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THE ORNL FOUR-LINEAR-ELEMENT PACKAGE ANALOG COMPUTER (FLEPAC) INSTRUCTION MANUAL

F. P. Green

OAK RIDGE NATIONAL LABORATORY U.S. ATOMIC ENERGY COMMISSION

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F. P. Green

ABSTRACT

A single-package analog computing device has been developed at ORNL and has been named the FLEPAC. The device consists of a power supply and four chopper-stabilized high-gain d-c amplifiers such as are used in large analog computer installations. The amplifiers are commercially available from the Reeves Instrument Corporation.

This package, the ORNL FLEPAC, is described briefly. Instructions for using the device in conjunction with other equipment as well as by itself are given here. What is considered a minimum amount of theory of the operation of the direct-coupled amplifier as a computer is also given for persons interested in a basic understanding of the device.

INTRODUCTION

An electronic analog computer is an engineering tool for the manipulation of electrical signals, both for the generation of transfer functions and for the solution of problems which are expressible as algebraic and differential equations in a single independent variable.

The independent variable is usually *time*, and, depending upon the application, the time scale is selected to be either "real" time or a compressed or expanded time to suit the range of variables and time constants involved. If real time is selected, the process is called "simulation," and the computer can thereby act as an artificial component of an actual system. Since the simulator may serve as a model of a specific kinetic process, it can be connected by suitable transducers to the very system it simulates for the purpose of controlling that system. By specifying the desired *demand* parameters, the controlling functions are defined by the difference between the measured variables and their demand settings.

The magnitudes of *currents* represent the magnitudes of the *dependent* input variables, regardless of their dimensional description. The currents are produced by applying known or derived signal voltages across properly selected input impedances made up of one or more resistors and capacitors. The mathematical operations upon these variables produce the desired unknown variables as output *voltages* whose magnitudes vary with time and which may feed transducers, indicators, and recorders.

The FLEPAC was designed by R. A. Dandl of the Instrumentation and Controls Division, and several have been constructed for use both by the Division and by other research and development divisions of the Laboratory. It is thought to be the first four-amplifier portable, self-contained, high-stability, general-purpose analog computer in service in the United States. The computer can perform linear operations with an accuracy of about 2% over a period of 5 min. It is capable of performing the kinetics of systems whose variables do not exceed a range of 10^4.

APPLICATION

An analog computer of four-amplifier simplicity can be given a great multitude of assignments which can be described most conveniently in three categories. First, it can be applied as a *measuring instrument*. Any variable which is in the form of an electrical signal or which may be converted with a transducer to an electrical signal can be transformed to a calibrated voltage by this device. One useful example is the ballistic galvanometer, an electrical-charge measuring device. The second application category is a logical extension of the first. It can be applied as a *data handler*.

The problem of collecting and analyzing data from an experimental installation can become burdensome. This is likely to be the case where such data are derived from a test installation involving a relatively large system. It has been customary to install instruments with strip charts which record whatever variables are considered important. These charts are accumulated and the information analyzed to derive whatever conclusions can be drawn from the information.

Analysis of such data for large systems has prompted those responsible for such work to turn to the use of digital data-reduction systems and some form of digital computer which essentially takes the data and makes whatever computations are needed for the analysis of the data. In general, these data-reduction and computing systems are large and expensive.

The cost of a data-reduction system using digital techniques is often prohibitive for use on jobs which are small, however tedious the analysis may be without such aids. There is only one criterion for the use of such systems, and that is one by which the worth of the analysis is compared with the cost of doing it either by a substantial data-reduction system or by the slow and tedious manual method. If one method is substantially more economical than the other, then that method should be used.

Use of analog equipment for analyzing data from an experiment has not usually been practical. For experiments which last for days, analog equipment cannot be used because of the drift in the performance of the components. Were such equipment practical the large analog installation needed for some experiments would be expensive. The installation would be difficult to standardize. The large analog installation for such work is not very promising.

There are instances where a small analog installation can be of considerable value in analyzing experimental data. The merits of the installation become apparent when very few computations need to be made in order to evaluate the data. There are special-purpose instruments available which perform an arithmetical operation on the data and record or indicate the result of such a mathematical operation. An example is the wattmeter, which indicates the product of the voltage and current in an electric circuit.

During the past ten years some excellent analog computer components have been developed. These components are used in the larger installations such as the ORNL Analog Computer Facility. Our experience indicates that they are as reliable as the best strip-chart recorder available and in general give less trouble than does the recorder.

There are certain mathematical operations which can be done simply with these computer components and which can be found useful for analyzing experimental data as it is taken from the experiment. Some of these computer components are described below, and suggestions are made on how they may be used to perform the mathematical operations for which they were designed. A typical example of this application to data reduction is the use of one or more integrators to perform the function of a planimeter.

