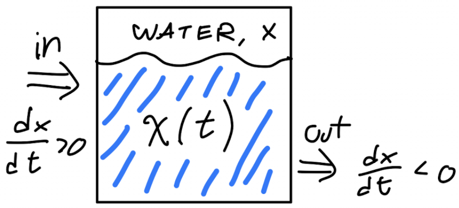
Stock-and-Flow Modelling / System Dynamics

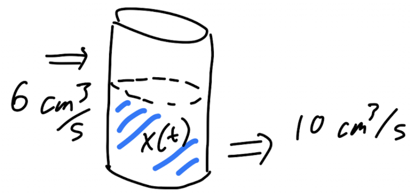
So far, we have studied $\frac{dx}{dt} = f(x)$ where x increases if $f(x) > 0$, and x decreases if $f(x) < 0$.

Consider:

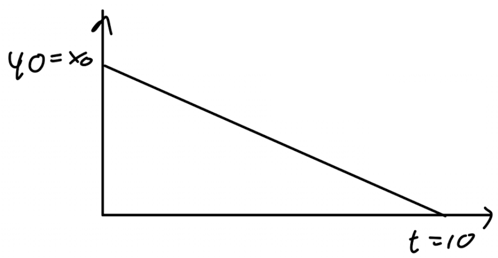


$$\frac{dx}{dt} = \text{net flow rate} = \text{rate in} - \text{rate out}$$

For example,



The total change in the system is: $\frac{dx}{dt} = 6 - 10 = -4$. If $x(0) = 40$,

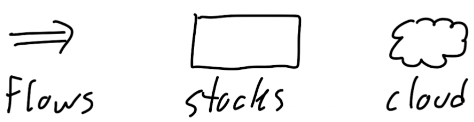


This system displays linear dynamics. We can see this from the graph since it's a straight line, as well as the DE $\frac{dx}{dt} = -4$ since it is constant.

Here, lets call water, $x(t)$ the STOCK (units are cm^3). We will call the water flowing in / out the FLOW (units are $\frac{\text{cm}^3}{s}$). The flow is always in units of $\frac{\text{units of stock}}{\text{time}}$.

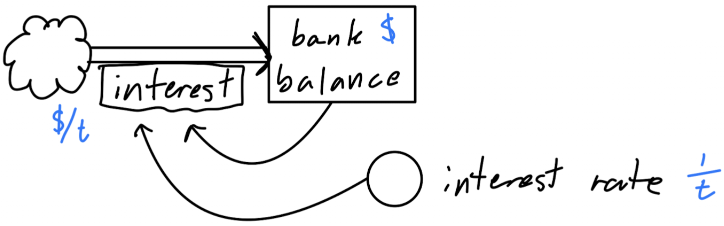
Let's develop a picture based framework to help develop models. Here, we would draw:

The drawing above is the schematic drawing.



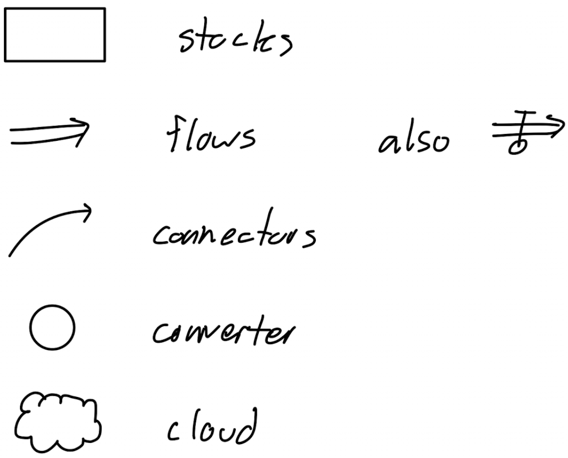
The cloud represents components outside the model.

Example: Bank Account



We use the curves single arrows to represent model dependency. Here, we need the interest rate to determine interest, so we will create a converter (a piece of information or parameter that aids in model description).

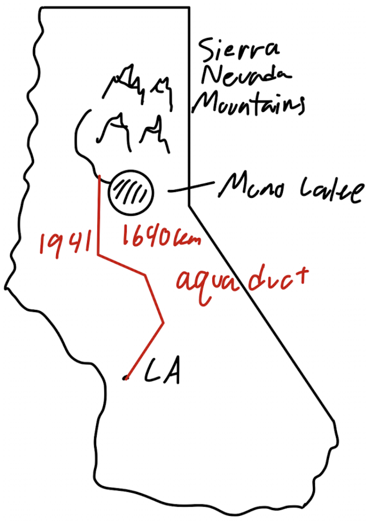
In summary:



After drawing, we use the diagram to develop equations for inflows / outflows based on model dependencies in the diagram.

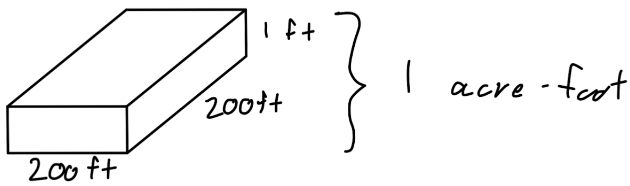
Mono Lake

In 1900, the population of Los Angeles was 100,000, and in 1930, the population of Los Angeles was up to 1,000,000. By 1981, (40 years later), the volume of Lake Mono was cut in half, and the salinity nearly doubled (55 $g/L \rightarrow 100\ g/L$).



Let's model this.

1. Units: Flows of KAF/yr (KAF is thousand acre feet)



   1000 of the above picture is a KAF.

2. Timescale : Measure time in years, model over decades