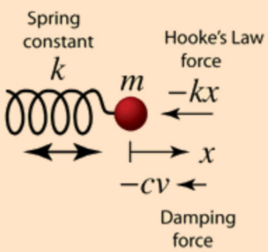
Driven Oscillator

If a damped oscillator is driven by an external force, the solution to the motion equation has two parts, a transient and a steady-state part, which must be used together to fit the physical boundary conditions of the problem.

The motion equation is of the form



Newton's 2nd Law terms — Sinusoidal driving force

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F_0\cos(\omega t + \varphi_d)$$

and has a general solution

$$x(t) = x_{transient} + x_{steady\ \ state}$$

In the underdamped case this solution takes the form

Transient solution — Steady-state solution

$$x(t) = A_h e^{-\gamma t}\sin(\omega' t + \varphi_h) + A\cos(\omega t - \varphi)$$

Determined by initial position and velocity — Determined by driving force

The initial behavior of a damped, driven oscillator can be quite complex. The parameters in the above solution depend upon the initial conditions and the nature of the driving force, but deriving the detailed form is an involved algebra problem. The form of the parameters is shown below.

Details of parameter evaluation

Examples of driven oscillators

Transient solution	Steady-state solution	Expansion of above terms

Index

Periodic motion concepts

HyperPhysics***** Mechanics *R Nave* Go Back

Underdamped Driven Oscillator

The expanded expressions for the underdamped oscillator in terms of the mass, spring constant, damping, and driving force. They are also affected by the initial values of position and velocity.

Index

Periodic motion concepts

Reference

Barger & Olsson

$$\varphi = \tan^{-1}\left[\frac{c\omega}{k - m\omega^2}\right] - \varphi_d$$

$$\gamma = \frac{c}{2m}$$

$$A = \frac{F_0 / m}{\sqrt{\left[\omega_0^2 - \omega^2\right]^2 - 4\gamma^2\omega^2}}$$

$$x(t) = A_h e^{-\gamma t}\sin(\omega' t + \varphi_h) + A\cos(\omega t - \varphi)$$

$$A_h = \frac{x_0 - A\cos\varphi}{\sin\varphi_h}$$

$$\omega' = \sqrt{\omega_0^2 - \gamma^2}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\varphi_h = \tan^{-1}\left[\frac{\omega'(x_0 - A\cos\varphi)}{v_0 + \gamma(x_0 - A\cos\varphi) - A\omega\sin\varphi}\right]$$

HyperPhysics***** Mechanics *R Nave* Go Back

Transient Solution, Driven Oscillator

The solution to the driven harmonic oscillator has a transient and a steady-state part. The transient solution is the solution to the homogeneous differential equation of motion which has been combined with the particular solution and forced to fit the physical boundary conditions of the problem at hand. The form of this transient solution is that of the undriven damped oscillator and as such can be underdamped, overdamped, or critically damped.

Examples of driven oscillators

Index

Periodic motion concepts

HyperPhysics***** Mechanics *R Nave* Go Back

Steady-State Solution, Driven Oscillator

The solution to the driven harmonic oscillator has a transient and a steady-state part. The steady-state solution is the particular solution to the inhomogeneous differential equation of motion. It is determined by the driving force and is independent of the initial conditions of motion.

Examples of driven oscillators

Index

Periodic motion concepts

HyperPhysics***** Mechanics R Nave Go Back