The third field of application of FLEPAC is certainly a most valuable and versatile one – its use as a *function generator*. Examples of this use are practically innumerable and include such self-contained devices as a very-low-frequency oscillator, a nuclear reactor kinetics simulator, an adjustable time constant with linear calibration, a precision exponential generator, and, when used with simple auxiliary devices such as a relay or gas tube, a saw-tooth generator. An additional group of applications in this category come to mind when the unit is considered as an element in a closed-loop system such as a servo-controller. The advantages of using an analog computer in servomechanism design and analysis are: (1) responses to step, sinusoidal, and complex inputs are readily observed, (2) changes in parts of the system can be made rapidly, and (3) nonlinear conditions or multiloop feedbacks which are often used to improve system operation and which are formidable to study mathematically can be handled easily.

In addition to the primary output variable, FLEPAC simultaneously presents at the output of each amplifier in use an intermediate variable which may be of considerable interest in the elucidation of system operation and in the selection of or design of components in the system.

Applications requiring two or more amplifiers contain a sequence of mathematical operations which can usually be set up in an arbitrary order. This arbitrary sequence inevitably provides several adequate component arrangements for solution of a given task. However, certain physical equipment limitations, such as a computer linear voltage range and a finite number of amplifier access points, curtail the sequence orders available. The length of setup time and the validity of output data are greatly improved by keeping these points in mind.

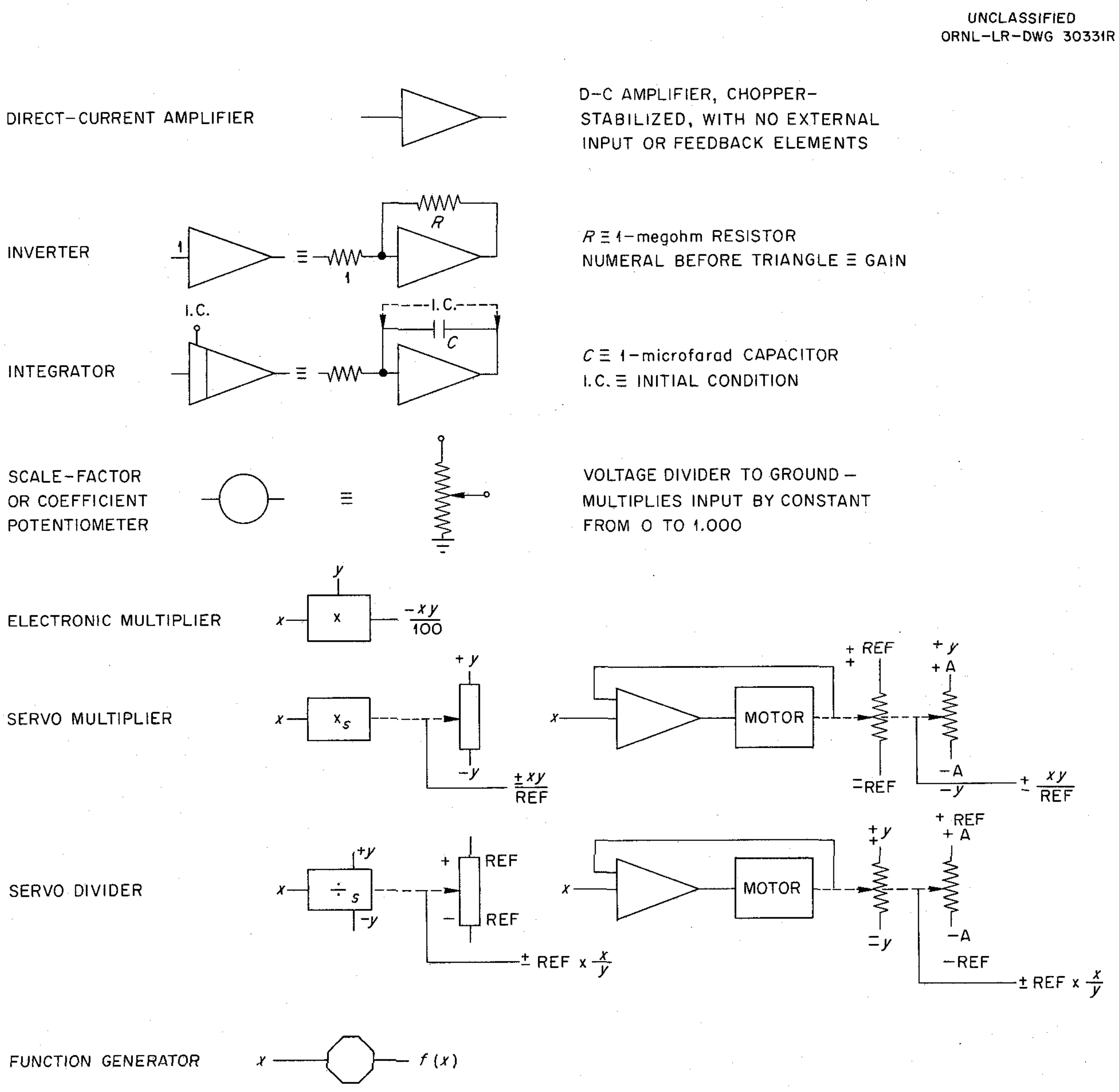
SYMBOLS

A discussion of analog computing circuits involves an individual designation of a large number of different quantities. The wide range of subject matter requires repetition of some letter symbols in different sections to represent different physical quantities. The following conventions are utilized in this manual.

