Cardinality

Recall from last time:

- Two sets have the same size if and only if there is a bijection between them
- The size of a set gives an equivalence relation on sets
- The sets $\mathbb{N}, \mathbb{Z}$, and $\mathbb{Q}$ all have the same size.
- $|\mathbb{N}| = \aleph_0$
- Any set that is the same size as $\mathbb{N}$ is countably infinite, or listable. $A = \{a_1, a_2, \ldots\}$
- $|\mathbb{R}| > |\mathbb{N}|$, since we can't list all the reals, we assumed we could list them, but we were able to construct a real that isn't in our list using Cantor's Diagonalization proof.
- $|\mathbb{R}| = c$, for continuum

Question: is there a set with it's size between $\aleph_0$ and c? That is, is there a set A such that $|\mathbb{N}| < |A| < |\mathbb{R}|$

Definition : Continuum Hypothesis

There is no set whose cardinality is strictly between $|\mathbb{N}|$ and $|\mathbb{R}|$.

Based on the axioms of set theory, this is unprovable.

Example

Find a bijection between each of the following sets:

1. $(0, \infty)$ to $(1, \infty)$
2. $[0, 2]$ to $[-3, 5]$
3. $\mathbb{R}$ to $(0, \infty)$
4. $(1, \infty)$ to $(0, 1)$

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1. $f(x) = x + 1$
2. $f(x) = 4x - 3$
3. $f(x) = e^x$
4. $f(x) = \frac{1}{x}$