Exponential Moving Average

This emphasizes the values closer to the current day.

$$\text{EMA} = \text{value}_{\text{today}} \times \left(\frac{\text{smoothing}}{1+\text{days}}\right) + \text{EMA}_{\text{yesterday}} \times \left(1 - \left(\frac{\text{smoothing}}{1+\text{days}}\right)\right)$$

Smoothing is predetermined (often 2). Cold start problem: How to calculate the first EMA when EMA of previous day is needed? Calculate the first EMA by using SMA as the previous day's EMA.

Day	Cost	4-day EMA	4-day SMA
1	10		
2	9		
3	11		
4	12	10.5 (SMA)	10.5
5	6	7.9	9.5
6	13	9.94	10.5

$$\begin{aligned}
\text{EMA} &= \text{value}_{\text{today}} \times \left(\frac{\text{smoothing}}{1+\text{days}}\right) + \text{EMA}_{\text{yesterday}} \times \left(1 - \left(\frac{\text{smoothing}}{1+\text{days}}\right)\right) \\
\text{EMA}_t &= \left(\frac{\text{smoothing}}{1+\text{days}}\right) \times \text{value}_t + \left(1 - \left(\frac{\text{smoothing}}{1+\text{days}}\right)\right) \times \text{EMA}_{t-1} \\
&= \alpha \times \text{value}_t + (1-\alpha) \times \text{EMA}_{t-1} \\
&= \alpha \times \text{value}_t + (1-\alpha) \times (\alpha \times \text{value}_{t-1} + (1-\alpha) \times \text{EMA}_{t-2})
\end{aligned}$$

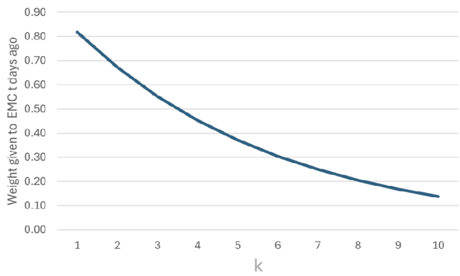
How much weight is given to the EMA two days ago?

$$(1-\alpha) \times (1-\alpha) - (1-\alpha)^2$$

Remember that $(1-\alpha)$ will be a fraction so, if smoothing $= 2$ and days $= 10$.

$$\alpha = \frac{2}{11}, \qquad 1-\alpha = \frac{9}{11}$$

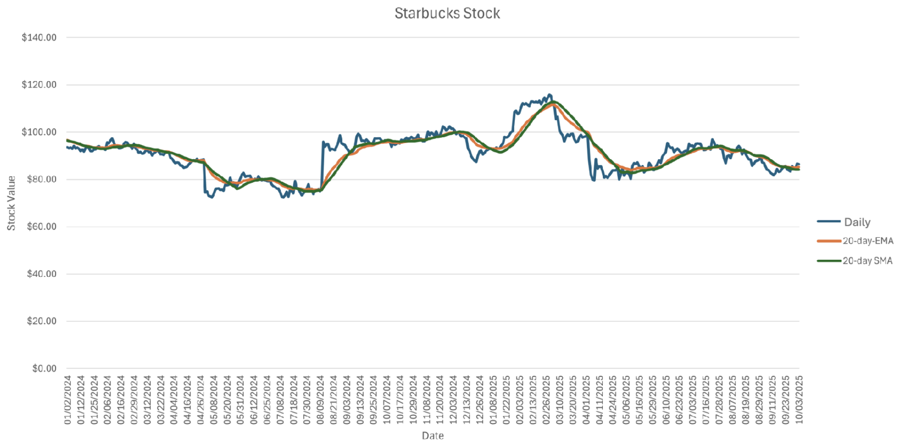
The weight given to EMA_{t-1} is $\frac{9}{11} = 0.82$. The weight given to EMA_{t-2} is $\left(\frac{9}{11}\right)^2 = 0.67$. We can generalize this to k days ago. The weight given to $\text{EMA}_k = (1-\alpha)^k$. Let $\alpha = \frac{2}{11}$

