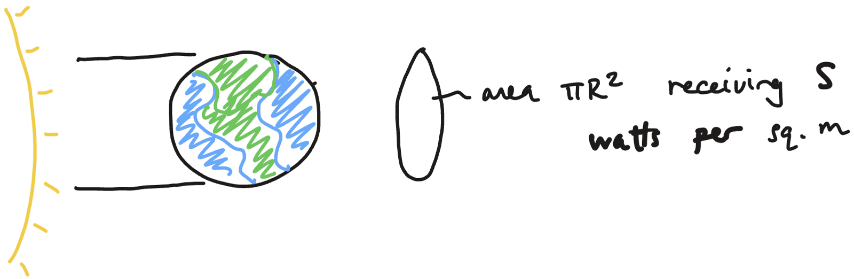
Energy Balance on Earth

Climate modelling can be highly complex and detailed. Here, we study a simple energy balance model for one part of the climate picture.

The unit of energy is Joules, J, and the rate of energy is J/sec (Watts)



Not all the energy is absorbed, some is reflected back. Consider $1 - \alpha(T)$, T is the temperature of Earth. Here, $\alpha(T)$ is the reflected energy, and $1 - \alpha(T)$ is the proportion of energy absorbed.

Total Energy In:

$$E_{\text{in}} = (1 - \alpha(T)) \cdot \pi R^2 \cdot S$$

Total Energy Out: The Earth radiates like a black body according to Stefan's Law: energy radiates out $\propto T^4$.

$$E_{\text{out}} = \varepsilon \cdot 4\pi R^2 \cdot \sigma T^4$$

Now put it together:

$$C \cdot \frac{dT}{dt} = E_{\text{in}} - E_{\text{out}} \qquad E_{\text{in}} = E_{\text{out}} \Rightarrow \frac{dT}{dt} = 0 \Rightarrow \text{equilibrium}$$

$$C \cdot \frac{dT}{dt} = \left[(1 - \alpha(T))\, \pi R^2 S\right] - \left[\varepsilon \cdot 4\pi R^2 \sigma T^4\right]$$

Now solve for the equilibria: $\frac{dT}{dt} = 0$.

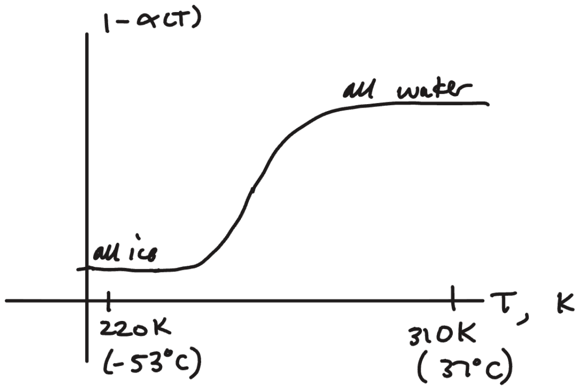
$$\begin{aligned} (1 - \alpha(T))\, \pi R^2 S &= \varepsilon \cdot 4\pi R^2 \sigma T^4 \\ (1 - \alpha(T))\, \frac{S}{4} &= \varepsilon \sigma T^4 \qquad \text{Let } Q = \frac{S}{4} \\ (1 - \alpha(T))\, Q &= \varepsilon \sigma T^4 \end{aligned}$$

What does $(1 - \alpha(T))$ look like? It is the energy reflection as a function of global temperature.

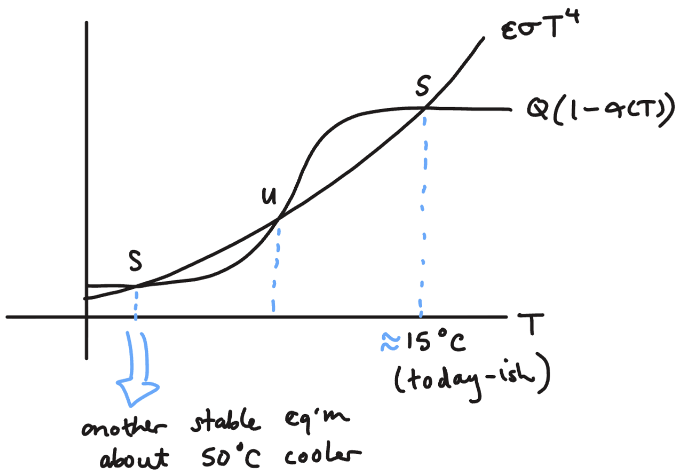
Note: Ice reflects more energy than water does

$$\begin{array}{c} \text{all ice} \Rightarrow \text{small } T \ ; \ \alpha(T) \text{ large} \ ; \ 1-\alpha(T) \text{ small} \\ \downarrow \\ \text{increase } T \ ; \ \text{lower } \alpha(T) \ ; \ 1-\alpha(T) \text{ larger} \\ \downarrow \\ \text{all water} \Rightarrow \text{large } T \ ; \ \alpha(T) \text{ saturates} \ ; \ 1-\alpha(T) \text{ largest; saturated} \end{array}$$

