate  $f$  on  $I$  by a polynomial with an error  $\epsilon = pw - f$  whose infinity norm not greater than  $\epsilon$ . More precisely, it finds  $n$  minimal such that there exists a polynomial  $p$  of degree  $n$  such that  $\|pw - f\|_\infty < \epsilon$ .
- **guessdegree** returns an interval: for common cases, this interval is reduced to a single number (i.e. the minimal degree). But in certain cases, **guessdegree** does not succeed in finding the minimal degree. In such cases the returned interval is of the form  $[n, p]$  such that:
  - no polynomial of degree  $n - 1$  gives an error less than  $\epsilon$ .
  - there exists a polynomial of degree  $p$  giving an error less than  $\epsilon$ .
- The fifth optional argument  $bound$  is used to prevent **guessdegree** from trying to find too large degrees. If **guessdegree** does not manage to find a degree  $n$  satisfying the error and such that  $n \leq bound$ , an interval of the form  $[\cdot, +\infty]$  is returned. Note that  $bound$  must be a positive integer.

Example 1:

```
> guessdegree(exp(x), [-1;1], 1e-10);
[10;10]
```

Example 2:

```
> guessdegree(exp(x), [-1;1], 1e-10, default, 9);
[10;@Inf@]
```

Example 3:

```
> guessdegree(1, [-1;1], 1e-8, 1/exp(x));
[8;9]
```

See also: **dirtyinfnorm** (8.42), **remez** (8.147), **fpminimax** (8.68), **degree** (8.36)

## 8.75 halfprecision

Names: **halfprecision**, **HP**

rounding to the nearest IEEE 754 half-precision number (binary16).

Library names:

```
sollya_obj_t sollya_lib_halfprecision(sollya_obj_t)
sollya_obj_t sollya_lib_halfprecision_obj()
int sollya_lib_is_halfprecision_obj(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_halfprecision(sollya_obj_t)
#define SOLLYA_HP(x) sollya_lib_build_function_halfprecision(x)
```

Description:

- **halfprecision** is both a function and a constant.
- As a function, it rounds its argument to the nearest IEEE 754 half-precision (i.e. IEEE754-2008 binary16) number. Subnormal numbers are supported as well as standard numbers: it is the real rounding described in the standard.
- As a constant, it symbolizes the half-precision format. It is used in contexts when a precision format is necessary, e.g. in the commands **round** and **roundcoefficients**. It is not supported for **implementpoly**. See the corresponding help pages for examples.

Example 1:

```
> display=binary!;
> HP(0.1);
1.100110011_2 * 2^(-4)
> HP(4.17);
1.00001011_2 * 2^(2)
> HP(1.011_2 * 2^(-23));
1.1_2 * 2^(-23)
```

See also: **single** (8.165), **double** (8.46), **doubleextended** (8.48), **doubledouble** (8.47), **quad** (8.138), **tripledouble** (8.184), **roundcoefficients** (8.154), **fpminimax** (8.68), **implementpoly** (8.83), **round** (8.153), **printsingl** (8.133)

## 8.76 head

Name: **head**

gives the first element of a list.

Library name:

```
sollya_obj_t sollya_lib_head(sollya_obj_t)
```

Usage:

**head**( $L$ ) : list  $\rightarrow$  any type

Parameters:

- $L$  is a list.

Description:

- **head**( $L$ ) returns the first element of the list  $L$ . It is equivalent to  $L[0]$ .
- If  $L$  is empty, the command will fail with an error.

Example 1:

```
> head([|1,2,3|]);  
1  
> head([|1,2...|]);  
1
```

See also: **tail** (8.175)

## 8.77 hexadecimal

Name: **hexadecimal**

special value for global state **display**

Library names:

```
sollya_obj_t sollya_lib_hexadecimal()  
int sollya_lib_is_hexadecimal(sollya_obj_t)
```

Description:

- **hexadecimal** is a special value used for the global state **display**. If the global state **display** is equal to **hexadecimal**, all data will be output in hexadecimal C99/ IEEE 754-2008 notation.

As any value it can be affected to a variable and stored in lists.

See also: **decimal** (8.34), **dyadic** (8.49), **powers** (8.126), **binary** (8.18), **display** (8.44)

## 8.78 honorcoeffprec

Name: **honorcoeffprec**

indicates the (forced) honoring the precision of the coefficients in **implementpoly**

Library names:

```
sollya_obj_t sollya_lib_honorcoeffprec()  
int sollya_lib_is_honorcoeffprec(sollya_obj_t)
```

Usage:

**honorcoeffprec** : honorcoeffprec

Description:

- Used with command **implementpoly**, **honorcoeffprec** makes **implementpoly** honor the precision of the given polynomial. This means if a coefficient needs a double-double or a triple-double to be exactly stored, **implementpoly** will allocate appropriate space and use a double-double or triple-double operation even if the automatic (heuristic) determination implemented in command **implementpoly** indicates that the coefficient could be stored on less precision or, respectively, the operation could be performed with less precision. See **implementpoly** for details.

Example 1:



```

> verbosity = 1!;
> q = implementpoly(1 - simplify(TD(1/6)) * x^2, [-1b-10;1b-10], 1b-60, DD, "p", "imp
lementation.c");
Warning: at least one of the coefficients of the given polynomial has been round
ed in a way
that the target precision can be achieved at lower cost. Nevertheless, the imple
mented polynomial
is different from the given one.
> printexpansion(q);
0x3ff0000000000000 + x^2 * 0xbfc5555555555555
> r = implementpoly(1 - simplify(TD(1/6)) * x^2, [-1b-10;1b-10], 1b-60, DD, "p", "imp
lementation.c", honorcoeffprec);
Warning: the inferred precision of the 2th coefficient of the polynomial is great
er than
the necessary precision computed for this step. This may make the automatic dete
rmination
of precisions useless.
> printexpansion(r);
0x3ff0000000000000 + x^2 * (0xbfc5555555555555 + 0xbc65555555555555 + 0xb9055555
55555555)

```

See also: **implementpoly** (8.83), **printexpansion** (8.132), **fpminimax** (8.68)

## 8.79 hopitalrecursions

Name: **hopitalrecursions**

controls the number of recursion steps when applying L'Hopital's rule.

Library names:

```

void sollya_lib_set_hopitalrecursions_and_print(sollya_obj_t)
void sollya_lib_set_hopitalrecursions(sollya_obj_t)
sollya_obj_t sollya_lib_get_hopitalrecursions()

```

Usage:

```

hopitalrecursions =  $n$  : integer  $\rightarrow$  void
hopitalrecursions =  $n$  ! : integer  $\rightarrow$  void
hopitalrecursions : integer

```

Parameters:

- $n$  represents the number of recursions

Description:

- **hopitalrecursions** is a global variable. Its value represents the number of steps of recursion that are tried when applying L'Hopital's rule. This rule is applied by the interval evaluator present in the core of **Sollya** (and particularly visible in commands like **infnorm**).
- If an expression of the form  $f/g$  has to be evaluated by interval arithmetic on an interval  $I$  and if  $f$  and  $g$  have a common zero in  $I$ , a direct evaluation leads to NaN. **Sollya** implements a safe heuristic to avoid this, based on L'Hopital's rule: in such a case, it can be shown that  $(f/g)(I) \subseteq (f'/g')(I)$ . Since the same problem may exist for  $f'/g'$ , the rule is applied recursively. The number of step in this recursion process is controlled by **hopitalrecursions**.
- Setting **hopitalrecursions** to 0 makes **Sollya** use this rule only once; setting it to 1 makes **Sollya** use the rule twice, and so on. In particular: the rule is always applied at least once, if necessary.

Example 1:

```

> hospitalrecursions=0;
The number of recursions for Hopital's rule has been set to 0.
> evaluate(log(1+x)^2/x^2,[-1/2; 1]);
[-@Inf@;@Inf@]
> hospitalrecursions=1;
The number of recursions for Hopital's rule has been set to 1.
> evaluate(log(1+x)^2/x^2,[-1/2; 1]);
[-2.52258872223978123766892848583270627230200053744108;6.77258872223978123766892
84858327062723020005374411]

```

See also: **taylorrecursions** (8.180), **infnorm** (8.86), **findzeros** (8.64), **evaluate** (8.54)

## 8.80 horner

Name: **horner**

brings all polynomial subexpressions of an expression to Horner form

Library name:

`sollya_obj_t sollya_lib_horner(sollya_obj_t)`

Usage:

**horner**(*function*) : function  $\rightarrow$  function

Parameters:

- *function* represents the expression to be rewritten in Horner form

Description:

- The command **horner** rewrites the expression representing the function *function* in a way such that all polynomial subexpressions (or the whole expression itself, if it is a polynomial) are written in Horner form. The command **horner** does not endanger the safety of computations even in **Sollya**'s floating-point environment: the function returned is mathematically equal to the function *function*.

Example 1:

```

> print(horner(1 + 2 * x + 3 * x^2));
1 + x * (2 + x * 3)
> print(horner((x + 1)^7));
1 + x * (7 + x * (21 + x * (35 + x * (35 + x * (21 + x * (7 + x))))))

```

Example 2:

```

> print(horner(exp((x + 1)^5) - log(asin(x + x^3) + x)));
exp(1 + x * (5 + x * (10 + x * (10 + x * (5 + x)))) - log(asin(x * (1 + x^2)) +
x)

```

See also: **canonical** (8.21), **print** (8.130), **coeff** (8.25), **degree** (8.36), **autosimplify** (8.15), **simplifysafe** (8.163)

## 8.81 HP

Name: **HP**

short form for **halfprecision**

See also: **halfprecision** (8.75)

## 8.82 `implementconstant`

Name: **implementconstant**

implements a constant in arbitrary precision

Library names:

```
void sollya_lib_implementconstant(sollya_obj_t, ...);  
void sollya_lib_v_implementconstant(sollya_obj_t, va_list);
```

Usage:

```
implementconstant(expr) : constant → void  
implementconstant(expr, filename) : (constant, string) → void  
implementconstant(expr, filename, functionname) : (constant, string, string) → void
```

Description:

- The command **implementconstant** implements the constant expression *expr* in arbitrary precision. More precisely, it generates the source code (written in C, and using MPFR) of a C function `const_something` with the following signature:

```
void const_something (mpfr_ptr y, mp_prec_t prec)
```

Let us denote by  $c$  the exact mathematical value of the constant defined by the expression *expr*. When called with arguments  $y$  and *prec* (where the variable  $y$  is supposed to be already initialized), the function `mpfr_const_something` sets the precision of  $y$  to a suitable precision and stores in it an approximate value of  $c$  such that

$$|y - c| \leq |c| 2^{1-\text{prec}}.$$

- When no filename *filename* is given or if **default** is given as *filename*, the source code produced by **implementconstant** is printed on standard output. Otherwise, when *filename* is given as a string of characters, the source code is output to a file named *filename*. If that file cannot be opened and/or written to, **implementconstant** fails and has no other effect.
- When *functionname* is given as an argument to **implementconstant** and *functionname* evaluates to a string of characters, the default name for the C function `const_something` is replaced by *functionname*. When **default** is given as *functionname*, the default name is used nevertheless, as if no *functionname* argument were given. When choosing a character sequence for *functionname*, the user should keep attention to the fact that *functionname* must be a valid C identifier in order to enable error-free compilation of the produced code.
- If *expr* refers to a constant defined with **libraryconstant**, the produced code uses the external code implementing this constant. The user should keep in mind that it is up to them to make sure the symbol for that external code can get resolved when the newly generated code is to be loaded.
- If a subexpression of *expr* evaluates to 0, **implementconstant** will most likely fail with an error message.
- **implementconstant** is unable to implement constant expressions *expr* that contain procedure-based functions, i.e. functions created from **Sollya** procedures using the **function** construct. If *expr* contains such a procedure-based function, **implementconstant** prints a warning and fails silently. The reason for this lack of functionality is that the produced C source code, which is supposed to be compiled, would have to call back to the **Sollya** interpreter in order to evaluate the procedure-based function.
- Similarly, **implementconstant** is currently unable to implement constant expressions *expr* that contain library-based functions, i.e. functions dynamically bound to **Sollya** using the **library** construct. If *expr* contains such a library-based function, **implementconstant** prints a warning and fails silently. Support for this feature is in principle feasible from a technical standpoint and might be added in a future release of **Sollya**.

- Currently, non-differentiable functions such as **double**, **doubledouble**, **tripledouble**, **single**, **halfprecision**, **quad**, **doubleextended**, **floor**, **ceil**, **nearestint** are not supported by **implementconstant**. If **implementconstant** encounters one of them, a warning message is displayed and no code is produced. However, if **autosimplify** equals on, it is possible that **Sollya** silently simplifies subexpressions of *expr* containing such functions and that **implementconstant** successfully produces code for evaluating *expr*.
- While it produces an MPFR-based C source code for *expr*, **implementconstant** takes architectural and system-dependent parameters into account. For example, it checks whether literal constants figuring in *expr* can be represented on a C `long int` type or if they must be stored in a different manner not to affect their accuracy. These tests, performed by **Sollya** during execution of **implementconstant**, depend themselves on the architecture **Sollya** is running on. Users should keep this matter in mind, especially when trying to compile source code on one machine whilst it has been produced on another.

Example 1:

```

> implementconstant(exp(1)+log(2)/sqrt(1/10));
#include <mpfr.h>

void
const_something (mpfr_ptr y, mp_prec_t prec)
{
    /* Declarations */
    mpfr_t tmp1;
    mpfr_t tmp2;
    mpfr_t tmp3;
    mpfr_t tmp4;
    mpfr_t tmp5;
    mpfr_t tmp6;
    mpfr_t tmp7;

    /* Initializations */
    mpfr_init2 (tmp2, prec+5);
    mpfr_init2 (tmp1, prec+3);
    mpfr_init2 (tmp4, prec+8);
    mpfr_init2 (tmp3, prec+7);
    mpfr_init2 (tmp6, prec+11);
    mpfr_init2 (tmp7, prec+11);
    mpfr_init2 (tmp5, prec+11);

    /* Core */
    mpfr_set_prec (tmp2, prec+4);
    mpfr_set_ui (tmp2, 1, MPFR_RNDN);
    mpfr_set_prec (tmp1, prec+3);
    mpfr_exp (tmp1, tmp2, MPFR_RNDN);
    mpfr_set_prec (tmp4, prec+8);
    mpfr_set_ui (tmp4, 2, MPFR_RNDN);
    mpfr_set_prec (tmp3, prec+7);
    mpfr_log (tmp3, tmp4, MPFR_RNDN);
    mpfr_set_prec (tmp6, prec+11);
    mpfr_set_ui (tmp6, 1, MPFR_RNDN);
    mpfr_set_prec (tmp7, prec+11);
    mpfr_set_ui (tmp7, 10, MPFR_RNDN);
    mpfr_set_prec (tmp5, prec+11);
    mpfr_div (tmp5, tmp6, tmp7, MPFR_RNDN);
    mpfr_set_prec (tmp4, prec+7);
    mpfr_sqrt (tmp4, tmp5, MPFR_RNDN);
    mpfr_set_prec (tmp2, prec+5);
    mpfr_div (tmp2, tmp3, tmp4, MPFR_RNDN);
    mpfr_set_prec (y, prec+3);
    mpfr_add (y, tmp1, tmp2, MPFR_RNDN);

    /* Cleaning stuff */
    mpfr_clear(tmp1);
    mpfr_clear(tmp2);
    mpfr_clear(tmp3);
    mpfr_clear(tmp4);
    mpfr_clear(tmp5);
    mpfr_clear(tmp6);
    mpfr_clear(tmp7);
}

```

Example 2:

```
> implementconstant(sin(13/17),"sine_of_thirteen_seventeenth.c");
> readfile("sine_of_thirteen_seventeenth.c");
#include <mpfr.h>

void
const_something (mpfr_ptr y, mp_prec_t prec)
{
    /* Declarations */
    mpfr_t tmp1;
    mpfr_t tmp2;
    mpfr_t tmp3;

    /* Initializations */
    mpfr_init2 (tmp2, prec+6);
    mpfr_init2 (tmp3, prec+6);
    mpfr_init2 (tmp1, prec+6);

    /* Core */
    mpfr_set_prec (tmp2, prec+6);
    mpfr_set_ui (tmp2, 13, MPFR_RNDN);
    mpfr_set_prec (tmp3, prec+6);
    mpfr_set_ui (tmp3, 17, MPFR_RNDN);
    mpfr_set_prec (tmp1, prec+6);
    mpfr_div (tmp1, tmp2, tmp3, MPFR_RNDN);
    mpfr_set_prec (y, prec+2);
    mpfr_sin (y, tmp1, MPFR_RNDN);

    /* Cleaning stuff */
    mpfr_clear(tmp1);
    mpfr_clear(tmp2);
    mpfr_clear(tmp3);
}
```

Example 3:

```

> implementconstant(asin(1/3 * pi),default,"arcsin_of_one_third_pi");
#include <mpfr.h>

void
arcsin_of_one_third_pi (mpfr_ptr y, mp_prec_t prec)
{
    /* Declarations */
    mpfr_t tmp1;
    mpfr_t tmp2;
    mpfr_t tmp3;

    /* Initializations */
    mpfr_init2 (tmp2, prec+8);
    mpfr_init2 (tmp3, prec+8);
    mpfr_init2 (tmp1, prec+8);

    /* Core */
    mpfr_set_prec (tmp2, prec+8);
    mpfr_const_pi (tmp2, MPFR_RNDN);
    mpfr_set_prec (tmp3, prec+8);
    mpfr_set_ui (tmp3, 3, MPFR_RNDN);
    mpfr_set_prec (tmp1, prec+8);
    mpfr_div (tmp1, tmp2, tmp3, MPFR_RNDN);
    mpfr_set_prec (y, prec+2);
    mpfr_asin (y, tmp1, MPFR_RNDN);

    /* Cleaning stuff */
    mpfr_clear(tmp1);
    mpfr_clear(tmp2);
    mpfr_clear(tmp3);
}

```

Example 4:

```

> implementconstant(ceil(log(19 + 1/3)),"constant_code.c","magic_constant");
> readfile("constant_code.c");
#include <mpfr.h>

void
magic_constant (mpfr_ptr y, mp_prec_t prec)
{
    /* Initializations */

    /* Core */
    mpfr_set_prec (y, prec);
    mpfr_set_ui (y, 3, MPFR_RNDN);
}

```

Example 5:

```

> bashexecute("gcc -fPIC -Wall -c libraryconstantexample.c -I$HOME/.local/include");
> bashexecute("gcc -shared -o libraryconstantexample libraryconstantexample.o -lgmp -lmpfr");
> euler_gamma = libraryconstant("./libraryconstantexample");
> implementconstant(euler_gamma^(1/3));
#include <mpfr.h>

void
const_something (mpfr_ptr y, mp_prec_t prec)
{
    /* Declarations */
    mpfr_t tmp1;

    /* Initializations */
    mpfr_init2 (tmp1, prec+1);

    /* Core */
    euler_gamma (tmp1, prec+1);
    mpfr_set_prec (y, prec+2);
    mpfr_root (y, tmp1, 3, MPFR_RNDN);

    /* Cleaning stuff */
    mpfr_clear(tmp1);
}

```

See also: **implementpoly** (8.83), **libraryconstant** (8.94), **library** (8.93), **function** (8.70)

### 8.83 **implementpoly**

Name: **implementpoly**

implements a polynomial using double, double-double and triple-double arithmetic and generates a Gappa proof

Library names:

```

sollya_obj_t sollya_lib_implementpoly(sollya_obj_t, sollya_obj_t,
                                     sollya_obj_t, sollya_obj_t,
                                     sollya_obj_t, sollya_obj_t, ...)
sollya_obj_t sollya_lib_v_implementpoly(sollya_obj_t, sollya_obj_t,
                                       sollya_obj_t, sollya_obj_t,
                                       sollya_obj_t, sollya_obj_t, va_list)

```

Usage:

```

implementpoly(polynomial, range, error bound, format, functionname, filename) : (function, range,
constant, D|double|DD|doubledouble|TD|tripledouble, string, string) → function
implementpoly(polynomial, range, error bound, format, functionname, filename, honor coefficient
precisions) : (function, range, constant, D|double|DD|doubledouble|TD|tripledouble, string, string,
honorcoeffprec) → function
implementpoly(polynomial, range, error bound, format, functionname, filename, proof filename) :
(function, range, constant, D|double|DD|doubledouble|TD|tripledouble, string, string, string) → function
implementpoly(polynomial, range, error bound, format, functionname, filename, honor coefficient
precisions, proof filename) : (function, range, constant, D|double|DD|doubledouble|TD|tripledouble, string,
string, honorcoeffprec, string) → function

```

Description:

- The command **implementpoly** implements the polynomial *polynomial* in range *range* as a function called *functionname* in C code using double, double-double and triple-double arithmetic in a way



that the rounding error (estimated at its first order) is bounded by *error bound*. The produced code is output in a file named *filename*. The argument *format* indicates the double, double-double or triple-double format of the variable in which the polynomial varies, influencing also in the signature of the **C** function.

If a seventh or eighth argument *proof filename* is given and if this argument evaluates to a variable of type **string**, the command **implementpoly** will produce a **Gappa** proof that the rounding error is less than the given bound. This proof will be output in **Gappa** syntax in a file name *proof filename*.

The command **implementpoly** returns the polynomial that has been implemented. As the command **implementpoly** tries to adapt the precision needed in each evaluation step to its strict minimum and as it applies renormalization to double-double and triple-double precision coefficients to bring them to a round-to-nearest expansion form, the returned polynomial may differ from the polynomial *polynomial*. Nevertheless the difference will be small enough that the rounding error bound with regard to the polynomial *polynomial* (estimated at its first order) will be less than the given error bound.

If a seventh argument *honor coefficient precisions* is given and evaluates to a variable **honorcoeffprec** of type **honorcoeffprec**, **implementpoly** will honor the precision of the given polynomial *polynomials*. This means if a coefficient needs a double-double or a triple-double to be exactly stored, **implementpoly** will allocate appropriate space and use a double-double or triple-double operation even if the automatic (heuristic) determination implemented in command **implementpoly** indicates that the coefficient could be stored on less precision or, respectively, the operation could be performed with less precision. The use of **honorcoeffprec** has advantages and disadvantages. If the polynomial *polynomial* given has not been determined by a process considering directly polynomials with floating-point coefficients, **honorcoeffprec** should not be indicated. The **implementpoly** command can then determine the needed precision using the same error estimation as used for the determination of the precisions of the operations. Generally, the coefficients will get rounded to double, double-double and triple-double precision in a way that minimizes their number and respects the rounding error bound *error bound*. Indicating **honorcoeffprec** may in this case short-circuit most precision estimations leading to sub-optimal code. On the other hand, if the polynomial *polynomial* has been determined with floating-point precisions in mind, **honorcoeffprec** should be indicated because such polynomials often are very sensitive in terms of error propagation with regard to their coefficients' values. Indicating **honorcoeffprec** prevents the **implementpoly** command from rounding the coefficients and altering by many orders of magnitude the approximation error of the polynomial with regard to the function it approximates.

The implementer behind the **implementpoly** command makes some assumptions on its input and verifies them. If some assumption cannot be verified, the implementation will not succeed and **implementpoly** will evaluate to a variable **error** of type **error**. The same behaviour is observed if some file is not writable or some other side-effect fails, e.g. if the implementer runs out of memory.

As error estimation is performed only on the first order, the code produced by the **implementpoly** command should be considered valid iff a **Gappa** proof has been produced and successfully run in **Gappa**.

Example 1:

```
> implementpoly(1 - 1/6 * x^2 + 1/120 * x^4, [-1b-10;1b-10], 1b-30, D, "p","impl
ementation.c");
1 + x^2 * (-0.166666666666666657414808128123695496469736099243164 + x^2 * 8.3333
33333333332176851016015461937058717012405395e-3)
> readfile("implementation.c");
#define p_coeff_0h 1.00000000000000000000000000000000000000000000000000000000000000
0000000000000000000000e+00
#define p_coeff_2h -1.6666666666666665741480812812369549646973609924316406250000
0000000000000000000000e-01
#define p_coeff_4h 8.3333333333333321768510160154619370587170124053955078125000
0000000000000000000000e-03

void p(double *p_res, double x) {
double p_x_0_pow2h;

p_x_0_pow2h = x * x;

double p_t_1_0h;
double p_t_2_0h;
double p_t_3_0h;
double p_t_4_0h;
double p_t_5_0h;

p_t_1_0h = p_coeff_4h;
p_t_2_0h = p_t_1_0h * p_x_0_pow2h;
p_t_3_0h = p_coeff_2h + p_t_2_0h;
p_t_4_0h = p_t_3_0h * p_x_0_pow2h;
p_t_5_0h = p_coeff_0h + p_t_4_0h;
*p_res = p_t_5_0h;

}
```

Example 2:

```
> implementpoly(1 - 1/6 * x^2 + 1/120 * x^4, [-1b-10;1b-10], 1b-30, D, "p","impl
ementation.c","implementation.gappa");
1 + x^2 * (-0.166666666666666657414808128123695496469736099243164 + x^2 * 8.3333
33333333332176851016015461937058717012405395e-3)
```

Example 3:

```

> verbosity = 1!;
> q = implementpoly(1 - simplify(TD(1/6)) * x^2, [-1b-10;1b-10], 1b-60, DD, "p", "implementation.c");
Warning: at least one of the coefficients of the given polynomial has been rounded in a way
that the target precision can be achieved at lower cost. Nevertheless, the implemented polynomial
is different from the given one.
> printexpansion(q);
0x3ff0000000000000 + x^2 * 0xbfc5555555555555
> r = implementpoly(1 - simplify(TD(1/6)) * x^2, [-1b-10;1b-10], 1b-60, DD, "p", "implementation.c", honorcoeffprec);
Warning: the inferred precision of the 2th coefficient of the polynomial is greater than
the necessary precision computed for this step. This may make the automatic determination
of precisions useless.
> printexpansion(r);
0x3ff0000000000000 + x^2 * (0xbfc5555555555555 + 0xbc65555555555555 + 0xb905555555555555)

```

Example 4:

```

> p = 0x3ff0000000000000 + x * (0x3ff0000000000000 + x * (0x3fe0000000000000 + x
* (0x3fc5555555555559 + x * (0x3fa55555555555bd + x * (0x3f811111111106e2 + x
* (0x3f56c16c16bf5eb7 + x * (0x3f2a01a01a292dcd + x * (0x3efa01a0218a016a + x
* (0x3ec71de360331aad + x * (0x3e927e42e3823bf3 + x * (0x3e5ae6b2710c2c9a + x
* (0x3e2203730c0a7c1d + x * 0x3de5da557e0781df)))))))));
> q = implementpoly(p, [-1/2;1/2], 1b-60, D, "p", "implementation.c", honorcoeffprec, "implementation.gappa");
> if (q != p) then print("During implementation, rounding has happened.") else print("Polynomial implemented as given.");
Polynomial implemented as given.

```

See also: **honorcoeffprec** (8.78), **roundcoefficients** (8.154), **double** (8.46), **doubledouble** (8.47), **tripleddouble** (8.184), **readfile** (8.144), **printexpansion** (8.132), **error** (8.53), **remez** (8.147), **fpminimax** (8.68), **taylor** (8.178), **implementconstant** (8.82)

## 8.84 in

Name: **in**

containment test operator

Library name:

`sollya_obj_t sollya_lib_cmp_in(sollya_obj_t, sollya_obj_t)`

Usage:

*expr* **in** *range1* : (constant, range) → boolean  
*range1* **in** *range2* : (range, range) → boolean

Parameters:

- *expr* represents a constant expression
- *range1* and *range2* represent ranges (intervals)

Description:

- When its first operand is a constant expression *expr*, the operator **in** evaluates to true iff the constant value of the expression *expr* is contained in the interval *range1*.
- When both its operands are ranges (intervals), the operator **in** evaluates to true iff all values in *range1* are contained in the interval *range2*.
- **in** is also used as a keyword for loops over the different elements of a list.

Example 1:

```
> 5 in [-4;7];
true
> 4 in [-1;1];
false
> 0 in sin([-17;17]);
true
```

Example 2:

```
> [5;7] in [2;8];
true
> [2;3] in [4;5];
false
> [2;3] in [2.5;5];
false
```

Example 3:

```
> for i in [|1,...,5|] do print(i);
1
2
3
4
5
```

See also: `==` (8.50), `!=` (8.109), `>=` (8.71), `>` (8.73), `<=` (8.91), `<` (8.100), `!` (8.111), `&&` (8.6), `||` (8.116), `prec` (8.127), `print` (8.130)

## 8.85 inf

Name: **inf**

gives the lower bound of an interval.

Library name:

`sollya_obj_t sollya_lib_inf(sollya_obj_t)`

Usage:

**inf**(*I*) : range → constant  
**inf**(*x*) : constant → constant

Parameters:

- *I* is an interval.
- *x* is a real number.

Description:

- Returns the lower bound of the interval *I*. Each bound of an interval has its own precision, so this command is exact, even if the current precision is too small to represent the bound.

- When called on a real number  $x$ , **inf** behaves like the identity.

Example 1:

```
> inf([1;3]);
1
> inf(0);
0
```

Example 2:

```
> display=binary!;
> I=[0.111110000011111_2; 1];
> inf(I);
1.11110000011111_2 * 2^(-1)
> prec=12!;
> inf(I);
1.11110000011111_2 * 2^(-1)
```

See also: **mid** (8.103), **sup** (8.172), **max** (8.102), **min** (8.105)

## 8.86 infnorm

Name: **infnorm**

computes an interval bounding the infinity norm of a function on an interval.

Library names:

```
sollya_obj_t sollya_lib_infnorm(sollya_obj_t, sollya_obj_t, ...)
sollya_obj_t sollya_lib_v_infnorm(sollya_obj_t, sollya_obj_t, va_list)
```

Usage:

**infnorm**( $f, I, filename, Ilist$ ) : (function, range, string, list)  $\rightarrow$  range

Parameters:

- $f$  is a function.
- $I$  is an interval.
- $filename$  (optional) is the name of the file into a proof will be saved.
- $Ilist$  (optional) is a list of intervals to be excluded.

Description:

- **infnorm**( $f, range$ ) computes an interval bounding the infinity norm of the given function  $f$  on the interval  $I$ , e.g. computes an interval  $J$  such that  $\max_{x \in I} \{|f(x)|\} \subseteq J$ .
- If  $filename$  is given, a proof in English will be produced (and stored in file called  $filename$ ) proving that  $\max_{x \in I} \{|f(x)|\} \subseteq J$ .
- If a list  $Ilist$  of intervals  $I_1, \dots, I_n$  is given, the infinity norm will be computed on  $I \setminus (I_1 \cup \dots \cup I_n)$ .
- The function  $f$  is assumed to be at least twice continuous on  $I$ . More generally, if  $f$  is  $\mathcal{C}^k$ , global variables **hopitalrecursions** and **taylorrecursions** must have values not greater than  $k$ .
- If the interval is reduced to a single point, the result of **infnorm** is an interval containing the exact absolute value of  $f$  at this point.
- If the interval is not bound, the result will be  $[0, +\infty]$  which is correct but perfectly useless. **infnorm** is not meant to be used with infinite intervals.

- The result of this command depends on the global variables **prec**, **diam**, **taylorrecursions** and **hopitalrecursions**. The contribution of each variable is not easy even to analyse.
  - The algorithm uses interval arithmetic with precision **prec**. The precision should thus be set high enough to ensure that no critical cancellation will occur.
  - When an evaluation is performed on an interval  $[a, b]$ , if the result is considered being too large, the interval is split into  $[a, \frac{a+b}{2}]$  and  $[\frac{a+b}{2}, b]$  and so on recursively. This recursion step is not performed if the  $(b - a) < \delta \cdot |I|$  where  $\delta$  is the value of variable **diam**. In other words, **diam** controls the minimum length of an interval during the algorithm.
  - To perform the evaluation of a function on an interval, Taylor's rule is applied, e.g.  $f([a, b]) \subseteq f(m) + [a - m, b - m] \cdot f'([a, b])$  where  $m = \frac{a+b}{2}$ . This rule is recursively applied  $n$  times where  $n$  is the value of variable **taylorrecursions**. Roughly speaking, the evaluations will avoid decorrelation up to order  $n$ .
  - When a function of the form  $\frac{g}{h}$  has to be evaluated on an interval  $[a, b]$  and when  $g$  and  $h$  vanish at a same point  $z$  of the interval, the ratio may be defined even if the expression  $\frac{g(z)}{h(z)} = \frac{0}{0}$  does not make any sense. In this case, L'Hopital's rule may be used and  $(\frac{g}{h})([a, b]) \subseteq (\frac{g'}{h'})([a, b])$ . Since the same can occur with the ratio  $\frac{g'}{h'}$ , the rule is applied recursively. The variable **hopitalrecursions** controls the number of recursion steps.
- The algorithm used for this command is quite complex to be explained here. Please find a complete description in the following article:  
 S. Chevillard and C. Lauter  
 A certified infinity norm for the implementation of elementary functions  
 LIP Research Report number RR2007-26  
<http://prunel.ccsd.cnrs.fr/ensl-00119810>
- Users should be aware about the fact that the algorithm behind **infnorm** is inefficient in most cases and that other, better suited algorithms, such as **supnorm**, are available inside **Sollya**. As a matter of fact, while **infnorm** is maintained for compatibility reasons with legacy **Sollya** codes, users are advised to avoid using **infnorm** in new **Sollya** scripts and to replace it, where possible, by the **supnorm** command.

Example 1:

```
> infnorm(exp(x), [-2;3]);
[2.00855369231876677409285296545817178969879078385537e1;2.0085536923187667740928
5296545817178969879078385544e1]
```

Example 2:

```
> infnorm(exp(x), [-2;3], "proof.txt");
[2.00855369231876677409285296545817178969879078385537e1;2.0085536923187667740928
5296545817178969879078385544e1]
```

Example 3:

```
> infnorm(exp(x), [-2;3], [| [0;1], [2;2.5] |]);
[2.00855369231876677409285296545817178969879078385537e1;2.0085536923187667740928
5296545817178969879078385544e1]
```

Example 4:

```
> infnorm(exp(x), [-2;3], "proof.txt", [| [0;1], [2;2.5] |]);
[2.00855369231876677409285296545817178969879078385537e1;2.0085536923187667740928
5296545817178969879078385544e1]
```

Example 5:

```
> infnorm(exp(x), [1;1]);  
[2.71828182845904523536028747135266249775724709369989;2.718281828459045235360287  
47135266249775724709369998]
```

Example 6:

```
> infnorm(exp(x), [log(0);log(1)]);  
[0;@Inf@]
```

See also: **prec** (8.127), **diam** (8.38), **hopitalrecursions** (8.79), **taylorrecursions** (8.180), **dirty-infnorm** (8.42), **checkinfnorm** (8.24), **supnorm** (8.173), **findzeros** (8.64), **diff** (8.40), **taylorrecursions** (8.180), **autodiff** (8.14), **numberroots** (8.112), **taylorform** (8.179)

## 8.87 integer

Name: **integer**

keyword representing a machine integer type

Usage:

**integer** : type type

Description:

- **integer** represents the machine integer type for declarations of external procedures **externalproc**.  
Remark that in contrast to other indicators, type indicators like **integer** cannot be handled outside the **externalproc** context. In particular, they cannot be assigned to variables.

