5. Abstract Data Types (ADTs), Stacks, and Queues

- What is an abstract data type (ADT)?
- Stack as an ADT, its data structure, operations, and error conditions.
- Stack implementations and applications.
- Queue as an ADT, its data structure, operations, and error conditions.
- Queue implementations and applications.

Abstract Data Types (ADTs)

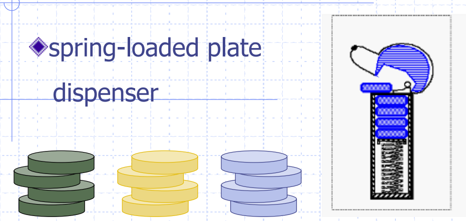
- An abstract data type (ADT) is an abstraction of a data structure.
- ADT refers to a way of packaging some intermediate-level data structures and their operations into a useful collection whose properties have been carefully studied.
- An ADT has a clean and simple interface.
- An ADT specifies:
  - Data stored
  - Operations on the data (clean, simple interface)
  - Error conditions associated with operations

An ADT Example

- Example: ADT modeling a simple stock trading system
  - The data stored are buy/sell orders
  - The operations supported are
    - order buy(stock, shares, price)
    - order sell(stock, shares, price)
    - void cancel(order)
  - Error conditions:
    - Buy/sell a nonexistent stock
    - Cancel a nonexistent order

Stacks

- spring-loaded plate dispenser



The Stack ADT

- The Stack ADT stores arbitrary elements
- Insertions and deletions follow the last-in first-out scheme
- Main stack operations:
  - push(element): inserts an element
  - element pop(): removes and returns the last inserted element
- Auxiliary stack operations:
  - element top(): returns the last inserted element without removing it
  - integer size(): returns the number of elements stored
  - boolean isEmpty(): indicates whether no elements are stored

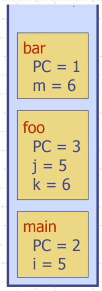
Applications of Stacks

- Direct applications
  - Undo sequence in a text editor
  - Chain of function calls in any language runtime system
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

Execution Stack in C

- The C runtime system keeps track of the chain of active functions with a stack
- When a function is called, the runtime system pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed
- When a function ends, its frame is popped from the stack and control is passed to the function on top of the stack

```
main() {
  int i = 5;
  foo(i);
}
foo(int j) {
  int k;
  k = j+1;
  bar(k);
}
bar(int m) {
  …
}
```

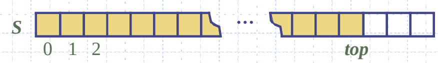


Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element

function *size*()
  return ***top*** + 1;

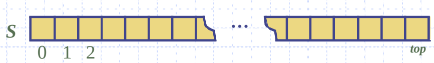
function *pop*(S)
  if (*isEmpty*(S))
    error (*"EmptyStack"*);
  else
    top = top – 1;
    return *S*[*top* + 1];



Array-based Stack (cont.)

- The array storing the stack elements may become full
- A push operation will then send FullStack error message
  - Limitation of the array-based implementation
  - Not intrinsic to the Stack ADT

function *push*(S, *val*)
  if (*isFull*(S))
    error(*"FullStack");*
  else {
    top = *top* + 1;
    S[*top*] = *val;*
  }



Array-based Stack

```
typedef struct {
  int array[MAX];
  int top;
} stack;

  void push(stack *s, int val){
      if (s->top >= MAX-1) error("Stack is full");
      s->top++;
      s->array[s->top] = val;
  }

  void pop(stack *s, int *val){
      if (s->top < 0) error("Stack is empty");
      *val = s->array[s->top];
      s->top--;
  }
```


Performance and Limitations

- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$
- Limitations
  - The maximum size of the stack must be defined a priori and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific error

Stack ---- behavior

- A stack is defined by how it is used, not by its underlying structure.
- We can implement a stack by different data structures.
  - linked lists
  - arrays
  - what else?
- The only requirement for a stack is the ability to store elements in order of insertion, so that we can get the **LIFO** behavior.