Let's summarize this in a plot

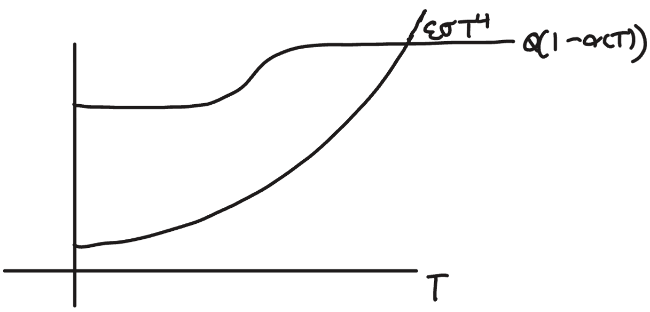


So the equilibria are intersections in the graph below

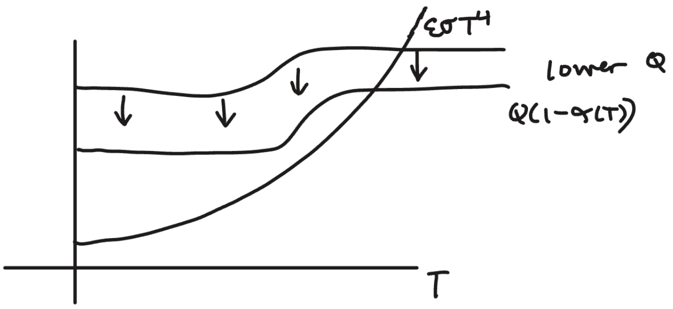


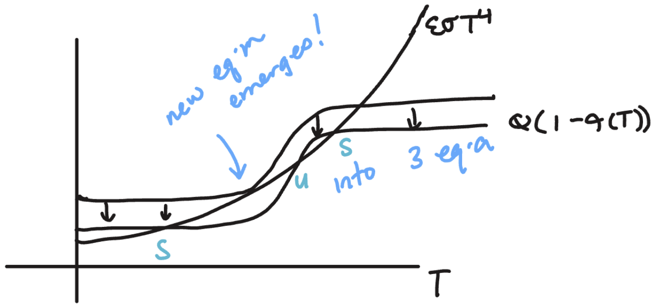
Study system by varying $Q = \frac{S}{4} \rightarrow$ solar radiation (W/m^2)

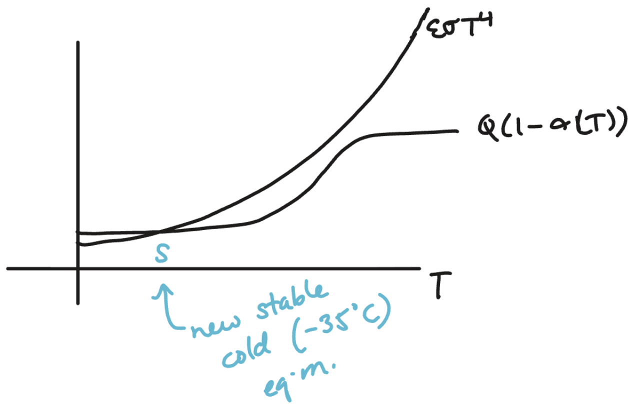
High Q



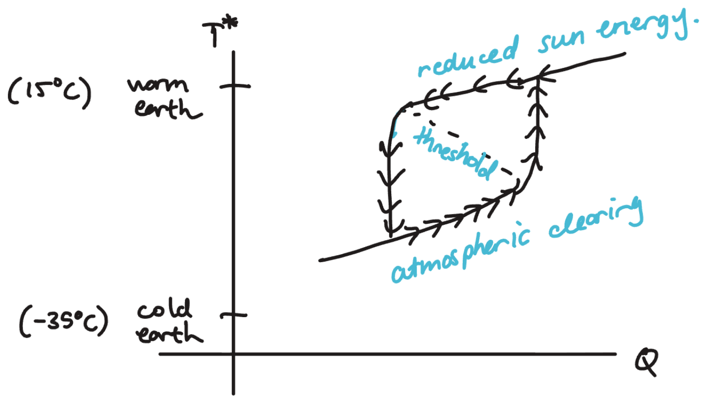
Now, imagine a volcano erupts







Let's put it all together in a bifurcation plot



The cold earth is known as the "icehouse Earth". The places where the the equilibria "jump" up or down are called tipping points. Tipping points are where a small parameter change leads to a large system change.

As in the budworm model, we see hysteresis. The system has a "memory", changing a parameter and observing the system change in one direction. The system change is not reversible by reversing the parameter.