See also: **externalproc** (8.61), **boolean** (8.20), **constant** (8.28), **function** (8.70), **list of** (8.95), **range** (8.140), **string** (8.169)

## 8.88 integral

Name: **integral**

computes an interval bounding the integral of a function on an interval.

Library name:

`sollya_obj_t sollya_lib_integral(sollya_obj_t, sollya_obj_t)`

Usage:

**integral**( $f, I$ ) : (function, range)  $\rightarrow$  range

Parameters:

- $f$  is a function.
- $I$  is an interval.

Description:

- **integral**( $f, I$ ) returns an interval  $J$  such that the exact value of the integral of  $f$  on  $I$  lies in  $J$ .
- This command is safe but very inefficient. Use **dirtyintegral** if you just want an approximate value.
- The result of this command depends on the global variable **diam**. The method used is the following:  $I$  is cut into intervals of length not greater than  $\delta \cdot |I|$  where  $\delta$  is the value of global variable **diam**. On each small interval  $J$ , an evaluation of  $f$  by interval is performed. The result is multiplied by the length of  $J$ . Finally all values are summed.

Example 1:

```
> sin(10);
-0.54402111088936981340474766185137728168364301291621
> integral(cos(x), [0;10]);
[-0.54710197983579690224097637163525943075698599257332; -0.5409401513001318384815
0540881373370744053741191728]
> diam=1e-5!;
> integral(cos(x), [0;10]);
[-0.54432915685955427101857780295936956775293876382777; -0.5437130640124996950803
9644221927489010425803173555]
```

See also: **diam** (8.38), **dirtyintegral** (8.43), **prec** (8.127)

## 8.89 isbound

Name: **isbound**

indicates whether a variable is bound or not.

Usage:

**isbound**(*ident*) : boolean

Parameters:

- *ident* is a name.

Description:

- **isbound**(*ident*) returns a boolean value indicating whether the name *ident* is used or not to represent a variable. It returns true when *ident* is the name used to represent the global variable or if the name is currently used to refer to a (possibly local) variable.
- When a variable is defined in a block and has not been defined outside, **isbound** returns true when called inside the block, and false outside. Note that **isbound** returns true as soon as a variable has been declared with **var**, even if no value is actually stored in it.
- If *ident1* is bound to a variable and if *ident2* refers to the global variable, the command **re-name**(*ident2*, *ident1*) hides the value of *ident1* which becomes the global variable. However, if the global variable is again renamed, *ident1* gets its value back. In this case, **isbound**(*ident1*) returns true. If *ident1* was not bound before, **isbound**(*ident1*) returns false after that *ident1* has been renamed.

Example 1:

```
> isbound(x);
false
> isbound(f);
false
> isbound(g);
false
> f=sin(x);
> isbound(x);
true
> isbound(f);
true
> isbound(g);
false
```

Example 2:



```

> isbound(a);
false
> { var a; isbound(a); };
true
> isbound(a);
false

```

Example 3:

```

> f=sin(x);
> isbound(x);
true
> rename(x,y);
> isbound(x);
false

```

Example 4:

```

> x=1;
> f=sin(y);
> rename(y,x);
> f;
sin(x)
> x;
x
> isbound(x);
true
> rename(x,y);
> isbound(x);
true
> x;
1

```

See also: **rename** (8.148)

## 8.90 isevaluable

Name: **isevaluable**

tests whether a function can be evaluated at a point

Usage:

**isevaluable**(*function*, *constant*) : (function, constant) → boolean

Parameters:

- *function* represents a function
- *constant* represents a constant point

Description:

- **isevaluable** applied to function *function* and a constant *constant* returns a boolean indicating whether or not a subsequent call to **evaluate** on the same function *function* and constant *constant* will produce a numerical result or NaN. This means **isevaluable** returns false iff **evaluate** will return NaN.
- The command **isevaluable** is now considered DEPRECATED in Sollya. As checks for NaNs are now possible in Sollya, the command **isevaluable** can be fully emulated with a call to **evaluate** and a couple of tests, as shown below in the last example.

Example 1:

```
> isevaluable(sin(pi * 1/x), 0.75);
true
> print(evaluate(sin(pi * 1/x), 0.75));
-0.86602540378443864676372317075293618347140262690518
```

Example 2:

```
> isevaluable(sin(pi * 1/x), 0.5);
true
> print(evaluate(sin(pi * 1/x), 0.5));
[-1.72986452514381269516508615031098129542836767991679e-12715;7.5941198201187963
145069564314525661706039084390067e-12716]
```

Example 3:

```
> isevaluable(sin(pi * 1/x), 0);
false
> print(evaluate(sin(pi * 1/x), 0));
[@NaN@;@NaN@]
```

Example 4:

```
> procedure isEvaluableEmulation(f, c) {
    return match evaluate(f, c) with
        NaN : (false)
        [NaN;NaN] : (false)
        default : (true);
};
> isEvaluableEmulation(sin(pi * 1/x), 0.75);
true
> isEvaluableEmulation(sin(pi * 1/x), 0.5);
true
> isEvaluableEmulation(sin(pi * 1/x), 0);
false
```

See also: **evaluate** (8.54)

## 8.91 <=

Name: <=

less-than-or-equal-to operator

Library name:

`sollya_obj_t sollya_lib_cmp_less_equal(sollya_obj_t, sollya_obj_t)`

Usage:

$$expr1 \leq expr2 : (\text{constant}, \text{constant}) \rightarrow \text{boolean}$$

Parameters:

- *expr1* and *expr2* represent constant expressions

Description:

- The operator <= evaluates to true iff its operands *expr1* and *expr2* evaluate to two floating-point numbers  $a_1$  respectively  $a_2$  with the global precision **prec** and  $a_1$  is less than or equal to  $a_2$ . The user should be aware of the fact that because of floating-point evaluation, the operator <= is not exactly the same as the mathematical operation *less-than-or-equal-to*.

Example 1:

```
> 5 <= 4;
false
> 5 <= 5;
true
> 5 <= 6;
true
> exp(2) <= exp(1);
false
> log(1) <= exp(2);
true
```

Example 2:

```
> prec = 12;
The precision has been set to 12 bits.
> 16385.1 <= 16384.1;
true
```

See also: `==` (8.50), `!=` (8.109), `>=` (8.71), `>` (8.73), `<` (8.100), `in` (8.84), `!` (8.111), `&&` (8.6), `||` (8.116), `prec` (8.127), `max` (8.102), `min` (8.105)

## 8.92 length

Name: **length**

computes the length of a list or string.

Library name:

`sollya_obj_t sollya_lib_length(sollya_obj_t)`

Usage:

**length**( $L$ ) : list  $\rightarrow$  integer  
**length**( $s$ ) : string  $\rightarrow$  integer

Parameters:

- $L$  is a list.
- $s$  is a string.

Description:

- **length** returns the length of a list or a string, e.g. the number of elements or letters.
- The empty list or string have length 0. If  $L$  is an end-elliptic list, **length** returns  $+\text{Inf}$ .

Example 1:

```
> length("Hello World!");
12
```

Example 2:

```
> length([1,...,5]);
5
```

Example 3:

```
> length([| |]);
0
```

Example 4:

```
> length([1,2...]);  
@Inf@
```

## 8.93 library

Name: **library**

binds an external mathematical function to a variable in **Sollya**

Library names:

```
sollya_obj_t sollya_lib_libraryfunction(sollya_obj_t, char *,  
                                         int (*)(mpfi_t, mpfi_t, int))  
sollya_obj_t sollya_lib_build_function_libraryfunction(sollya_obj_t, char *,  
                                                         int (*)(mpfi_t,  
                                                         mpfi_t, int))
```

Usage:

**library**(*path*) : string → function

Description:

- The command **library** lets you extend the set of mathematical functions known to **Sollya**. By default, **Sollya** knows the most common mathematical functions such as **exp**, **sin**, **erf**, etc. Within **Sollya**, these functions may be composed. This way, **Sollya** should satisfy the needs of a lot of users. However, for particular applications, one may want to manipulate other functions such as Bessel functions, or functions defined by an integral or even a particular solution of an ODE.
- **library** makes it possible to let **Sollya** know about new functions. In order to let it know, you have to provide an implementation of the function you are interested in. This implementation is a C file containing a function of the form:

```
int my_ident(sollya_mpfi_t result, sollya_mpfi_t op, int n)
```

The semantic of this function is the following: it is an implementation of the function and its derivatives in interval arithmetic. **my\_ident**(**result**, **I**, **n**) shall store in **result** an enclosure of the image set of the  $n$ -th derivative of the function  $f$  over  $I$ :  $f^{(n)}(I) \subseteq \text{result}$ .

- The integer value returned by the function implementation currently has no meaning.
- You do not need to provide a working implementation for any **n**. Most functions of **Sollya** requires a relevant implementation only for  $f$ ,  $f'$  and  $f''$ . For higher derivatives, it is not so critical and the implementation may just store  $[-\infty, +\infty]$  in **result** whenever  $n > 2$ .
- Note that you should respect somehow interval-arithmetic standards in your implementation: **result** has its own precision and you should perform the intermediate computations so that **result** is as tight as possible.
- You can include **sollya.h** in your implementation and use library functionalities of **Sollya** for your implementation. However, this requires to have compiled **Sollya** with **-fPIC** in order to make the **Sollya** executable code position independent and to use a system on with programs, using **dlopen** to open dynamic routines can dynamically open themselves.
- To bind your function into **Sollya**, you must use the same identifier as the function name used in your implementation file (**my\_ident** in the previous example). Once the function code has been bound to an identifier, you can use a simple assignment to assign the bound identifier to yet another identifier. This way, you may use convenient names inside **Sollya** even if your implementation environment requires you to use a less convenient name.

Example 1:

```

> bashexecute("gcc -fPIC -Wall -c libraryexample.c -I$HOME/.local/include");
> bashexecute("gcc -shared -o libraryexample libraryexample.o -lgmp -lmpfr");
> myownlog = library("./libraryexample");
> evaluate(log(x), 2);
0.69314718055994530941723212145817656807550013436024
> evaluate(myownlog(x), 2);
0.69314718055994530941723212145817656807550013436024

```

See also: **function** (8.70), **bashexecute** (8.17), **externalproc** (8.61), **externalplot** (8.60), **diff** (8.40), **evaluate** (8.54), **libraryconstant** (8.94)

## 8.94 libraryconstant

Name: **libraryconstant**

binds an external mathematical constant to a variable in **Sollya**

Library names:

```

sollya_obj_t sollya_lib_libraryconstant(char *, void (*)(mpfr_t, mp_prec_t))
sollya_obj_t sollya_lib_build_function_libraryconstant(char *,
                                                         void (*)(mpfr_t,
                                                         mp_prec_t))

```

Usage:

**libraryconstant**(*path*) : string → function

Description:

- The command **libraryconstant** lets you extend the set of mathematical constants known to **Sollya**. By default, the only mathematical constant known by **Sollya** is **pi**. For particular applications, one may want to manipulate other constants, such as Euler's gamma constant, for instance.
- **libraryconstant** makes it possible to let **Sollya** know about new constants. In order to let it know, you have to provide an implementation of the constant you are interested in. This implementation is a C file containing a function of the form:

```
void my_ident(mpfr_t result, mp_prec_t prec)
```

The semantic of this function is the following: it is an implementation of the constant in arbitrary precision. **my\_ident(result, prec)** shall set the precision of the variable **result** to a suitable precision (the variable is assumed to be already initialized) and store in **result** an approximate value of the constant with a relative error not greater than  $2^{1-\text{prec}}$ . More precisely, if  $c$  is the exact value of the constant, the value stored in **result** should satisfy

$$|\text{result} - c| \leq |c| 2^{1-\text{prec}}.$$

- You can include **sollya.h** in your implementation and use library functionalities of **Sollya** for your implementation. However, this requires to have compiled **Sollya** with **-fPIC** in order to make the **Sollya** executable code position independent and to use a system on which programs, using **dlopen** to open dynamic routines can dynamically open themselves.
- To bind your constant into **Sollya**, you must use the same identifier as the function name used in your implementation file (**my\_ident** in the previous example). Once the function code has been bound to an identifier, you can use a simple assignment to assign the bound identifier to yet another identifier. This way, you may use convenient names inside **Sollya** even if your implementation environment requires you to use a less convenient name.
- Once your constant is bound, it is considered by **Sollya** as an infinitely accurate constant (i.e. a 0-ary function, exactly like **pi**).

Example 1:

```
> bashexecute("gcc -fPIC -Wall -c libraryconstantexample.c -I$HOME/.local/include");
> bashexecute("gcc -shared -o libraryconstantexample libraryconstantexample.o -lgmp -lmpfr");
> euler_gamma = libraryconstant("./libraryconstantexample");
> prec = 20!;
> euler_gamma;
0.577215
> prec = 100!;
> euler_gamma;
0.577215664901532860606512090082
> midpointmode = on;
Midpoint mode has been activated.
> [euler_gamma];
0.57721566490153286060651209008~2/4~
```

See also: **bashexecute** (8.17), **externalproc** (8.61), **externalplot** (8.60), **pi** (8.119), **library** (8.93), **evaluate** (8.54), **implementconstant** (8.82)

## 8.95 list of

Name: **list of**

keyword used in combination with a type keyword

Description:

- **list of** is used in combination with one of the following keywords for indicating lists of the respective type in declarations of external procedures using **externalproc**: **boolean**, **constant**, **function**, **integer**, **range** and **string**.

See also: **externalproc** (8.61), **boolean** (8.20), **constant** (8.28), **function** (8.70), **integer** (8.87), **range** (8.140), **string** (8.169)

## 8.96 log

Name: **log**

natural logarithm.

Library names:

```
sollya_obj_t sollya_lib_log(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_log(sollya_obj_t)
#define SOLLYA_LOG(x) sollya_lib_build_function_log(x)
```

Description:

- **log** is the natural logarithm defined as the inverse of the exponential function:  $\log(y)$  is the unique real number  $x$  such that  $\exp(x) = y$ .
- It is defined only for  $y \in [0; +\infty]$ .

See also: **exp** (8.56), **log2** (8.99), **log10** (8.97)

## 8.97 log10

Name: **log10**

decimal logarithm.

Library names:

```

sollya_obj_t sollya_lib_log10(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_log10(sollya_obj_t)
#define SOLLYA_LOG10(x) sollya_lib_build_function_log10(x)

```

Description:

- **log10** is the decimal logarithm defined by:  $\log_{10}(x) = \log(x)/\log(10)$ .
- It is defined only for  $x \in [0; +\infty]$ .

See also: **log** (8.96), **log2** (8.99)

## 8.98 log1p

Name: **log1p**

translated logarithm.

Library names:

```

sollya_obj_t sollya_lib_log1p(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_log1p(sollya_obj_t)
#define SOLLYA_LOG1P(x) sollya_lib_build_function_log1p(x)

```

Description:

- **log1p** is the function defined by  $\log_{1p}(x) = \log(1 + x)$ .
- It is defined only for  $x \in [-1; +\infty]$ .

See also: **log** (8.96)

## 8.99 log2

Name: **log2**

binary logarithm.

Library names:

```

sollya_obj_t sollya_lib_log2(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_log2(sollya_obj_t)
#define SOLLYA_LOG2(x) sollya_lib_build_function_log2(x)

```

Description:

- **log2** is the binary logarithm defined by:  $\log_2(x) = \log(x)/\log(2)$ .
- It is defined only for  $x \in [0; +\infty]$ .

See also: **log** (8.96), **log10** (8.97)

## 8.100 <

Name: **<**

less-than operator

Library name:

```

sollya_obj_t sollya_lib_cmp_less(sollya_obj_t, sollya_obj_t)

```

Usage:

$$expr1 < expr2 : (\text{constant}, \text{constant}) \rightarrow \text{boolean}$$

Parameters:

- *expr1* and *expr2* represent constant expressions

Description:

- The operator `<` evaluates to true iff its operands *expr1* and *expr2* evaluate to two floating-point numbers  $a_1$  respectively  $a_2$  with the global precision **prec** and  $a_1$  is less than  $a_2$ . The user should be aware of the fact that because of floating-point evaluation, the operator `<` is not exactly the same as the mathematical operation *less-than*.

Example 1:

```
> 5 < 4;
false
> 5 < 5;
false
> 5 < 6;
true
> exp(2) < exp(1);
false
> log(1) < exp(2);
true
```

Example 2:

```
> prec = 12;
The precision has been set to 12 bits.
> 16384.1 < 16385.1;
false
```

See also: `==` (8.50), `!=` (8.109), `>=` (8.71), `>` (8.73), `<=` (8.91), `in` (8.84), `!` (8.111), `&&` (8.6), `||` (8.116), **prec** (8.127), **max** (8.102), **min** (8.105)

## 8.101 mantissa

Name: **mantissa**

returns the integer mantissa of a number.

Library name:

```
sollya_obj_t sollya_lib_mantissa(sollya_obj_t)
```

Usage:

**mantissa**( $x$ ) : constant  $\rightarrow$  integer

Parameters:

- $x$  is a dyadic number.

Description:

- **mantissa**( $x$ ) is by definition  $x$  if  $x$  equals 0, NaN, or Inf.
- If  $x$  is not zero, it can be uniquely written as  $x = m \cdot 2^e$  where  $m$  is an odd integer and  $e$  is an integer. **mantissa**( $x$ ) returns  $m$ .

Example 1:

```
> a=round(Pi,20,RN);
> e=exponent(a);
> m=mantissa(a);
> m;
411775
> a-m*2^e;
0
```

See also: **exponent** (8.59), **precision** (8.128)



## 8.102 max

Name: **max**

determines which of given constant expressions has maximum value

Library names:

```
sollya_obj_t sollya_lib_max(sollya_obj_t, ...)
sollya_obj_t sollya_lib_v_max(sollya_obj_t, va_list)
```

Usage:

$$\mathbf{max}(expr1, expr2, \dots, exprn) : (\text{constant}, \text{constant}, \dots, \text{constant}) \rightarrow \text{constant}$$
$$\mathbf{max}(l) : \text{list} \rightarrow \text{constant}$$

Parameters:

- *expr* are constant expressions.
- *l* is a list of constant expressions.

Description:

- **max** determines which of a given set of constant expressions *expr* has maximum value. To do so, **max** tries to increase the precision used for evaluation until it can decide the ordering or some maximum precision is reached. In the latter case, a warning is printed indicating that there might actually be another expression that has a greater value.
- Even though **max** determines the maximum expression by evaluation, it returns the expression that is maximum as is, i.e. as an expression tree that might be evaluated to any accuracy afterwards.
- **max** can be given either an arbitrary number of constant expressions in argument or a list of constant expressions. The list however must not be end-elliptic.
- Users should be aware that the behavior of **max** follows the IEEE 754-2008 standard with respect to NaNs. In particular, **max** evaluates to NaN if and only if all arguments of **max** are NaNs. This means that NaNs may disappear during computations.

Example 1:

```
> max(1,2,3,exp(5),log(0.25));
1.48413159102576603421115580040552279623487667593878e2
> max(17);
17
```

Example 2:

```
> l = [|1,2,3,exp(5),log(0.25)|];
> max(l);
1.48413159102576603421115580040552279623487667593878e2
```

Example 3:

```
> print(max(exp(17),sin(62)));
exp(17)
```

Example 4:

```
> verbosity = 1!;
> print(max(17 + log2(13)/log2(9),17 + log(13)/log(9)));
Warning: the tool is unable to decide a maximum computation by evaluation even t
hough faithful evaluation of the terms has been possible. The terms will be cons
idered to be equal.
17 + log2(13) / log2(9)
```

See also: **min** (8.105), **==** (8.50), **!=** (8.109), **>=** (8.71), **>** (8.73), **<** (8.100), **<=** (8.91), **in** (8.84), **inf** (8.85), **sup** (8.172)

### 8.103 mid

Name: **mid**

gives the middle of an interval.

Library name:

`sollya_obj_t sollya_lib_mid(sollya_obj_t)`

Usage:

$$\begin{aligned} \mathbf{mid}(I) &: \text{range} \rightarrow \text{constant} \\ \mathbf{mid}(x) &: \text{constant} \rightarrow \text{constant} \end{aligned}$$

Parameters:

- $I$  is an interval.
- $x$  is a real number.

Description:

- Returns the middle of the interval  $I$ . If the middle is not exactly representable at the current precision, the value is returned as an unevaluated expression.
- When called on a real number  $x$ , **mid** behaves like the identity.

Example 1:

```
> mid([1;3]);
2
> mid(17);
17
```

See also: **inf** (8.85), **sup** (8.172)

### 8.104 midpointmode

Name: **midpointmode**

global variable controlling the way intervals are displayed.

Library names:

```
void sollya_lib_set_midpointmode_and_print(sollya_obj_t)
void sollya_lib_set_midpointmode(sollya_obj_t)
sollya_obj_t sollya_lib_get_midpointmode()
```

Usage:

$$\begin{aligned} \mathbf{midpointmode} &= \text{activation value} : \text{on|off} \rightarrow \text{void} \\ \mathbf{midpointmode} &= \text{activation value} ! : \text{on|off} \rightarrow \text{void} \\ \mathbf{midpointmode} &: \text{on|off} \end{aligned}$$

Parameters:

- *activation value* enables or disables the mode.

Description:

- **midpointmode** is a global variable. When its value is **off**, intervals are displayed as usual (in the form  $[a; b]$ ). When its value is **on**, and if  $a$  and  $b$  have the same first significant digits, the interval is displayed in a way that lets one immediately see the common digits of the two bounds.
- This mode is supported only with **display** set to **decimal**. In other modes of display, **midpointmode** value is simply ignored.

Example 1:

```
> a = round(Pi,30,RD);
> b = round(Pi,30,RU);
> d = [a,b];
> d;
[3.1415926516056060791015625;3.1415926553308963775634765625]
> midpointmode=on!;
> d;
0.314159265~1/6~e1
```

See also: **on** (8.115), **off** (8.114), **roundingwarnings** (8.156), **display** (8.44), **decimal** (8.34)

## 8.105 min

Name: **min**

determines which of given constant expressions has minimum value

Library names:

```
sollya_obj_t sollya_lib_min(sollya_obj_t, ...)
sollya_obj_t sollya_lib_v_min(sollya_obj_t, va_list)
```

Usage:

$$\mathbf{min}(expr1, expr2, \dots, exprn) : (\text{constant}, \text{constant}, \dots, \text{constant}) \rightarrow \text{constant}$$
$$\mathbf{min}(l) : \text{list} \rightarrow \text{constant}$$

Parameters:

- *expr* are constant expressions.
- *l* is a list of constant expressions.

Description:

- **min** determines which of a given set of constant expressions *expr* has minimum value. To do so, **min** tries to increase the precision used for evaluation until it can decide the ordering or some maximum precision is reached. In the latter case, a warning is printed indicating that there might actually be another expression that has a lesser value.
- Even though **min** determines the minimum expression by evaluation, it returns the expression that is minimum as is, i.e. as an expression tree that might be evaluated to any accuracy afterwards.
- **min** can be given either an arbitrary number of constant expressions in argument or a list of constant expressions. The list however must not be end-elliptic.
- Users should be aware that the behavior of **min** follows the IEEE 754-2008 standard with respect to NaNs. In particular, **min** evaluates to NaN if and only if all arguments of **min** are NaNs. This means that NaNs may disappear during computations.

Example 1:

```
> min(1,2,3,exp(5),log(0.25));
-1.3862943611198906188344642429163531361510002687205
> min(17);
17
```

Example 2:

```
> l = [|1,2,3,exp(5),log(0.25)|];
> min(l);
-1.3862943611198906188344642429163531361510002687205
```

Example 3:

```
> print(min(exp(17),sin(62)));  
sin(62)
```

Example 4:

```
> verbosity = 1!;  
> print(min(17 + log2(13)/log2(9),17 + log(13)/log(9)));  
Warning: the tool is unable to decide a minimum computation by evaluation even t  
hough faithful evaluation of the terms has been possible. The terms will be cons  
idered to be equal.  
17 + log(13) / log(9)
```

See also: **max** (8.102), **==** (8.50), **!=** (8.109), **>=** (8.71), **>** (8.73), **<** (8.100), **<=** (8.91), **in** (8.84), **inf** (8.85), **sup** (8.172)

## 8.106 —

Name: —

subtraction function

Library names:

```
sollya_obj_t sollya_lib_sub(sollya_obj_t, sollya_obj_t)  
sollya_obj_t sollya_lib_build_function_sub(sollya_obj_t, sollya_obj_t)  
#define SOLLYA_SUB(x,y) sollya_lib_build_function_sub((x), (y))  
sollya_obj_t sollya_lib_neg(sollya_obj_t)  
sollya_obj_t sollya_lib_build_function_neg(sollya_obj_t)  
#define SOLLYA_NEG(x) sollya_lib_build_function_neg(x)
```

Usage:

$$\begin{aligned} &function1 - function2 : (function, function) \rightarrow function \\ &interval1 - interval2 : (range, range) \rightarrow range \\ &interval1 - constant : (range, constant) \rightarrow range \\ &interval1 - constant : (constant, range) \rightarrow range \\ &- function1 : function \rightarrow function \\ &- interval1 : range \rightarrow range \end{aligned}$$

Parameters:

- *function1* and *function2* represent functions
- *interval1* and *interval2* represent intervals (ranges)
- *constant* represents a constant or constant expression

Description:

- — represents the subtraction (function) on reals. The expression *function1 - function2* stands for the function composed of the subtraction function and the two functions *function1* and *function2*, where *function1* is the subtrahend and *function2* the subtrahend.
- — can be used for interval arithmetic on intervals (ranges). — will evaluate to an interval that safely encompasses all images of the subtraction function with arguments varying in the given intervals. Any combination of intervals with intervals or constants (resp. constant expressions) is supported. However, it is not possible to represent families of functions using an interval as one argument and a function (varying in the free variable) as the other one.
- — stands also for the negation function.

Example 1:

```
> 5 - 2;  
3
```

Example 2:

```
> x - 2;  
-2 + x
```

Example 3:

```
> x - x;  
0
```

Example 4:

```
> diff(sin(x) - exp(x));  
cos(x) - exp(x)
```

Example 5:

```
> [1;2] - [3;4];  
[-3;-1]  
> [1;2] - 17;  
[-16;-15]  
> 13 - [-4;17];  
[-4;17]
```

Example 6:

```
> -exp(x);  
-exp(x)  
> -13;  
-13  
> -[13;17];  
[-17;-13]
```

See also: + (8.121), \* (8.107), / (8.45), ^ (8.125)

## 8.107 \*

Name: \*

multiplication function

Library names:

```
sollya_obj_t sollya_lib_mul(sollya_obj_t, sollya_obj_t)  
sollya_obj_t sollya_lib_build_function_mul(sollya_obj_t, sollya_obj_t)  
#define SOLLYA_MUL(x,y) sollya_lib_build_function_mul((x), (y))
```

Usage:

$$\begin{aligned} \text{function1} * \text{function2} &: (\text{function}, \text{function}) \rightarrow \text{function} \\ \text{interval1} * \text{interval2} &: (\text{range}, \text{range}) \rightarrow \text{range} \\ \text{interval1} * \text{constant} &: (\text{range}, \text{constant}) \rightarrow \text{range} \\ \text{interval1} * \text{constant} &: (\text{constant}, \text{range}) \rightarrow \text{range} \end{aligned}$$

Parameters:

- *function1* and *function2* represent functions

- *interval1* and *interval2* represent intervals (ranges)
- *constant* represents a constant or constant expression

Description:

- `*` represents the multiplication (function) on reals. The expression *function1* \* *function2* stands for the function composed of the multiplication function and the two functions *function1* and *function2*.
- `*` can be used for interval arithmetic on intervals (ranges). `*` will evaluate to an interval that safely encompasses all images of the multiplication function with arguments varying in the given intervals. Any combination of intervals with intervals or constants (resp. constant expressions) is supported. However, it is not possible to represent families of functions using an interval as one argument and a function (varying in the free variable) as the other one.

Example 1:

```
> 5 * 2;
10
```

Example 2:

```
> x * 2;
x * 2
```

Example 3:

```
> x * x;
x^2
```

Example 4:

```
> diff(sin(x) * exp(x));
sin(x) * exp(x) + exp(x) * cos(x)
```

Example 5:

```
> [1;2] * [3;4];
[3;8]
> [1;2] * 17;
[17;34]
> 13 * [-4;17];
[-52;221]
```

See also: `+` (8.121), `-` (8.106), `/` (8.45), `^` (8.125)

## 8.108 nearestint

Name: **nearestint**

the function mapping the reals to the integers nearest to them.

Library names:

```
sollya_obj_t sollya_lib_nearestint(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_nearestint(sollya_obj_t)
#define SOLLYA_NEARESTINT(x) sollya_lib_build_function_nearestint(x)
```

Description:

- **nearestint** is defined as usual: **nearestint**(*x*) is the integer nearest to *x*, with the special rule that the even integer is chosen if there exist two integers equally near to *x*.
- It is defined for every real number *x*.

See also: **ceil** (8.22), **floor** (8.67), **round** (8.153), **RN** (8.152)

## 8.109 !=

Name: !=

negated equality test operator

Library name:

`sollya_obj_t sollya_lib_cmp_not_equal(sollya_obj_t, sollya_obj_t)`

Usage:

$expr1 \text{ != } expr2 : (\text{any type, any type}) \rightarrow \text{boolean}$

Parameters:

- *expr1* and *expr2* represent expressions

Description:

- The operator != evaluates to true iff its operands *expr1* and *expr2* are syntactically unequal and both different from **error** or constant expressions that are not constants and that evaluate to two different floating-point number with the global precision **prec**. The user should be aware of the fact that because of floating-point evaluation, the operator != is not exactly the same as the negation of the mathematical equality.

Note that the expressions  $!(expr1 \text{ != } expr2)$  and  $expr1 == expr2$  do not evaluate to the same boolean value. See **error** for details.

Example 1:

```
> "Hello" != "Hello";
false
> "Hello" != "Salut";
true
> "Hello" != 5;
true
> 5 + x != 5 + x;
false
```

Example 2:

```
> 1 != exp(0);
false
> asin(1) * 2 != pi;
false
> exp(5) != log(4);
true
```

Example 3:

```
> sin(pi/6) != 1/2 * sqrt(3);
true
```

Example 4:

```
> prec = 12;
The precision has been set to 12 bits.
> 16384.1 != 16385.1;
false
```

Example 5:

```
> error != error;
false
```

See also: `==` (8.50), `>` (8.73), `>=` (8.71), `<=` (8.91), `<` (8.100), `in` (8.84), `!` (8.111), `&&` (8.6), `||` (8.116), **error** (8.53), **prec** (8.127)

### 8.110 **nop**

Name: **nop**  
no operation

Usage:

**nop** : void  $\rightarrow$  void  
**nop**() : void  $\rightarrow$  void  
**nop**(*n*) : integer  $\rightarrow$  void

Description:

- The command **nop** does nothing. This means it is an explicit parse element in the **Sollya** language that finally does not produce any result or side-effect.
- The command **nop** may take an optional positive integer argument *n*. The argument controls how much (useless) multiprecision floating-point multiplications **Sollya** performs while doing nothing. With this behaviour, **nop** can be used for calibration of timing tests.
- The keyword **nop** is implicit in some procedure definitions. Procedures without imperative body get parsed as if they had an imperative body containing one **nop** statement.

Example 1:

```
> nop;
```

Example 2:

```
> nop(100);
```

Example 3:

```
> succ = proc(n) { return n + 1; };  
> succ;  
proc(n)  
{  
  nop;  
  return (n) + (1);  
}  
> succ(5);  
6
```

See also: **proc** (8.135), **time** (8.182)

### 8.111 **!**

Name: **!**  
boolean NOT operator

Library name:  
`sollya_obj_t sollya_lib_negate(sollya_obj_t)`

Usage:

**!** *expr* : boolean  $\rightarrow$  boolean

Parameters:

- *expr* represents a boolean expression

Description:



- `!` evaluates to the boolean NOT of the boolean expression *expr*. `! expr` evaluates to true iff *expr* does not evaluate to true.

Example 1:

```
> ! false;
true
```

Example 2:

```
> ! (1 == exp(0));
false
```

See also: `&&` (8.6), `||` (8.116)

## 8.112 numberroots

Name: **numberroots**

Computes the number of roots of a polynomial in a given range.

Library name:

```
sollya_obj_t sollya_lib_numberroots(sollya_obj_t, sollya_obj_t)
```

Usage:

**numberroots**(*p*, *I*) : (function, range) → integer

Parameters:

- *p* is a polynomial.
- *I* is an interval.

Description:

- **numberroots** rigorously computes the number of roots of polynomial the *p* in the interval *I*. The technique used is Sturm's algorithm. The value returned is not just a numerical estimation of the number of roots of *p* in *I*: it is the exact number of roots.
- The command **findzeros** computes safe enclosures of all the zeros of a function, without forgetting any, but it is not guaranteed to separate them all in distinct intervals. **numberroots** is more accurate since it guarantees the exact number of roots. However, it does not compute them. It may be used, for instance, to certify that **findzeros** did not put two distinct roots in the same interval.
- Multiple roots are counted only once.
- The interval *I* must be bounded. The algorithm cannot handle unbounded intervals. Moreover, the interval is considered as a closed interval: if one (or both) of the endpoints of *I* are roots of *p*, they are counted.
- The argument *p* can be any expression, but if **Sollya** fails to prove that it is a polynomial an error is produced. Also, please note that if the coefficients of *p* or the endpoints of *I* are not exactly representable, they are first numerically evaluated, before the algorithm is used. In that case, the counted number of roots corresponds to the rounded polynomial on the rounded interval **and not** to the exact parameters given by the user. A warning is displayed to inform the user.