List-based Stack:

- Linked list can be used to implement stack data structure: add and remove node from the "top".
- Property: Last In First Out (LIFO).
- Operations: is_empty, push, pop

An example of stack application:

$\pi*10^{128} + e*10^{128}$ Each operand has 128 digits

Read an operand(from left to right): 3141592653…..
27182818…..

Calculate the sum(from right to left): xx…xxxxx
+ yy…yyyyy

= ss…sssss

We need three stacks where two for operands and one for the sum.


```
typedef struct node  node;
struct node{
    int n;
    node* next;
};
#define empty(s)  (!(s))

void push(node** top, int n){
    node* new = malloc(sizeof(node));        // create a new node
    if (!new) exit(-1);
    new->n = n;
    new->next = *top;
    *top = new;                              // set up stack top
}

int pop(node** top){                         // return a value (not a node)
    int n;
    node* temp;
    if (empty(*top)) return 0;
    temp = *top;                             // save top node
    *top = temp->next;                       // set up stack top
    n = temp->n;
    free(temp);
    return n;
}
```

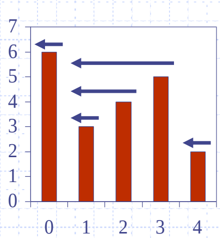


```
node* get_operand(){ // read an operand and store on a stack
    node* s = NULL;
    int n;
    while (1){
        n = getchar();
        if (n < '0' || n > '9') return s;      // return if input char is not a digit
        push(&s, n – '0');
    }
}

main(){
    node *a, *b, *sum = NULL;
    int sumdig, carry = 0;
    printf("input two operands\n");
    a = get_operand();
    b = get_operand();
    while (!empty(a) || !empty(b)){                  // perform addition
        sumdig = pop(&a) + pop(&b) + carry;
        push(&sum, sumdig%10);
        carry = sumdig/10;
    }
    if (carry != 0) push(&sum, carry);
    printf("the sum is:");
    while (!empty(sum)) printf("%c", pop(&sum));     // output result
}
```


Example: Computing Spans

- We show how to use a stack as an auxiliary data structure in an algorithm
- Given an array X, the span $S[i]$ of $X[i]$ is the maximum number of consecutive elements $X[j]$ immediately preceding $X[i]$ such that $X[j] \le X[i]$
- Spans have applications to financial analysis
  - E.g., stock at 52-week high



X	6	3	4	5	2
S	1	1	2	3	1

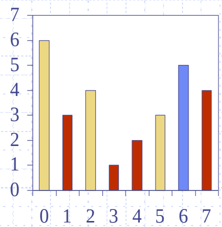
Quadratic Algorithm

function *spans1*(X, n)	
Input: $X[n]$ integers	
Output: array $S[n]$ of spans of X	**#**
int s;	
for (*int i* = 0; i < n; i++){	n
$s = 1$;	n
while ($s \le i$ **&&** $X[i - s] \le X[i]$){	$1 + 2 + \ldots + (n - 1)$
$s = s + 1$;	$1 + 2 + \ldots + (n - 1)$
}	
$S[i] = s$;	n
}	
return S	1
}	

- Algorithm ***spans1*** runs in $O(n^2)$ time

Computing Spans with a Stack

- We keep in a stack the indices of the elements visible when "looking back"
- We scan the array from left to right
  - Let i be the current index
  - We pop indices from the stack until we find index j such that $X[i] < X[j]$
  - We set $S[i] \leftarrow i - j$
  - We push i onto the stack