Lower-case letters denote the instantaneous value of a time variant function; thus e represents voltage; i, current; t, time; etc. When specific problem variables are referred to, subscripts are used to distinguish one quantity from another. These are defined as the discussion proceeds.

Upper-case letters denote fixed values of voltage, current, circuit parameters, and constants. Table 1 presents the graphic symbolism used in this manual. No industry-wide conventions on symbolism have been determined yet, so a careful observance of definitions should be made whenever reference material is studied.

Table 1. Symbolism



THE D-C OPERATIONAL AMPLIFIER

Addition, subtraction, multiplication by a constant, and integration are the only operations necessary for the solution of linear differential equations with constant coefficients. These operations are carried out by means of high-gain *d-c operational amplifiers* with appropriate input and feedback circuits. Four of these amplifiers with common power supplies and basic passive circuit elements are housed in one self-contained unit called the FLEPAC (Fig. 1).

A simplified schematic diagram[1] of a typical, single-ended, operational amplifier is shown in Fig. 2. It is a high-gain, d-c amplifier with an

[1]F. P. Green, *Reactor Controls Analog Facility (RCAF) Operations Manual*, ORNL-2405,(Aug. 20, 1958).

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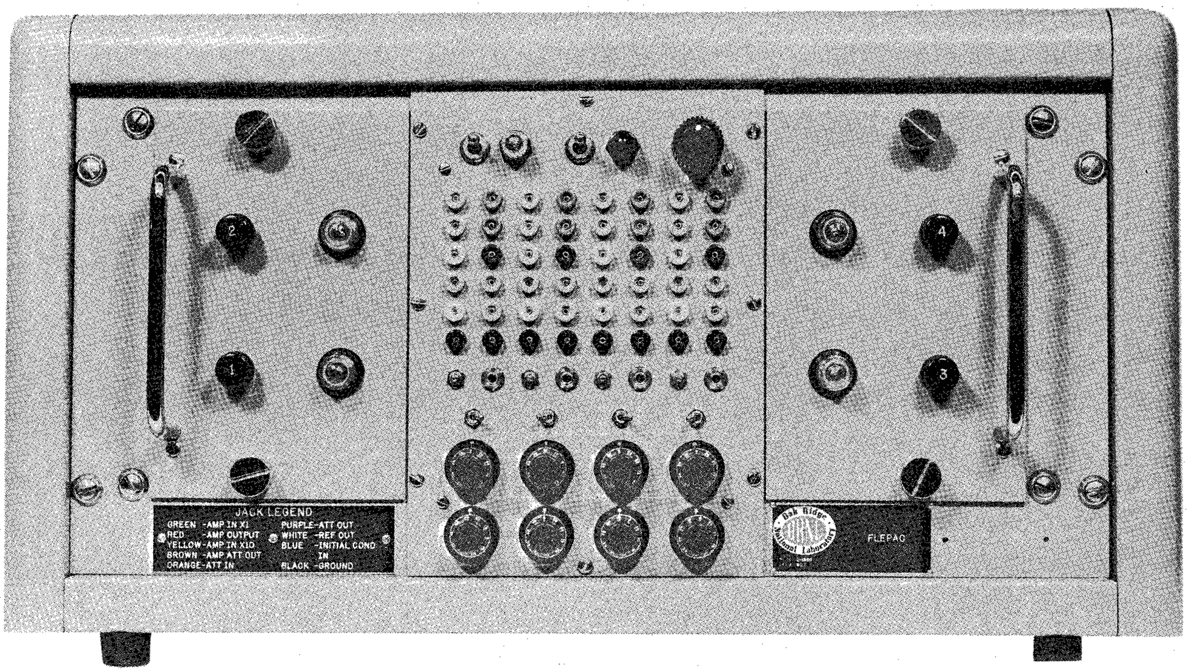

Fig. 1. Control Panel of the Four Linear Element Package Analog Computer (FLEPAC).

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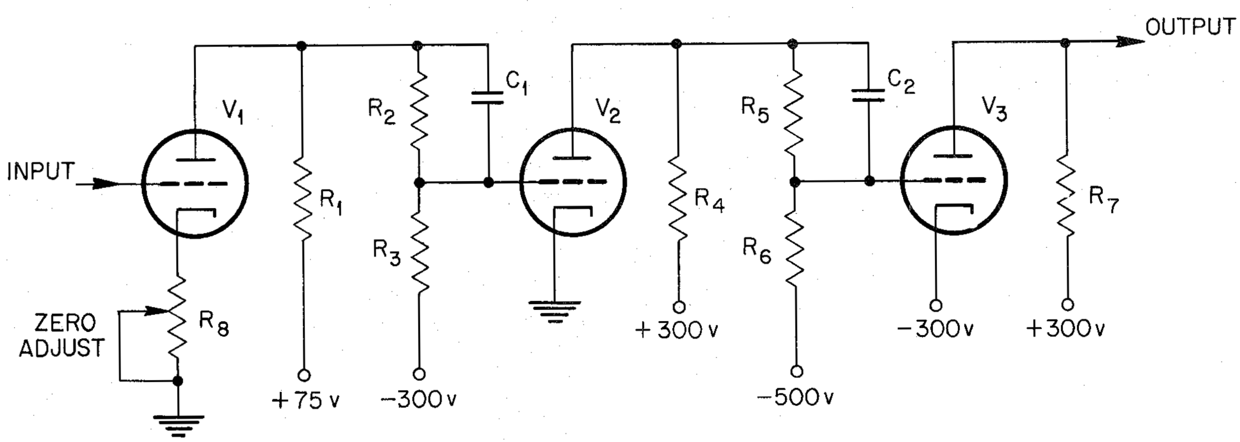

