# Coupling coefficient of spiral resonators used for wireless power transfer

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Abstract — Coupling coefficient between two spiral resonators used for wireless power transfer is studied. It is pointed out that spiral resonators have different coupling coefficient corresponding to the mirror co-directional or inversely wound configurations alignment. The reason is clarified by calculating the coupling coefficient separately as the electric and magnetic parts. Helical resonators are also studied for comparison showing negligible difference between the two configurations.

### I. INTRODUCTION

After the MIT group proposed a wireless power transfer system, many groups have tried to develop the system into practical use. We have found the system is neither more nor less than 2-stage band pass filter (BPF) [1][2]. Since resonators constitute BPF, the properties of the resonator are crucial for the constructed power transfer system. Among them, the coupling coefficient determines the outreach of the power transfer as well as the transfer efficiently. The present article "elucidates" the content of coupling, in other words, decomposes it into the electric and magnetic components. The decomposition will show whether the widely taken understanding that the coupling between coils should be magnetic is correct or not.

Secondly, it may give a new insight into the search for stronger coupling scheme. The fact that the coupling coefficient is made by the subtraction of both components and they can be either positive or negative [3], will suggest a principle to increase the coupling. It is to find a structure/configuration of the resonators which gives the opposite sign to each together with large magnitude as possible.

In section 2, the experimental result for differently codirectional pairs of two spiral resonators will be shown, being compared with the solenoidal resonators. The reason for different properties for each of those examples will be explained by use of decomposition of coupling coefficient. Section 3 deals with the effect of rotation along the axis of spiral coils, being compared with the microwave open ring resonators. Averaging effect in spiral coils suppressed the drastic variation in the coupling coefficient observed in open ring resonators.

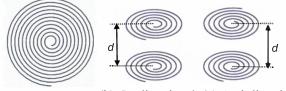
Strange discrepancy between theoretical and experimental results for the coupling coefficient of spiral coils is worth to be noted. It keeps decreasing more slowly for the longer distance than the theory even after deliberate experimental setup. Its elucidation would contribute to more extension of the outreach of power transfer.

# II. CO-DIRECTIONAL AND ANTI-DIRECTIONAL ALIGNMENTS

Fig.1(a) shows a spiral resonator constructed by a 1mm diameter wire. Its diameter is kept 245mm and pitch is 10mm with the resonant frequency around 25MHz throughout Section 2. Such two spiral resonators are arranged in codirectional or anti-directional alignments, as shown in Fig.1(b) and Fig.1(c), respectively. Coupling strength between the resonators is described by the coupling coefficient. The conventional method to calculate the coupling coefficient k is the frequency method using the equation as follows,

$$k = \frac{2(f_{\rm h} - f_{\rm l})}{(f_{\rm h} + f_{\rm l})} \tag{1}$$

,where  $f_h$  and  $f_l$  are the higher and lower split frequencies of the coupled resonators, respectively.



(a) Spiral resonator (b) Co-directional (c) Anti-directional Diameter: 245mm Pitch: 10mm

Fig.1 Spiral resonator, co-directional and anti-directional alignment of spiral resonators

The measured coupling coefficients for the co-directional and anti-directional alignment are shown in Fig.2. Coupling for the anti-directional alignment is stronger than that for co-directional alignment. This can be explained by separation of the coupling coefficient into the magnetic and electric components, which is quite useful for understanding and predicting the coupling of resonators.

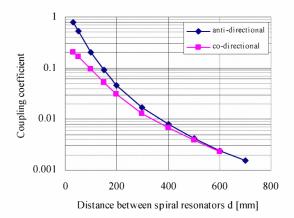


Fig.2 Experimental result of coupling coefficient for the two spiral resonators

The electric and magnetic components of coupling coefficient for the resonators in co-directional alignment can be separated by the perturbation method proposed by one of the present authors [4]. The perturbation method calculates coupling coefficient using the following equation,

$$k = \frac{\int_{ev} \mu |H_1|^2 dv - \int_{ev} \varepsilon |E_1|^2 dv}{\int_{ev} |E_1|^2 dv}$$
(2)