Example 1:

```
> numberroots(1+x-x^2, [1,2]);
1
> findzeros(1+x-x^2, [1,2]);
[[1.617919921875;1.6180419921875] |]
```

Example 2:

```
> numberroots((1+x)*(1-x), [-1,1]);
2
> numberroots(x^2, [-1,1]);
1
```

Example 3:

```
> verbosity = 1!;
> numberroots(x-pi, [0,4]);
Warning: the 0th coefficient of the polynomial is neither a floating point
constant nor can be evaluated without rounding to a floating point constant.
Will faithfully evaluate it with the current precision (165 bits)
1
```

Example 4:

```
> verbosity = 1!;
> numberroots(1+x-x^2, [0, @Inf@]);
Warning: the given interval must have finite bounds.
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
error
> numberroots(exp(x), [0, 1]);
Warning: the given function must be a polynomial in this context.
Warning: at least one of the given expressions or a subexpression is not correct
ly typed
or its evaluation has failed because of some error on a side-effect.
error
```

See also: **dirtyfindzeros** (8.41), **findzeros** (8.64)

### 8.113 numerator

Name: **numerator**

gives the numerator of an expression

Library name:

`sollya_obj_t sollya_lib_numerator(sollya_obj_t)`

Usage:

**numerator**(*expr*) : function  $\rightarrow$  function

Parameters:

- *expr* represents an expression

Description:

- If *expr* represents a fraction *expr1*/*expr2*, **numerator**(*expr*) returns the numerator of this fraction, i.e. *expr1*.

If *expr* represents something else, **numerator**(*expr*) returns the expression itself, i.e. *expr*.

Note that for all expressions *expr*, **numerator**(*expr*) / **denominator**(*expr*) is equal to *expr*.

Example 1:

```
> numerator(5/3);
5
```

Example 2:

```
> numerator(exp(x));
exp(x)
```

Example 3:

```
> a = 5/3;
> b = numerator(a)/denominator(a);
> print(a);
5 / 3
> print(b);
5 / 3
```

Example 4:

```
> a = exp(x/3);
> b = numerator(a)/denominator(a);
> print(a);
exp(x / 3)
> print(b);
exp(x / 3)
```

See also: **denominator** (8.37), **rationalmode** (8.142)

## 8.114 off

Name: **off**

special value for certain global variables.

Library names:

```
sollya_obj_t sollya_lib_off()
int sollya_lib_is_off(sollya_obj_t)
```

Description:

- **off** is a special value used to deactivate certain functionalities of **Sollya**.
- As any value it can be affected to a variable and stored in lists.

Example 1:

```
> canonical=on;
Canonical automatic printing output has been activated.
> p=1+x+x^2;
> mode=off;
> p;
1 + x + x^2
> canonical=mode;
Canonical automatic printing output has been deactivated.
> p;
1 + x * (1 + x)
```

See also: **on** (8.115), **autosimplify** (8.15), **canonical** (8.21), **timing** (8.183), **fullparentheses** (8.69), **midpointmode** (8.104), **rationalmode** (8.142), **roundingwarnings** (8.156), **timing** (8.183), **dieonerrormode** (8.39)

### 8.115 on

Name: **on**

special value for certain global variables.

Library names:

```
sollya_obj_t sollya_lib_on()
int sollya_lib_is_on(sollya_obj_t)
```

Description:

- **on** is a special value used to activate certain functionalities of Sollya.
- As any value it can be affected to a variable and stored in lists.

Example 1:

```
> p=1+x+x^2;
> mode=on;
> p;
1 + x * (1 + x)
> canonical=mode;
Canonical automatic printing output has been activated.
> p;
1 + x + x^2
```

See also: **off** (8.114), **autosimplify** (8.15), **canonical** (8.21), **timing** (8.183), **fullparentheses** (8.69), **midpointmode** (8.104), **rationalmode** (8.142), **roundingwarnings** (8.156), **timing** (8.183), **dieonerrormode** (8.39)

### 8.116 ||

Name: **||**

boolean OR operator

Library name:

```
sollya_obj_t sollya_lib_or(sollya_obj_t, sollya_obj_t)
```

Usage:

$$expr1 \parallel expr2 : (\text{boolean}, \text{boolean}) \rightarrow \text{boolean}$$

Parameters:

- *expr1* and *expr2* represent boolean expressions

Description:

- **||** evaluates to the boolean OR of the two boolean expressions *expr1* and *expr2*. **||** evaluates to true iff at least one of *expr1* or *expr2* evaluates to true.

Example 1:

```
> false || false;
false
```

Example 2:

```
> (1 == exp(0)) || (0 == log(1));
true
```

See also: **&&** (8.6), **!** (8.111)

## 8.117 parse

Name: **parse**

parses an expression contained in a string

Library name:

`sollya_obj_t sollya_lib_parse(sollya_obj_t)`

Usage:

**parse**(*string*) : string  $\rightarrow$  function | error

Parameters:

- *string* represents a character sequence

Description:

- **parse**(*string*) parses the character sequence *string* containing an expression built on constants and base functions.

If the character sequence does not contain a well-defined expression, a warning is displayed indicating a syntax error and **parse** returns a **error** of type **error**.

- The character sequence to be parsed by **parse** may contain commands that return expressions, including **parse** itself. Those commands get executed after the string has been parsed. **parse**(*string*) will return the expression computed by the commands contained in the character sequence *string*.

Example 1:

```
> parse("exp(x)");  
exp(x)
```

Example 2:

```
> text = "remez(exp(x),5,[-1;1])";  
> print("The string", text, "gives", parse(text));  
The string remez(exp(x),5,[-1;1]) gives 8.73819098827562036768683157316876049039  
64388498642e-3 * x^5 + 4.3793696379596015478233171265365272893795005588381e-2 *  
x^4 + 0.16642465614952768185129433844012193925654065755905 * x^3 + 0.49919698262  
963614991826575452094101562044819693772 * x^2 + 1.000038346505998154663400680582  
31011540878088492516 * x + 1.00004475029559502606203712816558243384077522932213
```

Example 3:

```
> verbosity = 1!;  
> parse("5 + * 3");  
Warning: syntax error, unexpected *. Will try to continue parsing (expecting ";"  
) . May leak memory.  
Warning: the string "5 + * 3" could not be parsed by the miniparser.  
Warning: at least one of the given expressions or a subexpression is not correct  
ly typed  
or its evaluation has failed because of some error on a side-effect.  
error
```

See also: **execute** (8.55), **readfile** (8.144), **print** (8.130), **error** (8.53), **dieonerrormode** (8.39)

## 8.118 perturb

Name: **perturb**

indicates random perturbation of sampling points for **externalplot**

Library names:

```
sollya_obj_t sollya_lib_perturb()
int sollya_lib_is_perturb(sollya_obj_t)
```

Usage:

**perturb** : perturb

Description:

- The use of **perturb** in the command **externalplot** enables the addition of some random noise around each sampling point in **externalplot**.

See **externalplot** for details.

Example 1:

```
> bashexecute("gcc -fPIC -c externalplotexample.c");
> bashexecute("gcc -shared -o externalplotexample externalplotexample.o -lgmp -lmpfr");
> externalplot("./externalplotexample",relative,exp(x),[-1/2;1/2],12,perturb);
```

See also: **externalplot** (8.60), **absolute** (8.2), **relative** (8.146), **bashexecute** (8.17)

## 8.119 pi

Name: **pi**

the constant  $\pi$ .

Library names:

```
sollya_obj_t sollya_lib_pi()
int sollya_lib_is_pi(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_pi()
#define SOLLYA_PI (sollya_lib_build_function_pi())
```

Description:

- **pi** is the constant  $\pi$ , defined as half the period of sine and cosine.
- In **Sollya**, **pi** is considered a 0-ary function. This way, the constant is not evaluated at the time of its definition but at the time of its use. For instance, when you define a constant or a function relating to  $\pi$ , the current precision at the time of the definition does not matter. What is important is the current precision when you evaluate the function or the constant value.
- Remark that when you define an interval, the bounds are first evaluated and then the interval is defined. In this case, **pi** will be evaluated as any other constant value at the definition time of the interval, thus using the current precision at this time.

Example 1:

```

> verbosity=1!; prec=12!;
> a = 2*pi;
> a;
Warning: rounding has happened. The value displayed is a faithful rounding to 12
bits of the true result.
6.283
> prec=20!;
> a;
Warning: rounding has happened. The value displayed is a faithful rounding to 20
bits of the true result.
6.283187

```

Example 2:

```

> display=binary;
Display mode is binary numbers.
> prec=12!;
> d = [pi; 5];
> d;
[1.1001001_2 * 2^(1); 1.01_2 * 2^(2)]
> prec=20!;
> d;
[1.1001001_2 * 2^(1); 1.01_2 * 2^(2)]

```

See also: `cos` (8.29), `sin` (8.164), `tan` (8.176), `asin` (8.10), `acos` (8.4), `atan` (8.12), `evaluate` (8.54), `prec` (8.127), `libraryconstant` (8.94)

## 8.120 plot

Name: **plot**

plots one or several functions

Library names:

```

void sollya_lib_plot(sollya_obj_t, sollya_obj_t, ...)
void sollya_lib_v_plot(sollya_obj_t, sollya_obj_t, va_list)

```

Usage:

```

      plot(f1, ... ,fn, I) : (function, ... ,function, range) → void
      plot(f1, ... ,fn, I, file, name) : (function, ... ,function, range, file, string) → void
      plot(f1, ... ,fn, I, postscript, name) : (function, ... ,function, range, postscript, string) → void
      plot(f1, ... ,fn, I, postscriptfile, name) : (function, ... ,function, range, postscriptfile, string) → void
      plot(L, I) : (list, range) → void
      plot(L, I, file, name) : (list, range, file, string) → void
      plot(L, I, postscript, name) : (list, range, postscript, string) → void
      plot(L, I, postscriptfile, name) : (list, range, postscriptfile, string) → void

```

Parameters:

- *f1*, ..., *fn* are functions to be plotted.
- *L* is a list of functions to be plotted.
- *I* is the interval where the functions have to be plotted.
- *name* is a string representing the name of a file.

Description:

- This command plots one or several functions  $f_1, \dots, f_n$  on an interval  $I$ . Functions can be either given as parameters of **plot** or as a list  $L$  which elements are functions. The functions are drawn on the same plot with different colors.
- If  $L$  contains an element that is not a function (or a constant), an error occurs.
- **plot** relies on the value of global variable **points**. Let  $n$  be the value of this variable. The algorithm is the following: each function is evaluated at  $n$  evenly distributed points in  $I$ . At each point, the computed value is a faithful rounding of the exact value with a sufficiently high precision. Each point is finally plotted. This should avoid numerical artefacts such as critical cancellations.
- You can save the function plot either as a data file or as a postscript file.
- If you use argument **file** with a string *name*, **Sollya** will save a data file called *name.dat* and a gnuplot directives file called *name.p*. Invoking gnuplot on *name.p* will plot the data stored in *name.dat*.
- If you use argument **postscript** with a string *name*, **Sollya** will save a postscript file called *name.eps* representing your plot.
- If you use argument **postscriptfile** with a string *name*, **Sollya** will produce the corresponding *name.dat*, *name.p* and *name.eps*.
- This command uses gnuplot to produce the final plot. If your terminal is not graphic (typically if you use **Sollya** through ssh without -X) gnuplot should be able to detect that and produce an ASCII-art version on the standard output. If it is not the case, you can either store the plot in a postscript file to view it locally, or use **asciiplot** command.
- If every function is constant, **plot** will not plot them but just display their value.
- If the interval is reduced to a single point, **plot** will just display the value of the functions at this point.

Example 1:

```
> plot(sin(x),0,cos(x),[-Pi,Pi]);
```

Example 2:

```
> plot(sin(x),0,cos(x),[-Pi,Pi],postscriptfile,"plotSinCos");
```

Example 3:

```
> plot(exp(0), sin(1), [0;1]);
1
0.84147098480789650665250232163029899962256306079837
```

Example 4:

```
> plot(sin(x), cos(x), [1;1]);
0.84147098480789650665250232163029899962256306079837
0.54030230586813971740093660744297660373231042061792
```

See also: **externalplot** (8.60), **asciiplot** (8.9), **file** (8.63), **postscript** (8.123), **postscriptfile** (8.124), **points** (8.122)



## 8.121 +

Name: +  
addition function

Library names:

```
sollya_obj_t sollya_lib_add(sollya_obj_t, sollya_obj_t)
sollya_obj_t sollya_lib_build_function_add(sollya_obj_t, sollya_obj_t)
#define SOLLYA_ADD(x,y) sollya_lib_build_function_add((x), (y))
```

Usage:

$$\begin{aligned} \textit{function1} + \textit{function2} &: (\textit{function}, \textit{function}) \rightarrow \textit{function} \\ \textit{interval1} + \textit{interval2} &: (\textit{range}, \textit{range}) \rightarrow \textit{range} \\ \textit{interval1} + \textit{constant} &: (\textit{range}, \textit{constant}) \rightarrow \textit{range} \\ \textit{interval1} + \textit{constant} &: (\textit{constant}, \textit{range}) \rightarrow \textit{range} \end{aligned}$$

Parameters:

- *function1* and *function2* represent functions
- *interval1* and *interval2* represent intervals (ranges)
- *constant* represents a constant or constant expression

Description:

- + represents the addition (function) on reals. The expression *function1* + *function2* stands for the function composed of the addition function and the two functions *function1* and *function2*.
- + can be used for interval arithmetic on intervals (ranges). + will evaluate to an interval that safely encompasses all images of the addition function with arguments varying in the given intervals. Any combination of intervals with intervals or constants (resp. constant expressions) is supported. However, it is not possible to represent families of functions using an interval as one argument and a function (varying in the free variable) as the other one.

Example 1:

```
> 1 + 2;
3
```

Example 2:

```
> x + 2;
2 + x
```

Example 3:

```
> x + x;
x * 2
```

Example 4:

```
> diff(sin(x) + exp(x));
cos(x) + exp(x)
```

Example 5:

```
> [1;2] + [3;4];
[4;6]
> [1;2] + 17;
[18;19]
> 13 + [-4;17];
[9;30]
```

See also: − (8.106), \* (8.107), / (8.45), ^ (8.125)

## 8.122 points

Name: **points**

controls the number of points chosen by Sollya in certain commands.

Library names:

```
void sollya_lib_set_points_and_print(sollya_obj_t)
void sollya_lib_set_points(sollya_obj_t)
sollya_obj_t sollya_lib_get_points()
```

Usage:

```
points =  $n$  : integer  $\rightarrow$  void
points =  $n$  ! : integer  $\rightarrow$  void
points : constant
```

Parameters:

- $n$  represents the number of points

Description:

- **points** is a global variable. Its value represents the number of points used in numerical algorithms of Sollya (namely **dirtyinfnorm**, **dirtyintegral**, **dirtyfindzeros**, **plot**).

Example 1:

```
> f=x^2*sin(1/x);
> points=10;
The number of points has been set to 10.
> dirtyfindzeros(f, [0;1]);
[|0, 0.318309886183790671537767526745028724068919291480918|]
> points=100;
The number of points has been set to 100.
> dirtyfindzeros(f, [0;1]);
[|0, 2.4485375860291590118289809749617594159147637806224e-2, 3.97887357729738339
422209408431285905086149114351147e-2, 4.5472840883398667362538218106432674866988
4702115589e-2, 5.3051647697298445256294587790838120678153215246819e-2, 6.3661977
236758134307553505349005744813783858296183e-2, 7.9577471545947667884441881686257
181017229822870229e-2, 0.106103295394596890512589175581676241356306430493638, 0.
159154943091895335768883763372514362034459645740459, 0.3183098861837906715377675
26745028724068919291480918|]
```

See also: **dirtyinfnorm** (8.42), **dirtyintegral** (8.43), **dirtyfindzeros** (8.41), **plot** (8.120), **diam** (8.38), **prec** (8.127)

## 8.123 postscript

Name: **postscript**

special value for commands **plot** and **externalplot**

Library names:

```
sollya_obj_t sollya_lib_postscript()
int sollya_lib_is_postscript(sollya_obj_t)
```

Description:

- **postscript** is a special value used in commands **plot** and **externalplot** to save the result of the command in a postscript file.
- As any value it can be affected to a variable and stored in lists.

Example 1:

```
> savemode=postscript;  
> name="plotSinCos";  
> plot(sin(x),0,cos(x),[-Pi,Pi],savemode, name);
```

See also: **externalplot** (8.60), **plot** (8.120), **file** (8.63), **postscriptfile** (8.124)

## 8.124 postscriptfile

Name: **postscriptfile**

special value for commands **plot** and **externalplot**

Library names:

```
sollya_obj_t sollya_lib_postscriptfile()  
int sollya_lib_is_postscriptfile(sollya_obj_t)
```

Description:

- **postscriptfile** is a special value used in commands **plot** and **externalplot** to save the result of the command in a data file and a postscript file.
- As any value it can be affected to a variable and stored in lists.

Example 1:

```
> savemode=postscriptfile;  
> name="plotSinCos";  
> plot(sin(x),0,cos(x),[-Pi,Pi],savemode, name);
```

See also: **externalplot** (8.60), **plot** (8.120), **file** (8.63), **postscript** (8.123)

## 8.125 ^

Name: ^

power function

Library names:

```
sollya_obj_t sollya_lib_pow(sollya_obj_t, sollya_obj_t)  
sollya_obj_t sollya_lib_build_function_pow(sollya_obj_t, sollya_obj_t)  
#define SOLLYA_POW(x,y) sollya_lib_build_function_pow((x), (y))
```

Usage:

$$\begin{aligned} function1 \wedge function2 &: (function, function) \rightarrow function \\ interval1 \wedge interval2 &: (range, range) \rightarrow range \\ interval1 \wedge constant &: (range, constant) \rightarrow range \\ interval1 \wedge constant &: (constant, range) \rightarrow range \end{aligned}$$

Parameters:

- *function1* and *function2* represent functions
- *interval1* and *interval2* represent intervals (ranges)
- *constant* represents a constant or constant expression

Description:

- ^ represents the power (function) on reals. The expression *function1* ^ *function2* stands for the function composed of the power function and the two functions *function1* and *function2*, where *function1* is the base and *function2* the exponent. If *function2* is a constant integer, ^ is defined on negative values of *function1*. Otherwise ^ is defined as  $e^{y \cdot \ln x}$ .



## 8.126 powers

Name: **powers**

special value for global state **display**

Library names:

```
sollya_obj_t sollya_lib_powers()
int sollya_lib_is_powers(sollya_obj_t)
```

Description:

- **powers** is a special value used for the global state **display**. If the global state **display** is equal to **powers**, all data will be output in dyadic notation with numbers displayed in a Maple and PARI/GP compatible format.

As any value it can be affected to a variable and stored in lists.

See also: **decimal** (8.34), **dyadic** (8.49), **hexadecimal** (8.77), **binary** (8.18), **display** (8.44)

## 8.127 prec

Name: **prec**

controls the precision used in numerical computations.

Library names:

```
void sollya_lib_set_prec_and_print(sollya_obj_t)
void sollya_lib_set_prec(sollya_obj_t)
sollya_obj_t sollya_lib_get_prec()
```

Description:

- **prec** is a global variable. Its value represents the precision of the floating-point format used in numerical computations.
- Many commands try to adapt their working precision in order to have approximately  $n$  correct bits in output, where  $n$  is the value of **prec**.

Example 1:

```
> display=binary!;
> prec=50;
The precision has been set to 50 bits.
> dirtyinfnorm(exp(x),[1;2]);
1.110110001110011001001011100011010100110111011011_2 * 2^(2)
> prec=100;
The precision has been set to 100 bits.
> dirtyinfnorm(exp(x),[1;2]);
1.110110001110011001001011100011010100110111011010110111001100001100111010001110
11101000100000011011_2 * 2^(2)
```

See also: **evaluate** (8.54), **diam** (8.38)

## 8.128 precision

Name: **precision**

returns the precision necessary to represent a number.

Library name:

```
sollya_obj_t sollya_lib_precision(sollya_obj_t)
```

Usage:

**precision**( $x$ ) : constant  $\rightarrow$  integer

Parameters:

- $x$  is a dyadic number.

Description:

- **precision**( $x$ ) is by definition  $|x|$  if  $x$  equals 0, NaN, or Inf.
- If  $x$  is not zero, it can be uniquely written as  $x = m \cdot 2^e$  where  $m$  is an odd integer and  $e$  is an integer. **precision**( $x$ ) returns the number of bits necessary to write  $m$  in binary (i.e.  $\lceil \log_2(m) \rceil$ ).

Example 1:

```
> a=round(Pi,20,RN);
> precision(a);
19
> m=mantissa(a);
> ceil(log2(m));
19
```

See also: **mantissa** (8.101), **exponent** (8.59), **round** (8.153)

## 8.129 **::**

Name: **::**

add an element at the beginning of a list.

Library name:

```
sollya_obj_t sollya_lib_prepend(sollya_obj_t, sollya_obj_t)
```

Usage:

$$x::L : (\text{any type, list}) \rightarrow \text{list}$$

Parameters:

- $x$  is an object of any type.
- $L$  is a list (possibly empty).

Description:

- **::** adds the element  $x$  at the beginning of the list  $L$ .
- Note that since  $x$  may be of any type, it can be in particular a list.

Example 1:

```
> 1.: [|2,3,4|];
[|1, 2, 3, 4|]
```

Example 2:

```
> [|1,2,3|].: [|4,5,6|];
[| [|1, 2, 3|], 4, 5, 6|]
```

Example 3:

```
> 1.: [| |];
[|1|]
```

See also: **::** (8.7), **@** (8.27)

## 8.130 print

Name: **print**

prints an expression

Usage:

```
print(expr1,...,exprn) : (any type,..., any type) → void
print(expr1,...,exprn) > filename : (any type,..., any type, string) → void
print(expr1,...,exprn) >> filename : (any type,...,any type, string) → void
```

Parameters:

- *expr* represents an expression
- *filename* represents a character sequence indicating a file name

Description:

- **print**(*expr1*,...,*exprn*) prints the expressions *expr1* through *exprn* separated by spaces and followed by a newline.

If a second argument *filename* is given after a single ">", the displaying is not output on the standard output of Sollya but if in the file *filename* that get newly created or overwritten. If a double ">>" is given, the output will be appended to the file *filename*.

The global variables **display**, **midpointmode** and **fullparentheses** have some influence on the formatting of the output (see **display**, **midpointmode** and **fullparentheses**).

Remark that if one of the expressions *expri* given in argument is of type **string**, the character sequence *expri* evaluates to is displayed. However, if *expri* is of type **list** and this list contains a variable of type **string**, the expression for the list is displayed, i.e. all character sequences get displayed surrounded by double quotes ("). Nevertheless, escape sequences used upon defining character sequences are interpreted immediately.

Example 1:

```
> print(x + 2 + exp(sin(x)));
x + 2 + exp(sin(x))
> print("Hello","world");
Hello world
> print("Hello","you", 4 + 3, "other persons.");
Hello you 7 other persons.
```

Example 2:

```
> print("Hello");
Hello
> print(["Hello"]);
["Hello"]
> s = "Hello";
> print(s,[s]);
Hello ["Hello"]
> t = "Hello\tyou";
> print(t,[t]);
Hello    you ["Hello\tyou"]
```

Example 3:

```
> print(x + 2 + exp(sin(x))) > "foo.sol";
> readfile("foo.sol");
x + 2 + exp(sin(x))
```

Example 4:

```
> print(x + 2 + exp(sin(x))) >> "foo.sol";
```

Example 5:



```

> display = decimal;
Display mode is decimal numbers.
> a = evaluate(sin(pi * x), 0.25);
> b = evaluate(sin(pi * x), [0.25; 0.25 + 1b-50]);
> print(a);
0.70710678118654752440084436210484903928483593768847
> display = binary;
Display mode is binary numbers.
> print(a);
1.011010100000100111100110011001111111001110111100110010010000100010110010111110
1100010011011001101110101010010101011110100111110001110101101111011000001011101
010001_2 * 2^(-1)
> display = hexadecimal;
Display mode is hexadecimal numbers.
> print(a);
0xb.504f333f9de6484597d89b3754abe9f1d6f60ba88p-4
> display = dyadic;
Display mode is dyadic numbers.
> print(a);
33070006991101558613323983488220944360067107133265b-165
> display = powers;
Display mode is dyadic numbers in integer-power-of-2 notation.
> print(a);
33070006991101558613323983488220944360067107133265 * 2^(-165)
> display = decimal;
Display mode is decimal numbers.
> midpointmode = off;
Midpoint mode has been deactivated.
> print(b);
[0.70710678118654752440084436210484903928483593768844;0.707106781186549497437217
82517557347782646274417048]
> midpointmode = on;
Midpoint mode has been activated.
> print(b);
0.7071067811865~4/5~
> display = dyadic;
Display mode is dyadic numbers.
> print(b);
[2066875436943847413332748968013809022504194195829b-161;165350034955508254441962
37019385936414432675156571b-164]
> display = decimal;
Display mode is decimal numbers.
> autosimplify = off;
Automatic pure tree simplification has been deactivated.
> fullparentheses = off;
Full parentheses mode has been deactivated.
> print(x + x * ((x + 1) + 1));
x + x * (x + 1 + 1)
> fullparentheses = on;
Full parentheses mode has been activated.
> print(x + x * ((x + 1) + 1));
x + (x * ((x + 1) + 1))

```

See also: **write** (8.191), **printexpansion** (8.132), **printdouble** (8.131), **printsingl** (8.133), **printxml** (8.134), **readfile** (8.144), **autosimplify** (8.15), **display** (8.44), **midpointmode** (8.104), **fullparentheses** (8.69), **evaluate** (8.54), **rationalmode** (8.142)

### 8.131 printdouble

Name: **printdouble**

prints a constant value as a hexadecimal double precision number

Library name:

```
void sollya_lib_printdouble(sollya_obj_t)
```

Usage:

**printdouble**(*constant*) : constant  $\rightarrow$  void

Parameters:

- *constant* represents a constant

Description:

- Prints a constant value as a hexadecimal number on 16 hexadecimal digits. The hexadecimal number represents the integer equivalent to the 64 bit memory representation of the constant considered as a double precision number.

If the constant value does not hold on a double precision number, it is first rounded to the nearest double precision number before displayed. A warning is displayed in this case.

Example 1:

```
> printdouble(3);  
0x4008000000000000
```

Example 2:

```
> prec=100!;  
> verbosity = 1!;  
> printdouble(exp(5));  
Warning: the given expression is not a constant but an expression to evaluate. A  
faithful evaluation to 100 bits will be used.  
Warning: rounding down occurred before printing a value as a double.  
0x40628d389970338f
```

See also: **printsingle** (8.133), **printexpansion** (8.132), **double** (8.46)

### 8.132 printexpansion

Name: **printexpansion**

prints a polynomial in Horner form with its coefficients written as a expansions of double precision numbers

Library name:

```
void sollya_lib_printexpansion(sollya_obj_t)
```

Usage:

**printexpansion**(*polynomial*) : function  $\rightarrow$  void

Parameters:

- *polynomial* represents the polynomial to be printed

Description:

- The command **printexpansion** prints the polynomial *polynomial* in Horner form writing its coefficients as expansions of double precision numbers. The double precision numbers themselves are displayed in hexadecimal memory notation (see **printdouble**).

If some of the coefficients of the polynomial *polynomial* are not floating-point constants but constant expressions, they are evaluated to floating-point constants using the global precision **prec**. If a rounding occurs in this evaluation, a warning is displayed.

If the exponent range of double precision is not sufficient to display all the mantissa bits of a coefficient, the coefficient is displayed rounded and a warning is displayed.

If the argument *polynomial* does not a polynomial, nothing but a warning or a newline is displayed. Constants can be displayed using **printexpansion** since they are polynomials of degree 0.

Example 1:

```
> printexpansion(roundcoefficients(taylor(exp(x),5,0),[|DD...|]));
0x3ff0000000000000 + x * (0x3ff0000000000000 + x * (0x3fe0000000000000 + x * ((0
x3fc5555555555555 + 0x3c65555555555555) + x * ((0x3fa5555555555555 + 0x3c4555555
5555555) + x * (0x3f81111111111111 + 0x3c01111111111111))))))
```

Example 2:

```
> printexpansion(remez(exp(x),5,[-1;1]));
(0x3ff0002eec90e5a6 + 0x3c9ea6a6a0087757 + 0xb8eb3e644ef44998) + x * ((0x3ff0002
8358fd3ac + 0x3c8ffa7d96c95f7a + 0xb91da9809b13dd54 + 0x35c0000000000000) + x *
((0x3fdff2d7e6a9fea5 + 0x3c74460e4c0e4fe2 + 0x38fcd1b6b4e85bb0 + 0x35900000000000
000) + x * ((0x3fc54d6733b4839e + 0x3c6654e4d8614a44 + 0xb905c7a26b66ea92 + 0xb5
9800000000000000) + x * ((0x3fa66c209b7150a8 + 0x3c34b1bba8f78092 + 0xb8c75f6eb90d
ae02 + 0x356000000000000000) + x * (0x3f81e554242ab128 + 0xbc23e920a76e760c + 0x38
c0589c2cae6caf + 0x356400000000000000))))))
```

Example 3:

```
> verbosity = 1!;
> prec = 3500!;
> printexpansion(pi);
(0x400921fb54442d18 + 0x3ca1a62633145c07 + 0xb92f1976b7ed8fbc + 0x35c4cf98e80417
7d + 0x32631d89cd9128a5 + 0x2ec0f31c6809bbdf + 0x2b5519b3cd3a431b + 0x27e8158536
f92f8a + 0x246ba7f09ab6b6a9 + 0xa0eedd0dbd2544cf + 0x1d779fb1bd1310ba + 0x1a1a63
7ed6b0bfff6 + 0x96aa485fca40908e + 0x933e501295d98169 + 0x8fd160dbee83b4e0 + 0x8c
59b6d799ae131c + 0x08f6cf70801f2e28 + 0x05963bf0598da483 + 0x023871574e69a459 +
0x8000000005702db3 + 0x800000000000000000)
Warning: the expansion is not complete because of the limited exponent range of
double precision.
Warning: rounding occurred while printing.
```

See also: **printdouble** (8.131), **horner** (8.80), **print** (8.130), **prec** (8.127), **remez** (8.147), **taylor** (8.178), **roundcoefficients** (8.154), **fpminimax** (8.68), **implementpoly** (8.83)

## 8.133 printsingle

Name: **printsingle**

prints a constant value as a hexadecimal single precision number

Library name:

```
void sollya_lib_printsingle(sollya_obj_t)
```

Usage:

**printsingle**(*constant*) : constant → void

Parameters:

- *constant* represents a constant

Description:

- Prints a constant value as a hexadecimal number on 8 hexadecimal digits. The hexadecimal number represents the integer equivalent to the 32 bit memory representation of the constant considered as a single precision number.

If the constant value does not hold on a single precision number, it is first rounded to the nearest single precision number before it is displayed. A warning is displayed in this case.

Example 1:

```
> printsingle(3);  
0x40400000
```

Example 2:

```
> prec=100!;  
> verbosity = 1!;  
> printsingle(exp(5));  
Warning: the given expression is not a constant but an expression to evaluate. A  
faithful evaluation to 100 bits will be used.  
Warning: rounding down occurred before printing a value as a single.  
0x431469c5
```

See also: **printdouble** (8.131), **single** (8.165)

## 8.134 printxml

Name: **printxml**

prints an expression as an MathML-Content-Tree

Library names:

```
void sollya_lib_printxml(sollya_obj_t)  
void sollya_lib_printxml_newfile(sollya_obj_t, sollya_obj_t)  
void sollya_lib_printxml_appendfile(sollya_obj_t, sollya_obj_t)
```

Usage:

```
printxml(expr) : function → void  
printxml(expr) > filename : (function, string) → void  
printxml(expr) > > filename : (function, string) → void
```

Parameters:

- *expr* represents a functional expression
- *filename* represents a character sequence indicating a file name

Description:

- **printxml**(*expr*) prints the functional expression *expr* as a tree of MathML Content Definition Markups. This XML tree can be re-read in external tools or by usage of the **readxml** command.

If a second argument *filename* is given after a single >, the MathML tree is not output on the standard output of **Sollya** but if in the file *filename* that get newly created or overwritten. If a double > > is given, the output will be appended to the file *filename*.

Example 1:

```

> printxml(x + 2 + exp(sin(x)));

<?xml version="1.0" encoding="UTF-8"?>
<!-- generated by sollya: http://sollya.gforge.inria.fr/ -->
<!-- syntax: printxml(...);   example: printxml(x^2-2*x+5); -->
<?xml-stylesheet type="text/xsl" href="http://sollya.gforge.inria.fr/mathmlc2p-web.xsl"?>
<?xml-stylesheet type="text/xsl" href="mathmlc2p-web.xsl"?>
<!-- This stylesheet allows direct web browsing of MathML-c XML files (http:// o
r file://) -->

<math xmlns="http://www.w3.org/1998/Math/MathML">
<semantics>
<annotation-xml encoding="MathML-Content">
<lambda>
<bvar><ci> x </ci></bvar>
<apply>
<apply>
<plus/>
<apply>
<plus/>
<ci> x </ci>
<cn type="integer" base="10"> 2 </cn>
</apply>
<apply>
<exp/>
<apply>
<sin/>
<ci> x </ci>
</apply>
</apply>
</apply>
</apply>
</lambda>
</annotation-xml>
<annotation encoding="sollya/text">(x + 1b1) + exp(sin(x))</annotation>
</semantics>
</math>

```

Example 2:

```

> printxml(x + 2 + exp(sin(x))) > "foo.xml";

```

Example 3:

```

> printxml(x + 2 + exp(sin(x))) >> "foo.xml";

```

See also: **readxml** (8.145), **print** (8.130), **write** (8.191)

## 8.135 **proc**

Name: **proc**

defines a Sollya procedure

Usage:

```

proc(formal parameter1, formal parameter2,..., formal parameter n) { procedure body } : void →
    procedure
proc(formal parameter1, formal parameter2,..., formal parameter n) { procedure body return
    expression; } : void → procedure
    proc(formal list parameter = ...) { procedure body } : void → procedure
proc(formal list parameter = ...) { procedure body return expression; } : void → procedure

```

Parameters:

- *formal parameter1*, *formal parameter2* through *formal parameter n* represent identifiers used as formal parameters
- *formal list parameter* represents an identifier used as a formal parameter for the list of an arbitrary number of parameters
- *procedure body* represents the imperative statements in the body of the procedure
- *expression* represents the expression **proc** shall evaluate to

Description:

- The **proc** keyword allows for defining procedures in the Sollya language. These procedures are common Sollya objects that can be applied to actual parameters after definition. Upon such an application, the Sollya interpreter applies the actual parameters to the formal parameters *formal parameter1* through *formal parameter n* (resp. builds up the list of arguments and applies it to the list *formal list parameter*) and executes the *procedure body*. The procedure applied to actual parameters evaluates then to the expression *expression* in the **return** statement after the *procedure body* or to **void**, if no return statement is given (i.e. a **return void** statement is implicitly given).
- Sollya procedures defined by **proc** have no name. They can be bound to an identifier by assigning the procedure object a **proc** expression produces to an identifier. However, it is possible to use procedures without giving them any name. For instance, Sollya procedures, i.e. procedure objects, can be elements of lists. They can even be given as an argument to other internal Sollya procedures. See also **procedure** on this subject.
- Upon definition of a Sollya procedure using **proc**, no type check is performed. More precisely, the statements in *procedure body* are merely parsed but not interpreted upon procedure definition with **proc**. Type checks are performed once the procedure is applied to actual parameters or to **void**. At this time, if the procedure was defined using several different formal parameters *formal parameter 1* through *formal parameter n*, it is checked whether the number of actual parameters corresponds to the number of formal parameters. If the procedure was defined using the syntax for a procedure with an arbitrary number of parameters by giving a *formal list parameter*, the number of actual arguments is not checked but only a list [formal list parameter] of appropriate length is built up. Type checks are further performed upon execution of each statement in *procedure body* and upon evaluation of the expression *expression* to be returned.