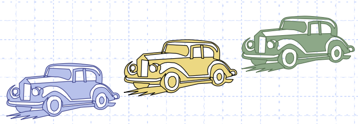


Linear Algorithm

- Each index of the array
  - Is pushed into the stack exactly once
  - Is popped from the stack at most once
- The statements in the while-loop are executed at most n times
- Algorithm *spans2* runs in $O(n)$ time

```
function spans2(X, n)                        #
Input and output: same as span1
   stack A;
   for (i =0; i < n; i++){                   n
      while (!A.isEmpty() &&
             X[A.top()] ≤ X[i] ) {           n
         A.pop();                            n
      }
      if (A.isEmpty())                       n
         S[i] = i + 1;                       n
      else
         S[i] = i – A.top();                 n
      A.push(i);                             n
   }
   return S                                  1
```


Queues



The Queue ADT

- The Queue ADT stores arbitrary elements
- Insertions and deletions follow the first-in first-out scheme
- Insertions are at the rear of the queue and removals are at the front of the queue
- Main queue operations:
  - enqueue(element): inserts an element at the end of the queue
  - element dequeue(): removes and returns the element at the front of the queue
- Auxiliary queue operations:
  - element front(): returns the element at the front without removing it
  - int size(): returns the number of elements stored
  - boolean isEmpty(): indicates whether no elements are stored
- Errors
  - Attempting the execution of dequeue or front on an empty queue, or enqueue on a full queue(?)

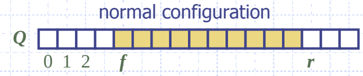
Applications of Queues

- Direct applications
  - Waiting lists
  - Access to shared resources (e.g., printer)
  - Multiprogramming
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

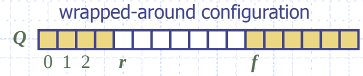
Array-based Queue

- Use an array of size N in a circular fashion
- Two variables keep track of the front and rear
  - f index of the front element
  - r index immediately past the rear element
- Array location r is kept empty

normal configuration



wrapped-around configuration

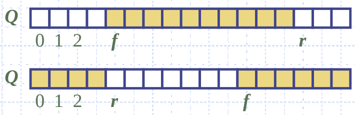


Queue Operations

- We use the modulo operator (remainder of division)

function *size*()
  return (*N* – *f* + *r*) mod *N*

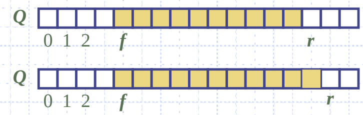
function *isEmpty*()
  return (*f* = *r*)



Queue Operations (cont.)

- Operation enqueue reports an error if the array is full
- This error is implementation-dependent

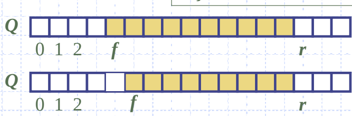
function *enqueue*(*val*)
  if (*size*() == *N* – 1)
    error(“*Full Queue*”);
  else{
    Q[*r*] = val;
    r = (***r*** + 1) mod ***N*;**
  }



Queue Operations (cont.)

- Operation dequeue reports an error if the queue is empty
- This error is specified in the queue ADT

function *dequeue*()
  if (*isEmpty*()) **then**
    error(“*Empty Queue*”);
  else{
    val = *Q*[*f*];
    f = (*f* + 1) mod ***N*;**
    return *val;*
  }



Array-based Queue Declarations

```
#define MAX 1000

typedef struct {
            int count;
            int front;
            int rear;
            int data[MAX];
} queue;

void createQueue(queue *q)
{
    q->front   = 0;
    q->rear    = 0;
    q->count   = 0;
};
```


Circular-Array-based: Enqueue

```
int queueFull(queue *q)
{
   return q->count >= MAX;
}

void enqueue(int x, queue *q)
{
     if (queueFull(q))
        error("QUEUE IS FULL");
     q->count++;
     q->data[q->rear] = x;
     /* Move to next open position */
     q->rear = (q->rear + 1) % MAX;
}
```


Circular-Array-based: Dequeue

```
void dequeue(int *x,  queue *q)
{
    if (queueEmpty(q))
       error("QUEUE IS EMPTY");
    q->count--;
    *x = q->data[q->front]; /* data from front */
    /* Move to the next slot to dequeue */
    q->front = (q->front + 1)  % MAX;
}
```