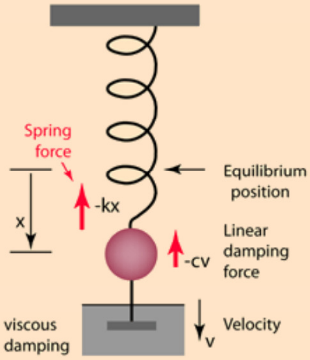
Damped Harmonic Oscillator



Damping coefficient
Undamped oscillator
Driven oscillator

The Newton's 2nd Law motion equation is

$$ma + cv + kx = 0$$

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

This is in the form of a homogeneous second order differential equation and has a solution of the form

$$x = e^{\lambda t}$$

Substituting this form gives an auxiliary equation for λ

$$m\lambda^2 + c\lambda + k = 0$$

The roots of the quadratic auxiliary equation are

$$\lambda = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

The three resulting cases for the damped oscillator are

$c^2 - 4mk > 0$	Overdamped
$c^2 - 4mk = 0$	Critical damping
$c^2 - 4mk < 0$	Underdamped

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Damping Coefficient

When a damped oscillator is subject to a damping force which is linearly dependent upon the velocity, such as viscous damping, the oscillation will have exponential decay terms which depend upon a damping coefficient. If the damping force is of the form

$$F_{damping} = -cv$$

then the damping coefficient is given by

$$\gamma = \frac{c}{2m}$$

This will seem logical when you note that the damping force is proportional to c, but its influence inversely proportional to the mass of the oscillator.

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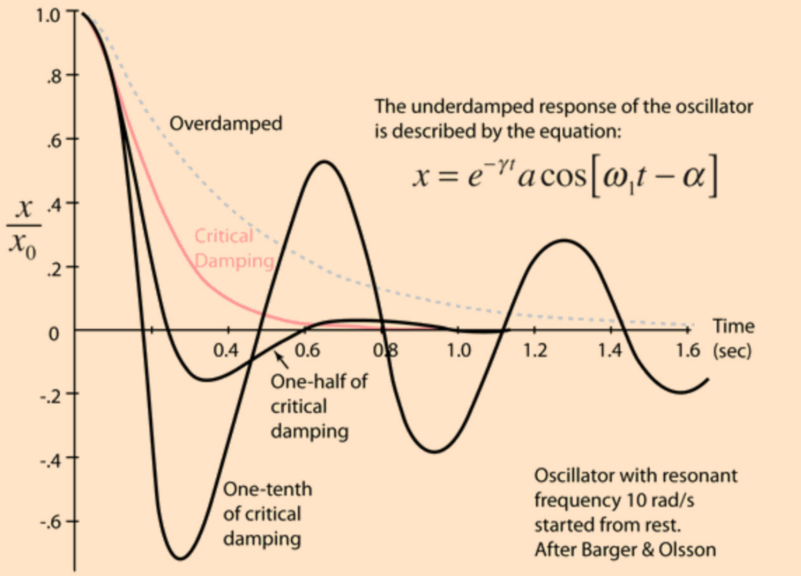

For any value of the damping coefficient γ less than the critical damping factor the mass will overshoot the zero point and oscillate about x=0. The behavior is shown for one-half and one-tenth of the critical damping factor. Also shown is an example of the overdamped case with twice the critical damping factor.

Damped Oscillator | Further details for underdamped case

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Reference

Barger & Olsson

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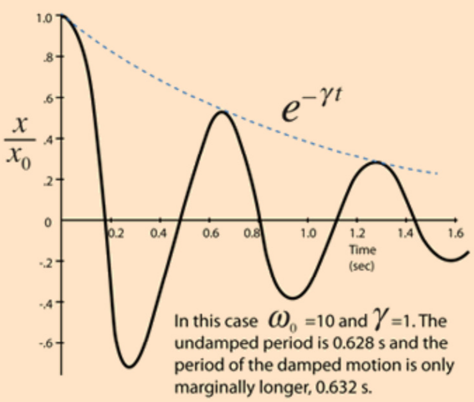
Underdamped Oscillator

When a damped oscillator is underdamped, it approaches zero faster than in the case of critical damping, but oscillates about that zero.



The equation is that of an exponentially decaying sinusoid.

$$x = e^{-\gamma t} a \cos[\omega_1 t - \alpha]$$

The damping coefficient is less than the undamped resonant frequency . The sinusoid frequency is given by

$$\omega_1 = \sqrt{\omega_0^2 - \gamma^2}$$

but the motion is not strictly periodic.

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