Fig. 2. Conventional Single-Ended Operational Amplifier.

odd number of stages of gain. The odd number of stages produces a polarity reversal through the circuit so that feedback networks connecting the output to the input will be degenerative. The over-all gain is the product of the individual gains of each of the three stages and is usually near 60,000. A triangle, such as that shown in Fig. 3, is frequently used as a schematic symbol of an operational amplifier and will be so employed in this report. The n denotes the number assigned to a particular amplifier in a circuit employing more than one such unit. The triangle represents the amplifier only; the feedback and input networks are shown separately.

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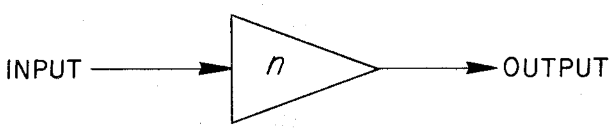

Fig. 3. Block-Diagram Representation of Operational Amplifier.

Ideally, the output voltage of such a circuit would equal the input times the gain of the amplifier, provided that the zero adjustment had been made properly. However, numerous effects may alter this condition. In the circuit of Fig. 4, a feedback impedance, Z_2, degeneratively couples the output, e_2, to the input, while an input impedance, Z_{11}, connects the input to a driving source, e_{11}; e_g represents the normal input signal e developed by e_{11}.

An operating condition of the amplifier shown schematically in Fig. 2 is that the input grid current in the vacuum tube V_1 is less than 10^{-10} amp, which is essentially zero when compared

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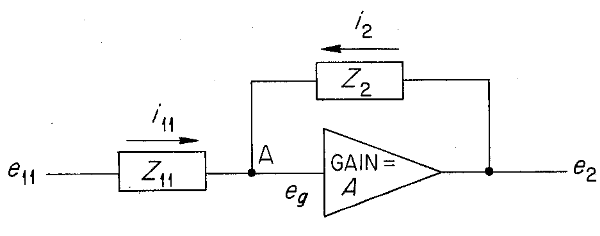

Fig. 4. Operational Amplifier with Feedback and Input Impedances Connected.

with the currents i_{11} and i_2. Accordingly, the current i_{11} through Z_{11} is equal and opposite to the current i_2 through Z_2, and the following relations exist between i_{11}, i_2, e_{11}, e_2, Z_{11}, Z_2, and e_g:

$$i_{11} = -i_2 \tag{1}$$

$$= \frac{e_{11} - e_g}{Z_{11}} \,, \tag{2}$$

$$i_2 = \frac{e_2 - e_g}{Z_2} \,. \tag{3}$$

By using Eqs. (2) and (3) in Eq. (1),

$$\frac{e_{11} - e_g}{Z_{11}} = -\frac{e_2 - e_g}{Z_2} \,, \tag{4}$$

and by solving for e_2,

$$e_2 = -\frac{Z_2}{Z_{11}} e_{11} + e_g \left(1 + \frac{Z_2}{Z_{11}}\right) \,. \tag{5}$$

The second term on the right of Eq. (5) is neglected in all applications of the operational amplifier, since this term represents the output error and should be small when compared with the first term on the right of this equation for a well-designed amplifier:

$$\text{Error} = e_g \left(1 + \frac{Z_2}{Z_{11}}\right) \,. \tag{6}$$

Over the useful range of the amplifier output,

$$e_g = \frac{e_2}{A} \,. \tag{7}$$

Since the gain, A, is quite large, the error will normally be negligible except when the ratio of impedances is very large. An example for which the latter condition is true is the condition when Z_2 is capacitance only and the operational amplifier with pure reactance feedback Z_2 is operating as an integrator.

A dynamic operating range of three decades is normally desirable. With a maximum value in e_2 of 100 v, the minimum limit of 0.1 v will be too low for accuracy in the conventional single-ended d-c amplifier because of the input drift signal. To extend the operating range to three decades, the FLEPAC employs a stabilized single-ended operational amplifier which is manufactured by the Reeves Instrument Corporation. Figure 5 shows a simplified schematic diagram of this amplifier, which is composed of a conventional, single-ended amplifier with a no-drift chopper amplifier added to the input circuit. The chopper amplifier has no high-frequency response; so high-frequency components in the input signal are amplified exactly as they would be in the conventional amplifier. Very low frequencies, however, will be amplified by the chopper circuit, and the behavior of the amplifier will be modified by the feedback from the chopper amplifier.