Procedures defined by **proc** containing a **quit** or **restart** command cannot be executed (i.e. applied). Upon application of a procedure, the Sollya interpreter checks beforehand for such a statement. If one is found, the application of the procedure to its arguments evaluates to **error**. A warning is displayed. Remark that in contrast to other type or semantic correctness checks, this check is really performed before interpreting any other statement in the body of the procedure.

- Through the **var** keyword it is possible to declare local variables and thus to have full support of recursive procedures. This means a procedure defined using **proc** may contain in its *procedure body* an application of itself to some actual parameters: it suffices to assign the procedure (object) to an identifier with an appropriate name.
- Sollya procedures defined using **proc** may return other procedures. Further *procedure body* may contain assignments of locally defined procedure objects to identifiers. See **var** for the particular behaviour of local and global variables.

- The expression *expression* returned by a procedure is evaluated with regard to **Sollya** commands, procedures and external procedures. Simplification may be performed. However, an application of a procedure defined by **proc** to actual parameters evaluates to the expression *expression* that may contain the free global variable or that may be composed.

Example 1:

```
> succ = proc(n) { return n + 1; };
> succ(5);
6
> 3 + succ(0);
4
> succ;
proc(n)
{
  nop;
  return (n) + (1);
}
```

Example 2:

```
> add = proc(m,n) { var res; res := m + n; return res; };
> add(5,6);
11
> add;
proc(m, n)
{
  var res;
  res := (m) + (n);
  return res;
}
> verbosity = 1!;
> add(3);
Warning: at least one of the given expressions or a subexpression is not correctly typed
or its evaluation has failed because of some error on a side-effect.
error
> add(true,false);
Warning: at least one of the given expressions or a subexpression is not correctly typed
or its evaluation has failed because of some error on a side-effect.
Warning: the given expression or command could not be handled.
error
```

Example 3:

```
> succ = proc(n) { return n + 1; };
> succ(5);
6
> succ(x);
1 + x
```

Example 4:

```

> hey = proc() { print("Hello world."); };
> hey();
Hello world.
> print(hey());
Hello world.
void
> hey;
proc()
{
print("Hello world.");
return void;
}

```

Example 5:

```

> fac = proc(n) { var res; if (n == 0) then res := 1 else res := n * fac(n - 1);
  return res; };
> fac(5);
120
> fac(11);
39916800
> fac;
proc(n)
{
var res;
if (n) == (0) then
res := 1
else
res := (n) * (fac((n) - (1)));
return res;
}

```

Example 6:

```

> myprocs = [| proc(m,n) { return m + n; }, proc(m,n) { return m - n; } |];
> (myprocs[0])(5,6);
11
> (myprocs[1])(5,6);
-1
> succ = proc(n) { return n + 1; };
> pred = proc(n) { return n - 1; };
> applier = proc(p,n) { return p(n); };
> applier(succ,5);
6
> applier(pred,5);
4

```

Example 7:



```

> verbosity = 1!;
> myquit = proc(n) { print(n); quit; };
> myquit;
proc(n)
{
print(n);
quit;
return void;
}
> myquit(5);
Warning: a quit or restart command may not be part of a procedure body.
The procedure will not be executed.
Warning: an error occurred while executing a procedure.
Warning: the given expression or command could not be handled.
error

```

Example 8:

```

> printsucc = proc(n) { var succ; succ = proc(n) { return n + 1; }; print("Succe
ssor of",n,"is",succ(n)); };
> printsucc(5);
Successor of 5 is 6

```

Example 9:

```

> makeadd = proc(n) { var add; print("n =",n); add = proc(m,n) { return n + m; }
; return add; };
> makeadd(4);
n = 4
proc(m, n)
{
nop;
return (n) + (m);
}
> (makeadd(4))(5,6);
n = 4
11

```

Example 10:

```

> sumall = proc(L = ...) { var acc, i; acc = 0; for i in L do acc = acc + i; ret
urn acc; };
> sumall;
proc(L = ...)
{
var acc, i;
acc = 0;
for i in L do
acc = (acc) + (i);
return acc;
}
> sumall();
0
> sumall(2);
2
> sumall(2,5);
7
> sumall(2,5,7,9,16);
39
> sumall @ [|1,...,10|];
55

```

See also: **return** (8.150), **externalproc** (8.61), **void** (8.189), **quit** (8.139), **restart** (8.149), **var** (8.187), **@** (8.27), **bind** (8.19), **error** (8.53)

## 8.136 procedure

Name: **procedure**

defines and assigns a Sollya procedure

Usage:

```

procedure identifier(formal parameter1, formal parameter2,..., formal parameter n) { procedure body }
                                     : void → void
procedure identifier(formal parameter1, formal parameter2,..., formal parameter n) { procedure body
                                     return expression; } : void → void
procedure identifier(formal list parameter = ...) { procedure body } : void → void
procedure identifier(formal list parameter = ...) { procedure body return expression; } : void → void

```

Parameters:

- *identifier* represents the name of the procedure to be defined and assigned
- *formal parameter1*, *formal parameter2* through *formal parameter n* represent identifiers used as formal parameters
- *formal list parameter* represents an identifier used as a formal parameter for the list of an arbitrary number of parameters
- *procedure body* represents the imperative statements in the body of the procedure
- *expression* represents the expression **procedure** shall evaluate to

Description:

- The **procedure** keyword allows for defining and assigning procedures in the Sollya language. It is an abbreviation to a procedure definition using **proc** with the same formal parameters, procedure body and return-expression followed by an assignment of the procedure (object) to the identifier *identifier*. In particular, all rules concerning local variables declared using the **var** keyword apply for **procedure**.

Example 1:

```
> procedure succ(n) { return n + 1; };
> succ(5);
6
> 3 + succ(0);
4
> succ;
proc(n)
{
  nop;
  return (n) + (1);
}
```

Example 2:

```
> procedure myprint(L = ...) { var i; for i in L do i; };
> myprint("Lyon","Nancy","Beaverton","Coye-la-Forêt","Amberg","Nizhny Novgorod",
"Cluj-Napoca");
Lyon
Nancy
Beaverton
Coye-la-Forêt
Amberg
Nizhny Novgorod
Cluj-Napoca
```

See also: **proc** (8.135), **var** (8.187), **bind** (8.19)

## 8.137 QD

Name: **QD**

short form for **quad**

See also: **quad** (8.138)

## 8.138 quad

Names: **quad**, **QD**

rounding to the nearest IEEE 754 quad (binary128).

Library names:

```
sollya_obj_t sollya_lib_quad(sollya_obj_t)
sollya_obj_t sollya_lib_quad_obj()
int sollya_lib_is_quad_obj(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_quad(sollya_obj_t)
#define SOLLYA_QD(x) sollya_lib_build_function_quad(x)
```

Description:

- **quad** is both a function and a constant.
- As a function, it rounds its argument to the nearest IEEE 754 quad precision (i.e. IEEE754-2008 binary128) number. Subnormal numbers are supported as well as standard numbers: it is the real rounding described in the standard.
- As a constant, it symbolizes the quad precision format. It is used in contexts when a precision format is necessary, e.g. in the commands **round** and **roundcoefficients**. It is not supported for **implementpoly**. See the corresponding help pages for examples.

Example 1:

```
> display=binary!;  
> QD(0.1);  
1.1001100110011001100110011001100110011001100110011001100110011001100110011001100110  
011001100110011001100110011001101_2 * 2^(-4)  
> QD(4.17);  
1.000010101110000101000111101011100001010001111010111000010100011110101110000101  
000111101011100001010001111010111_2 * 2^(2)  
> QD(1.011_2 * 2^(-16493));  
1.1_2 * 2^(-16493)
```

See also: **halfprecision** (8.75), **single** (8.165), **double** (8.46), **doubleextended** (8.48), **doubledouble** (8.47), **tripledouble** (8.184), **roundcoefficients** (8.154), **implementpoly** (8.83), **fminimax** (8.68), **round** (8.153), **printsingl** (8.133)

## 8.139 quit

Name: **quit**  
quits Sollya

Usage:

**quit** : void → void

Description:

- The command **quit**, when executed, stops the execution of a **Sollya** script and leaves the **Sollya** interpreter unless the **quit** command is executed in a **Sollya** script read into a main **Sollya** script by **execute** or **#include**.

Upon exiting the **Sollya** interpreter, all state is thrown away, all memory is deallocated, all bound libraries are unbound and the temporary files produced by **plot** and **externalplot** are deleted.

If the **quit** command does not lead to a halt of the **Sollya** interpreter, a warning is displayed.

Example 1:

```
> quit;
```

See also: **restart** (8.149), **execute** (8.55), **plot** (8.120), **externalplot** (8.60), **return** (8.150)

## 8.140 range

Name: **range**  
keyword representing a **range** type

Usage:

**range** : type type

Description:

- **range** represents the **range** type for declarations of external procedures by means of **externalproc**.

Remark that in contrast to other indicators, type indicators like **range** cannot be handled outside the **externalproc** context. In particular, they cannot be assigned to variables.

See also: **externalproc** (8.61), **boolean** (8.20), **constant** (8.28), **function** (8.70), **integer** (8.87), **list of** (8.95), **string** (8.169)

## 8.141 rationalapprox

Name: **rationalapprox**

returns a fraction close to a given number.

Library name:

`sollya_obj_t sollya_lib_rationalapprox(sollya_obj_t, sollya_obj_t)`

Usage:

**rationalapprox**( $x, n$ ) : (constant, integer)  $\rightarrow$  function

Parameters:

- $x$  is a number to approximate.
- $n$  is a integer (representing a format).

Description:

- **rationalapprox**( $x, n$ ) returns a constant function of the form  $a/b$  where  $a$  and  $b$  are integers. The value  $a/b$  is an approximation of  $x$ . The quality of this approximation is determined by the parameter  $n$  that indicates the number of correct bits that  $a/b$  should have.
- The command is not safe in the sense that it is not ensured that the error between  $a/b$  and  $x$  is less than  $2^{-n}$ .
- The following algorithm is used:  $x$  is first rounded downwards and upwards to a format of  $n$  bits, thus obtaining an interval  $[x_l, x_u]$ . This interval is then developed into a continued fraction as far as the representation is the same for every elements of  $[x_l, x_u]$ . The corresponding fraction is returned.
- Since rational numbers are not a primitive object of **Sollya**, the fraction is returned as a constant function. This can be quite amazing, because **Sollya** immediately simplifies a constant function by evaluating it when the constant has to be displayed. To avoid this, you can use **print** (that displays the expression representing the constant and not the constant itself) or the commands **numerator** and **denominator**.

Example 1:

```
> pi10 = rationalapprox(Pi,10);
> pi50 = rationalapprox(Pi,50);
> pi100 = rationalapprox(Pi,100);
> print( pi10, ": ", simplify(floor(-log2(abs(pi10-Pi)/Pi))), "bits." );
3.140625 : 11 bits.
> print( pi50, ": ", simplify(floor(-log2(abs(pi50-Pi)/Pi))), "bits." );
85563208 / 27235615 : 51 bits.
> print( pi100, ": ", simplify(floor(-log2(abs(pi100-Pi)/Pi))), "bits." );
4422001152019829 / 1407566683404023 : 100 bits.
```

Example 2:

```
> a=0.1;
> b=rationalapprox(a,4);
> numerator(b); denominator(b);
1
10
> print(simplify(floor(-log2(abs((b-a)/a)))), "bits.");
166 bits.
```

See also: **print** (8.130), **numerator** (8.113), **denominator** (8.37), **rationalmode** (8.142)

## 8.142 rationalmode

Name: **rationalmode**

global variable controlling if rational arithmetic is used or not.

Library names:

```
void sollya_lib_set_rationalmode_and_print(sollya_obj_t)
void sollya_lib_set_rationalmode(sollya_obj_t)
sollya_obj_t sollya_lib_get_rationalmode()
```

Usage:

```
rationalmode = activation value : on|off → void
rationalmode = activation value ! : on|off → void
rationalmode : on|off
```

Parameters:

- *activation value* controls if rational arithmetic should be used or not

Description:

- **rationalmode** is a global variable. When its value is **off**, which is the default, **Sollya** will not use rational arithmetic to simplify expressions. All computations, including the evaluation of constant expressions given on the **Sollya** prompt, will be performed using floating-point and interval arithmetic. Constant expressions will be approximated by floating-point numbers, which are in most cases faithful roundings of the expressions, when shown at the prompt.
- When the value of the global variable **rationalmode** is **on**, **Sollya** will use rational arithmetic when simplifying expressions. Constant expressions, given at the **Sollya** prompt, will be simplified to rational numbers and displayed as such when they are in the set of the rational numbers. Otherwise, floating-point and interval arithmetic will be used to compute a floating-point approximation, which is in most cases a faithful rounding of the constant expression.

Example 1:

```
> rationalmode=off!;
> 19/17 + 3/94;
1.1495619524405506883604505632040050062578222778473
> rationalmode=on!;
> 19/17 + 3/94;
1837 / 1598
```

Example 2:

```
> rationalmode=off!;
> exp(19/17 + 3/94);
3.15680977395514136754709208944824276340328162814418
> rationalmode=on!;
> exp(19/17 + 3/94);
3.15680977395514136754709208944824276340328162814418
```

See also: **on** (8.115), **off** (8.114), **numerator** (8.113), **denominator** (8.37), **simplifysafe** (8.163), **rationalapprox** (8.141), **autosimplify** (8.15)

## 8.143 RD

Name: **RD**

constant representing rounding-downwards mode.

Library names:

```
sollya_obj_t sollya_lib_round_down()
int sollya_lib_is_round_down(sollya_obj_t)
```

Description:

- **RD** is used in command **round** to specify that the value  $x$  must be rounded to the greatest floating-point number  $y$  such that  $y \leq x$ .

Example 1:

```
> display=binary!;
> round(Pi,20,RD);
1.1001001000011111101_2 * 2^(1)
```

See also: **RZ** (8.158), **RU** (8.157), **RN** (8.152), **round** (8.153), **floor** (8.67)

## 8.144 readfile

Name: **readfile**

reads the content of a file into a string variable

Usage:

**readfile**(*filename*) : string → string

Parameters:

- *filename* represents a character sequence indicating a file name

Description:

- **readfile** opens the file indicated by *filename*, reads it and puts its contents in a character sequence of type **string** that is returned.

If the file indicated by *filename* cannot be opened for reading, a warning is displayed and **readfile** evaluates to an **error** variable of type **error**.

Example 1:

```
> print("Hello world") > "myfile.txt";
> t = readfile("myfile.txt");
> t;
Hello world
```

Example 2:

```
> verbosity=1!;
> readfile("afile.txt");
Warning: the file "afile.txt" could not be opened for reading.
Warning: at least one of the given expressions or a subexpression is not correctly typed
or its evaluation has failed because of some error on a side-effect.
error
```

See also: **parse** (8.117), **execute** (8.55), **write** (8.191), **print** (8.130), **bashexecute** (8.17), **error** (8.53)

### 8.145 readxml

Name: **readxml**

reads an expression written as a MathML-Content-Tree in a file

Library name:

`sollya_obj_t sollya_lib_readxml(sollya_obj_t)`

Usage:

**readxml**(*filename*) : string → function | error

Parameters:

- *filename* represents a character sequence indicating a file name

Description:

- **readxml**(*filename*) reads the first occurrence of a lambda application with one bounded variable on applications of the supported basic functions in file *filename* and returns it as a **Sollya** functional expression.

If the file *filename* does not contain a valid MathML-Content tree, **readxml** tries to find an "annotation encoding" markup of type "sollya/text". If this annotation contains a character sequence that can be parsed by **parse**, **readxml** returns that expression. Otherwise **readxml** displays a warning and returns an **error** variable of type **error**.

Example 1:

```
> readxml("readxmlexample.xml");  
2 + _x_ + exp(sin(_x_))
```

See also: **printxml** (8.134), **readfile** (8.144), **parse** (8.117), **error** (8.53)

### 8.146 relative

Name: **relative**

indicates a relative error for **externalplot**, **fpminimax** or **supnorm**

Library names:

`sollya_obj_t sollya_lib_relative()  
int sollya_lib_is_relative(sollya_obj_t)`

Usage:

**relative** : absolute|relative

Description:

- The use of **relative** in the command **externalplot** indicates that during plotting in **externalplot** a relative error is to be considered.  
See **externalplot** for details.
- Used with **fpminimax**, **relative** indicates that **fpminimax** must try to minimize the relative error.  
See **fpminimax** for details.
- When given in argument to **supnorm**, **relative** indicates that a relative error is to be considered for supremum norm computation.  
See **supnorm** for details.

Example 1:



```

> bashexecute("gcc -fPIC -c externalplotexample.c");
> bashexecute("gcc -shared -o externalplotexample externalplotexample.o -lgmp -l
mpfr");
> externalplot("./externalplotexample",absolute,exp(x),[-1/2;1/2],12,perturb);

```

See also: **externalplot** (8.60), **fpminimax** (8.68), **absolute** (8.2), **bashexecute** (8.17), **supnorm** (8.173)

## 8.147 remez

Name: **remez**

computes the minimax of a function on an interval.

Library names:

```

sollya_obj_t sollya_lib_remez(sollya_obj_t, sollya_obj_t, sollya_obj_t, ...)
sollya_obj_t sollya_lib_v_remez(sollya_obj_t, sollya_obj_t, sollya_obj_t,
                                va_list)

```

Usage:

**remez**( $f$ ,  $n$ ,  $range$ ,  $w$ ,  $quality$ ,  $bounds$ ) : (function, integer, range, function, constant, range)  $\rightarrow$  function  
**remez**( $f$ ,  $L$ ,  $range$ ,  $w$ ,  $quality$ ,  $bounds$ ) : (function, list, range, function, constant, range)  $\rightarrow$  function

Parameters:

- $f$  is the function to be approximated
- $n$  is the degree of the polynomial that must approximate  $f$
- $L$  is a list of integers or a list of functions and indicates the basis for the approximation of  $f$
- $range$  is the interval where the function must be approximated
- $w$  (optional) is a weight function. Default is 1.
- $quality$  (optional) is a parameter that controls the quality of the returned polynomial  $p$ , with respect to the exact minimax  $p^*$ . Default is  $1e-5$ .
- $bounds$  (optional) is a parameter that allows the user to make the algorithm stop earlier, whenever a given accuracy is reached or a given accuracy is proved unreachable. Default is  $[0, +\infty]$ .

Description:

- **remez** computes an approximation of the function  $f$  with respect to the weight function  $w$  on the interval  $range$ . More precisely, it searches  $p$  such that  $\|pw - f\|_\infty$  is (almost) minimal among all  $p$  of a certain form. The norm is the infinity norm, e.g.  $\|g\|_\infty = \max\{|g(x)|, x \in range\}$ .
- If  $w = 1$  (the default case), it consists in searching the best polynomial approximation of  $f$  with respect to the absolute error. If  $f = 1$  and  $w$  is of the form  $1/g$ , it consists in searching the best polynomial approximation of  $g$  with respect to the relative error.
- If  $n$  is given,  $p$  is searched among the polynomials with degree not greater than  $n$ . If  $L$  is given and is a list of integers,  $p$  is searched as a linear combination of monomials  $X^k$  where  $k$  belongs to  $L$ . In the case when  $L$  is a list of integers, it may contain ellipses but cannot be end-elliptic. If  $L$  is given and is a list of functions  $g_k$ ,  $p$  is searched as a linear combination of the  $g_k$ . In that case  $L$  cannot contain ellipses. It is the user responsibility to check that the  $g_k$  are linearly independent over the interval  $range$ . Moreover, the functions  $w \cdot g_k$  must be at least twice differentiable over  $range$ . If these conditions are not fulfilled, the algorithm might fail or even silently return a result as if it successfully found the minimax, though the returned  $p$  is not optimal.

- The polynomial is obtained by a convergent iteration called Remez' algorithm (and an extension of this algorithm, due to Stiefel). The algorithm computes a sequence  $p_1, \dots, p_k, \dots$  such that  $e_k = \|p_k w - f\|_\infty$  converges towards the optimal value  $e$ . The algorithm is stopped when the relative error between  $e_k$  and  $e$  is less than *quality*.
- The optional argument *bounds* is an interval  $[\varepsilon_\ell, \varepsilon_u]$  with the following behavior:
  - if, during the algorithm, we manage to prove that  $\varepsilon_u$  is unreachable, we stop the algorithm returning the last computed polynomial.
  - if, during the algorithm, we obtain a polynomial with an error smaller than  $\varepsilon_\ell$ , we stop the algorithm returning that polynomial.
  - otherwise we loop until we find an optimal polynomial with the required quality, as usual.

Examples of use:

$[0, +\infty]$  (compute the optimal polynomial with the required quality)

$[\varepsilon_u]$  (stops as soon as a polynomial achieving  $\varepsilon_u$  is obtained or as soon as such a polynomial is proved not to exist).

$[0, \varepsilon_u]$  (finds the optimal polynomial, but provided that its error is smaller than  $\varepsilon_u$ ).

$[\varepsilon_\ell, +\infty]$  (stops as soon as a polynomial achieving  $\varepsilon_\ell$  is obtained. If such a polynomial does not exist, returns the optimal polynomial).

Example 1:

```
> p = remez(exp(x),5,[0;1]);
> degree(p);
5
> dirtyinfnorm(p-exp(x),[0;1]);
1.12956981510961487071711938292660776072226345893629e-6
```

Example 2:

```
> p = remez(1,[|0,2,4,6,8|],[0,Pi/4],1/cos(x));
> canonical=on!;
> p;
0.99999999994393732180959690352543887130348096061124 + -0.49999999957155685776877
20530637215446709494672222587 * x^2 + 4.1666613233473633009941059480570275870113
220089059e-2 * x^4 + -1.3886529147145693651355523880319714051047635695061e-3 * x
^6 + 2.4372679177224179934800328511009205218114284220126e-5 * x^8
```

Example 3:

```
> p1 = remez(exp(x),5,[0;1],default,1e-5);
> p2 = remez(exp(x),5,[0;1],default,1e-10);
> p3 = remez(exp(x),5,[0;1],default,1e-15);
> dirtyinfnorm(p1-exp(x),[0;1]);
1.12956981510961487071711938292660776072226345893629e-6
> dirtyinfnorm(p2-exp(x),[0;1]);
1.12956980227478675612619255125474525171079325793124e-6
> dirtyinfnorm(p3-exp(x),[0;1]);
1.12956980227478675612619255125474525171079325793124e-6
```

Example 4:

```

> L = [exp(x), sin(x), cos(x)-1, sin(x^3)]];
> g = (2^x-1)/x;
> p1 = remez(g, L, [-1/16;1/16]);
> p2 = remez(g, 3, [-1/16;1/16]);
> dirtyinfnorm(p1 - g, [-1/16;1/16]);
9.8841323829271038137685646777951687620288462194745e-8
> dirtyinfnorm(p2 - g, [-1/16;1/16]);
2.54337800593461418356437401152248866818783932027105e-9

```

Example 5:

```

> f = sin(x);
> I = [-3b-5;-1b-1074];
> time(popt = remez(1, [|1, 3, 4, 5, 7, 8, 9|], I, 1/f));
0.574958000000000000010667855487866972907795570790767
> time(p1 = remez(1, [|1, 3, 4, 5, 7, 8, 9|], I, 1/f, default, [0, 1b-73]));
0.3666929999999999999960649532670942107870359905064106
> time(p2 = remez(1, [|1, 3, 4, 5, 7, 8, 9|], I, 1/f, default, [3b-72, +@Inf@]));
;
0.4501669999999999999982659704134135836284258402884007
> dirtyinfnorm(popt/f-1, I);
2.06750931454112835098093903810531156576504665659064e-22
> dirtyinfnorm(p1/f-1, I);
2.49711266837493110470637913808914046704452778960875e-22
> dirtyinfnorm(p2/f-1, I);
5.4567247553615435246376977231253834265248756996947e-22
> 1b-73;
1.05879118406787542383540312584955245256423950195312e-22
> 3b-72;
6.3527471044072525430124187550973147153854370117187e-22

```

See also: **dirtyinfnorm** (8.42), **infnorm** (8.86), **fpminimax** (8.68), **guessdegree** (8.74), **taylorform** (8.179), **taylor** (8.178)

## 8.148 rename

Name: **rename**

rename the free variable.

Library name:

```
void sollya_lib_name_free_variable(const char *)
```

Usage:

```
rename(ident1,ident2) : void
```

Parameters:

- *ident1* is the current name of the free variable.
- *ident2* is a fresh name.

Description:

- **rename** permits a change of the name of the free variable. **Sollya** can handle only one free variable at a time. The first time in a session that an unbound name is used in a context where it can be interpreted as a free variable, the name is used to represent the free variable of **Sollya**. In the following, this name can be changed using **rename**.

- Be careful: if *ident2* has been set before, its value will be lost. Use the command **isbound** to know if *ident2* is already used or not.
- If *ident1* is not the current name of the free variable, an error occurs.
- If **rename** is used at a time when the name of the free variable has not been defined, *ident1* is just ignored and the name of the free variable is set to *ident2*.
- It is always possible to use the special keyword `_x_` to denote the free variable. Hence *ident1* can be `_x_`.

Example 1:

```
> f=sin(x);
> f;
sin(x)
> rename(x,y);
> f;
sin(y)
```

Example 2:

```
> a=1;
> f=sin(x);
> rename(x,a);
> a;
a
> f;
sin(a)
```

Example 3:

```
> verbosity=1!;
> f=sin(x);
> rename(y, z);
Warning: the current free variable is named "x" and not "y". Can only rename the
free variable.
The last command will have no effect.
> rename(_x_, z);
Information: the free variable has been renamed from "x" to "z".
```

Example 4:

```
> verbosity=1!;
> rename(x,y);
Information: the free variable has been named "y".
> isbound(x);
false
> isbound(y);
true
```

See also: **isbound** (8.89)

## 8.149 restart

Name: **restart**

brings Sollya back to its initial state

Usage:

**restart** : void → void

Description:

- The command **restart** brings **Sollya** back to its initial state. All current state is abandoned, all libraries unbound and all memory freed.

The **restart** command has no effect when executed inside a **Sollya** script read into a main **Sollya** script using **execute**. It is executed in a **Sollya** script included by a **#include** macro.

Using the **restart** command in nested elements of imperative programming like for or while loops is possible. Since in most cases abandoning the current state of **Sollya** means altering a loop invariant, warnings for the impossibility of continuing a loop may follow unless the state is rebuilt.

Example 1:

```
> print(exp(x));
exp(x)
> a = 3;
> restart;
The tool has been restarted.
> print(x);
x
> a;
Warning: the identifier "a" is neither assigned to, nor bound to a library function nor external procedure, nor equal to the current free variable.
Will interpret "a" as "x".
x
```

Example 2:

```
> print(exp(x));
exp(x)
> for i from 1 to 10 do {
    print(i);
    if (i == 5) then restart;
};
1
2
3
4
5
The tool has been restarted.
Warning: the tool has been restarted inside a for loop.
The for loop will no longer be executed.
```

Example 3:

```

> print(exp(x));
exp(x)
> a = 3;
> for i from 1 to 10 do {
    print(i);
    if (i == 5) then {
        restart;
        i = 7;
    };
};
1
2
3
4
5
The tool has been restarted.
8
9
10
> print(x);
x
> a;
Warning: the identifier "a" is neither assigned to, nor bound to a library function nor external procedure, nor equal to the current free variable.
Will interpret "a" as "x".
x

```

See also: **quit** (8.139), **execute** (8.55)

## 8.150 **return**

Name: **return**

indicates an expression to be returned in a procedure

Usage:

**return** *expression* : void

Parameters:

- *expression* represents the expression to be returned

Description:

- The keyword **return** allows for returning the (evaluated) expression *expression* at the end of a begin-end-block (-block) used as a **Sollya** procedure body. See **proc** for further details concerning **Sollya** procedure definitions.

Statements for returning expressions using **return** are only possible at the end of a begin-end-block used as a **Sollya** procedure body. Only one **return** statement can be given per begin-end-block.

- If at the end of a procedure definition using **proc** no **return** statement is given, a **return void** statement is implicitly added. Procedures, i.e. procedure objects, when printed out in **Sollya** defined with an implicit **return void** statement are displayed with this statement explicitly given.

Example 1:

```

> succ = proc(n) { var res; res := n + 1; return res; };
> succ(5);
6
> succ;
proc(n)
{
var res;
res := (n) + (1);
return res;
}

```

Example 2:

```

> hey = proc(s) { print("Hello",s); };
> hey("world");
Hello world
> hey;
proc(s)
{
print("Hello", s);
return void;
}

```

See also: **proc** (8.135), **void** (8.189)

## 8.151 revert

Name: **revert**

reverts a list.

Library name:

`sollya_obj_t sollya_lib_revert(sollya_obj_t)`

Usage:

**revert**( $L$ ) : list  $\rightarrow$  list

Parameters:

- $L$  is a list.

Description:

- **revert**( $L$ ) returns the same list, but with its elements in reverse order.
- If  $L$  is an end-elliptic list, **revert** will fail with an error.

Example 1:

```

> revert([| |]);
[| |]

```

Example 2:

```

> revert([|2,3,5,2,1,4|]);
[|4, 1, 2, 5, 3, 2|]

```

## 8.152 RN

Name: **RN**

constant representing rounding-to-nearest mode.

Library names:

```
sollya_obj_t sollya_lib_round_to_nearest()  
int sollya_lib_is_round_to_nearest(sollya_obj_t)
```

Description:

- **RN** is used in command **round** to specify that the value must be rounded to the nearest representable floating-point number.

Example 1:

```
> display=binary!;  
> round(Pi,20,RN);  
1.100100100001111111_2 * 2^(1)
```

See also: **RD** (8.143), **RU** (8.157), **RZ** (8.158), **round** (8.153), **nearestint** (8.108)

## 8.153 round

Name: **round**

rounds a number to a floating-point format.

Library name:

```
sollya_obj_t sollya_lib_round(sollya_obj_t, sollya_obj_t, sollya_obj_t)
```

Usage:

**round**( $x, n, mode$ ) : (constant, integer, RN|RZ|RU|RD) → constant  
**round**( $x, format, mode$ ) : (constant,  
HP|halfprecision|SG|single|D|double|DE|doubleextended|DD|doubledouble|QD|quad|TD|tripleddouble,  
RN|RZ|RU|RD) → constant

Parameters:

- $x$  is a constant to be rounded.
- $n$  is the precision of the target format.
- $format$  is the name of a supported floating-point format.
- $mode$  is the desired rounding mode.

Description:

- If used with an integer parameter  $n$ , **round**( $x, n, mode$ ) rounds  $x$  to a floating-point number with precision  $n$ , according to rounding-mode  $mode$ .
- If used with a format parameter  $format$ , **round**( $x, format, mode$ ) rounds  $x$  to a floating-point number in the floating-point format  $format$ , according to rounding-mode  $mode$ .
- Subnormal numbers are handled for the case when  $format$  is one of **halfprecision**, **single**, **double**, **doubleextended**, **doubledouble**, **quad** or **tripleddouble**. Otherwise, when  $format$  is an integer, **round** does not take any exponent range into consideration, i.e. typically uses the full exponent range of the underlying MPFR library.

Example 1:

```
> display=binary!;  
> round(Pi,20,RN);  
1.100100100001111111_2 * 2^(1)
```



Example 2:

```
> printdouble(round(exp(17),53,RU));  
0x417709348c0ea4f9  
> printdouble(D(exp(17)));  
0x417709348c0ea4f9
```

Example 3:

```
> display=binary!;  
> a=2^(-1100);  
> round(a,53,RN);  
1_2 * 2^(-1100)  
> round(a,D,RN);  
0  
> double(a);  
0
```

See also: **RN** (8.152), **RD** (8.143), **RU** (8.157), **RZ** (8.158), **halfprecision** (8.75), **single** (8.165), **double** (8.46), **doubleextended** (8.48), **doubledouble** (8.47), **quad** (8.138), **tripledouble** (8.184), **roundcoefficients** (8.154), **roundcorrectly** (8.155), **printdouble** (8.131), **printsingl** (8.133), **ceil** (8.22), **floor** (8.67), **nearestint** (8.108)

## 8.154 roundcoefficients

Name: **roundcoefficients**

rounds the coefficients of a polynomial to classical formats.

Library name:

`sollya_obj_t sollya_lib_roundcoefficients(sollya_obj_t, sollya_obj_t)`

Usage:

**roundcoefficients**( $p,L$ ) : (function, list)  $\rightarrow$  function

Parameters:

- $p$  is a function. Usually a polynomial.
- $L$  is a list of formats.

Description:

- If  $p$  is a polynomial and  $L$  a list of floating-point formats, **roundcoefficients**( $p,L$ ) rounds each coefficient of  $p$  to the corresponding format in  $L$ .
- If  $p$  is not a polynomial, **roundcoefficients** does not do anything.
- If  $L$  contains other elements than **HP**, **halfprecision**, **SG**, **single**, **D**, **double**, **DE**, **doubleextended**, **DD**, **doubledouble**, **QD**, **quad**, **TD** and **tripledouble**, an error occurs.
- The coefficients in  $p$  corresponding to  $X^i$  is rounded to the format  $L[i]$ . If  $L$  does not contain enough elements (e.g. if  $\text{length}(L) < \text{degree}(p)+1$ ), a warning is displayed. However, the coefficients corresponding to an element of  $L$  are rounded. The trailing coefficients (that do not have a corresponding element in  $L$ ) are kept with their own precision. If  $L$  contains too much elements, the trailing useless elements are ignored. In particular  $L$  may be end-elliptic in which case **roundcoefficients** has the natural behavior.