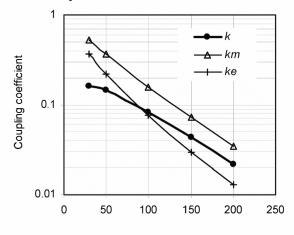
where  $E_1$ ,  $H_1$  are the evanescent fields of resonator 1 extending outside of the symmetry plane between the two resonators [4]. For the coupling between the resonators in an anti-directional alignment, on the other hand, the fast calculation method above cannot be applied. Thus, it is carried out by more complicated Overlap integral method proposed also by one of the present authors [3]. It is based on the coupled mode theory, and calculates the coupling coefficient as follows,

$$k = \frac{\int \mu H_1^* \cdot H_2 dv - \int \varepsilon E_1^* \cdot E_2 dv}{\left[\varepsilon |E|^2 dv\right]}$$
(3)

,where  $E_1$ ,  $E_2$  are the electric fields of two resonators before coupling, respectively, and  $H_1$ ,  $H_2$  are the corresponding magnetic fields [3].

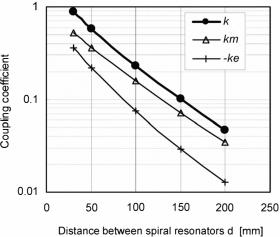
Fig.3 shows the calculated results for the coupling of spiral resonators in the co-directional alignment, and Fig.4 shows those in anti-directional alignment. Note that the electrical component in Fig.4 is with "-" sign. The results evidently indicate that larger coupling coefficient is obtained for the anti-directional alignment, while the co-directional alignment

has weaker coupling due to cancellation of magnetic and electric components.



Distance between spiral resonators d [mm]

Fig.3 Coupling coefficient for the co-directional pair of spiral resonators



Distance Semicon opinion (communication)

Fig. 4 Coupling coefficient for the anti-directional of spiral resonators

There are three significant features here.

- (1) Coupling coefficient differs according to the winding direction of spiral coils. Inversely wound coils couple strongly especially in the short distance.
- (2) Electric coupling is too strong to classify the spiral coil as magnetically coupling resonator. The mutual inductance-coupled equivalent circuit may have to be reconsidered for the spiral coils.
- (3) Since electric coupling decays faster than the magnetic counter-part, almost pure magnetic coupling prevails in the long distance range.

Comparisons of measurement and simulation results of coupling coefficient for anti-directional and co-directional alignment are shown in Fig.5 and Fig.6, respectively.

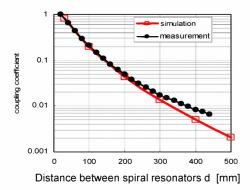


Fig.5 Simulated and measured coupling coefficient for the anti-directional pair of spiral resonators

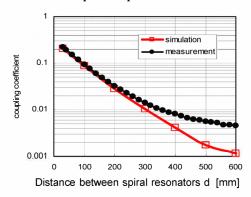


Fig.6 Simulated and measured coupling coefficient for the co-directional pair of spiral resonators

The difference in the long distance range was first attributed to the incomplete experiment. But recent deliberate experiment that is carried out on the big aluminum ground plane has shown the same result as the cursory experiment done before. And thus, we are now focusing on the extension of analysis area in the E/M wave simulation. Too close metal walls seem to affect the computed result.

For comparison, the measured result of coupling coefficient for the solenoidal resonators (Fig.7) is shown in Fig.8. Their dimensions are also similar to the spiral coil as shown in the figure and the resonant frequency is kept around 25MHz. The simulation result is shown in Fig.9.



(a) Solenoidal resonator (b) Co-directional (c) Anti-directional Diameter: 245mm Pitch : 10mm

Fig.7 Solenoidal resonator, and winding of two solenoidal resonator

Both the measurement and simulation results indicate that the coupling for co-directionally wound pair is close to that for the anti-directional pair. It is considered that the total coupling is almost made of the magnetic component, and there is very weak electric coupling between the solenoidal resonators. The further numerical analysis will be carried out.

