Spruce Budworm

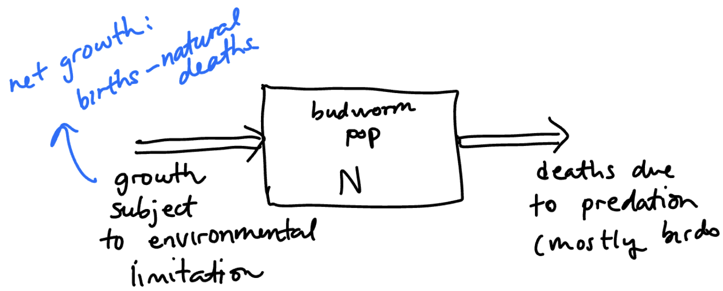
Spruce budworms are one of the most destructive insects for spruce / fir forests in Atlantic Canada. There is always a base population (endemic). Every so often though, there is an outbreak (epidemic). These happen on a roughly 30 - 40 year cycle.

Let's model this to understand how cyclical population emerges.

Idea:

- Low Budworm Population
  - Natural Predators: birds. These provide considerable mortality, creating a balance.
- High Budworm Population
  - Birds have a limited proportional effect.

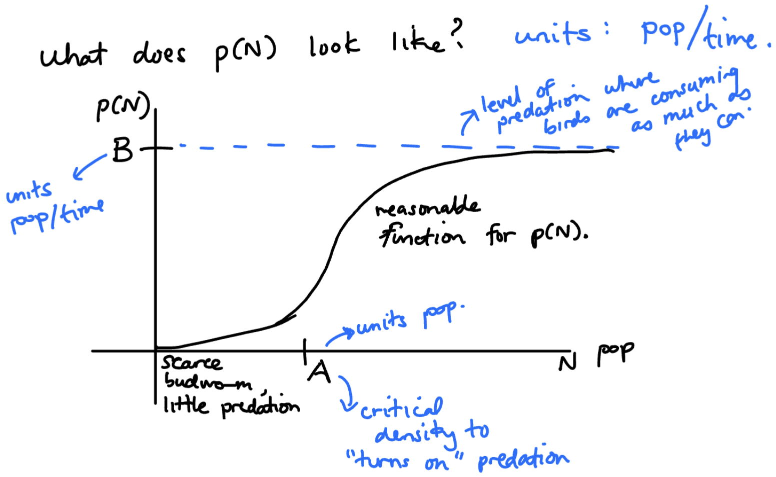
A sketch of our model is shown below:



We can use a rate in / out architecture to study this. Our rate in is going to be modelling by logistic growth, and we have an unknown function $P(N)$ representing bird predation.

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - P(N)$$

What does $P(N)$ look like? We know that it has units of population / time.



As drawn, $P(N)$ looks like a Hill function,

$$P(N) = \frac{BN^2}{A^2 + N}, A, B \neq 0$$

So the full budworm model is:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - \frac{BN^2}{A^2 + N}$$

- R: competition-free growth rate
- K: carrying capacity
- B: maximum predation rate
- A: critical population to initiate predator response

We have a problem here, there are 4 parameters. A phase portrait is pretty much hopeless, and a bifurcation diagram is only good for 1 parameter.

Dealing with many parameters:

- Do we know the parameters?
  - Yes: Am I looking to just simulate a solution as close to reality as possible?
    - Yes: Fix the parameters
- If the answer to either is no, then we can either numerically simulate over parameter combinations, or we can get rid of some parameters. To do this, we can perform nondimensionalization.

Follow the same approach as logistic model nondimensionalization. (skipping the details)

$$\frac{\hat{N}}{B\hat{T}}\dot{x} = \frac{R\hat{N}}{B}x\left(1 - \frac{\hat{N}x}{k}\right) - \frac{\left(\hat{N}x\right)^2}{A^2 + \left(\hat{N}x\right)^3}$$

Let $r = \frac{RA}{B}$, $k = \frac{K}{A}$, then letting $\hat{N} = A$, $\hat{T} = \frac{A}{B}$, this give us:

$$\dot{x} = rx\left(1 - \frac{x}{k}\right) - \frac{x^2}{1 + x^2}$$