Example 1:

```

> p=exp(1) + x*(exp(2) + x*exp(3));
> display=binary!;
> roundcoefficients(p,[|DD,D,D|]);
1.01011011111000010101000101100010100010101110110100101010011010101111101110
001010110001000000010011101_2 * 2^(1) + x * (1.110110001110011001001011100011010
100110111011010111_2 * 2^(2) + x * (1.01000001010111100101101111110110111101100
010000011_2 * 2^(4)))
> roundcoefficients(p,[|DD,D...|]);
1.010110111111000010101000101100010100010101110110100101010011010101111101110
001010110001000000010011101_2 * 2^(1) + x * (1.110110001110011001001011100011010
100110111011010111_2 * 2^(2) + x * (1.01000001010111100101101111110110111101100
010000011_2 * 2^(4)))

```

Example 2:

```

> f=sin(exp(1)*x);
> display=binary!;
> f;
sin(x * (1.010110111111000010101000101100010100010101110110100101010011010101011
11110111000101011000100000001001110011110100111100111100011101100010111001110001
01100000111101_2 * 2^(1)))
> roundcoefficients(f,[|D...|]);
sin(x * (1.010110111111000010101000101100010100010101110110100101010011010101011
11110111000101011000100000001001110011110100111100111100011101100010111001110001
01100000111101_2 * 2^(1)))

```

Example 3:

```

> p=exp(1) + x*(exp(2) + x*exp(3));
> verbosity=1!;
> display=binary!;
> roundcoefficients(p,[|DD,D|]);
Warning: the number of the given formats does not correspond to the degree of th
e given polynomial.
Warning: the 0th coefficient of the given polynomial does not evaluate to a floa
ting-point constant without any rounding.
Will evaluate the coefficient in the current precision in floating-point before
rounding to the target format.
Warning: the 1th coefficient of the given polynomial does not evaluate to a floa
ting-point constant without any rounding.
Will evaluate the coefficient in the current precision in floating-point before
rounding to the target format.
Warning: rounding may have happened.
1.010110111111000010101000101100010100010101110110100101010011010101111101110
001010110001000000010011101_2 * 2^(1) + x * (1.110110001110011001001011100011010
100110111011010111_2 * 2^(2) + x * (1.01000001010111100101101111110110111101100
010000010111110010110101001011101111110001010011011101000100110000111010001110
010000010110000101100000111001011100101001_2 * 2^(4)))

```

See also: **halfprecision** (8.75), **single** (8.165), **double** (8.46), **doubleextended** (8.48), **doubledouble** (8.47), **quad** (8.138), **tripledouble** (8.184), **fpminimax** (8.68), **remez** (8.147), **implementpoly** (8.83), **subpoly** (8.170)

## 8.155 roundcorrectly

Name: **roundcorrectly**

rounds an approximation range correctly to some precision

Library name:

```
sollya_obj_t sollya_lib_roundcorrectly(sollya_obj_t)
```

Usage:

**roundcorrectly**(*range*) : *range* → constant

Parameters:

- *range* represents a range in which an exact value lies

Description:

- Let *range* be a range of values, determined by some approximation process, safely bounding an unknown value *v*. The command **roundcorrectly**(*range*) determines a precision such that for this precision, rounding to the nearest any value in *range* yields to the same result, i.e. to the correct rounding of *v*.

If no such precision exists, a warning is displayed and **roundcorrectly** evaluates to NaN.

Example 1:

```
> printbinary(roundcorrectly([1.010001_2; 1.0101_2]));  
1.01_2  
> printbinary(roundcorrectly([1.000001_2; 1.001_2]));  
1_2
```

Example 2:

```
> roundcorrectly([-1; 1]);  
@NaN@
```

See also: **round** (8.153), **mantissa** (8.101), **exponent** (8.59), **precision** (8.128)

## 8.156 roundingwarnings

Name: **roundingwarnings**

global variable controlling whether or not a warning is displayed when roundings occur.

Library names:

```
void sollya_lib_set_roundingwarnings_and_print(sollya_obj_t)  
void sollya_lib_set_roundingwarnings(sollya_obj_t)  
sollya_obj_t sollya_lib_get_roundingwarnings()
```

Usage:

**roundingwarnings** = *activation value* : on|off → void  
**roundingwarnings** = *activation value* ! : on|off → void  
**roundingwarnings** : on|off

Parameters:

- *activation value* controls if warnings should be shown or not

Description:

- **roundingwarnings** is a global variable. When its value is **on**, warnings are emitted in appropriate verbosity modes (see **verbosity**) when roundings occur. When its value is **off**, these warnings are suppressed.
- This mode depends on a verbosity of at least 1. See **verbosity** for more details.
- Default is **on** when the standard input is a terminal and **off** when Sollya input is read from a file.

Example 1:

```
> verbosity=1!;
> roundingwarnings = on;
Rounding warning mode has been activated.
> exp(0.1);
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 165 bits.
If safe computation is needed, try to increase the precision.
Warning: rounding has happened. The value displayed is a faithful rounding to 16
5 bits of the true result.
1.1051709180756476248117078264902466682245471947375
> roundingwarnings = off;
Rounding warning mode has been deactivated.
> exp(0.1);
1.1051709180756476248117078264902466682245471947375
```

See also: **on** (8.115), **off** (8.114), **verbosity** (8.188), **midpointmode** (8.104), **rationalmode** (8.142), **suppressmessage** (8.174), **unsuppressmessage** (8.186), **showmessagenumbers** (8.161), **getsuppressedmessages** (8.72)

## 8.157 RU

Name: **RU**

constant representing rounding-upwards mode.

Library names:

```
sollya_obj_t sollya_lib_round_up()
int sollya_lib_is_round_up(sollya_obj_t)
```

Description:

- **RU** is used in command **round** to specify that the value  $x$  must be rounded to the smallest floating-point number  $y$  such that  $x \leq y$ .

Example 1:

```
> display=binary!;
> round(Pi,20,RU);
1.100100100001111111_2 * 2^(1)
```

See also: **RZ** (8.158), **RD** (8.143), **RN** (8.152), **round** (8.153), **ceil** (8.22)

## 8.158 RZ

Name: **RZ**

constant representing rounding-to-zero mode.

Library names:

```
sollya_obj_t sollya_lib_round_towards_zero()
int sollya_lib_is_round_towards_zero(sollya_obj_t)
```

Description:

- **RZ** is used in command **round** to specify that the value must be rounded to the closest floating-point number towards zero. It just consists in truncating the value to the desired format.

Example 1:

```

> display=binary!;
> round(Pi,20,RZ);
1.1001001000011111101_2 * 2^(1)

```

See also: **RD** (8.143), **RU** (8.157), **RN** (8.152), **round** (8.153), **floor** (8.67), **ceil** (8.22)

## 8.159 searchgal

Name: **searchgal**

searches for a preimage of a function such that the rounding the image yields an error smaller than a constant

Library name:

```

sollya_obj_t sollya_lib_searchgal(sollya_obj_t, sollya_obj_t, sollya_obj_t,
                                sollya_obj_t, sollya_obj_t, sollya_obj_t)

```

Usage:

**searchgal**(*function*, *start*, *preimage precision*, *steps*, *format*, *error bound*) : (function, constant, integer, integer, HP|halfprecision|SG|single|D|double|DE|doubleextended|DD|doubledouble|QD|quad|TD|tripledouble, constant) → list

**searchgal**(*list of functions*, *start*, *preimage precision*, *steps*, *list of format*, *list of error bounds*) : (list, constant, integer, integer, list, list) → list

Parameters:

- *function* represents the function to be considered
- *start* represents a value around which the search is to be performed
- *preimage precision* represents the precision (discretization) for the eligible preimage values
- *steps* represents the binary logarithm ( $\log_2$ ) of the number of search steps to be performed
- *format* represents the format the image of the function is to be rounded to
- *error bound* represents a upper bound on the relative rounding error when rounding the image
- *list of functions* represents the functions to be considered
- *list of formats* represents the respective formats the images of the functions are to be rounded to
- *list of error bounds* represents a upper bound on the relative rounding error when rounding the image

Description:

- The command **searchgal** searches for a preimage  $z$  of function *function* or a list of functions *list of functions* such that  $z$  is a floating-point number with *preimage precision* significant mantissa bits and the image  $y$  of the function, respectively each image  $y_i$  of the functions, rounds to format *format* respectively to the corresponding format in *list of format* with a relative rounding error less than *error bound* respectively the corresponding value in *list of error bounds*. During this search, at most  $2^{\text{steps}}$  attempts are made. The search starts with a preimage value equal to *start*. This value is then increased and decreased by 1 ulp in precision *preimage precision* until a value is found or the step limit is reached.

If the search finds an appropriate preimage  $z$ , **searchgal** evaluates to a list containing this value. Otherwise, **searchgal** evaluates to an empty list.

Example 1:

```
> searchgal(log(x),2,53,15,DD,1b-112);
[| |]
> searchgal(log(x),2,53,18,DD,1b-112);
[|2.0000000000384972054234822280704975128173828125|]
```

Example 2:

```
> f = exp(x);
> s = searchgal(f,2,53,18,DD,1b-112);
> if (s != [| |]) then {
    v = s[0];
    print("The rounding error is 2^(",evaluate(log2(abs(DD(f)/f - 1)),v),")");
} else print("No value found");
The rounding error is 2^(-1.12106878438809380148206984258358542322113874177832e
2 )
```

Example 3:

```
> searchgal([|sin(x),cos(x)|],1,53,15,[|D,D|],[|1b-62,1b-60|]);
[|1.00000000000159494639717649988597258925437927246094|]
```

See also: **round** (8.153), **double** (8.46), **doubledouble** (8.47), **tripledouble** (8.184), **evaluate** (8.54), **worstcase** (8.190)

## 8.160 SG

Name: **SG**

short form for **single**

See also: **single** (8.165)

## 8.161 showmessagenumbers

Name: **showmessagenumbers**

activates, deactivates or inspects the state variable controlling the displaying of numbers for messages

Library names:

```
void sollya_lib_set_showmessagenumbers_and_print(sollya_obj_t)
void sollya_lib_set_showmessagenumbers(sollya_obj_t)
sollya_obj_t sollya_lib_get_showmessagenumbers()
```

Usage:

```
showmessagenumbers = activation value : on|off → void
showmessagenumbers = activation value ! : on|off → void
showmessagenumbers : on|off
```

Parameters:

- *activation value* represents **on** or **off**, i.e. activation or deactivation

Description:

- An assignment **showmessagenumbers** = *activation value*, where *activation value* is one of **on** or **off**, activates respectively deactivates the displaying of numbers for warning and information messages. Every Sollya warning or information message (that is not fatal to the tool's execution) has a message number. By default, these numbers are not displayed when a message is output. When message number displaying is activated, the message numbers are displayed together with the message. This allows the user to recover the number of a particular message in order to suppress resp. unsuppress the displaying of this particular message (see **suppressmessage** and **unsuppressmessage**).

- The user should be aware of the fact that message number display activation resp. deactivation through **showmessagenumbers** does not affect message displaying in general. For instance, even with message number displaying activated, messages with only displayed when general verbosity and rounding warning mode are set accordingly.
- If the assignment **showmessagenumbers** = *activation value* is followed by an exclamation mark, no message indicating the new state is displayed. Otherwise the user is informed of the new state of the global mode by an indication.

Example 1:

```
> verbosity = 1;
The verbosity level has been set to 1.
> 0.1;
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 165 bits.
If safe computation is needed, try to increase the precision.
0.1
> showmessagenumbers = on;
Displaying of message numbers has been activated.
> 0.1;
Warning (174): Rounding occurred when converting the constant "0.1" to floating-
point with 165 bits.
If safe computation is needed, try to increase the precision.
0.1
> showmessagenumbers;
on
> showmessagenumbers = off!;
> 0.1;
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 165 bits.
If safe computation is needed, try to increase the precision.
0.1
```

Example 2:

```

> showmessagenumbers = on;
Displaying of message numbers has been activated.
> verbosity = 1;
The verbosity level has been set to 1.
> diff(0.1 * x + 1.5 * x^2);
Warning (174): Rounding occurred when converting the constant "0.1" to floating-
point with 165 bits.
If safe computation is needed, try to increase the precision.
0.1 + 3 * x
> verbosity = 0;
The verbosity level has been set to 0.
> diff(0.1 * x + 1.5 * x^2);
0.1 + 3 * x
> verbosity = 12;
The verbosity level has been set to 12.
> diff(0.1 * x + 1.5 * x^2);
Warning (174): Rounding occurred when converting the constant "0.1" to floating-
point with 165 bits.
If safe computation is needed, try to increase the precision.
Information (196): formally differentiating a function.
Information (197): differentiating the expression '0.1 * x + 1.5 * x^2'
Information (205): extraction of coefficient terms from a polynomial uses a spec
ial algorithm for canonical forms.
Information (195): expression '0.1 + 2 * 1.5 * x' has been simplified to express
ion '0.1 + 3 * x'.
Information (207): no Horner simplification will be performed because the given
tree is already in Horner form.
0.1 + 3 * x

```

See also: `getsuppressedmessages` (8.72), `suppressmessage` (8.174), `unsuppressmessage` (8.186), `verbosity` (8.188), `roundingwarnings` (8.156)

## 8.162 simplify

Name: **simplify**

simplifies an expression representing a function

Library name:

`sollya_obj_t sollya_lib_simplify(sollya_obj_t)`

Usage:

**simplify**(*function*) : function → function

Parameters:

- *function* represents the expression to be simplified

Description:

- The command **simplify** simplifies constant subexpressions of the expression given in argument representing the function *function*. Those constant subexpressions are evaluated using floating-point arithmetic with the global precision **prec**.

Example 1:

```

> print(simplify(sin(pi * x)));
sin(3.14159265358979323846264338327950288419716939937508 * x)
> print(simplify(erf(exp(3) + x * log(4))));
erf(2.00855369231876677409285296545817178969879078385544e1 + x * 1.3862943611198
906188344642429163531361510002687205)

```



Example 2:

```
> prec = 20!;  
> t = erf(0.5);  
> s = simplify(erf(0.5));  
> prec = 200!;  
> t;  
0.5204998778130465376827466538919645287364515757579637000588058  
> s;  
0.52050018310546875
```

See also: **simplifysafe** (8.163), **autosimplify** (8.15), **prec** (8.127), **evaluate** (8.54), **horner** (8.80), **rationalmode** (8.142)

## 8.163 simplifysafe

Name: **simplifysafe**

simplifies an expression representing a function

Library name:

`sollya_obj_t sollya_lib_simplifysafe(sollya_obj_t)`

Usage:

**simplifysafe**(*function*) : function  $\rightarrow$  function

Parameters:

- *function* represents the expression to be simplified

Description:

- The command **simplifysafe** simplifies the expression given in argument representing the function *function*. The command **simplifysafe** does not endanger the safety of computations even in Sollya's floating-point environment: the function returned is mathematically equal to the function *function*.

Remark that the simplification provided by **simplifysafe** is not perfect: they may exist simpler equivalent expressions for expressions returned by **simplifysafe**.

Example 1:

```
> print(simplifysafe((6 + 2) + (5 + exp(0)) * x));  
8 + 6 * x
```

Example 2:

```
> print(simplifysafe((log(x - x + 1) + asin(1))));  
(pi) / 2
```

Example 3:

```
> print(simplifysafe((log(x - x + 1) + asin(1)) - (atan(1) * 2)));  
(pi) / 2 - (pi) / 4 * 2
```

See also: **simplify** (8.162), **autosimplify** (8.15), **rationalmode** (8.142), **horner** (8.80)

## 8.164 sin

Name: **sin**  
the sine function.

Library names:

```
sollya_obj_t sollya_lib_sin(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_sin(sollya_obj_t)
#define SOLLYA_SIN(x) sollya_lib_build_function_sin(x)
```

Description:

- **sin** is the usual sine function.
- It is defined for every real number  $x$ .

See also: **asin** (8.10), **cos** (8.29), **tan** (8.176)

## 8.165 single

Names: **single**, **SG**  
rounding to the nearest IEEE 754 single (binary32).

Library names:

```
sollya_obj_t sollya_lib_single(sollya_obj_t)
sollya_obj_t sollya_lib_single_obj()
int sollya_lib_is_single_obj(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_single(sollya_obj_t)
#define SOLLYA_SG(x) sollya_lib_build_function_single(x)
```

Description:

- **single** is both a function and a constant.
- As a function, it rounds its argument to the nearest IEEE 754 single precision (i.e. IEEE754-2008 binary32) number. Subnormal numbers are supported as well as standard numbers: it is the real rounding described in the standard.
- As a constant, it symbolizes the single precision format. It is used in contexts when a precision format is necessary, e.g. in the commands **round** and **roundcoefficients**. It is not supported for **implementpoly**. See the corresponding help pages for examples.

Example 1:

```
> display=binary!;
> SG(0.1);
1.10011001100110011001101_2 * 2^(-4)
> SG(4.17);
1.000010101110000101001_2 * 2^(2)
> SG(1.011_2 * 2^(-1073));
0
```

See also: **halfprecision** (8.75), **double** (8.46), **doubleextended** (8.48), **doubledouble** (8.47), **quad** (8.138), **tripledouble** (8.184), **roundcoefficients** (8.154), **implementpoly** (8.83), **round** (8.153), **printsingl**e (8.133)

## 8.166 sinh

Name: **sinh**  
the hyperbolic sine function.

Library names:

```

sollya_obj_t sollya_lib_sinh(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_sinh(sollya_obj_t)
#define SOLLYA_SINH(x) sollya_lib_build_function_sinh(x)

```

Description:

- **sinh** is the usual hyperbolic sine function:  $\sinh(x) = \frac{e^x - e^{-x}}{2}$ .
- It is defined for every real number  $x$ .

See also: **asinh** (8.11), **cosh** (8.30), **tanh** (8.177)

## 8.167 sort

Name: **sort**

sorts a list of real numbers.

Library name:

```
sollya_obj_t sollya_lib_sort(sollya_obj_t)
```

Usage:

**sort**( $L$ ) : list  $\rightarrow$  list

Parameters:

- $L$  is a list.

Description:

- If  $L$  contains only constant values, **sort**( $L$ ) returns the same list, but sorted in increasing order.
- If  $L$  contains at least one element that is not a constant, the command fails with a type error.
- If  $L$  is an end-elliptic list, **sort** will fail with an error.

Example 1:

```

> sort([| |]);
[| |]
> sort([|2,3,5,2,1,4|]);
[|1, 2, 2, 3, 4, 5|]

```

## 8.168 sqrt

Name: **sqrt**

square root.

Library names:

```

sollya_obj_t sollya_lib_sqrt(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_sqrt(sollya_obj_t)
#define SOLLYA_SQRT(x) sollya_lib_build_function_sqrt(x)

```

Description:

- **sqrt** is the square root, e.g. the inverse of the function square:  $\sqrt{y}$  is the unique positive  $x$  such that  $x^2 = y$ .
- It is defined only for  $x$  in  $[0; +\infty]$ .

## 8.169 string

Name: **string**

keyword representing a **string** type

Usage:

**string** : type type

Description:

- **string** represents the **string** type for declarations of external procedures by means of **externalproc**.

Remark that in contrast to other indicators, type indicators like **string** cannot be handled outside the **externalproc** context. In particular, they cannot be assigned to variables.

See also: **externalproc** (8.61), **boolean** (8.20), **constant** (8.28), **function** (8.70), **integer** (8.87), **list of** (8.95), **range** (8.140)

## 8.170 subpoly

Name: **subpoly**

restricts the monomial basis of a polynomial to a list of monomials

Library name:

`sollya_obj_t sollya_lib_subpoly(sollya_obj_t, sollya_obj_t)`

Usage:

**subpoly**(*polynomial*, *list*) : (function, list) → function

Parameters:

- *polynomial* represents the polynomial the coefficients are taken from
- *list* represents the list of monomials to be taken

Description:

- **subpoly** extracts the coefficients of a polynomial *polynomial* and builds up a new polynomial out of those coefficients associated to monomial degrees figuring in the list *list*.

If *polynomial* represents a function that is not a polynomial, **subpoly** returns 0.

If *list* is a list that is end-elliptic, let be *j* the last value explicitly specified in the list. All coefficients of the polynomial associated to monomials greater or equal to *j* are taken.

Example 1:

```
> p = taylor(exp(x),5,0);
> s = subpoly(p,[1,3,5]);
> print(p);
1 + x * (1 + x * (0.5 + x * (1 / 6 + x * (1 / 24 + x / 120))))
> print(s);
x * (1 + x^2 * (1 / 6 + x^2 / 120))
```

Example 2:

```
> p = remez(atan(x),10,[-1,1]);
> subpoly(p,[1,3,5...]);
x * (0.99986632941452949026018468446163586361700915018231 + x^2 * (-0.3303047850
2455936362667794059988443130926433421739 + x^2 * (0.1801592931781875646289423703
7824735129130095574422 + x * (2.284558411542478828511250156535857664242985696307
19e-9 + x * (-8.5156349064111377895500552996061844977507560037484e-2 + x * (-2.7
1756340962775019916818769239340943524383018921799e-9 + x * (2.084511343071147293
73239910549169872454686955894998e-2 + x * 1.108898611811290576571996643868266300
81793400489512e-9))))))
```

Example 3:

```
> subpoly(exp(x),[1,2,3]);  
0
```

See also: **roundcoefficients** (8.154), **taylor** (8.178), **remez** (8.147), **fpminimax** (8.68), **implement-poly** (8.83)

## 8.171 substitute

Name: **substitute**

replace the occurrences of the free variable in an expression.

Library name:

```
sollya_obj_t sollya_lib_substitute(sollya_obj_t, sollya_obj_t)
```

Usage:

**substitute**( $f,g$ ) : (function, function)  $\rightarrow$  function  
**substitute**( $f,t$ ) : (function, constant)  $\rightarrow$  constant

Parameters:

- $f$  is a function.
- $g$  is a function.
- $t$  is a real number.

Description:

- **substitute**( $f, g$ ) produces the function  $(f \circ g) : x \mapsto f(g(x))$ .
- **substitute**( $f, t$ ) is the constant  $f(t)$ . Note that the constant is represented by its expression until it has been evaluated (exactly the same way as if you type the expression  $f$  replacing instances of the free variable by  $t$ ).
- If  $f$  is stored in a variable  $F$ , the effect of the commands **substitute**( $F,g$ ) or **substitute**( $F,t$ ) is absolutely equivalent to writing  $F(g)$  resp.  $F(t)$ .

Example 1:

```
> f=sin(x);  
> g=cos(x);  
> substitute(f,g);  
sin(cos(x))  
> f(g);  
sin(cos(x))
```

Example 2:

```
> a=1;  
> f=sin(x);  
> substitute(f,a);  
0.84147098480789650665250232163029899962256306079837  
> f(a);  
0.84147098480789650665250232163029899962256306079837
```

See also: **evaluate** (8.54), **composepolynomials** (8.26)

## 8.172 sup

Name: **sup**

gives the upper bound of an interval.

Library name:

`sollya_obj_t sollya_lib_sup(sollya_obj_t)`

Usage:

$$\begin{aligned}\mathbf{sup}(I) &: \text{range} \rightarrow \text{constant} \\ \mathbf{sup}(x) &: \text{constant} \rightarrow \text{constant}\end{aligned}$$

Parameters:

- $I$  is an interval.
- $x$  is a real number.

Description:

- Returns the upper bound of the interval  $I$ . Each bound of an interval has its own precision, so this command is exact, even if the current precision is too small to represent the bound.
- When called on a real number  $x$ , **sup** behaves like the identity.

Example 1:

```
> sup([1;3]);
3
> sup(5);
5
```

Example 2:

```
> display=binary!;
> I=[0; 0.111110000011111_2];
> sup(I);
1.11110000011111_2 * 2^(-1)
> prec=12!;
> sup(I);
1.11110000011111_2 * 2^(-1)
```

See also: **inf** (8.85), **mid** (8.103), **max** (8.102), **min** (8.105)

## 8.173 supnorm

Name: **supnorm**

computes an interval bounding the supremum norm of an approximation error (absolute or relative) between a given polynomial and a function.

Library name:

`sollya_obj_t sollya_lib_supnorm(sollya_obj_t, sollya_obj_t, sollya_obj_t, sollya_obj_t, sollya_obj_t)`

Usage:

**supnorm**( $p, f, I, errorType, accuracy$ ) : (function, function, range, absolute|relative, constant)  $\rightarrow$  range

Parameters:

- $p$  is a polynomial.

- $f$  is a function.
- $I$  is an interval.
- $errorType$  is the type of error to be considered: **absolute** or **relative** (see details below).
- $accuracy$  is a constant that controls the relative tightness of the interval returned.

Description:

- **supnorm**( $p, f, I, errorType, accuracy$ ) tries to compute an interval bound  $r = [\ell, u]$  for the supremum norm of the error function  $\varepsilon_{\text{absolute}} = p - f$  (when  $errorType$  evaluates to **absolute**) or  $\varepsilon_{\text{relative}} = p/f - 1$  (when  $errorType$  evaluates to **relative**), over the interval  $I$ , such that  $\sup_{x \in I} \{|\varepsilon(x)|\} \subseteq r$  and  $0 \leq |u - \ell| \leq accuracy$ . If **supnorm** succeeds in computing a suitable interval  $r$ , which it returns, that interval is guaranteed to contain the supremum norm value and to satisfy the required quality. Otherwise, **supnorm** evaluates to **error**, displaying a corresponding error message. These failure cases are rare and basically happen only for functions which are too complicated.
- Roughly speaking, **supnorm** is based on **taylorform** to obtain a higher degree polynomial approximation for  $f$ . This process is coupled with an a posteriori validation of a potential supremum norm upper bound. The validation is based on showing a certain polynomial the problem gets reduced to does not vanish. In cases when this process alone does not succeed, for instance because **taylorform** is unable to compute a sufficiently good approximation to  $f$ , **supnorm** falls back to bisecting the working interval until safe supremum norm bounds can be computed with the required accuracy or until the width of the subintervals becomes less than **diam** times the original interval  $I$ , in which case **supnorm** fails.
- The algorithm used for **supnorm** is quite complex, which makes it impossible to explain it here in further detail. Please find a complete description in the following article:

Sylvain Chevillard, John Harrison, Mioara Joldes, Christoph Lauter  
Efficient and accurate computation of upper bounds of approximation errors  
Journal of Theoretical Computer Science (TCS), 2010  
LIP Research Report number RR LIP2010-2  
<http://prunel.ccsd.cnrs.fr/ensl-00445343/fr/>

- In practical cases, **supnorm** should be able to automatically handle removable discontinuities that relative errors might have. This means that usually, if  $f$  vanishes at a point  $x_0$  in the interval considered, the approximation polynomial  $p$  is designed such that it also vanishes at the same point with a multiplicity large enough. Hence, although  $f$  vanishes,  $\varepsilon_{\text{relative}} = p/f - 1$  may be defined by continuous extension at such points  $x_0$ , called removable discontinuities (see Example 3).

Example 1:

```
> p = remez(exp(x), 5, [-1;1]);
> midpointmode=on!;
> supnorm(p, exp(x), [-1;1], absolute, 2^(-40));
0.45205513967~0/2~e-4
```

Example 2:

```

> prec=200!;
> midpointmode=on!;
> d = [1;2];
> f = exp(cos(x)^2 + 1);
> p = remez(1,15,d,1/f,1e-40);
> theta=1b-60;
> prec=default!;
> mode=relative;
> supnorm(p,f,d,mode,theta);
0.30893006200251428~5/6~e-13

```

Example 3:

```

> midpointmode=on!;
> mode=relative;
> theta=1b-135;
> d = [-1b-2;1b-2];
> f = expm1(x);
> p = x * (1 + x * ( 2097145 * 2^(-22) + x * ( 349527 * 2^(-21) + x * (87609 *
2^(-21) + x * 4369 * 2^(-19)))));
> theta=1b-40;
> supnorm(p,f,d,mode,theta);
0.98349131972~2/3~e-7

```

See also: **dirtyinfnorm** (8.42), **infnorm** (8.86), **checkinfnorm** (8.24), **absolute** (8.2), **relative** (8.146), **taylorform** (8.179), **autodiff** (8.14), **numberroots** (8.112), **diam** (8.38)

## 8.174 suppressmessage

Name: **suppressmessage**

suppresses the displaying of messages with a certain number

Library name:

```
void sollya_lib_suppressmessage(sollya_obj_t, ...);
```

Usage:

```

suppressmessage(msg num 1, ..., msg num n) : (integer, ..., integer) → void
suppressmessage(msg list) : list → void

```

Parameters:

- *msg num 1* thru *msg num n* represent the numbers of *n* messages to be suppressed
- *msg list* represents a list with numbers of messages to be suppressed

Description:

- The **suppressmessage** command allows particular warning and information messages to be suppressed from message output, while maintaining global verbosity levels (see **verbosity**) high. Every **Sollya** warning or information message (that is not fatal to the tool's execution) has a message number. When these message numbers *msg num 1* thru *msg num n* are given to **suppressmessage**, the corresponding message are no longer displayed. The **unsuppressmessage** command reverts this suppression from output for a particular message.
- Instead of giving **suppressmessage** several message numbers *msg num 1* thru *msg num n* or calling **suppressmessage** several times, it is possible to give a whole list *msg list* of message numbers to **suppressmessage**.



- The user should be aware that **suppressmessage** presents sticky behavior for the warning and information messages suppressed from output. This means that even if subsequent calls to **suppressmessage** occur, a message suppressed by a call to **suppressmessage** stays suppressed until it is unsuppressed using **unsuppressmessage** or the tool is restarted. This behavior distinguishes message suppression from other global states of the **Sollya** tool. The user may use **getsuppressedmessages** to obtain a list of currently suppressed messages.
- When **suppressmessage** is used on message numbers that do not exist in the current version of the tool, a warning is displayed. The call has no other effect though.

Example 1:

```
> verbosity = 1;
The verbosity level has been set to 1.
> 0.1;
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 165 bits.
If safe computation is needed, try to increase the precision.
0.1
> suppressmessage(174);
> 0.1;
0.1
> suppressmessage(407);
> 0.1;
0.1
> verbosity = 12;
The verbosity level has been set to 12.
> showmessagenumbers = on;
Displaying of message numbers has been activated.
> diff(exp(x * 0.1));
Information (196): formally differentiating a function.
Information (197): differentiating the expression 'exp(x * 0.1)'
Information (204): extraction of coefficient terms from a polynomial uses a spec
ial algorithm for Horner forms.
Information (207): no Horner simplification will be performed because the given
tree is already in Horner form.
exp(x * 0.1) * 0.1
> suppressmessage(207, 196);
> diff(exp(x * 0.1));
Information (197): differentiating the expression 'exp(x * 0.1)'
Information (204): extraction of coefficient terms from a polynomial uses a spec
ial algorithm for Horner forms.
exp(x * 0.1) * 0.1
> unsuppressmessage(174);
> 0.1;
Warning (174): Rounding occurred when converting the constant "0.1" to floating-
point with 165 bits.
If safe computation is needed, try to increase the precision.
0.1
```

Example 2:

```

> verbosity = 12;
The verbosity level has been set to 12.
> showmessagenumbers = on;
Displaying of message numbers has been activated.
> diff(exp(x * 0.1));
Warning (174): Rounding occurred when converting the constant "0.1" to floating-
point with 165 bits.
If safe computation is needed, try to increase the precision.
Information (196): formally differentiating a function.
Information (197): differentiating the expression 'exp(x * 0.1)'
Information (204): extraction of coefficient terms from a polynomial uses a spec
ial algorithm for Horner forms.
Information (207): no Horner simplification will be performed because the given
tree is already in Horner form.
exp(x * 0.1) * 0.1
> suppressmessage([| 174, 207, 196 |]);
> diff(exp(x * 0.1));
Information (197): differentiating the expression 'exp(x * 0.1)'
Information (204): extraction of coefficient terms from a polynomial uses a spec
ial algorithm for Horner forms.
exp(x * 0.1) * 0.1

```

See also: **getsuppressedmessages** (8.72), **suppressmessage** (8.174), **unsuppressmessage** (8.186), **verbosity** (8.188), **roundingwarnings** (8.156)

## 8.175 tail

Name: **tail**

gives the tail of a list.

Library name:

`sollya_obj_t sollya_lib_tail(sollya_obj_t)`

Usage:

**tail**( $L$ ) : list  $\rightarrow$  list

Parameters:

- $L$  is a list.

Description:

- **tail**( $L$ ) returns the list  $L$  without its first element.
- If  $L$  is empty, the command will fail with an error.
- **tail** can also be used with end-elliptic lists. In this case, the result of **tail** is also an end-elliptic list.

Example 1:

```

> tail([|1,2,3|]);
[|2, 3|]
> tail([|1,2...|]);
[|2...|]

```

See also: **head** (8.76)

## 8.176 tan

Name: **tan**

the tangent function.

Library names:

```
sollya_obj_t sollya_lib_tan(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_tan(sollya_obj_t)
#define SOLLYA_TAN(x) sollya_lib_build_function_tan(x)
```

Description:

- **tan** is the tangent function, defined by  $\tan(x) = \sin(x)/\cos(x)$ .
- It is defined for every real number  $x$  that is not of the form  $n\pi + \pi/2$  where  $n$  is an integer.

See also: **atan** (8.12), **cos** (8.29), **sin** (8.164)

## 8.177 tanh

Name: **tanh**

the hyperbolic tangent function.

Library names:

```
sollya_obj_t sollya_lib_tanh(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_tanh(sollya_obj_t)
#define SOLLYA_TANH(x) sollya_lib_build_function_tanh(x)
```

Description:

- **tanh** is the hyperbolic tangent function, defined by  $\tanh(x) = \sinh(x)/\cosh(x)$ .
- It is defined for every real number  $x$ .

See also: **atanh** (8.13), **cosh** (8.30), **sinh** (8.166)

## 8.178 taylor

Name: **taylor**

computes a Taylor expansion of a function in a point

Library name:

```
sollya_obj_t sollya_lib_taylor(sollya_obj_t, sollya_obj_t, sollya_obj_t)
```

Usage:

**taylor**(*function*, *degree*, *point*) : (function, integer, constant) → function

Parameters:

- *function* represents the function to be expanded
- *degree* represents the degree of the expansion to be delivered
- *point* represents the point in which the function is to be developed

Description:

- The command **taylor** returns an expression that is a Taylor expansion of function *function* in point *point* having the degree *degree*.

Let  $f$  be the function *function*,  $t$  be the point *point* and  $n$  be the degree *degree*. Then, **taylor**(*function*, *degree*, *point*) evaluates to an expression mathematically equal to

$$\sum_{i=0}^n \frac{f^{(i)}(t)}{i!} x^i.$$

In other words, if  $p(x)$  denotes the polynomial returned by **taylor**,  $p(x-t)$  is the Taylor polynomial of degree  $n$  of  $f$  developed at point  $t$ .

Remark that **taylor** evaluates to 0 if the degree *degree* is negative.

Example 1:

```
> print(taylor(exp(x),3,1));
exp(1) + x * (exp(1) + x * (0.5 * exp(1) + x * exp(1) / 6))
```

Example 2:

```
> print(taylor(asin(x),7,0));
x * (1 + x^2 * (1 / 6 + x^2 * (9 / 120 + x^2 * 225 / 5040)))
```

Example 3:

```
> print(taylor(erf(x),6,0));
x * (1 / sqrt((pi) / 4) + x^2 * ((sqrt((pi) / 4) * 4 / (pi) * (-2)) / 6 + x^2 *
(sqrt((pi) / 4) * 4 / (pi) * 12) / 120))
```

See also: **remez** (8.147), **fpminimax** (8.68), **taylorform** (8.179)

## 8.179 taylorform

Name: **taylorform**

computes a rigorous polynomial approximation (polynomial, interval error bound) for a function, based on Taylor expansions.