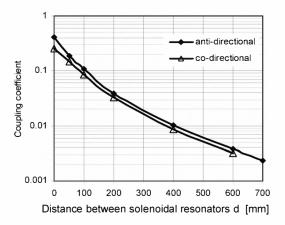


Fig.8 Measurement result of coupling coefficient for two configuration of solenoidal resonator pair

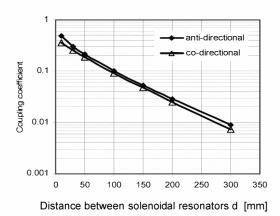


Fig.9 Simulation result of coupling coefficient for the two configuration of solenoidal resonator pair

# III. ROTATING RESONATOR

This Section deals with the effect of rotation along the axis of spiral coils, to find the optimum coupling coefficient between the spiral resonators. It is because a drastic variation in the coupling coefficient was observed in open ring resonators [5]. In Fig.10, a pair of open-ring resonators is shown to be coupled with the broad-side. When one of the resonators rotates as shown in the figure, the electric coupling changes quite much from positive to negative values and the total coupling coefficient ranges from 0.1 to 0.6, giving 6 times difference by rotation (Fig.11). The qualitative explanation in Fig.12 will assist to understand the wide range variance of the coupling. Each resonator is extended straight for easier comparison of the electromagnetic field distribution. Multiplication of magnetic fields for two resonators and its integration along the longitudinal direction is compared with that for the electric fields. In the case  $\theta$ =0, they are subtracted

each other, but when  $\theta$ =180 degree, they are added, on the contrary, due to the opposite signs.

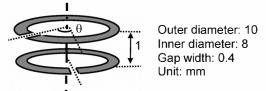


Fig.10 Mutual rotation angle of broad-side coupled open ring resonators

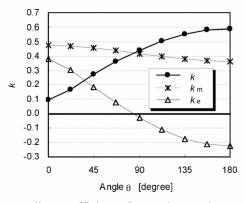


Fig.11 Coupling coefficient of rotated open ring resonators

Figure 13 shows the measured coupling coefficient of spiral resonators. The resonators are in co-directional alignment at 0 degree, and one spiral resonator was rotated to 0, 90, 180 and 270 degree with respect to another spiral resonator. However, little variation in coefficient was observed, and the similar result was obtained for the inversely wound alignment, too.

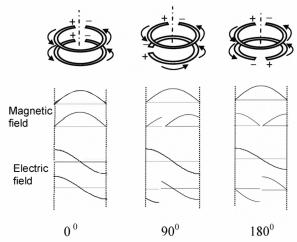


Fig.12 Electromagnetic field distribution along open ring resonators

It is considered that the electric coupling does not change according to the rotation angle due to the averaging effect between multiply wound lines. The magnetic coupling should be almost constant as in the case of the open ring resonator shown in Fig.11.

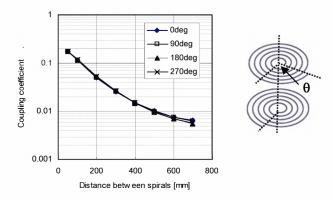


Fig.13 Coupling coefficient of rotated spiral resonators

### IV. CONCLUSION

Spiral coils are studied in terms of coupling coefficient for application to the wireless power transfer. Separation of coupling coefficient into the magnetic and electric components clarifies the physical images for the unique features of the coupling. It was found that

- (1) Electric coupling is fairly strong in the short distance, and hence the magnetically-coupled model may be modified
- (2) Inversely wound coils couple strongly each other. Way of winding would be an important issue for the stronger coupling
- (3) Solenoidal coils do not show the noticeable difference due to the winding probably because the coupling is predominantly magnetic
- (4) Rotation of coils also does not change the coupling coefficient appreciably due to the averaging effect of the multiply wound lines.

## REFERENCES

- [1] Ikuo Awai, Takuya Komori, "A Simple and Versatile Design Method of Resonator-coupled Wireless Power Transfer System", Proc. of 2010 International Conference on Communications, Circuits and Systems, Chengdu, China, July 2010, to be published.
- [2] Ikuo Awai, "Design Theory of Wireless Power Transfer System Based on Magnetically Coupled Resonators", Proc. Of 2010 IEEE International Conference on Wireless Information Technology and Systems, Honolulu, USA, Aug. 2010, to be published
- [3] Ikuo Awai, "New Expressions for Coupling Coefficient between Resonators", IEICE Trans. Electron., E88C, No.12, pp.2295-2301, Dec. 2005.
- [4] Ikuo Awai, Shintaro Iwamura, Hiroshi Kubo, Atsushi Sanada, "Separation of coupling coefficient between resonators into electric and magnetic contributions" IEICE, Vol.J88-C, No.12, pp.1033-1039, 2005. (in Japanese)
- [5] Ikuo Awai, Yangjun Zhang, "Separation of Coupling Coefficient between Resonators into Magnetic and Electric omponents toward Its Application to BPF Development", Proc. CJMW 2008, pp.61-65, Sept. 2008.