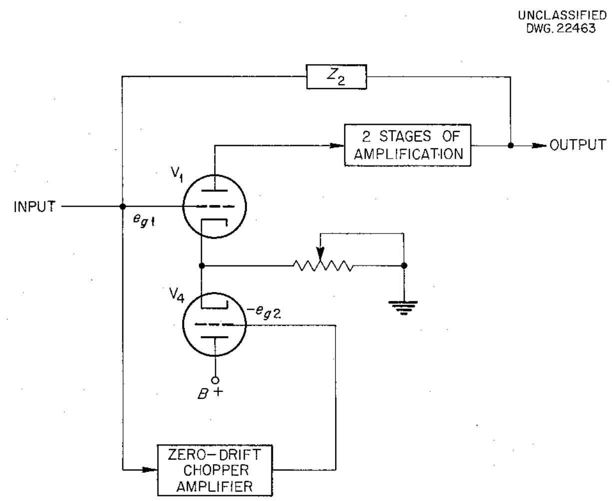

Fig. 5. Simplified Block Diagram of Stabilized Operational Amplifier.

A drift in any portion of the conventional circuit appears as a d-c error offset at the input, shown as e_{g1} in Fig. 5. The actual voltage at the summing junction, e_g, is amplified by the chopper amplifier by a factor of $-A_1$ and applied to V_4 as $-e_{g2}$. The cathode connection of V_4 to V_1 couples $-e_{g2}$ to V_1 so that the effective input to the first tube equals $e_g + e_{g1} + e_{g2}$. Therefore, in Eq. (5), e_g represents the total input required for the conventional circuit; so it follows that

$$e_2 = (e_g + e_{g1} + e_{g2})\,A \quad , \qquad (8)$$

and by following relations analogous to Eqs. (1) through (5),

$$e_2 \cong -\frac{Z_2}{Z_{11}}\,e_{11} - \left(1+\frac{Z_2}{Z_{11}}\right)\frac{e_{g1}}{A_1} \; . \qquad (9)$$

Therefore the chopper amplifier reduces the zero offset voltage, e_{g1}, by the factor $1/A_1$. The zero frequency gain of the chopper amplifier used is approximately 1000, so that the drift signal of the conventional portion of the circuit is reduced by a factor of approximately 10^{-3}. Thus, if the drift of the conventional amplifier were 100 mv, it would be reduced to 0.10 mv by the chopper amplifier. Such a signal is negligible when compared with the normal input signal, e, and will be neglected in later analyses involving the operational amplifier. Since the chopper amplifier is in series with the d-c amplifier at very low frequencies, a d-c input of 1 μv across the input impedance will give an output of 60 v in the absence of negative feedback.

Characteristics of this amplifier are the following:

Chopper amplifier gain	1,000
D-C amplifier gain	60,000
Zero frequency gain	60,000,000
D-C amplifier frequency response	to 10 kc
Chopper amplifier frequency response	0.5 cps
Drift – high-frequency components	<2 mv
Drift – low-frequency components	<0.1 mv

The functional behavior of this amplifier as a basic unit of the simulator is determined by the input networks, Z_{11}, and feedback networks, Z_{22}, that are employed. These networks vary considerably from stage to stage. To provide flexibility, neither the input nor the feedback networks are included within the amplifier but can be coupled to the amplifier by means of the pin jacks located on the front panel.

LINEAR OPERATIONS

A d-c amplifier maintains, throughout its operating range, an input voltage which is negligible in comparison with the output. *Consequently, the potential at point A (Fig. 4) may be considered zero.* Furthermore, since input grid current of the amplifier is vanishingly small, current i_{11} is equal in magnitude and opposite in sign to i_2. The error introduced by these assumptions for the stabilized d-c amplifier is less than the probable error of the measuring equipment except for extremely small values of e_2 and for cases in which a time derivative or integration of the input grid voltage is involved. For the latter two cases further discussion of the probable error will be made. For all other applications this error will be ignored in the following explanations. Thus, in general, referring to Fig. 4,

$$i_{11} = \frac{e_{11}}{Z_{11}} , \quad (10)$$

$$i_2 = \frac{e_2}{Z_2} . \quad (11)$$

It follows that

$$\frac{e_{11}}{Z_{11}} = -\frac{e_2}{Z_2} , \quad (12)$$

or

$$e_2 = \frac{Z_2}{Z_{11}} e_{11} . \quad (13)$$

The output, e_2, equals the negative of the input, e_{11}, modified by the ratio Z_2/Z_{11}. In general, these impedances are complex and therefore frequency-sensitive. When e_{11} is specified, e_2 may be computed. A sufficient description of the system's kinetic response for many purposes may be obtained by specifying either a step change or a periodic function of time for e_{11} and by describing the behavior of e_2. For arbitrary variations of e_{11}, the impedance ratio, Z_2/Z_{11}, denotes an operation performed on e_{11} to obtain e_2. Operations include scaling, differentiating, integrating, and combinations of them.