Library names:

```
sollya_obj_t sollya_lib_taylorform(sollya_obj_t, sollya_obj_t,
                                   sollya_obj_t, ...)
sollya_obj_t sollya_lib_v_taylorform(sollya_obj_t, sollya_obj_t,
                                     sollya_obj_t, va_list)
```

Usage:

```
taylorform( $f, n, x_0, I, errorType$ ) : (function, integer, constant, range, absolute|relative) → list
taylorform( $f, n, x_0, I, errorType$ ) : (function, integer, range, range, absolute|relative) → list
taylorform( $f, n, x_0, errorType$ ) : (function, integer, constant, absolute|relative) → list
taylorform( $f, n, x_0, errorType$ ) : (function, integer, range, absolute|relative) → list
```

Parameters:

- $f$  is the function to be approximated.
- $n$  is the degree of the polynomial that must approximate  $f$ .
- $x_0$  is the point (it can be a real number or an interval) where the Taylor expansion of the function is to be considered.
- $I$  is the interval over which the function is to be approximated. If this parameter is omitted, the behavior is changed (see detailed description below).
- *errorType* (optional) is the type of error to be considered. See the detailed description below. Default is **absolute**.

Description:

- **WARNING:** **taylorform** is a certified command, not difficult to use but not completely straightforward to use either. In order to be sure to use it correctly, the reader is invited to carefully read this documentation entirely.

- **taylorform** computes an approximation polynomial and an interval error bound for function  $f$ . More precisely, it returns a list  $L = [p, \text{coeffErrors}, \Delta]$  where:
  - $p$  is an approximation polynomial of degree  $n$  such that  $p(x - x_0)$  is roughly speaking a numerical Taylor expansion of  $f$  at the point  $x_0$ .
  - $\text{coeffErrors}$  is a list of  $n + 1$  intervals. Each interval  $\text{coeffErrors}[i]$  contains an enclosure of all the errors accumulated when computing the  $i$ -th coefficient of  $p$ .
  - $\Delta$  is an interval that provides a bound for the approximation error between  $p$  and  $f$ . Its significance depends on the *errorType* considered.
- The polynomial  $p$  and the bound  $\Delta$  are obtained using Taylor Models principles.
- Please note that  $x_0$  can be an interval. In general, it is meant to be a small interval approximating a non representable value. If  $x_0$  is given as a constant expression, it is first numerically evaluated (leading to a faithful rounding  $\tilde{x}_0$  at precision **prec**), and it is then replaced by the (exactly representable) point-interval  $[\tilde{x}_0, \tilde{x}_0]$ . In particular, it is not the same to call **taylorform** with  $x_0 = \mathbf{pi}$  and with  $x_0 = [\mathbf{pi}]$ , for instance. In general, if the point around which one desires to compute the polynomial is not exactly representable, one should preferably use a small interval for  $x_0$ .
- More formally, the mathematical property ensured by the algorithm may be stated as follows. For all  $\xi_0$  in  $x_0$ , there exist (small) values  $\varepsilon_i \in \text{coeffErrors}[i]$  such that:
 

If *errorType* is **absolute**,  $\forall x \in I, \exists \delta \in \Delta, f(x) - p(x - \xi_0) = \sum_{i=0}^n \varepsilon_i (x - \xi_0)^i + \delta$ .

If *errorType* is **relative**,  $\forall x \in I, \exists \delta \in \Delta, f(x) - p(x - \xi_0) = \sum_{i=0}^n \varepsilon_i (x - \xi_0)^i + \delta (x - \xi_0)^{n+1}$ .
- It is also possible to use a large interval for  $x_0$ , though it is not obvious to give an intuitive sense to the result of **taylorform** in that case. A particular case that might be interesting is when  $x_0 = I$  in relative mode. In that case, denoting by  $p_i$  the coefficient of  $p$  of order  $i$ , the interval  $p_i + \text{coeffError}[i]$  gives an enclosure of  $f^{(i)}(I)/i!$ . However, the command **autodiff** is more convenient for computing such enclosures.
- When the interval  $I$  is not given, the approximated Taylor polynomial is computed but no remainder is produced. In that case the returned list is  $L = [p, \text{coeffErrors}]$ .
- The relative case is especially useful when functions with removable singularities are considered. In such a case, this routine is able to compute a finite remainder bound, provided that the expansion point given is the problematic removable singularity point.
- The algorithm does not guarantee that by increasing the degree of the approximation, the remainder bound will become smaller. Moreover, it may even become larger due to the dependency phenomenon present with interval arithmetic. In order to reduce this phenomenon, a possible solution is to split the definition domain  $I$  into several smaller intervals.
- The command **taylor** also computes a Taylor polynomial of a function. However it does not provide a bound on the remainder. Besides, **taylor** is a somehow symbolic command: each coefficient of the Taylor polynomial is computed exactly and returned as an expression tree exactly equal to theoretical value. It is henceforth much more inefficient than **taylorform** and **taylorform** should be preferred if only numerical (yet safe) computations are required. The same difference exists between commands **diff** and **autodiff**.

Example 1:



```

> TL1 = taylorform(exp(x), 3, 0, [0,1], relative);
> TL2 = taylorform(exp(x), 3, 0, relative);
> TL1[0]==TL2[0];
true
> TL1[1]==TL2[1];
true
> length(TL1);
3
> length(TL2);
2

```

Example 5:

```

> f = exp(cos(x)); x0 = 0;
> TL = taylorform(f, 3, x0);
> T1 = TL[0];
> T2 = taylor(f, 3, x0);
> print(coeff(T1, 2));
-1.35914091422952261768014373567633124887862354684999
> print(coeff(T2, 2));
0.5 * exp(1)

```

See also: **diff** (8.40), **autodiff** (8.14), **taylor** (8.178), **remez** (8.147)

## 8.180 taylorrecursions

Name: **taylorrecursions**

controls the number of recursion steps when applying Taylor's rule.

Library names:

```

void sollya_lib_set_taylorrecursions_and_print(sollya_obj_t)
void sollya_lib_set_taylorrecursions(sollya_obj_t)
sollya_obj_t sollya_lib_get_taylorrecursions()

```

Usage:

```

taylorrecursions =  $n$  : integer  $\rightarrow$  void
taylorrecursions =  $n$  ! : integer  $\rightarrow$  void
taylorrecursions : integer

```

Parameters:

- $n$  represents the number of recursions

Description:

- **taylorrecursions** is a global variable. Its value represents the number of steps of recursion that are used when applying Taylor's rule. This rule is applied by the interval evaluator present in the core of **Sollya** (and particularly visible in commands like **infnorm**).
- To improve the quality of an interval evaluation of a function  $f$ , in particular when there are problems of decorrelation), the evaluator of **Sollya** uses Taylor's rule:  $f([a, b]) \subseteq f(m) + [a - m, b - m] \cdot f'([a, b])$  where  $m = \frac{a+b}{2}$ . This rule can be applied recursively. The number of step in this recursion process is controlled by **taylorrecursions**.
- Setting **taylorrecursions** to 0 makes **Sollya** use this rule only once; setting it to 1 makes **Sollya** use the rule twice, and so on. In particular: the rule is always applied at least once.

Example 1:

```
> f=exp(x);
> p=remez(f,3,[0;1]);
> taylorrecursions=0;
The number of recursions for Taylor evaluation has been set to 0.
> evaluate(f-p, [0;1]);
[-0.46839364816268368775174657814112460243249079671039;0.46947781754646820647293
019728402934746974652584671]
> taylorrecursions=1;
The number of recursions for Taylor evaluation has been set to 1.
> evaluate(f-p, [0;1]);
[-0.138131114954063839905475752120786856031651747712954;0.1392152843378483586266
5937126369160106890747684927]
```

See also: **hopitalrecursions** (8.79), **evaluate** (8.54), **infnorm** (8.86)

## 8.181 TD

Name: **TD**

short form for **tripledouble**

See also: **tripledouble** (8.184)

## 8.182 time

Name: **time**

procedure for timing Sollya code.

Usage:

$$\mathbf{time}(code) : code \rightarrow \text{constant}$$

Parameters:

- *code* is the code to be timed.

Description:

- **time** permits timing a **Sollya** instruction, resp. a begin-end block of **Sollya** instructions. The timing value, measured in seconds, is returned as a **Sollya** constant (and not merely displayed as for **timing**). This permits performing computations of the timing measurement value inside **Sollya**.
- The extended **nop** command permits executing a defined number of useless instructions. Taking the ratio of the time needed to execute a certain **Sollya** instruction and the time for executing a **nop** therefore gives a way to abstract from the speed of a particular machine when evaluating an algorithm's performance.

Example 1:

```
> t = time(p=remez(sin(x),10,[-1;1]));  
> write(t,"s were spent computing p = ",p,"\n");  
0.213180999999999997262606354908598405017983168364s were spent computing p =  
9.0486898749977990986908851357759191711354777014602e-17 * x^10 + 2.6876259511512  
3596299959320959141640012683406736586e-6 * x^9 + -2.4247978492521313349073232289  
246205727856268698001e-16 * x^8 + -1.9834486302096592970124560650358646122613093  
7598776e-4 * x^7 + 2.2748214757753544349162426281857910162575492126267e-16 * x^6  
+ 8.3333037186560980567697821420813799547276481409702e-3 * x^5 + -8.57471519897  
20669741706961303549531312110511218869e-17 * x^4 + -0.16666666138601323707621656  
6493953847771564552744173 * x^3 + 1.05699558969863875841493332282097022580493449  
058156e-17 * x^2 + 0.9999999973628365676559825181776417246038944720794 * x + (-  
3.12065309566018830243163208536426045628106466008778e-19)
```



Example 2:

```
> write(time({ p=remez(sin(x),10,[ -1;1]); write("The error is 2^(", log2(dirtyin  
fnorm(p-sin(x),[-1;1])), ")\n"); } ), " s were spent\n");  
The error is 2^(log2(2.39601979446524486606649528289933482070294808074097e-11))  
0.39035199999999999999977837172870920312561793252825737 s were spent
```

Example 3:

```
> t = time(basheexecute("sleep 10"));
> write(~(t-10),"s of execution overhead.\n");
2.378000000000000016402434965812062728218734264373779e-3s of execution overhead.
```

Example 4:

```
> ratio := time(p=remez(sin(x),10,[-1;1]))/time(nop(10));
> write("This ratio = ", ratio, " should somehow be independent of the type of machine.\n");
This ratio = 6.561889634074597010654339089621421685931814496494 should somehow be independent of the type of machine.
```

See also: **timing** (8.183), **nop** (8.110)

### 8.183 timing

Name: **timing**

global variable controlling timing measures in Sollya.

Library names:

```
void sollya_lib_set_timing_and_print(sollya_obj_t)
void sollya_lib_set_timing(sollya_obj_t)
sollya_obj_t sollya_lib_get_timing()
```

Usage:

```

timing = activation value : on|off → void
timing = activation value ! : on|off → void
timing : on|off

```

Parameters:

- *activation value* controls if timing should be performed or not

Description:

- **timing** is a global variable. When its value is **on**, the time spent in each command is measured and displayed (for **verbosity** levels higher than 1).

Example 1:

```
> verbosity=1!;  
> timing=on;  
Timing has been activated.  
> p=remez(sin(x),10,[-1;1]);  
Information: Remez: computing the matrix spent 3 ms  
Information: Remez: computing the quality of approximation spent 16 ms  
Information: Remez: computing the matrix spent 1 ms  
Information: Remez: computing the quality of approximation spent 10 ms  
Information: Remez: computing the matrix spent 1 ms  
Information: Remez: computing the quality of approximation spent 10 ms  
Information: computing a minimax approximation spent 185 ms  
Information: assignment spent 197 ms  
Information: full execution of the last parse chunk spent 215 ms
```

See also: **on** (8.115), **off** (8.114), **time** (8.182)

## 8.184 tripledouble

Names: **tripledouble**, **TD**

represents a number as the sum of three IEEE doubles.

Library names:

```
sollya_obj_t sollya_lib_triple_double(sollya_obj_t)
sollya_obj_t sollya_lib_triple_double_obj()
int sollya_lib_is_triple_double_obj(sollya_obj_t)
sollya_obj_t sollya_lib_build_function_triple_double(sollya_obj_t)
#define SOLLYA_TD(x) sollya_lib_build_function_triple_double(x)
```

Description:

- **tripledouble** is both a function and a constant.
- As a function, it rounds its argument to the nearest number that can be written as the sum of three double precision numbers.
- The algorithm used to compute **tripledouble**( $x$ ) is the following: let  $x_h = \mathbf{double}(x)$ , let  $x_m = \mathbf{double}(x - x_h)$  and let  $x_l = \mathbf{double}(x - x_h - x_m)$ . Return the number  $x_h + x_m + x_l$ . Note that if the current precision is not sufficient to represent exactly  $x_h + x_m + x_l$ , a rounding will occur and the result of **tripledouble**( $x$ ) will be useless.
- As a constant, it symbolizes the triple-double precision format. It is used in contexts when a precision format is necessary, e.g. in the commands **roundcoefficients** and **implementpoly**. See the corresponding help pages for examples.

Example 1:

```
> verbosity=1!;
> a = 1+ 2^(-55)+2^(-115);
> TD(a);
1.00000000000000002775557561562891353466491600711096
> prec=110!;
> TD(a);
1.00000000000000002775557561562891353
```

See also: **halfprecision** (8.75), **single** (8.165), **double** (8.46), **doubleextended** (8.48), **doubledouble** (8.47), **quad** (8.138), **roundcoefficients** (8.154), **implementpoly** (8.83), **fpminimax** (8.68), **print-expansion** (8.132)

## 8.185 true

Name: **true**

the boolean value representing the truth.

Library names:

```
sollya_obj_t sollya_lib_true()
int sollya_lib_is_true(sollya_obj_t)
```

Description:

- **true** is the usual boolean value.

Example 1:

```
> true && false;
false
> 2>1;
true
```

See also: **false** (8.62), **&&** (8.6), **||** (8.116)

## 8.186 **unsuppressmessage**

Name: **unsuppressmessage**

unsuppresses the displaying of messages with a certain number

Library name:

```
void sollya_lib_unsuppressmessage(sollya_obj_t, ...);
```

Usage:

```
unsuppressmessage(msg num 1, ..., msg num n) : (integer, ..., integer) → void  
unsuppressmessage(msg list) : list → void
```

Parameters:

- *msg num 1* thru *msg num n* represent the numbers of *n* messages to be suppressed
- *msg list* represents a list with numbers of messages to be suppressed

Description:

- The **unsuppressmessage** command allows particular warning and information messages that have been suppressed from message output to be unsuppressed, i.e. activated for display again. Every **Sollya** warning or information message (that is not fatal to the tool's execution) has a message number. When these message numbers *msg num 1* thru *msg num n* are given to **unsuppressmessage**, the corresponding message are displayed again, as they are by default at according verbosity levels. Actually, the **unsuppressmessage** command just reverts the effects of the **suppressmessage** command.
- Instead of giving **unsuppressmessage** several message numbers *msg num 1* thru *msg num n* or calling **unsuppressmessage** several times, it is possible to give a whole list *msg list* of message numbers to **unsuppressmessage**.
- The user should be aware that **unsuppressmessage** presents sticky behavior for the warning and information messages suppressed from output. In fact, **unsuppressmessage** just unsuppresses the warning or information messages given in argument. All other suppressed messages stay suppressed until they get unsuppressed by subsequent calls to **unsuppressmessage** or the **Sollya** tool is restarted. This behavior distinguishes message suppression from other global states of the **Sollya** tool. The user may use **getsuppressedmessages** to obtain a list of currently suppressed messages. In particular, in order to unsuppressed all currently suppressed warning or information messages, the user may feed the output of **getsuppressedmessages** (a list) into **unsuppressmessage**.
- The user should also note that unsuppressing warning or information messages with **unsuppressmessage** just reverts the effects of the **suppressmessage** command but that other conditions exist that affect the actual displaying of a message, such as global verbosity (see **verbosity**) and modes like rounding warnings (see **roundingwarnings**). A message will not just get displayed because it was unsuppressed with **unsuppressmessage**.
- When **unsuppressmessage** is used on message numbers that do not exist in the current version of the tool, a warning is displayed. The call has no other effect though.

Example 1:

```

> verbosity = 1;
The verbosity level has been set to 1.
> 0.1;
Warning: Rounding occurred when converting the constant "0.1" to floating-point
with 165 bits.
If safe computation is needed, try to increase the precision.
0.1
> suppressmessage(174);
> 0.1;
0.1
> suppressmessage(174);
> 0.1;
0.1

```

Example 2:

```

> verbosity = 12;
The verbosity level has been set to 12.
> showmessagenumbers = on;
Displaying of message numbers has been activated.
> diff(exp(x * 0.1));
Warning (174): Rounding occurred when converting the constant "0.1" to floating-
point with 165 bits.
If safe computation is needed, try to increase the precision.
Information (196): formally differentiating a function.
Information (197): differentiating the expression 'exp(x * 0.1)'
Information (204): extraction of coefficient terms from a polynomial uses a spec
ial algorithm for Horner forms.
Information (207): no Horner simplification will be performed because the given
tree is already in Horner form.
exp(x * 0.1) * 0.1
> suppressmessage([174, 207, 196]);
> diff(exp(x * 0.1));
Information (197): differentiating the expression 'exp(x * 0.1)'
Information (204): extraction of coefficient terms from a polynomial uses a spec
ial algorithm for Horner forms.
exp(x * 0.1) * 0.1
> unsuppressmessage([174, 196]);

```

Example 3:

```

> verbosity = 12;
The verbosity level has been set to 12.
> showmessagenumbers = on;
Displaying of message numbers has been activated.
> suppressmessage(207, 387, 390, 388, 391, 196, 195, 197, 205);
> getsuppressedmessages();
[[195, 196, 197, 205, 207, 387, 388, 390, 391]]
> evaluate(x/sin(x) - 1, [-1;1]);
[0;0.85081571768092561791175324139865019347039665509409]
> unsuppressmessage(getsuppressedmessages());
> getsuppressedmessages();
[]

```

See also: `getsuppressedmessages` (8.72), `suppressmessage` (8.174), `unsuppressmessage` (8.186), `verbosity` (8.188), `roundingwarnings` (8.156)

## 8.187 var

Name: **var**

declaration of a local variable in a scope

Usage:

**var** *identifier1, identifier2,... , identifiern* : void

Parameters:

- *identifier1, identifier2,... , identifiern* represent variable identifiers

Description:

- The keyword **var** allows for the declaration of local variables *identifier1* through *identifiern* in a begin-end-block ({}-block). Once declared as a local variable, an identifier will shadow identifiers declared in higher scopes and undeclared identifiers available at top-level.

Variable declarations using **var** are only possible in the beginning of a begin-end-block. Several **var** statements can be given. Once another statement is given in a begin-end-block, no more **var** statements can be given.

Variables declared by **var** statements are dereferenced as **error** until they are assigned a value.

Example 1:

```
> exp(x);
exp(x)
> a = 3;
> {var a, b; a=5; b=3; {var a; var b; b = true; a = 1; a; b;}; a; b; };
1
true
5
3
> a;
3
```

See also: **error** (8.53), **proc** (8.135)

## 8.188 verbosity

Name: **verbosity**

global variable controlling the amount of information displayed by commands.

Library names:

```
void sollya_lib_set_verbosity_and_print(sollya_obj_t)
void sollya_lib_set_verbosity(sollya_obj_t)
sollya_obj_t sollya_lib_get_verbosity()
```

Usage:

**verbosity** = *n* : integer → void  
**verbosity** = *n* ! : integer → void  
**verbosity** : integer

Parameters:

- *n* controls the amount of information to be displayed

Description:

- **verbosity** accepts any integer value. At level 0, commands do not display anything on standard output. Note that very critical information may however be displayed on standard error.

- Example 1:

See also: **roundingwarnings** (8.156), **suppressmessage** (8.174), **unsuppressmessage** (8.186), **showmessagesnumbers** (8.161), **getsuppressedmessages** (8.72)

Name: **void**

Library names:

Usage:

Description:

- The **void** result produced by a procedure or an external procedure is not printed at the prompt. However, it is possible to print it out in a print statement or in complex data types such as lists.

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this case, an implicit **void** as the only parameter to a call of such a procedure gets converted into an empty list of arguments, an explicit **void** gets passed as-is in the formal list of parameters the procedure receives.

- **void** is used also as a type identifier for **externalproc**. Typically, an external procedure taking **void** as an argument or returning **void** is bound with a signature **void** –  $\rightarrow$  some type or some type –  $\rightarrow$  **void**. See **externalproc** for more details.

Example 1:

```
> print(void);  
void  
> void;
```

Example 2:

```
> hey = proc() { print("Hello world."); };  
> hey;  
proc()  
{  
  print("Hello world.");  
  return void;  
}  
> hey();  
Hello world.  
> hey(void);  
Hello world.  
> print(hey());  
Hello world.  
void
```

Example 3:

```
> bashexecute("gcc -fPIC -Wall -c externalprocvoidexample.c");  
> bashexecute("gcc -fPIC -shared -o externalprocvoidexample externalprocvoidexample.o");  
> externalproc(foo, "./externalprocvoidexample", void -> void);  
> foo;  
foo(void) -> void  
> foo();  
Hello from the external world.  
> foo(void);  
Hello from the external world.  
> print(foo());  
Hello from the external world.  
void
```

Example 4:

```
> procedure blub(L = ...) { print("Argument list:", L); };  
> blub(1);  
Argument list: [|1|]  
> blub();  
Argument list: [| |]  
> blub(void);  
Argument list: [|void|]
```

See also: **error** (8.53), **proc** (8.135), **externalproc** (8.61)

## 8.190 worstcase

Name: **worstcase**

searches for hard-to-round cases of a function

Library names:

```
void sollya_lib_worstcase(sollya_obj_t, sollya_obj_t, sollya_obj_t,  
                        sollya_obj_t, sollya_obj_t, ...)  
void sollya_lib_v_worstcase(sollya_obj_t, sollya_obj_t, sollya_obj_t,  
                          sollya_obj_t, sollya_obj_t, va_list)
```

Usage:

```
worstcase(function, preimage precision, preimage exponent range, image precision, error bound) :  
          (function, integer, range, integer, constant) → void  
worstcase(function, preimage precision, preimage exponent range, image precision, error bound,  
          filename) : (function, integer, range, integer, constant, string) → void
```

Parameters:

- *function* represents the function to be considered
- *preimage precision* represents the precision of the preimages
- *preimage exponent range* represents the exponents in the preimage format
- *image precision* represents the precision of the format the images are to be rounded to
- *error bound* represents the upper bound for the search w.r.t. the relative rounding error
- *filename* represents a character sequence containing a filename

Description:

- The **worstcase** command is deprecated. It searches for hard-to-round cases of a function. The command **searchgal** has a comparable functionality.

Example 1:

```
> worstcase(exp(x),24,[1,2],24,1b-26);  
prec = 165  
x = 1.99999988079071044921875      f(x) = 7.3890552520751953125      eps = 4  
.5998601423446695596184695493764120138001954979037e-9 = 2^(-27.695763)  
x = 2      f(x) = 7.38905620574951171875      eps = 1.4456360874967301812222  
8379395533417878125150587072e-8 = 2^(-26.043720)
```

See also: **round** (8.153), **searchgal** (8.159), **evaluate** (8.54)

## 8.191 write

Name: **write**

prints an expression without separators

Usage:

```
write(expr1,...,exprn) : (any type,..., any type) → void  
write(expr1,...,exprn) > filename : (any type,..., any type, string) → void  
write(expr1,...,exprn) >> filename : (any type,...,any type, string) → void
```

Parameters:

- *expr* represents an expression



- *filename* represents a character sequence indicating a file name

Description:

- **write**(*expr1*,...,*exprn*) prints the expressions *expr1* through *exprn*. The character sequences corresponding to the expressions are concatenated without any separator. No newline is displayed at the end. In contrast to **print**, **write** expects the user to give all separators and newlines explicitly.

If a second argument *filename* is given after a single ">", the displaying is not output on the standard output of Sollya but if in the file *filename* that get newly created or overwritten. If a double ">>" is given, the output will be appended to the file *filename*.

The global variables **display**, **midpointmode** and **fullparentheses** have some influence on the formatting of the output (see **display**, **midpointmode** and **fullparentheses**).

Remark that if one of the expressions *expri* given in argument is of type **string**, the character sequence *expri* evaluates to is displayed. However, if *expri* is of type **list** and this list contains a variable of type **string**, the expression for the list is displayed, i.e. all character sequences get displayed surrounded by quotes ("). Nevertheless, escape sequences used upon defining character sequences are interpreted immediately.

Example 1:

```
> write(x + 2 + exp(sin(x)));
> write("Hello\n");
x + 2 + exp(sin(x))Hello
> write("Hello","world\n");
Helloworld
> write("Hello","you", 4 + 3, "other persons.\n");
Helloyou7other persons.
```

Example 2:

```
> write("Hello","\n");
Hello
> write(["Hello"],"\n");
["Hello"]
> s = "Hello";
> write(s,[s],"\n");
Hello["Hello"]
> t = "Hello\tyou";
> write(t,[t],"\n");
Hello    you["Hello\tyou"]
```

Example 3:

```
> write(x + 2 + exp(sin(x))) > "foo.sol";
> readfile("foo.sol");
x + 2 + exp(sin(x))
```

Example 4:

```
> write(x + 2 + exp(sin(x))) >> "foo.sol";
```

See also: **print** (8.130), **printexpansion** (8.132), **printdouble** (8.131), **printsingl** (8.133), **printxml** (8.134), **readfile** (8.144), **autosimplify** (8.15), **display** (8.44), **midpointmode** (8.104), **fullparentheses** (8.69), **evaluate** (8.54), **roundingwarnings** (8.156), **autosimplify** (8.15)

## 9 Appendix: interval arithmetic philosophy in Sollya

Although it is currently based on the MPFI library, **Sollya** has its own way of interpreting interval arithmetic when infinities or NaN occur, or when a function is evaluated on an interval containing points out of its domain, etc. This philosophy may differ from the one applied in MPFI. It is also possible that the behavior of **Sollya** does not correspond to the behavior that one would expect, e.g. as a natural consequence of the IEEE-754 standard.

The topology that we consider is always the usual topology of  $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$ . For any function, if one of its arguments is empty (respectively NaN), we return empty (respectively NaN).

### 9.1 Univariate functions

Let  $f$  be a univariate basic function and  $I$  an interval. We denote by  $J$  the result of the interval evaluation of  $f$  over  $I$  in **Sollya**. If  $I$  is completely included in the domain of  $f$ ,  $J$  will usually be the smallest interval (at the current precision) containing the exact image  $f(I)$ . However, in some cases, it may happen that  $J$  is not as small as possible. It is guaranteed however, that  $J$  tends to  $f(I)$  when the precision of the tool tends to infinity.

When  $f$  is not defined at some point  $x$  but is defined on a neighborhood of  $x$ , we consider that the “value” of  $f$  at  $x$  is the convex hull of the limit points of  $f$  around  $x$ . For instance, consider the evaluation of  $f = \tan$  on  $[0, \pi]$ . It is not defined at  $\pi/2$  (and only at this point). The limit points of  $f$  around  $\pi/2$  are  $-\infty$  and  $+\infty$ , so, we return  $[-\infty, \infty]$ . Another example:  $f = \sin$  on  $[+\infty]$ . The function has no limit at this point, but all points of  $[-1, 1]$  are limit points. So, we return  $[-1, 1]$ .

Finally, if  $I$  contains a subinterval on which  $f$  is not defined, we return  $[\text{NaN}, \text{NaN}]$  (example:  $\sqrt{[-1, 2]}$ ).

### 9.2 Bivariate functions

Let  $f$  be a bivariate function and  $I_1$  and  $I_2$  be intervals. If  $I_1 = [x]$  and  $I_2 = [y]$  are both point-intervals, we return the convex hull of the limit points of  $f$  around  $(x, y)$  if it exists. In particular, if  $f$  is defined at  $(x, y)$  we return its value (or a small interval around it, if it is not exactly representable). As an example  $[1]/[+\infty]$  returns  $[0]$ . Also,  $[1]/[0]$  returns  $[-\infty, +\infty]$  (note that **Sollya** does not consider signed zeros). If it is not possible to give a meaning to the expression  $f(I_1, I_2)$ , we return NaN: for instance  $[0]/[0]$  or  $[0] * [+\infty]$ .

If one and only one of the intervals is a point-interval (say  $I_1 = [x]$ ), we consider the partial function  $g : y \mapsto f(x, y)$  and return the value that would be obtained when evaluating  $g$  on  $I_2$ . For instance, in order to evaluate  $[0]/I_2$ , we consider the function  $g$  defined for every  $y \neq 0$  by  $g(y) = 0/y = 0$ . Hence,  $g(I_2) = [0]$  (even if  $I_2$  contains 0, by the argument of limit-points). In particular, please note that  $[0]/[-1, 1]$  returns  $[0]$  even though  $[0]/[0]$  returns NaN. This rule even holds when  $g$  can only be defined as limit points: for instance, in the case  $I_1/[0]$  we consider  $g : x \mapsto x/0$ . This function cannot be defined *stricto sensu*, but we can give it a meaning by considering 0 as a limit. Hence  $g$  is multivalued and its value is  $\{-\infty, +\infty\}$  for every  $x$ . Hence,  $I_1/[0]$  returns  $[-\infty, +\infty]$  when  $I_1$  is not a point-interval.

Finally, if neither  $I_1$  nor  $I_2$  are point-intervals, we try to give a meaning to  $f(I_1, I_2)$  by an argument of limit-points when possible. For instance  $[1, 2]/[0, 1]$  returns  $[1, +\infty]$ .

As a special exception to these rules,  $[0]^{[0]}$  returns  $[1]$ .

## 10 Appendix: the Sollya library

### 10.1 Introduction

The header file of the Sollya library is `sollya.h`. Its inclusion may provoke the inclusion of other header files, such as `gmp.h`, `mpfr.h` or `mpfi.h`.

The library provides a virtual Sollya session that is perfectly similar to an interactive session: global variables such as `verbosity`, `prec`, `display`, `midpointmode`, etc. are maintained and affect the behavior of the library, warning messages are displayed when something is not exact, etc. Please notice that the Sollya library currently is **not** re-entrant and can only be opened once. A process using the library must hence not be multi-threaded and is limited to one single virtual Sollya session.

In order to get started with the Sollya library, the first thing to do is hence to initialize this virtual session. This is performed with the `sollya_lib_init` function. Accordingly, one should close the session at the end of the program (which has the effect of releasing all the memory used by Sollya). Please notice that Sollya uses its own allocation functions and registers them to GMP using the custom allocation functions provided by GMP. Particular precautions should hence be taken when using the Sollya library in a program that also registers its own functions to GMP: in that case `sollya_lib_init_with_custom_memory_functions` should be used instead of `sollya_lib_init` for initializing the library. This is discussed in Section 10.17.

In the usual case when Sollya is used in a program that does not register allocation functions to GMP, a minimal file using the library is hence the following.

```
#include <sollya.h>

int main(void) {
    sollya_lib_init();

    /* Functions of the library can be called here */

    sollya_lib_close();
    return 0;
}
```

Suppose that this code is saved in a file called `foo.c`. The compilation is performed as usual without forgetting to link against `libsollya` (since the libraries `libgmp`, `libmpfr` and `libmpfi` are dependencies of Sollya, it might also be necessary to explicitly link against them):

```
~/ % cc foo.c -c
~/ % cc foo.o -o foo -lsollya -lmpfi -lmpfr -lgmp
```

### 10.2 Sollya object data-type

The library provides a single data type called `sollya_obj_t` that can contain any Sollya object (a Sollya object is anything that can be stored in a variable within the interactive tool. See Section 5 of the present documentation for details). Please notice that `sollya_obj_t` is in fact a pointer type; this has two consequences:

- `NULL` is a placeholder that can be used as a `sollya_obj_t` in some contexts. This placeholder is particularly useful as an end marker for functions with a variable number of arguments (see Sections 10.5.5 and 10.15).
- An assignment with the “=” sign does not copy an object but only copies the reference to it. In order to perform a (deep) copy, the `sollya_lib_copy_obj()` function is available.

Except for a few functions for which the contrary is explicitly specified, the following conventions are used:

- A function does not touch its arguments. Hence if `sollya_lib.foo` is a function of the library, a call to `sollya_lib.foo(a)` leaves the object referenced by `a` unchanged (the notable exceptions to that rule are the functions containing `build` in their name, e.g., `sollya_lib.build.foo`).
- A function that returns a `sollya_obj_t` creates a new object (this means that memory is dynamically allocated for that object). The memory allocated for that object should manually be cleared when the object is no longer used and all references to it (on the stack) get out of reach, e.g. on a function return: this is performed by the `sollya_lib.clear_obj()` function. By convenience `sollya_lib.clear_obj(NULL)` is a valid call, which just does nothing.

In general, except if the user perfectly knows what they are doing, the following rules should be applied (here `a` and `b` are C variables of type `sollya_obj_t`, and `sollya_lib.foo` and `sollya_lib.bar` are functions of the library):

- One should never write `a = b`. Instead, use `a = sollya_lib.copy_obj(b)`.
- One should never write `a = sollya_lib.foo(a)` because one loses the reference to the object initially referenced by the variable `a` (which is hence not cleared).
- One should never chain function calls such as, e.g., `a = sollya_lib.foo(sollya_lib.bar(b))` (the reference to the object created by `sollya_lib.bar(b)` would be lost and hence not cleared).
- A variable `a` should never be used twice at the left-hand side of the “=” sign (or as an lvalue in general) without performing `sollya_lib.clear_obj(a)` in-between.
- In an assignment of the form “`a = ...`”, the right-hand side should always be a function call (i.e., something like `a = sollya_lib.foo(...)`);).

Please notice that `sollya_lib.close()` clears the memory allocated by the virtual Sollya session but not the objects that have been created and stored in C variables. All the `sollya_obj_t` created by function calls should be cleared manually.

We can now write a simple Hello World program using the Sollya library:

```
#include <sollya.h>

int main(void) {
    sollya_obj_t s1, s2, s;
    sollya_lib_init();

    s1 = sollya_lib_string("Hello ");
    s2 = sollya_lib_string("World!");
    s = sollya_lib_concat(s1, s2);
    sollya_lib_clear_obj(s1);
    sollya_lib_clear_obj(s2);

    sollya_lib_printf("%b\n", s);
    sollya_lib_clear_obj(s);
    sollya_lib_close();
    return 0;
}
```

A universal function allows the user to execute any expression, as if it were given at the prompt of the Sollya tool, and to get a `sollya_obj_t` containing the result of the evaluation: this function is `sollya_lib.parse_string("some expression here")`. This is very convenient, and indeed, any script written in the Sollya language, could easily be converted into a C program by intensively using `sollya_lib.parse_string`. However, this should not be the preferred way if efficiency is targeted because (as its name suggests) this function uses a parser to decompose its argument, then constructs intermediate data structures to store the abstract interpretation of the expression, etc. Low-level functions are provided for efficiently creating Sollya objects; they are detailed in the next Section.

