Recap

So far, when we have $\dot{x} = \frac{dx}{dt} = f(x)$, we can do a number of things

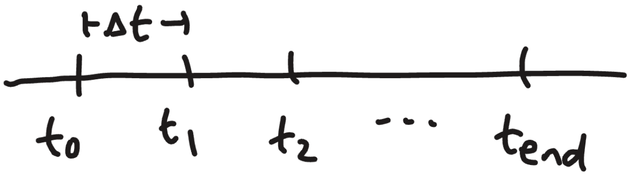
- Finding $\frac{dx}{dt} = 0$, gives us the equilibria
- Plotting $\dot{x}$ vs x, gives us the phase portrait
- We can plot trajectories
- We can do linear stability analysis, $f'(x^\star) > 0$ or $f'(x^\star) < 0$
- If there is a parameter, we can do bifurcation analysis

Numerically Solving DEs

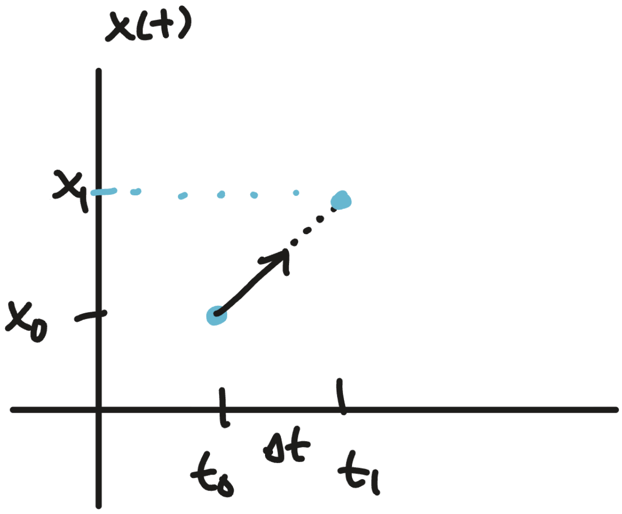
On the other side of things, sometimes:

- Math analysis is not clear
- We want to verify our results
- We just want a simulation, with known parameters

In these cases we can evaluate numerically. $\frac{dx}{dt} = f(t, x) \Rightarrow$ we will discretize the DE, and numerically solve of the time domain.



We start at some (t_0, x_0).



$$\begin{aligned}\frac{dx}{dt} &= f(t,x)\\ \frac{x_1-x_0}{t_1-t_0} &= f\left(t_0,x_0\right)\\ \frac{x_1-x_0}{\Delta t} &= f\left(t_0,x_0\right)\\ x_1 &= x_0+\Delta t\cdot f\left(t_0,x_0\right)\end{aligned}$$

Repeat to get x_2.

$$x_2 = x_1+\Delta t\cdot f\left(t_1,x_1\right)$$

$$\vdots$$

$$x_{n+1} = x_n+\Delta t\cdot f\left(t_n,x_n\right)$$

- x_{n+1} is the next approximate solution
- x_n is the previous approximate solution
- Δt is the time step
- $f\left(t_n,x_n\right)$ is the RHS of the DE evaluated at the last time step

This is a numerical method to solve a DE. To do this, we need

- A DE
- An initial value of t_0, x_0
- A choice of Δt

How do we choose Δt?

- Smaller $\rightarrow$ slower, more accurate, smoother curve
- Larger $\rightarrow$ faster, larger error

There are other ways to solve DEs numerically too

	Euler's Method	ODE23	ODE45
Global Error	$O(\Delta t)$	$O(\Delta t^2)$	$O(\Delta t^4)$
Local Error	$O(\Delta t^2)$	$O(\Delta t^3)$	$O(\Delta t^5)$

There is also ODE15s which is good for "stiff" DEs (DEs with a large spike / change in their value). ODE23 and ODE45 are called Runge-Kutta methods. Euler's method will be our focus for now.