If, in Eq. (13), Z_{11} and Z_2 are pure resistances, then

$$e_2 = -\frac{R_2}{R_{11}} e_{11} = -A_{11} e_{11} . \quad (14)$$

Here the output equals the input multiplied by a scalar A_{11} which is equal to the ratio of the two resistances. Such a circuit may be used for amplifying or attenuating a function for use at another point in the circuit. A reversal of polarity accompanies the magnitude change. The change in sign may be the sole purpose of such a circuit in a computer. Since the complexity of the problems which may be handled depends on the number of amplifiers available, the number of sign reversals should be kept low to leave as many amplifiers as possible available for more useful applications.

If, in the circuit shown in Fig. 4, Z_2 is a capacitance while Z_{11} remains a resistance,

$$i_{11} = \frac{e_{11}}{R_{11}} , \quad (15)$$

$$i_2 = C_2 \frac{de_2}{dt} ; \quad (16)$$

therefore

$$\frac{e_{11}}{R_{11}} = -C_2 \frac{de_2}{dt} , \quad (17)$$

and, by integrating each side,

$$e_2 = -\frac{1}{R_{11}C_2} \int e_{11}\, dt + C_1 . \quad (18)$$

The output is equal to the time integral of the input plus a constant of integration, C_1. The RC product is a scaling factor upon the output and is chosen in such a manner that the output does not exceed the voltage limit of the amplifier. The factor RC must also conform to the level of output required by the problem being solved. High-frequency noise and signals contribute little to the output e_2, thus permitting these circuits to be used in cascade without decreasing the signal-to-noise ratio.

There is an error in the output of an integrator which may cause difficulty in some applications. To show where this arises, a more exact derivation of the circuit operation follows.

The current i_{11} is

$$i_{11} = \frac{e_{11} - e_g}{R_{11}} , \quad (19)$$

$$i_2 = C_2 \frac{d(e_2 - e_g)}{dt} ; \quad (20)$$

therefore

$$\frac{e_{11} - e_g}{R_{11}} = -C_2 \frac{d(e_2 - e_g)}{dt} \quad . \qquad (21)$$

Upon integrating each side of Eq. (21),

$$e_2 = -\frac{1}{R_{11}C_2} \int e_{11}\, dt + C_1 + \\ + \frac{1}{R_{11}C_2} \int e_g\, dt + e_g \quad . \qquad (22)$$

The first two terms on the right of Eq. (22) constitute the desired integral, while the last two terms on the right are errors. The last term, an error, equals the output divided by the gain of the amplifier. For most purposes this will be negligible in comparison with e_2 because of the large gain (of the order of 60,000) of the amplifier. Although the magnitude of e_g may be ignored, the time integral of this term involved in the third term on the right may become appreciable. For short-time integrations this may be ignored, and in many other applications involving longer periods, the accumulation of error is resisted by feedback in the computer.

For an integrator using a 1-μf, polystyrene, Western Electric condenser and 1-megohm input resistance, the decay of the output occurs with a time constant of 3 hr with the use of the stabilized d-c amplifier.

Those familiar with digital computer techniques know that the process of integration by such techniques is one of summing the products of the width of the interval and some mean value of the function in that interval as denoted by the expression

$$\sum_{i=1}^{n} \bar{f}_i(t)\, \delta t \ ,$$

where $\bar{f}_i(t)$ is the mean value of the function in the interval δt. If the function $f(t)$ is integrable in the interval $t_0 < t < t_1$, then it should be possible to choose the δt in such a way that the expression

$$\sum_{i=1}^{n} \bar{f}_i(t)\, \delta t$$

differs from

$$\int_{t_0}^{t_1} f(t)\, dt$$

by an amount different from zero but otherwise as small in magnitude as desired.

Persons familiar with digital computer techniques sometimes have difficulty in grasping the analog conceptual technique of integration. The digital technique involves the arithmetic operations of multiplication and addition. In mathematical language it may be said that the amplifier gives an output which is proportional to the integral of the input to whatever degree of accuracy is desired by increasing the gain of the amplifier. Neither type of computer gives the mathematical definition of the integral of the input variable.

If in the circuit of Fig. 4, Z_{11} is a capacitance and Z_2 a resistance, then a detailed analysis gives

$$i_{11} = C_{11} \frac{d(e_{11} - e_g)}{dt} \quad , \qquad (23)$$

$$i_2 = \frac{e_2 - e_g}{R_2} \quad . \qquad (24)$$

Therefore

$$C_{11}\frac{de_{11}}{dt} - C_{11}\frac{de_g}{dt} = -\frac{e_2}{R_2} + \frac{e_g}{R_2} \ , \qquad (25)$$

or

$$e_2 = -R_2C_{11}\frac{de_{11}}{dt} + R_2C_{11}\frac{de_g}{dt} + e_g \quad . \qquad (26)$$

The first term of Eq. (26) is the derivative of the input voltage times a scalar, R_2C_{11}. The second two terms are errors. The third term may be neglected because it will always be negligible in comparison to the output. The second term, however, can be shown to introduce an error which increases with input signal frequency, so that the high gain of the amplifier at high frequencies is not effective in reducing the error. For a gain of 60,000 and a unit RC product, if the error is not to exceed 0.1% of the output, then

$$f < 9.6 \text{ cps} \quad . \qquad (27)$$

Cascading differentiators can easily degrade the output signal to the point where it is unreliable. For a limited use, therefore,

$$e_2 = -R_2C_{11}\frac{de_{11}}{dt} \ . \tag{28}$$

Figure 6 shows a more generalized circuit employing the operational amplifier. For this circuit, input networks connected to n sources provide currents common to the amplifier input whose sum must equal the current from the output through M parallel paths.