## 10.3 Conventions in use in the library

The library follows some conventions that are useful to remember:

- When a function is a direct transposition of a command or function available in the interactive tool, it returns a `sollya_obj_t`. This is true, even when it would sound natural to return, e.g. an `int`. For instance `sollya_lib_get_verbosity()` returns a `sollya_obj_t`, whose integer value must then be recovered with `sollya_lib_get_constant_as_int`. This forces the user to declare (and clear afterwards) a temporary `sollya_obj_t` to store the value, but this is the price of homogeneity in the library.
- When a function returns an integer, this integer generally is a boolean in the usual C meaning, i.e. 0 represents false and any non-zero value represents true. In many cases, the integer returned by the function indicates a status of success or failure: the convention is “false means failure” and “true means success”. In case of failure, the convention is that the function did not touch any of its arguments.
- When a function would need to return several things, or when a function would need to return something together with a status of failure or success, the convention is that pointers are given as the first arguments of the function. These pointers shall point to valid addresses where the function will store the results. This can sometimes give obscure signatures, when the function would in principle returns a pointer and actually takes as argument a pointer to a pointer (this typically happens when the function allocates a segment of memory and should return a pointer to that segment of memory).

## 10.4 Displaying Sollya objects and numerical values

Within the interactive tool, the most simple way of displaying the content of a variable or the value of an expression is to write the name of the variable or the expression, followed by the character “;”. As a result, Sollya evaluates the expression or the variable and displays the result. Alternatively, a set of objects can be displayed the same way, separating the objects with commas. In library mode, the same behavior can be reproduced using the function `void sollya_lib_autoprint(sollya_obj_t, ...)`. Please notice that this function has a variable number of arguments: they are all displayed, until an argument equal to `NULL` is found. The `NULL` argument is mandatory, even if only one object shall be displayed (the function has no other way to know if other arguments follow or not). So, if only one argument should be displayed, the correct function call is `sollya_lib_autoprint(arg, NULL)`. Accordingly, if two arguments should be displayed, the function call is `sollya_lib_autoprint(arg1, arg2, NULL)`, etc. The function `void sollya_lib_v_autoprint(sollya_obj_t, va_list)` is the same, but it takes a `va_list` argument instead of a variable number of arguments.

Further, there is another way of printing formatted strings containing Sollya objects, using a `printf`-like syntax. Eight functions are provided, namely:

- `sollya_lib_printf`, `sollya_lib_v_printf`,
- `sollya_lib_fprintf`, `sollya_lib_v_fprintf`,
- `sollya_lib_sprintf`, `sollya_lib_v_sprintf`,
- `sollya_lib_snprintf` and `sollya_lib_v_snprintf`.

Each one of these functions overloads the usual function (respectively, `printf`, `vprintf`, `fprintf`, `vfprintf`, `sprintf`, `vsprintf`, `snprintf` and `vsnprintf`). The full syntax of conversions specifiers supported with the usual functions is handled (please note that the style using ‘\$’ — as in `%3$` or `/*3$` — is not handled though. It is not included in the C99 standard anyway). Additionally, the following conversion specifiers are provided:

- `%b`: corresponds to a `sollya_obj_t` argument. There is no precision modifier support.
- `%v`: corresponds to a `mpfr_t` argument. An optional precision modifier can be applied (e.g. `%.5v`).
- `%w`: corresponds to a `mpfi_t` argument. An optional precision modifier can be applied (e.g. `%.5w`).

- `%r`: corresponds to a `mpq_t` argument. There is no precision modifier support.

When one of the above conversion specifiers is used, the corresponding argument is displayed as it would be within the interactive tool: i.e. the way the argument is displayed depends on `Sollya` environment variables, such as `prec`, `display`, `midpointmode`, etc. When a precision modifier  $n$  is used, the argument is first rounded to a binary precision of roughly  $\log_2(10) \times n$  bits (i.e. roughly equivalent to  $n$  decimal digits) before being displayed. As with traditional `printf`, the precision modifier can be replaced with `*` which causes the precision to be determined by an additional `int` argument.

Flag characters (e.g., `#`, `0`, etc.) are allowed but have no effect, except flag character `-` that is supported with its usual meaning of left-aligning the converted value. The full syntax for minimum field width is supported: it can be given directly as an integer in the format string (e.g., `%22b`) or it can be replaced with `*`, which causes the field width to be determined by an additional `int` argument. As usual, a negative field width is taken as a `-` flag followed by a positive width.

As a special (and sometimes convenient) case, `%b` accepts that its corresponding `sollya_obj_t` argument be `NULL`: in this particular case, the string “NULL” is used in the displayed string. Please notice that, except for the particular case of `NULL`, the behavior of `sollya_lib_printf` is completely undefined if the argument of `%b` is not a valid `Sollya` object.

The `sollya_lib_printf` functions return an integer with the same meaning as the traditional `printf` functions. It indicates the number of characters that have been output (excluding the final `\0` character). Similarly, the conversion specifier `%n` can be used, even together with the `Sollya` conversion specifiers `%b`, `%v`, `%w` and `%r`. The functions `sollya_lib_snprintf` and `sollya_lib_v_snprintf` will never write more characters than indicated by their size argument (including the final `\0` character). If the output gets truncated due to this limit, they will return the number of characters (excluding the final `\0` character) that would have been output if there had not been any truncation. In case of error, all `sollya_lib_printf` functions return a negative value.

## 10.5 Creating Sollya objects

`Sollya` objects conceptually fall into one of five categories: numerical constants (e.g. 1 or 1.5), functional expressions (they might contain numerical constants, e.g.,  $\sin(\cos(x + 1.5))$ ), other simple objects (intervals, strings, built-in constants such as `dyadic`, etc.), lists of objects (e.g., `[1, "Hello"]`) and structures (e.g., `{.a = 1; .b = "Hello"}`).

### 10.5.1 Numerical constants

Table 1 lists the different functions available to construct numerical constants. A `Sollya` constant is always created without rounding (whatever the value of global variable `prec` is at the moment of the function call): a sufficient precision is always allocated so that the constant is stored exactly. The objects returned by these functions are newly allocated and copies of the argument. For instance, after the instruction `a = sollya_lib_constant(b)`, the user will eventually have to clear `a` (with `sollya_lib_clear(a)`) and `b` (with `mpfr_clear(b)`).

The function `sollya_lib_constant_from_double` (or more conveniently its shortcut `SOLLYA_CONST`) is probably the preferred way for constructing numerical constants. As the name indicates it, its argument is a `double`; however, due to implicit casting in C, it is perfectly possible to give an `int` as argument: it will be converted into a `double` (without rounding if the integer fits on 53 bits) before being passed to `SOLLYA_CONST`. On the contrary, the user should be aware of the fact that if decimal non-integer constants are given, C rules of rounding (to double) are applied, regardless of the setting of the `Sollya` precision variable `prec`.

### 10.5.2 Functional expressions

Functional expressions are built by composition of basic functions with constants and the free mathematical variable. Since it is convenient to build such expressions by chaining function calls, the library provides functions that “eat up” their arguments (actually embedding them in a bigger expression). The convention is that functions that eat up their arguments are prefixed by `sollya_lib_build_`. For the purpose of building expressions, shortcut macros for the corresponding functions exist. They are all listed in Table 2.

Table 1: Creating numerical constants (Creates a fresh `sollya_obj_t`. Conversion is always exact)

Type of the argument	Name of the function	Shortcut macro
<code>double</code>	<code>sollya_lib_constant_from_double(x)</code>	<code>SOLLYA_CONST(x)</code>
<code>uint64_t</code>	<code>sollya_lib_constant_from_uint64(x)</code>	<code>SOLLYA_CONST_UI64(x)</code>
<code>int64_t</code>	<code>sollya_lib_constant_from_int64(x)</code>	<code>SOLLYA_CONST_SI64(x)</code>
<code>int</code>	<code>sollya_lib_constant_from_int(x)</code>	N/A
<code>mpfr_t</code>	<code>sollya_lib_constant(x)</code>	N/A

It is worth mentioning that, although `SOLLYA_X_` and `SOLLYA_PI` are used without parentheses (as if they denoted constants), they are in fact function calls that create a new object each time they are used. The absence of parentheses is just more convenient for constructing expressions, such as, e.g. `SOLLYA_COS(SOLLYA_X_)`.

For each function of the form `sollya_lib_build_function_foo`, there exists a function called `sollya_lib_foo`. There are two differences between them:

- First, `sollya_lib_foo` does not “eat up” its argument. This can sometimes be useful, e.g., if one has an expression stored in a variable `a` and one wants to build the expression `exp(a)` without loosing the reference to the expression represented by `a`.
- Second, while `sollya_lib_build_function_foo` mechanically constructs an expression, function `sollya_lib_foo` also evaluates it, as far as this is possible without rounding.  
For instance, after the instructions `a = SOLLYA_CONST(0); b = sollya_lib_exp(a);` the variable `b` contains the number 1, whereas it would have contained the expression “`exp(0)`” if it had been created by `b = sollya_lib_build_function(a)`.

Actually, `sollya_lib_foo` has exactly the same behavior as writing an expression at the prompt within the interactive tool. In particular, it is possible to give a range as an argument to `sollya_lib_foo`: the returned object will be the result of the evaluation of function `foo` on that range by interval arithmetic. In contrast, trying to use `sollya_lib_build_function_foo` on a range would result in a typing error.

### 10.5.3 Library functions, library constants and procedure functions

In addition to the mathematical base functions and constants provided by `Sollya` and listed in the Section above, the user may bind other mathematical functions and constants to `Sollya` objects under the condition that they can provide code to evaluate these functions or constants. The mechanism behind is similar to the one available in interactive `Sollya` through the `library`, `libraryconstant` and `function` commands (see Sections 8.93, 8.94 and 8.70).

With the `Sollya` library, this binding is done through one of the following functions:

- Binding of a (non-constant) mathematical function for which evaluation code is available through a C pointer to a function:

```
sollya_obj_t sollya_lib_libraryfunction(sollya_obj_t e,
                                       char *name,
                                       int (*f)(mpfi_t, mpfi_t, int));
sollya_obj_t sollya_lib_build_function_libraryfunction(sollya_obj_t e,
                                                      char *name,
                                                      int (*f)(mpfi_t, mpfi_t, int));
```

These two functions construct a `Sollya` object representing  $f(e)$  where  $e$  is given as the `Sollya` object `e` and  $f$  is given as the pointer to a function `f(mpfi_t y, mpfi_t x, int n)`. This code must evaluate the  $n$ -th derivative of  $f$  over the interval  $x$ , yielding  $y$ .

As usual, `sollya_lib_build_function_libraryfunction` “eats up” the object `e`, while the function `sollya_lib_libraryfunction` does not.

The `name` argument of the function is taken as a suggestion to the name the `Sollya` object representing the function should be printed as when displayed. The user may choose to provide `NULL`

Table 2: Building functional expressions (Eats up arguments, embedding them in the returned object.)

Name in the interactive tool	Function to build it	Shortcut macro
<code>_x_</code>	<code>sollya_lib_build_function_free_variable()</code>	<code>SOLLYA_X_</code>
<code>pi</code>	<code>sollya_lib_build_function_pi()</code>	<code>SOLLYA_PI</code>
<code>e1 + e2</code>	<code>sollya_lib_build_function_add(e1, e2)</code>	<code>SOLLYA_ADD(e1, e2)</code>
<code>e1 - e2</code>	<code>sollya_lib_build_function_sub(e1, e2)</code>	<code>SOLLYA_SUB(e1, e2)</code>
<code>e1 * e2</code>	<code>sollya_lib_build_function_mul(e1, e2)</code>	<code>SOLLYA_MUL(e1, e2)</code>
<code>e1 / e2</code>	<code>sollya_lib_build_function_div(e1, e2)</code>	<code>SOLLYA_DIV(e1, e2)</code>
<code>pow(e1, e2)</code>	<code>sollya_lib_build_function_pow(e1, e2)</code>	<code>SOLLYA_POW(e1, e2)</code>
<code>-e</code>	<code>sollya_lib_build_function_neg(e)</code>	<code>SOLLYA_NEG(e)</code>
<code>sqrt(e)</code>	<code>sollya_lib_build_function_sqrt(e)</code>	<code>SOLLYA_SQRT(e)</code>
<code>abs(e)</code>	<code>sollya_lib_build_function_abs(e)</code>	<code>SOLLYA_ABS(e)</code>
<code>erf(e)</code>	<code>sollya_lib_build_function_erf(e)</code>	<code>SOLLYA_ERF(e)</code>
<code>erfc(e)</code>	<code>sollya_lib_build_function_erfc(e)</code>	<code>SOLLYA_ERFC(e)</code>
<code>exp(e)</code>	<code>sollya_lib_build_function_exp(e)</code>	<code>SOLLYA_EXP(e)</code>
<code>expm1(e)</code>	<code>sollya_lib_build_function_expm1(e)</code>	<code>SOLLYA_EXPM1(e)</code>
<code>log(e)</code>	<code>sollya_lib_build_function_log(e)</code>	<code>SOLLYA_LOG(e)</code>
<code>log2(e)</code>	<code>sollya_lib_build_function_log2(e)</code>	<code>SOLLYA_LOG2(e)</code>
<code>log10(e)</code>	<code>sollya_lib_build_function_log10(e)</code>	<code>SOLLYA_LOG10(e)</code>
<code>log1p(e)</code>	<code>sollya_lib_build_function_log1p(e)</code>	<code>SOLLYA_LOG1P(e)</code>
<code>sin(e)</code>	<code>sollya_lib_build_function_sin(e)</code>	<code>SOLLYA_SIN(e)</code>
<code>cos(e)</code>	<code>sollya_lib_build_function_cos(e)</code>	<code>SOLLYA_COS(e)</code>
<code>tan(e)</code>	<code>sollya_lib_build_function_tan(e)</code>	<code>SOLLYA_TAN(e)</code>
<code>asin(e)</code>	<code>sollya_lib_build_function_asin(e)</code>	<code>SOLLYA_ASIN(e)</code>
<code>acos(e)</code>	<code>sollya_lib_build_function_acos(e)</code>	<code>SOLLYA_ACOS(e)</code>
<code>atan(e)</code>	<code>sollya_lib_build_function_atan(e)</code>	<code>SOLLYA_ATAN(e)</code>
<code>sinh(e)</code>	<code>sollya_lib_build_function_sinh(e)</code>	<code>SOLLYA_SINH(e)</code>
<code>cosh(e)</code>	<code>sollya_lib_build_function_cosh(e)</code>	<code>SOLLYA_COSH(e)</code>
<code>tanh(e)</code>	<code>sollya_lib_build_function_tanh(e)</code>	<code>SOLLYA_TANH(e)</code>
<code>asinh(e)</code>	<code>sollya_lib_build_function_asinh(e)</code>	<code>SOLLYA_ASINH(e)</code>
<code>acosh(e)</code>	<code>sollya_lib_build_function_acosh(e)</code>	<code>SOLLYA_ACOSH(e)</code>
<code>atanh(e)</code>	<code>sollya_lib_build_function_atanh(e)</code>	<code>SOLLYA_ATANH(e)</code>
<code>D(e), double(e)</code>	<code>sollya_lib_build_function_double(e)</code>	<code>SOLLYA_D(e)</code>
<code>SG(e), single(e)</code>	<code>sollya_lib_build_function_single(e)</code>	<code>SOLLYA_SG(e)</code>
<code>QD(e), quad(e)</code>	<code>sollya_lib_build_function_quad(e)</code>	<code>SOLLYA_QD(e)</code>
<code>HP(e), halfprecision(e)</code>	<code>sollya_lib_build_function_halfprecision(e)</code>	<code>SOLLYA_HP(e)</code>
<code>DD(e), doubledouble(e)</code>	<code>sollya_lib_build_function_double_double(e)</code>	<code>SOLLYA_DD(e)</code>
<code>TD(e), tripledouble(e)</code>	<code>sollya_lib_build_function_triple_double(e)</code>	<code>SOLLYA_TD(e)</code>
<code>DE(e), doubleextended(e)</code>	<code>sollya_lib_build_function_doubleextended(e)</code>	<code>SOLLYA_DE(e)</code>
<code>ceil(e)</code>	<code>sollya_lib_build_function_ceil(e)</code>	<code>SOLLYA_CEIL(e)</code>
<code>floor(e)</code>	<code>sollya_lib_build_function_floor(e)</code>	<code>SOLLYA_FLOOR(e)</code>
<code>nearestint(e)</code>	<code>sollya_lib_build_function_nearestint(e)</code>	<code>SOLLYA_NEARESTINT(e)</code>



instead. In any case, upon the binding, the **Sollya** library will determine a unique displaying name for the function. If it is not yet taken as a name (for some other **Sollya** object or **Sollya** keyword), the suggested name will be used. If no suggested name is provided, the name of the dynamic object behind the pointer to the function will be used if it can be determined. Otherwise, a more-or-less random name is used. If the (suggested) base name is already taken, the name is unified appending an underscore and a unique number to it. The **name** argument is never “eaten up”, i.e., it is up to the user to free any memory allocated to that pointer.

- Binding of a mathematical constant for which evaluation code is available through a **C** pointer to a function:

```
sollya_obj_t sollya_lib_libraryconstant(char *name,
                                         void (*c)(mpfr_t, mp_prec_t));
sollya_obj_t sollya_lib_build_function_libraryconstant(char *name,
                                                         void (*c)(mpfr_t, mp_prec_t));
```

These two functions construct a **Sollya** object representing the mathematical constant  $c$  for which a pointer to a function `c(mpfr_t rop, mp_prec_t prec)` is provided. This code must evaluate the constant to precision `prec` and affect the result to `rop`. See Section 8.94 for details with respect to `prec`.

The same remark as above concerning the suggested displaying name of the **Sollya** object applies for the **name** argument.

- Binding of a mathematical function for which evaluation code is available through a **Sollya** object representing a **Sollya** procedure:

```
sollya_obj_t sollya_lib_procedurefunction(sollya_obj_t e, sollya_obj_t f);
sollya_obj_t sollya_lib_build_function_procedurefunction(sollya_obj_t e,
                                                         sollya_obj_t f);
```

These two functions construct a **Sollya** library object representing  $f(e)$  where  $e$  corresponds to the mathematical function (or constant) given with argument  $e$  and where  $f$  is given as a **Sollya** procedure  $f(x, n, p)$  evaluating the  $n$ -th derivative of  $f$  over the interval  $x$  with precision  $p$ . See Section 8.70 concerning details of the arguments of that **Sollya** procedure.

As usual, `sollya_lib_build_function_procedurefunction` “eats up” its arguments  $e$  and  $f$  while `sollya_obj_t sollya_lib_procedurefunction` does not.

Currently, the only way of constructing a **Sollya** library object representing a **Sollya** procedure is to use `sollya_lib_parse_string`.

#### 10.5.4 Other simple objects

Other simple objects are created with functions listed in Table 3. The functions with a name of the form `sollya_lib_range_something` follow the same convention as `sollya_lib_constant`: they build a new object from a copy of their argument, and the conversion is always exact, whatever the value of `prec` is.

Please note that in the interactive tool, **D** either denotes the discrete mathematical function that maps a real number to its closest **double** number, or is used as a symbolic constant to indicate that the **double** format must be used (as an argument of `round` for instance). In the library, they are completely distinct objects, the mathematical function being obtained with `sollya_lib_build_function_double` and the symbolic constant with `sollya_lib_double_obj`. The same holds for other formats (**DD**, **SG**, etc.)

#### 10.5.5 Lists

There are actually two kinds of lists: regular lists (such as, e.g., `[1, 2, 3]`) and semi-infinite lists (such as, e.g. `[1, 2...]`). Withing the interactive tool, the ellipsis “...” can sometimes be used as a shortcut to define regular lists, e.g. `[1, 2, ..., 10]`.

In the library, there is no symbol for the ellipsis, and there are two distinct types: one for regular lists and one for semi-infinite lists (called end-elliptic). Defining a regular list with an ellipsis is not possible in the library (except of course with `sollya_lib_parse_string`).

Constructing regular lists is achieved through three functions:

Table 3: Creating Sollya objects from scratch (Returns a new `sollya_obj_t`)

Name in the interactive tool	Function to create it
<code>on</code>	<code>sollya_lib_on();</code>
<code>off</code>	<code>sollya_lib_off();</code>
<code>dyadic</code>	<code>sollya_lib_dyadic();</code>
<code>powers</code>	<code>sollya_lib_powers();</code>
<code>binary</code>	<code>sollya_lib_binary();</code>
<code>hexadecimal</code>	<code>sollya_lib_hexadecimal();</code>
<code>file</code>	<code>sollya_lib_file();</code>
<code>postscript</code>	<code>sollya_lib_postscript();</code>
<code>postscriptfile</code>	<code>sollya_lib_postscriptfile();</code>
<code>perturb</code>	<code>sollya_lib_perturb();</code>
<code>RD</code>	<code>sollya_lib_round_down();</code>
<code>RU</code>	<code>sollya_lib_round_up();</code>
<code>RZ</code>	<code>sollya_lib_round_towards_zero();</code>
<code>RN</code>	<code>sollya_lib_round_to_nearest();</code>
<code>honorcoeffprec</code>	<code>sollya_lib_honorcoeffprec();</code>
<code>true</code>	<code>sollya_lib_true();</code>
<code>false</code>	<code>sollya_lib_false();</code>
<code>void</code>	<code>sollya_lib_void();</code>
<code>default</code>	<code>sollya_lib_default();</code>
<code>decimal</code>	<code>sollya_lib_decimal();</code>
<code>absolute</code>	<code>sollya_lib_absolute();</code>
<code>relative</code>	<code>sollya_lib_relative();</code>
<code>fixed</code>	<code>sollya_lib_fixed();</code>
<code>floating</code>	<code>sollya_lib_floating();</code>
<code>error</code>	<code>sollya_lib_error();</code>
<code>D, double</code>	<code>sollya_lib_double_obj();</code>
<code>SG, single</code>	<code>sollya_lib_single_obj();</code>
<code>QD, quad</code>	<code>sollya_lib_quad_obj();</code>
<code>HP, halfprecision</code>	<code>sollya_lib_halfprecision_obj();</code>
<code>DE, doubleextended</code>	<code>sollya_lib_doubleextended_obj();</code>
<code>DD, doubledouble</code>	<code>sollya_lib_double_double_obj();</code>
<code>TD, tripledouble</code>	<code>sollya_lib_triple_double_obj();</code>
<code>"Hello"</code>	<code>sollya_lib_string("Hello")</code>
<code>[1, 3.5]</code>	<code>sollya_lib_range_from_interval(a);<sup>a</sup></code>
<code>[1, 3.5]</code>	<code>sollya_lib_range_from_bounds(b, c);<sup>b</sup></code>

<sup>a</sup>`a` is a `mpfi_t` containing the interval  $[1, 3.5]$ . Conversion is always exact.

<sup>b</sup>`b` and `c` are `mpfr_t` respectively containing the numbers 1 and 3.5. Conversion is always exact.

- `sollya_obj_t sollya_lib_list(sollya_obj_t[] L, int n)`: this function returns a new object that is a list the elements of which are copies of `L[0], ..., L[n-1]`.
- `sollya_obj_t sollya_lib_build_list(sollya_obj_t obj1, ...)`: this function accepts a variable number of arguments. The last one **must** be `NULL`. It “eats up” its arguments and returns a list containing the objects given as arguments. Since arguments are eaten up, they may be directly produced by function calls, without being stored in variables. A typical use could be

```
sollya_lib_build_list(SOLLYA_CONST(1), SOLLYA_CONST(2), SOLLYA_CONST(3), NULL);
```

- `sollya_obj_t sollya_lib_v_build_list(va_list)`: the same as the previous functions, but with a `va_list`.

Following the same conventions, end-elliptic lists can be constructed with the following functions:

- `sollya_obj_t sollya_lib_end_elliptic_list(sollya_obj_t[] L, int n)`.
- `sollya_obj_t sollya_lib_build_end_elliptic_list(sollya_obj_t obj1, ...)`.
- `sollya_obj_t sollya_lib_v_build_end_elliptic_list(va_list)`.

### 10.5.6 Structures

Sollya structures are also available in library mode as any other Sollya object. The support for Sollya structures is however minimal and creating them might seem cumbersome<sup>2</sup>. The only function available to create structures is

```
int sollya_lib_create_structure(sollya_obj_t *res, sollya_obj_t s, char *name,
                              sollya_obj_t val).
```

This function returns a boolean integer: false means failure, and true means success. Three cases of success are possible. In all cases, the function creates a new object and stores it at the address referred to by `res`.

- If `s` is `NULL`: `*res` is filled with a structure with only one field. This field is named after the string `name` and contains a copy of the object `val`.
- If `s` is an already existing structure that has a field named after the string `name`: `*res` is filled with a newly created structure. This structure is the same as `s` except that the field corresponding to `name` contains a copy of `val`.
- If `s` is an already existing structure that does **not** have a field named after the string `name`: `*res` is filled with a newly created structure. This structure is the same as `s` except that it has been augmented with a field corresponding to `name` and that contains a copy of `val`.

Please notice that `s` is not changed by this function: the structure stored in `*res` is a new one that does not refer to any of the components of `s`. As a consequence, one should not forget to explicitly clear `s` as well as `*res` when they become useless.

## 10.6 Getting the type of an object

Functions are provided that allow the user to test the type of a Sollya object. They are listed in Table 4. They all return an `int` interpreted as the boolean result of the test. Please note that from a typing point of view, a mathematical constant and a non-constant functional expression are both functions.

---

<sup>2</sup>Users are encouraged to make well-founded feature requests if they feel the need for better support of structures.

Table 4: Testing the type of a Sollya object (Returns non-zero if true, 0 otherwise)

sollya_lib_obj_is_function(obj) sollya_lib_obj_is_range(obj) sollya_lib_obj_is_string(obj) sollya_lib_obj_is_list(obj) sollya_lib_obj_is_end_elliptic_list(obj) sollya_lib_obj_is_structure(obj) sollya_lib_obj_is_error(obj)
sollya_lib_is_on(obj) sollya_lib_is_off(obj) sollya_lib_is_dyadic(obj) sollya_lib_is_powers(obj) sollya_lib_is_binary(obj) sollya_lib_is_hexadecimal(obj) sollya_lib_is_file(obj) sollya_lib_is_postscript(obj) sollya_lib_is_postscriptfile(obj) sollya_lib_is_perturb(obj) sollya_lib_is_round_down(obj) sollya_lib_is_round_up(obj) sollya_lib_is_round_towards_zero(obj) sollya_lib_is_round_to_nearest(obj) sollya_lib_is_honorcoeffprec(obj) sollya_lib_is_true(obj) sollya_lib_is_false(obj) sollya_lib_is_void(obj) sollya_lib_is_default(obj) sollya_lib_is_decimal(obj) sollya_lib_is_absolute(obj) sollya_lib_is_relative(obj) sollya_lib_is_fixed(obj) sollya_lib_is_floating(obj) sollya_lib_is_double_obj(obj) sollya_lib_is_single_obj(obj) sollya_lib_is_quad_obj(obj) sollya_lib_is_halfprecision_obj(obj) sollya_lib_is_doubleextended_obj(obj) sollya_lib_is_double_double_obj(obj) sollya_lib_is_triple_double_obj(obj) sollya_lib_is_pi(obj)

## 10.7 Recovering the value of a range

If a `sollya_obj_t` is a range, it is possible to recover the values corresponding to the bounds of the range. The range can be recovered either as a `mpfi_t` or as two `mpfr_t` (one per bound). This is achieved with the following conversion functions:

- `int sollya_lib_get_interval_from_range(mpfi_t res, sollya_obj_t arg),`
- `int sollya_lib_get_bounds_from_range(mpfr_t res_left, mpfr_t res_right, sollya_obj_t arg).`

They return a boolean integer: false means failure (i.e., if the `sollya_obj_t` is not a range) and true means success. These functions follow the same conventions as those of the MPFR and MPFI libraries: the variables `res`, `res_left` and `res_right` must be initialized beforehand, and are used to store the result of the conversion. Also, the functions `sollya_lib_get_something_from_range` **do not change the internal precision** of `res`, `res_left` and `res_right`. If the internal precision is sufficient to perform the conversion without rounding, then it is guaranteed to be exact. If, on the contrary, the internal precision is not sufficient, the actual bounds of the range stored in `arg` will be rounded at the target precision using a rounding mode that ensures that the inclusion property remains valid, i.e.  $\arg \subseteq \text{res}$  (resp.  $\arg \subseteq [\text{res\_left}, \text{res\_right}]$ ).

Function `int sollya_lib_get_prec_of_range(mp_prec_t *prec, sollya_obj_t arg)` stores at `*prec` a precision that is guaranteed to be sufficient to represent the range stored in `arg` without rounding. The returned value of this function is a boolean that follows the same convention as above. In conclusion, this is an example of a completely safe conversion:

```
...
mp_prec_t prec;
mpfr_t a, b;

if (!sollya_lib_get_prec_of_range(&prec, arg)) {
    sollya_lib_printf("Unexpected error: %b is not a range\n", arg);
}
else {
    mpfr_init2(a, prec);
    mpfr_init2(b, prec);
    sollya_lib_get_bounds_from_range(a, b, arg);

    /* Now [a, b] = arg exactly */
}
...
```

## 10.8 Recovering the value of a numerical constant or a constant expression

From a conceptual point of view, a numerical constant is nothing but a very simple constant functional expression. Hence there is no difference in `Sollya` between the way constants and constant expressions are handled. The functions presented in this Section allow one to recover the value of such constants or constant expressions into usual C data types.

A constant expression being given, three cases are possible:

- When naively evaluated at the current global precision, the expression always leads to provably exact computations (i.e., at each step of the evaluation, no rounding happens). For instance numerical constants or simple expressions such as  $(\exp(0) + 5)/16$  fall in this category.
- The constant expressions would be exactly representable at some precision but this is not straightforward from a naive evaluation at the current global precision. An example would be  $\sin(\pi/3)/\sqrt{3}$  or even  $1 + 2^{-\text{prec}-10}$ .
- Finally, a third possibility is that the value of the expression is not exactly representable at any precision on a binary floating-point number. Possible examples are  $\pi$  or  $1/10$ .

From now on, we suppose that `arg` is a `sollya_obj_t` that contains a constant expression (or, as a particular case, a numerical constant). The general scheme followed by the conversion functions is the following: **Sollya** chooses an initial working precision greater than the target precision. If the value of `arg` is easily proved to be exactly representable at that precision, **Sollya** first computes this exact value and then rounds it to the nearest number of the target format (ties-to-even). Otherwise, **Sollya** tries to adapt the working precision automatically in order to ensure that the result of the conversion is one of both floating-point numbers in the target format that are closest the exact value (a faithful rounding). A warning message indicates that the conversion is not exact and that a faithful rounding has been performed. In some cases really hard to evaluate, the algorithm can even fail to find a faithful rounding. In that case, too, a warning message is emitted indicating that the result of the conversion should not be trusted. Let us remark that these messages can be caught instead of being displayed and adapted handling can be provided by the user of the library at each emission of a warning (see Section 10.16).

The conversion functions are the following. They return a boolean integer: false means failure (i.e., `arg` is not a constant expression) and true means success.

- `int sollya_lib_get_constant_as_double(double *res, sollya_obj_t arg)`
- `int sollya_lib_get_constant_as_int(int *res, sollya_obj_t arg)`
- `int sollya_lib_get_constant_as_int64(int64_t *res, sollya_obj_t arg)`
- `int sollya_lib_get_constant_as_uint64(uint64_t *res, sollya_obj_t arg)`
- `int sollya_lib_get_constant(mpfr_t res, sollya_obj_t arg)`: the result of the conversion is stored in `res`. Please note that `res` must be initialized beforehand and that its internal precision is not modified by the algorithm.

Function `int sollya_lib_get_prec_of_constant(mp_prec_t *prec, sollya_obj_t arg)` tries to find a precision that would be sufficient to exactly represent the value of `arg` without rounding. If it manages to find such a precision, it stores it at `*prec` and returns true. If it does not manage to find such a precision, or if `arg` is not a constant expression, it returns false and `*prec` is left unchanged.

In conclusion, here is an example of use for converting a constant expression to a `mpfr_t`:

```
...
mp_prec_t prec;
mpfr_t a;
int test = 0;

test = sollya_lib_get_prec_of_constant(&prec, arg);
if (test) {
    mpfr_init2(a, prec);
    sollya_lib_get_constant(a, arg); /* Exact conversion */
}
else {
    mpfr_init2(a, 165); /* Initialization at some default precision */
    test = sollya_lib_get_constant(a, arg);
    if (!test) {
        sollya_lib_printf("Error: %b is not a constant expression\n", arg);
    }
}
...
```

## 10.9 Converting a string from Sollya to C

If `arg` is a `sollya_obj_t` that contains a string, that string can be recovered using

```
int sollya_lib_get_string(char **res, sollya_obj_t arg).
```

If **arg** really is a string, this function allocates enough memory on the heap to store the corresponding string, it copies the string at that newly allocated place, and sets **\*res** so that it points to it. The function returns a boolean integer: false means failure (i.e., **arg** is not a string) and true means success.

Since this function allocates memory on the heap, this memory should manually be cleared by the user with **sollya\_lib\_free** once it becomes useless.

## 10.10 Recovering the contents of a Sollya list

It is possible to recover the  $i$ -th element of a list **arg** (as one would do using **arg[i]** withing Sollya) with the following function:

```
int sollya_lib_get_element_in_list(sollya_obj_t *res, sollya_obj_t arg, int i).
```

It returns a boolean integer: false means failure (i.e. **arg** is not a list or the index is out of range) and true means success. In case of success, a copy of the  $i$ -th element of **arg** is stored at the address referred to by **res**. Since it is a copy, it should be cleared with **sollya\_lib\_clear\_obj** when it becomes useless. Please notice that this function works with regular lists as well as with end-elliptic lists, just as within the interactive tool.