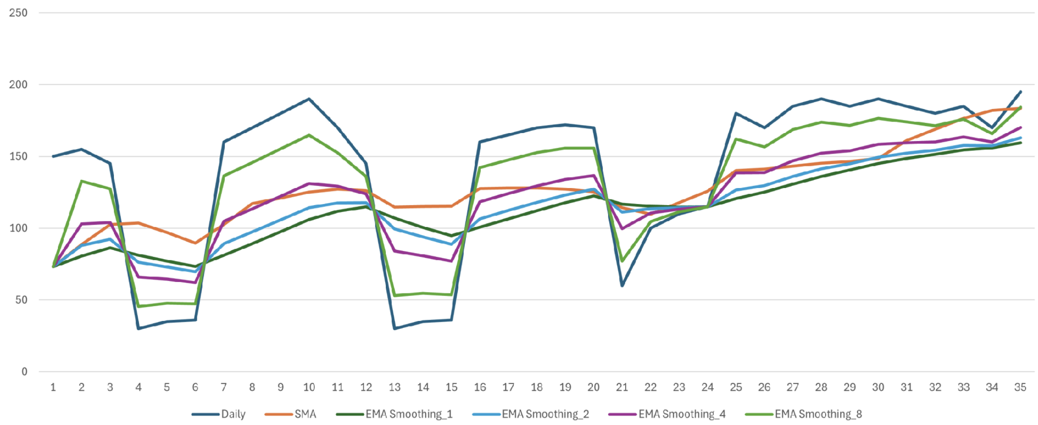


Consider $\text{EMA}_t = \left(\frac{2}{1+\text{days}}\right) \times \text{value}_t + \left(1 - \left(\frac{2}{1+\text{days}}\right)\right) \times \text{EMA}_{t-1}$

Days	Weight of value$_t$	Weight for EMA$_{t-1}$
1	1	0
2	0.67	0.33
10	0.18	0.82
20	0.10	0.90
50	0.04	0.96
200	0.01	0.99

Days	Weight of value$_t$ (EMA)	Weight of value$_t$ (SMA)
1	1	1
2	0.67	0.5
10	0.18	0.01
20	0.10	0.05
50	0.04	0.02
200	0.01	0.005





Variability

- Variability – A single number that represents the extent to which the values of a quantitative attribute in a dataset differ
    - Also called spread
    - A few ways to do this

- Range gives a quick simple sense of variability
  - range = max value − min value
- Variance gives a more detailed view of variability

Variance

$$\text{Variance} = \frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N - 1}$$

- $\bar{x}$ is the mean of the attribute, and N is the number of data points in the dataset.
- Variance is the average squared distance from each data point to the mean
- Variance does not keep the same units as the original dataset
- Squaring removes the issue of negative values and positive values canceling each other out.
- Consider:
  - Values = 19, 20, 20, 21, 22, $\bar{x} = 20.4$

$$\begin{aligned}\text{Variance} &= \frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N - 1} \\ &= \frac{(19 - 20.4)^2 + (20 - 20.4)^2 + (20 - 20.4)^2 + (21 - 20.4)^2 + (22 - 20.4)^2}{5 - 1} \\ &= 1.3\end{aligned}$$

- What we have been looking at so far is calculating the variance based on a sample from the entire population.

$$\text{Simple Variance } (S^2) = \frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N - 1}$$

- If we have every member of a population, we could calculate the population variance

$$\text{Population Variance } (\sigma^2) = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}$$

- Consider A: 1, 1, 99, 99, $\bar{x} = 50$

$$\begin{aligned}\text{Variance} &= \frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N - 1} \\ &= \frac{(1 - 50)^2 + (1 - 50)^2 + (99 - 50)^2 + (99 - 50)^2}{4 - 1} \\ &= 3201.33\end{aligned}$$

- Consider B: 41, 41, 59, 59, $\bar{x} = 50$

$$\begin{aligned}\text{Variance} &= \frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1} \\ &= \frac{(41-50)^2 + (41-50)^2 + (59-50)^2 + (59-50)^2}{4-1} \\ &= 108\end{aligned}$$

- The values don't appear to be related to the actual values in each dataset

Standard Deviation

- Standard Deviation of a Sample:

$$\text{SD} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}}$$

- Standard Deviation of a population:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}}$$

- Standard deviation is the square root of the variance
- This allows us to return to the original units of the dataset and makes it more interpretable
- This gives us a sense of how far, on average, each point deviates from the mean
- Consider the sets of values A and B from above.

$$\text{Standard Deviation of } A = \sqrt{3201.33} = 56.58$$

$$\text{Standard Deviation of } B = \sqrt{108} = 10.39$$