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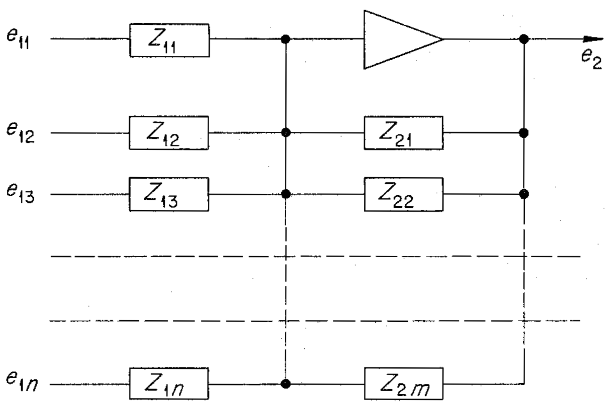

Fig. 6. Generalized Circuit Employing Operational Amplifier.

Since the sum of all input currents must equal the sum of all feedback currents, then

$$\sum_{i=1}^{i=n}\frac{e_{1i}}{Z_{1i}} = -\sum_{k=1}^{k=M}\frac{e_2}{Z_{2k}} \ . \tag{29}$$

The operations denoted by Eq. (29) may include any linear combination of those described in the preceding paragraphs. For example, suppose $n = 2$ and $M = 1$ and each is resistive. From Eq. (29),

$$\frac{e_{11}}{R_{11}} + \frac{e_{12}}{R_{12}} = -\frac{e_2}{R_{22}} \ , \tag{30}$$

or

$$e_2 = -\frac{R_{22}}{R_{11}}e_{11} - \frac{R_{22}}{R_{12}}e_{12} \ . \tag{31}$$

Thus a linear combination of two inputs is obtained at the output in what may be called an "adder."

If, in the above examples, Z_{12} were capacitive, the result, since it is a summation of currents as denoted by Eq. (29), is

$$e_2 = -\frac{R_{22}}{R_{11}}e_{11} - R_{22}C_{12}\frac{de_{12}}{dt} \ . \tag{32}$$

If

$$e_{12} = e_2 \ , \tag{33}$$

then

$$e_2 = -\frac{R_{22}}{R_{11}}e_{11} - R_{22}C_{12}\frac{de_2}{dt} \ , \tag{34}$$

which is a linear, first-order, differential equation with e_{11} as a "source" and $R_{22}C_{12}$ as the time constant. In other words, e_2 behaves as X in the following equation:

$$T\frac{dX}{dt} + X = Bf(t) \ , \tag{35}$$

where

$$T = R_{22}C_{12} \ , \tag{36}$$

$$B = -\frac{R_{22}}{R_{11}} \ . \tag{37}$$

The same result may also be obtained by treating the circuit as one with a single input and a parallel combination of two feedback impedances as shown in Fig. 7. Many more complicated mathematical

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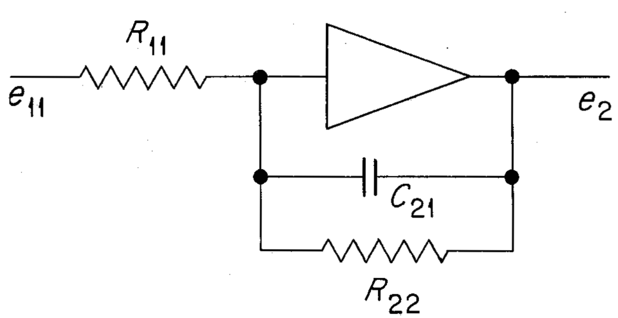

Fig. 7. First-Order Linear System.

operations with different input and feedback networks are tabulated in the RCAF operations manual.[2]

MULTIPLICATION

A scaling factor accompanies each of the linear operations just described, and this scalar depends upon the circuit parameters. A product of a time-dependent variable and a scalar is inherent in all operational amplifier applications. Each of these scalars is the result of a choice of circuit parameters and is a fixed quantity. A change of its value requires a change in the value of one or several of the circuit parameters.

Multiplication deals with those products in which both the multiplier and the multiplicand are time-dependent and will vary during the course of a single problem. A time-dependent variation of the scalar mentioned in the preceding paragraph is implied in such a multiplication. The manner by which this variation may be accomplished depends upon the nature of the time behavior. A step change in the scalar may be made by opening or closing a switch appropriately located in the circuit. Certain arbitrary variations may be provided by manually manipulating a potentiometer setting or a capacitor value. Such methods are frequently used for determining the behavior of a system which is operator-controlled. In general, however, much more extensive control over this scalar must be exercised than the human operator is capable of inserting manually.