Another function allows user to recover all elements of a list in a single call. This function returns a C array of **sollya\_obj\_t** objects and has the following signature:

```
int sollya_lib_get_list_elements(sollya_obj_t **L, int *n, int *end_ell,
                                sollya_obj_t arg).
```

Three cases are possible:

- If **arg** is a regular list of length  $N$ , the function allocates memory on the heap for  $N$  **sollya\_obj\_t**, sets **\*L** so that it points to that memory segment, and copies each of the elements  $N$  of **arg** to **(\*L)[0], ..., (\*L)[N-1]**. Finally, it sets **\*n** to  $N$ , **\*end\_ell** to zero and returns true. A particular case is when **arg** is the empty list: everything is the same except that no memory is allocated and **\*L** is left unchanged.
- If **arg** is an end-elliptic list containing  $N$  elements plus the ellipsis. The function allocates memory on the heap for  $N$  **sollya\_obj\_t**, sets **\*L** so that it points to that memory segment, and copies each of the elements  $N$  of **arg** at **(\*L)[0], ..., (\*L)[N-1]**. Finally, it sets **\*n** to  $N$ , **\*end\_ell** to a non-zero value and returns true. The only difference between a regular list and an end-elliptic list containing the same elements is hence that **\*end\_ell** is set to a non-zero value in the latter.
- If **arg** is neither a regular nor an end-elliptic list, **\*L**, **\*n** and **\*end\_ell** are left unchanged and the function returns false.

In case of success, please notice that **(\*L)[0], ..., (\*L)[N-1]** should manually be cleared with **sollya\_lib\_clear\_obj** when they become useless. Also, the pointer **\*L** itself should be cleared with **sollya\_lib\_free** since it points to a segment of memory allocated on the heap by Sollya.

## 10.11 Recovering the contents of a Sollya structure

If **arg** is a **sollya\_obj\_t** that contains a structure, the contents of a given field can be recovered using

```
int sollya_lib_get_element_in_structure(sollya_obj_t *res, char *name,
                                        sollya_obj_t arg).
```

If **arg** really is a structure and if that structure has a field named after the string **name**, this function copies the contents of that field into the Sollya object **\*res**. The function returns a boolean integer: false means failure (i.e., if **arg** is not a structure or if it does not have a field named after **name**) and true means success.

It is also possible to get all the field names and their contents. This is achieved through the function

```
int sollya_lib_get_structure_elements(char ***names, sollya_obj_t **objs, int *n,
                                     sollya_obj_t arg).
```

If `arg` really is a structure, say with  $N$  fields called “fieldA”, ..., “fieldZ”, this functions sets `*n` to  $N$ , allocates and fills an array of  $N$  strings and sets `*names` so that it points to that segment of memory (hence `(*names)[0]` is the string “fieldA”, ..., `(*names)[N-1]` is the string “fieldZ”). Moreover, it allocates memory for  $N$  `sollya_obj_t`, sets `*objs` so that it points on that memory segment, and copies the contents of each of the  $N$  fields at `(*objs)[0]`, ..., `(*objs)[N-1]`. Finally it returns true. If `arg` is not a structure, the function simply returns false without doing anything. Please note that since `*names` and `*objs` point to memory segments that have been dynamically allocated, they should manually be cleared by the user with `sollya_lib_free` once they become useless.

## 10.12 Decomposing a functional expression

If a `sollya_obj_t` contains a functional expression, one can decompose the expression tree using the following functions. These functions all return a boolean integer: true in case of success (i.e., if the `sollya_obj_t` argument really contains a functional expression) and false otherwise.

Table 5: List of values defined in type `sollya_base_function_t`

SOLLYA_BASE_FUNC_COS	SOLLYA_BASE_FUNC_DOUBLE	SOLLYA_BASE_FUNC_LOG
SOLLYA_BASE_FUNC_ACOS	SOLLYA_BASE_FUNC_DOUBLEDDOUBLE	SOLLYA_BASE_FUNC_LOG_2
SOLLYA_BASE_FUNC_ACOSH	SOLLYA_BASE_FUNC_DOUBLEEXTENDED	SOLLYA_BASE_FUNC_LOG_10
SOLLYA_BASE_FUNC_COSH	SOLLYA_BASE_FUNC_TRIPLEDDOUBLE	SOLLYA_BASE_FUNC_LOG_1P
SOLLYA_BASE_FUNC_SIN	SOLLYA_BASE_FUNC_HALFPRECISION	SOLLYA_BASE_FUNC_EXP
SOLLYA_BASE_FUNC_ASIN	SOLLYA_BASE_FUNC_SINGLE	SOLLYA_BASE_FUNC_EXP_M1
SOLLYA_BASE_FUNC_ASINH	SOLLYA_BASE_FUNC_QUAD	SOLLYA_BASE_FUNC_NEG
SOLLYA_BASE_FUNC_SINH	SOLLYA_BASE_FUNC_FLOOR	SOLLYA_BASE_FUNC_SUB
SOLLYA_BASE_FUNC_TAN	SOLLYA_BASE_FUNC_CEIL	SOLLYA_BASE_FUNC_ADD
SOLLYA_BASE_FUNC_ATAN	SOLLYA_BASE_FUNC_NEARESTINT	SOLLYA_BASE_FUNC_MUL
SOLLYA_BASE_FUNC_ATANH	SOLLYA_BASE_FUNC_LIBRARYCONSTANT	SOLLYA_BASE_FUNC_DIV
SOLLYA_BASE_FUNC_TANH	SOLLYA_BASE_FUNC_LIBRARYFUNCTION	SOLLYA_BASE_FUNC_POW
SOLLYA_BASE_FUNC_ERF	SOLLYA_BASE_FUNC_PROCEDUREFUNCTION	SOLLYA_BASE_FUNC_SQRT
SOLLYA_BASE_FUNC_ERFC	SOLLYA_BASE_FUNC_FREE_VARIABLE	SOLLYA_BASE_FUNC_PI
SOLLYA_BASE_FUNC_ABS	SOLLYA_BASE_FUNC_CONSTANT	

- `int sollya_lib_get_function_arity(int *n, sollya_obj_t f)`: it stores the arity of the head function in `f` at the address referred to by `n`. Currently, the mathematical functions handled in Sollya are at most dyadic. Mathematical constants are considered as 0-adic functions. The free variable is regarded as the identity function applied to the free variable: its arity is hence 1.
- `int sollya_lib_get_head_function(sollya_base_function_t *type, sollya_obj_t f)`: it stores the type of `f` at the address referred to by `type`. The `sollya_base_function_t` is an enum type listing all possible cases (see Table 5).
- `int sollya_lib_get_subfunctions(sollya_obj_t f, int *n, ...)`: let us denote by `g1`, ..., `gk` the arguments following the argument `n`. They must be of type `sollya_obj_t *`. The function stores the arity of `f` at the address referred to by `n` (except if `n` is NULL, in which case, `sollya_lib_get_subfunctions` simply ignores it and goes on). Suppose that `f` contains an expression of the form  $f_0(f_1, \dots, f_s)$  (as a particular case, if `f` is just the free variable, it is regarded in this context as the identity function applied to the free variable, so both  $f_0$  and  $f_1$  are the free variable). For each  $i$  from 1 to  $s$ , the expression corresponding to  $f_i$  is stored at the address referred to by `gi`, unless one of the `gi` is NULL in which case the function returns when encountering it. In practice, it means that the user should always put NULL as last argument, in order to



prevent the case when they would not provide enough variables `g_i`. They can check afterwards that they provided enough variables by checking the value contained at the address referred to by `n`. If the user does not put `NULL` as last argument and do not provide enough variables `g_i`, the algorithm will continue storing arguments at random places in the memory (on the contrary, providing more arguments than necessary does not harm: useless arguments are simply ignored and left unchanged). In the case when  $f_0$  is a library function, a library constant or a procedure function, and if the user provides a non-NULL argument `g_t` after `g_s`, additional information is returned in the remaining argument:

- If  $f_0$  is a library function, a `Sollya` object corresponding to the expression  $f_0(x)$  is stored at the address referred to by `g_t`. This allows the user to get a `Sollya` object corresponding to function  $f_0$ . This object can further be used to evaluate  $f_0$  at points or to build new expressions involving  $f_0$ . Please notice that a library function object is not necessarily the result of a call to the `library` command: it can also be, e.g., the derivative of a function created by a call to `library`.
- If  $f_0$  is a procedure function, a `Sollya` object corresponding to the expression  $f_0(x)$  is stored at the address referred to by `g_t`. The same remarks as above apply.
- If  $f_0$  is a library constant,  $f_0$  itself is stored at the address referred to by `g_t`. In this particular case,  $t = 1$  and the object referred to by `g_t` simply gets a copy of `f`. This (somehow useless) mechanism is made only to handle the cases of library functions, procedure functions and library constants in a unified way.

Please note that the objects that have been stored in variables `g_i` must manually be cleared once they become useless.

- `int sollya_lib_v_get_subfunctions(sollya_obj_t f, int *n, va_list va)`: the same as the previous function, but with a `va_list` argument.
- `int sollya_lib_decompose_function(sollya_obj_t f, sollya_base_function_t *type, int *n, ...)`:  
this function is a all-in-one function equivalent to using `sollya_lib_get_head_function` and `sollya_lib_get_subfunctions` in only one function call.
- `int sollya_lib_v_decompose_function(sollya_obj_t f, sollya_base_function_t *type, int *n, va_list va)`:  
the same as the previous function, but with a `va_list`.

As an example of use of these functions, the following code returns 1 if `f` denotes a functional expression made only of constants (i.e., without the free variable), and returns 0 otherwise:

```

#include <sollya.h>

/* Note: we suppose that the library has already been initialized */
int is_made_of_constants(sollya_obj_t f) {
    sollya_obj_t tmp1 = NULL;
    sollya_obj_t tmp2 = NULL;
    int n, r, res;
    sollya_base_function_t type;

    r = sollya_lib_decompose_function(f, &type, &n, &tmp1, &tmp2, NULL);
    if (!r) { sollya_lib_printf("Not a mathematical function\n"); res = 0; }
    else if (n >= 3) {
        sollya_lib_printf("Unexpected error: %b has more than two arguments.\n", f);
        res = 0;
    }
    else {
        switch (type) {
            case SOLLYA_BASE_FUNC_FREE_VARIABLE: res = 0; break;
            case SOLLYA_BASE_FUNC_PI: res = 1; break;
            case SOLLYA_BASE_FUNC_CONSTANT: res = 1; break;
            case SOLLYA_BASE_FUNC_LIBRARYCONSTANT: res = 1; break;
            default:
                res = is_made_of_constants(tmp1);
                if ((res) && (n==2)) res = is_made_of_constants(tmp2);
        }
    }

    if (tmp1) sollya_lib_clear_obj(tmp1);
    if (tmp2) sollya_lib_clear_obj(tmp2);

    return res;
}

```

Functions are provided to allow the user to retrieve further information from library function, library constant or procedure function objects:

- `int sollya_lib_decompose_libraryfunction(int (**f)(mpfi_t, mpfi_t, int),  
int *deriv, sollya_obj_t *e,  
sollya_obj_t g):`

assume that  $g$  represents an expression  $f_0(f_1)$  where  $f_0$  is a library function. Then,  $f_0$  is the  $n$ -th derivative (for some  $n$ ) of a function provided within Sollya via an external C function `int func(mpfi_t, mpfi_t, int)`.

As a result of a call to `sollya_lib_decompose_libraryfunction`, the value  $n$  is stored at the address referred to by `deriv`, a pointer to `func` is stored at the address referred to by `f` and a Sollya object representing  $f_1$  is stored at the address referred to by `e`. Please notice that the object stored in `e` must manually be cleared once it becomes useless. Upon success, a boolean integer representing true is returned. If  $g$  is not a library function object, nothing happens and false is returned.

- `int sollya_lib_decompose_procedurefunction(sollya_obj_t *f, int *deriv,  
sollya_obj_t *e, sollya_obj_t g):`

assume that  $g$  represents an expression  $f_0(f_1)$  where  $f_0$  is a procedure function. Then,  $f_0$  is the  $n$ -th derivative (for some  $n$ ) of a function provided within Sollya via a procedure `proc(X, n, p) {...}`. As a result of a call to `sollya_lib_decompose_procedurefunction`, the value  $n$  is stored at the address referred to by `deriv`, a Sollya object representing the procedure is stored at the address referred to by `f`, a Sollya object representing  $f_1$  is stored at the address referred to by `e`. Please

notice that the objects stored in **f** and **e** must manually be cleared once they become useless. Upon success, a boolean integer representing true is returned. If **g** is not a procedure function object, nothing happens and false is returned.

- `int sollya_lib_decompose_libraryconstant(void (**f)(mpfr_t, mp_prec_t),  
sollya_obj_t c):`  
assume that **c** is a constant provided via an external C function `void func(mpfr_t, mp_prec_t)`. As a result of a call to `sollya_lib_decompose_libraryconstant`, a pointer to `func` is stored at the address referred to by **f** and a boolean integer representing true is returned. Otherwise, nothing happens and false is returned.

### 10.13 Faithfully evaluate a functional expression

Let us suppose that **f** is a functional expression and **a** is a numerical value or a constant expression. One of the very convenient features of the interactive tool is that the user can simply write **f(a)** at the prompt: the tool automatically adapts its internal precision in order to compute a value that is a faithful rounding (at the current tool precision) of the true value  $f(a)$ . Sometimes it does not achieve to find a faithful rounding, but in any case, if the result is not proved to be exact, a warning is displayed explaining how confident one should be with respect to the returned value. This feature is made available within the library with the two following functions:

- `sollya_fp_result_t  
sollya_lib_evaluate_function_at_constant_expression(mpfr_t res, sollya_obj_t f,  
sollya_obj_t a,  
mpfr_t *cutoff),`
- `sollya_fp_result_t  
sollya_lib_evaluate_function_at_point(mpfr_t res, sollya_obj_t f,  
mpfr_t a, mpfr_t *cutoff).`

In the former, the argument **a** is any `sollya_obj_t` containing a numerical constant or a constant expression, while in the latter **a** is a constant already stored in a `mpfr_t`. These functions store the result in **res** and return a `sollya_fp_result_t` which is an enum type described in Table 6. In order to understand the role of the `cutoff` parameter and the value returned by the function, it is necessary to describe the algorithm in a nutshell:

---

**Input:** a functional expression **f**, a constant expression **a**, a target precision  $q$ , a parameter  $\varepsilon$ .

1. Choose an initial working precision  $p$ .
2. Evaluate **a** with interval arithmetic, performing the computations at precision  $p$ .
3. Replace the occurrences of the free variable in **f** by the interval obtained at step 2. Evaluate the resulting expression with interval arithmetic, performing the computations at precision  $p$ . This yields an interval  $I = [x, y]$ .
4. Examine the following cases successively (RN denotes rounding to nearest at precision  $q$ ):
  - (a) If  $\text{RN}(x) = \text{RN}(y)$ , set **res** to that value and return.
  - (b) If  $I$  does not contain any floating-point number at precision  $q$ , set **res** to one of both floating-point numbers enclosing  $I$  and return.
  - (c) If  $I$  contains exactly one floating-point number at precision  $q$ , set **res** to that number and return.
  - (d) If all numbers in  $I$  are smaller than  $\varepsilon$  in absolute value, then set **res** to 0 and return.
  - (e) If  $p$  has already been increased many times, then set **res** to some value in  $I$  and return.
  - (f) Otherwise, increase  $p$  and go back to step 2.

---

The target precision  $q$  is chosen to be the precision of the `mpfr_t` variable `res`. The parameter  $\varepsilon$  corresponds to the parameter `cutoff`. The reason why `cutoff` is a pointer is that, most of the time, the user may not want to provide it, and using a pointer makes it possible to pass `NULL` instead. So, if `NULL` is given,  $\varepsilon$  is set to 0. If `cutoff` is not `NULL`, the absolute value of `*cutoff` is used as value for  $\varepsilon$ . Using a non-zero value for  $\varepsilon$  can be useful when one does not care about the precise value of  $f(a)$  whenever its absolute value is below a given threshold. Typically, if one wants to compute the maximum of  $|f(a_1)|, \dots, |f(a_n)|$ , it is not necessary to spend too much effort on the computation of  $|f(a_i)|$  if one already knows that it is smaller than  $\varepsilon = \max\{|f(a_1)|, \dots, |f(a_{i-1})|\}$ .

In the interactive tool, it is also possible to write `f(a)` when `a` contains an interval: `Sollya` performs the evaluation using an enhanced interval arithmetic, e.g., using L'Hopital's rule to produce finite (yet valid of course) enclosures even in cases when  $f$  exhibits removable singularities (for instance  $\sin(x)/x$  over an interval containing 0). This feature is achieved in the library with the function

```
int sollya_lib_evaluate_function_over_interval(mpfi_t res, sollya_obj_t f, mpfi_t a).
```

This function returns a boolean integer: `false` means failure (i.e., `f` is not a functional expression), in which case `res` is left unchanged, and `true` means success, in which case `res` contains the result of the evaluation. The function might succeed, and yet `res` might contain something useless such as an unbounded interval or even `[NaN, NaN]` (this happens for instance when `a` contains points that lie in the interior of the complement of the definition domain of `f`). It is the user's responsibility to check afterwards whether the computed interval is bounded, unbounded or `NaN`.

## 10.14 Name of the free variable

The default name for the free variable is the same in the library and in the interactive tool: it is `_x_`. In the interactive tool, this name is automatically changed at the first use of an undefined symbol. Accordingly in library mode, if an object is defined by `sollya_lib_parse_string` with an expression containing an undefined symbol, that symbol will become the free variable name if it has not already been changed before. But what if one does not use `sollya_lib_parse_string` (because it is not efficient) but one wants to change the name of the free variable? The name can be changed with `sollya_lib_name_free_variable("some_name")`.

It is possible to get the current name of the free variable with `sollya_lib_get_free_variable_name()`. This function returns a `char *` containing the current name of the free variable. Please note that this `char *` is dynamically allocated on the heap and should be cleared after its use with `sollya_lib_free()` (see below).

## 10.15 Commands and functions

Besides some exceptions, every command and every function available in the `Sollya` interactive tool has its equivalent (with a very close syntax) in the library. Section 8 of the present documentation gives the library syntax as well as the interactive tool syntax of each commands and functions. The same information is available within the interactive tool by typing `help some_command`. So if one knows the name of a command or function in the interactive tool, it is easy to recover its library name and signature.

There are some commands and functions available in interactive mode which, for syntactical reasons, have a different function name in the `Sollya` library:

- The `Sollya` language construction `(obj1)(obj2, obj3, ...)` which applies the object `obj1` to the objects `obj2, obj3, etc.` is expressed in the `Sollya` library through a call to  
`sollya_obj_t sollya_lib_apply(sollya_obj_t obj1, sollya_obj_t obj2, ...)`  
resp. `sollya_obj_t sollya_lib_v_apply(sollya_obj_t obj1, sollya_obj_t obj2, va_list).`

A particular point is worth mentioning: some functions of the tool such as `remez` for instance have a variable number of arguments. For instance, one might call `remez(exp(x), 4, [0,1])` or `remez(1,`

Table 6: List of values defined in type `sollya_fp_result_t`

Value	Meaning
<code>SOLLYA_FP_OBJ_NO_FUNCTION</code>	<code>f</code> is not a functional expression.
<code>SOLLYA_FP_EXPRESSION_NOT_CONSTANT</code>	<code>a</code> is not a constant expression.
<code>SOLLYA_FP_FAILURE</code>	The algorithm ended up at step (e) and $I$ contained NaN. This typically happens when $a$ is not in the definition domain of $f$ .
<code>SOLLYA_FP_CUTOFF_IS_NAN</code>	<code>cutoff</code> was not NULL and the value of <code>*cutoff</code> is NaN.
<code>SOLLYA_FP_INFINITY</code>	The algorithm ended up at step (a) with $I$ of the form $[+\infty, +\infty]$ or $[-\infty, -\infty]$ . Hence $f(a)$ is proved to be an exact infinity.
<code>SOLLYA_FP_PROVEN_EXACT</code>	The algorithm ended up at step (a) with a finite value and $x = \text{RN}(x) = \text{RN}(y) = y$ .
<code>SOLLYA_FP_CORRECTLY_ROUNDED_PROVEN_INEXACT</code>	The algorithm ended up at step (b) with a finite value and $\text{res} < x \leq y$ or $x \leq y < \text{res}$ .
<code>SOLLYA_FP_CORRECTLY_ROUNDED</code>	The algorithm ended up at step (a) with a finite value and $x \leq \text{res} \leq y$ . <sup>a</sup>
<code>SOLLYA_FP_FAITHFUL_PROVEN_INEXACT</code>	The algorithm ended up at step (b) with a finite value and $\text{res} < x \leq y$ or $x \leq y < \text{res}$ .
<code>SOLLYA_FP_FAITHFUL</code>	The algorithm ended up at step (c) with a finite value. <sup>a</sup>
<code>SOLLYA_FP_BELOW_CUTOFF</code>	The algorithm ended up at step (d).
<code>SOLLYA_FP_NOT_FAITHFUL_ZERO_CONTAINED_BELOW_THRESHOLD</code>	The algorithm ended up at step (e) and $I$ was of the form $[-\delta_1, \delta_2]$ where $0 < \delta_i \ll 1$ (below some threshold of the algorithm). This typically happens when $f(a)$ exactly equals zero, but the algorithm does not manage to prove this exact equality.
<code>SOLLYA_FP_NOT_FAITHFUL_ZERO_CONTAINED_NOT_BELOW_THRESHOLD</code>	The algorithm ended up at step (e) with an interval $I$ containing 0 but too large to fall in the above case. <sup>b</sup>
<code>SOLLYA_FP_NOT_FAITHFUL_ZERO_NOT_CONTAINED</code>	The algorithm ended up at step (e) with an interval $I$ that does not contain 0. <sup>b</sup>
<code>SOLLYA_FP_NOT_FAITHFUL_INFINITY_CONTAINED</code>	The algorithm ended up at step (e) and (at least) one of the bounds of $I$ was infinite. This typically happens when the limit of $f(x)$ when $x$ goes to $a$ is infinite.

<sup>a</sup>Please notice that this means that the algorithm did not manage to conclude whether the result is exact or not. However, it might have been able to conclude if the working precision had been increased.

<sup>b</sup>In general, this should be considered as a case of failure and the value stored in `res` might be completely irrelevant.

4,  $[0,1]$ ,  $1/\exp(x)$ ). This feature is rendered in the C library by the use of variadic functions (functions with an arbitrary number of arguments), as they are permitted by the C standard. The notable difference is that there must **always be an explicit NULL argument** at the end of the function call. Hence one can write `sollya_lib_remez(a, b, c, NULL)` or `sollya_lib_remez(a, b, c, d, NULL)`. It is very easy to forget the NULL argument and to use for instance `sollya_lib_remez(a, b, c)`. This is **completely wrong** because the memory will be read until a NULL pointer is found. In the best case, this will lead to an error or a result obviously wrong, but it could also lead to subtle, not-easy-to-debug errors. The user is advised to be particularly careful with respect to this point.

Each command or function accepting a variable number of arguments comes in a `sollya_lib_v_` version accepting a `va_list` parameter containing the list of optional arguments. For instance, one might write a function that takes as arguments a function  $f$ , an interval  $I$ , optionally a weight function  $w$ , optionally a quality parameter  $q$ . That function would display the minimax obtained when approximating  $f$  over  $I$  (possibly with weight  $w$  and quality  $q$ ) by polynomials of degree  $n = 2$  to 20. So, that function would get a variable number of arguments (i.e. a `va_list` in fact) and pass them straight to `remez`. In that case, one needs to use the `v_remez` version, as the following code shows:

```
#include <sollya.h>
#include <stdarg.h>

/* Note: we suppose that the library has already been initialized */
void my_function(sollya_obj_t f, sollya_obj_t I, ...) {
    sollya_obj_t n, res;
    int i;
    va_list va;

    for(i=2;i<=20;i++) {
        n = SOLLYA_CONST(i);
        va_start(va, I);
        res = sollya_lib_v_remez(f, n, I, va);
        sollya_lib_printf("Approximation of degree %b is %b\n", n, res);
        va_end(va);
        sollya_lib_clear_obj(n);
        sollya_lib_clear_obj(res);
    }

    return;
}
```

## 10.16 Warning messages in library mode

The philosophy of Sollya is “whenever something is not exact, explicitly warn about that”. This is a nice feature since this ensures that the user always perfectly knows the degree of confidence they can have in a result (is it exact? or only faithful? or even purely numerical, without any warranty?) However, it is sometimes desirable to hide some (or all) of these messages. This is especially true in library mode where messages coming from Sollya are intermingled with the messages of the main program. The library hence provides a specific mechanism to catch all messages emitted by the Sollya core and handle each of them specifically: installation of a callback for messages.

Before describing the principle of the message callback, it seems appropriate to recall that several mechanisms are available in the interactive tool to filter the messages emitted by Sollya. These mechanisms are also available in library mode for completeness. When a message is emitted, it has two characteristics: a verbosity level and an id (a number uniquely identifying the message). After it has been emitted, it passes through the following steps where it can be filtered. If it has not been filtered (and only in this case) it is displayed.

1. If the verbosity level of the message is greater than the value of the environment variable `verbosity`, it is filtered.

2. If the environment variable `roundingwarnings` is set to `off` and if the message informs the user that a rounding occurred, it is filtered.
3. If the id of the message has been registered with the `suppressmessage` command, the message is filtered.
4. If a message callback has been installed and if the message has not been previously filtered, it is handled by the callback, which decides to filter it or to permit its displaying.

A message callback is a function of the form `int my_callback(sollya_msg_t msg, void *data)`. It receives as input an object representing the message and a user-defined pointer. It performs whatever treatment seems appropriate and returns an integer interpreted as a boolean. If the returned value is false, the message is not displayed. If, on the contrary, the returned value is true, the message is displayed as usual. By default, no callback is installed and all messages are displayed. To install a callback, use `sollya_lib_install_msg_callback(my_callback, data)`. The `(void *)` pointer `data` is arbitrary (it can be `NULL`) and is simply transmitted as second argument at each call of the callback. It can be used, e.g., to point to a segment of memory where some information should be stored from a call of the callback to another.

Please remember that, if a message is filtered because of one of the three other mechanisms, it will never be transmitted to the callback. Hence, in library mode, if one wants to catch every single message through the callback, one should set the value of `verbosity` to `MAX_INT`, set `roundingwarnings` to `on` (this is the default anyway) and one should not use the `suppressmessage` mechanism.

It is possible to come back to the default behavior, using `sollya_lib_uninstall_msg_callback()`. Please notice that callbacks do not stack over each other: i.e., if some callback `callback1` is installed, and if one installs another one `callback2`, then the effect of `sollya_lib_uninstall_msg_callback()` is to come back to the default behavior, **and not** to come back to callback `callback1`.

Both `sollya_lib_install_msg_callback` and `sollya_lib_uninstall_msg_callback` return an integer interpreted as a boolean: false means failure and true means success.

It is possible to get the current callback using `sollya_lib_get_msg_callback(cb_ptr, data_ptr)`. This stores the current callback at the address referred to by `cb_ptr` (the type of `cb_ptr` is hence `int (**)(sollya_msg_t, void *)`) and stores the current data pointer at the address referred to by `data_ptr` (which has hence `(void **)` type). The arguments `cb_ptr` and `data_ptr` can be `NULL` in which case the corresponding argument is not retrieved (please take care of the difference between `data_ptr` being `NULL` and `data_ptr` pointing to a `(void *)` pointer which value is `NULL`). If no callback is currently installed, the `NULL` value is stored at the addresses referred to by `cb_ptr` and `data_ptr`.

The type `sollya_msg_t` is indeed a pointer and its content is only accessible during the callback call: it does not make sense to keep it for further use after the callback call. Currently the type has only two accessors:

- `int sollya_lib_get_msg_id(sollya_msg_t msg)` returns an integer that identifies the type of the message. The message types are listed in the file `sollya-messages.h`. Please note that this file not only lists the possible identifiers but only defines meaningful names to each possible message number (e.g., `SOLLYA_MSG_UNDEFINED_ERROR` is an alias for the number 2 but is more meaningful to understand what the message is about). It is recommended to use these names instead of numerical values.
- `char *sollya_lib_msg_to_text(sollya_msg_t msg)` returns a generic string briefly summarizing the contents of the message. Please note that this `char *` is dynamically allocated on the heap and should manually be cleared with `sollya_lib_free` when it becomes useless.

In the future, other accessors could be added (to get the verbosity level at which the message has been emitted, to get data associated with the message, etc.) The developers of `Sollya` are open to suggestions and feature requests on this subject.

As an illustration let us give a few examples of possible use of callbacks:

**Example 1:** A callback that filters everything.

```

int hide_everything(sollya_msg_t msg, void *data) {
    return 0;
}

```

**Example 2:** filter everything but the messages indicating that a comparison is uncertain.

```

int keep_comparison_warnings(sollya_msg_t msg, void *data) {
    switch(sollya_lib_get_msg_id(msg)) {
        case SOLLYA_MSG_TEST_RELIES_ON_FP_RESULT_THAT_IS_NOT_FAITHFUL:
        case SOLLYA_MSG_TEST_RELIES_ON_FP_RESULT:
        case SOLLYA_MSG_TEST_RELIES_ON_FP_RESULT_FAITHFUL_BUT_UNDECIDED:
        case SOLLYA_MSG_TEST_RELIES_ON_FP_RESULT_FAITHFUL_BUT_NOT_REAL:
            return 1;
        default:
            return 0;
    }
}

```

**Example 3:** ensuring perfect silence for a particular function call (uses the callback defined in Example 1).

```

...
int (*old_callback)(sollya_msg_t, void *);
void *old_data;
sollya_lib_get_msg_callback(&old_callback, &old_data);
sollya_lib_install_msg_callback(hide_everything, NULL);
/* Here takes place the function call that must be completely silent */
if (old_callback) sollya_lib_install_msg_callback(old_callback, old_data);
...

```

**Example 4:** using the (void \*) data argument to store information from a call to another.

```

int set_flag_on_problem(sollya_msg_t msg, void *data) {
    switch(sollya_lib_get_msg_id(msg)) {
        case SOLLYA_MSG_DOUBLE_ROUNDING_ON_CONVERSION:
            *((int *) (data)) = 1;
    }
    return 1;
}

...

int main() {
    int flag_double_rounding = 0;
    ...
    sollya_lib_init();
    sollya_lib_install_msg_callback(set_flag_on_problem, &flag_double_rounding);
    ...
}

```

More involved examples are possible: for instance, instead of setting a flag, it is possible to keep in some variable what the last message was. One may even implement a stack mechanism and store the messages in a stack, in order to handle them later. (Please remember however that `sollya_msg_t` is a pointer type and that the `sollya_msg_t` object received as argument of a callback call has no more meaning once the callback call returned. If a stack mechanism is implemented it should store



information such as the message ID, or the message text, as given by `sollya_lib_get_msg_id` and `sollya_lib_msg_to_text`, but not the `sollya_msg_t` object itself.)

## 10.17 Using Sollya in a program that has its own allocation functions

Sollya uses its own allocation functions: as a consequence, pointers that have been allocated by Sollya functions must be freed using `sollya_lib_free` instead of the usual `free` function. Another consequence is that Sollya registers its own allocation functions to the GMP library, using the mechanism provided by GMP, so that GMP also uses Sollya allocation functions behind the scene, when the user performs a call to, e.g., `mpz_init`, `mpfr_init2`, etc.

In general, this is completely harmless and the user might even not notice it. However, this is a problem if Sollya is used in a program that also uses its own allocation functions and that has already registered these functions to GMP. Actually:

- If the main program has already registered allocation functions to GMP and if Sollya is naively initialized with `sollya_lib_init()`, Sollya will register its own allocation functions, thus overriding the previously registered functions.
- If the user initializes first Sollya, and then registers its own allocation functions to GMP, the exact opposite happens: Sollya allocation functions are overridden by those of the user, and this will likely cause Sollya to crash (or worst, silently behave not reliably).

In order to solve this issue, Sollya provides a chaining mechanism that we are now going to describe. The idea is the following: suppose that the main program should use a function `custom_malloc`. The user should not use `mp_set_memory_functions` as usual, but should instead initialize Sollya with the initializing function described above. This will cause Sollya to register an allocation function `sollya_lib_malloc` to GMP. This function overloads `custom_malloc`: when called, it uses `custom_malloc` to perform the actual allocation and does nothing else but some internal accounting and verification for that allocation. To repeat, the actual allocation is done by `custom_malloc`; hence from the point of view of the user, the mechanism is completely transparent and equivalent to directly registering `custom_malloc` to GMP. The same holds for all other allocation functions: in particular, this is true for `free` as well: if a function `custom_free` is given at the initialization of Sollya, then the function `sollya_lib_free` eventually uses `custom_free` to free the memory.

The initialization function providing this mechanism is:

```
int sollya_lib_init_with_custom_memory_functions(
    void *(*custom_malloc)(size_t),
    void *(*custom_calloc)(size_t, size_t),
    void *(*custom_realloc)(void *, size_t),
    void (*custom_free)(void *),
    void *(*custom_realloc_with_size)(void *, size_t, size_t),
    void (*custom_free_with_size)(void *, size_t)).
```

None of the arguments is mandatory: if the user does not want to provide an argument, they may use `NULL` as a placeholder for that argument. In that case, the corresponding Sollya default function will be used. Indeed, the default initializing function `sollya_lib_init()` is just an alias to `sollya_lib_init_with_custom_memory_functions(NULL, NULL, NULL, NULL, NULL, NULL)`.

Please notice, that if `custom_malloc` is provided, then the function `sollya_lib_malloc` will be defined as an overloaded version of `custom_malloc`. Hence, `custom_malloc` will eventually be used for all the allocations performed by Sollya (including the allocation of memory for its own purpose). This is true also for `custom_calloc`, `custom_realloc` and `custom_free`. However, this is not the case for `custom_realloc_with_size` and `custom_free_with_size`: these functions are only required for the registration to GMP and are not used by Sollya itself (except of course when Sollya allocates function through a call to a GMP, MPFR or MPFI function). Thus, to sum up:

- If the user only wants to register their own functions to GMP through Sollya, they only need to provide `custom_malloc`, `custom_realloc_with_size` and `custom_free_with_size` at the initialization of Sollya (actually an overloaded version will be registered to GMP but this is transparent for the user, as explained above).

- If the user also wants **Sollya** to use their custom allocation functions for all allocations of memory by **Sollya**, then they also need to provide `custom_calloc`, `custom_realloc` and `custom_free`.

Of course, even if the user registers `custom_malloc`, `custom_free`, etc., at the initialization of **Sollya**, they stay free to use them for their own allocation needs: only allocations performed by **GMP** (and consequently **MPFR** and **MPFI**) and allocations performed by **Sollya** have to use the chaining mechanism. However, for the convenience of the user, the library also provides access to the allocation functions of **Sollya**. They are the following:

- `void sollya_lib_free(void *)`
- `void *sollya_lib_malloc(size_t)`
- `void *sollya_lib_calloc(size_t, size_t)`
- `void *sollya_lib_realloc(void *, size_t).`

No access to the overloaded version of `custom_realloc_with_size` and `custom_free_with_size` is provided, but if the user really wants to retrieve them, they can do it with `mp_get_memory_functions` since they are registered to **GMP**.