Servocontrol of the circuit elements extends the range of multiplication beyond that provided by manual operation. Servomechanisms convert analog quantities to shaft rotations which, in turn, alter the value of circuit parameters. The response times of these systems are much shorter than those of a human operator; yet the motion of mass in such systems limits their ultimate response times also.

The schematic diagram, Fig. 8, illustrates electromechanical means for multiplying two time-varying functions. The Brown strip-chart recorder with an auxiliary slide-wire, perhaps consisting of a precision, 25,000-ohm potentiometer, can be used as a multiplier as shown in Fig. 8. If the variable y is placed across the potentiometer and the variable x is applied as the regular recorder input, and if the recorder full-scale indication in volts is $V_{F.S.}$, then the output taken from the slider of the potentiometer is the product $xy/V_{F.S.}$.

The d-c amplifier used in conjunction with some form of analog multiplier can also provide the operation of division, as shown in Fig. 9. Here the problem is to determine y such that

$$\frac{xy}{V_{F.S.}} = Z \ , \tag{38}$$

or

$$y = \frac{Z}{x} V_{F.S.} \ . \tag{39}$$

If it is assumed that means for multiplying two variables are available, the problem then is one of determining a function y such that

$$xy = Z \ . \tag{40}$$

The difference between the output of the multiplier and Z becomes the input of the operational amplifier. The output of this amplifier rises until the difference between the output of the multiplier and Z is very small – small enough to reduce the

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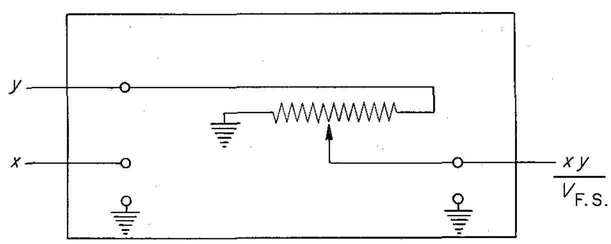

Fig. 8. Strip-Chart Recorder Used as a Multiplier.

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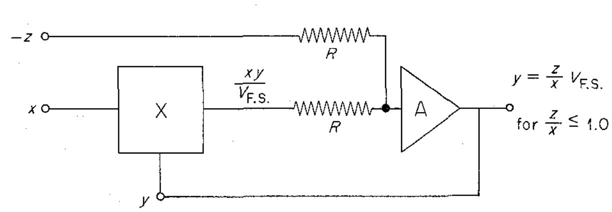

Fig. 9. Strip-Chart Recorder Used as a Divider.

[2]F. P. Green, *Reactor Controls Analog Facility (RCAF) Operations Manual*, ORNL-2405 (Aug. 20, 1958).

net current at the amplifier summing junction to less than 10^{-10} amp.

PANEL COMPONENT IDENTIFICATION

FLEPAC is a cabinet-mounted device with a front panel of standard 19-in. rack width and of $8\frac{3}{4}$-in. height. It may be removed from the cabinet temporarily for relay-rack mounting, when used on a loan basis.

The front panel is functionally divided into three areas. The left and right sections each contain a dual d-c amplifier module manufactured by Reeves Instrument Corporation, and the center section contains the controls and patch-board, Fig. 10. The amplifier modules are interchangeable and can be removed from the front. A spare module is available for on-the-spot maintenance checking and repair so that service continuity can be upheld in the more critical applications.

Across the top of the center section of the front panel is located one row of operating switches. At left is seen the AC POWER switch beside its clear pilot indicator using a GE-20 neon bulb. At center is seen the DC VOLT (plate voltage) switch beside its red pilot indicator using a GE-20 neon bulb. At right is seen the three-position control switch labeled RESET, HOLD, and OPERATE.

The remainder of the center section is functionally divided into four columns entitled AMP 1, AMP 2, AMP 3, and AMP 4, referring to each of the four d-c amplifiers' input, network, and output terminations. Associated with each amplifier are 12 varicolored pin jacks, each of which is identified in abbreviated form on the JACK LEGEND attached at the lower-left corner of the front panel. The number of pin jacks available for each amplifier, the color, the legend code, and the full terminal designations follow:

The patch-panel circuit for one of the four identical networks in FLEPAC is illustrated in Fig. 11.

The screw-driver-adjustable control located below the pin jacks, marked REF, is a single-turn potentiometer used to set the positive or negative reference voltage output to the white pin-jack above it in the same column. The cable jack immediately to the right of the potentiometer, labeled O-C, is a "normally closed" jack at the amplifier input grid which is used to "zero-check" the d-c amplifier by use of a specially designed plug assembly described in this report. It may also be used as the input jack for other special input networks to perform such mathematical operations as differentiation and others shown in the RCAF operations manual.[2]

The toggle switches marked SUM INT are normally thrown to the left to place the 1-megohm resistor in the amplifier feedback for summation or for maintaining stable output when the amplifier is not in use. The switches are thrown to the right to place the 1-μf capacitor in the amplifier feedback to perform integration.

The amplifier attenuator, below the SUM INT switch, is a 10-turn 30,000-ohm potentiometer, one end of which is grounded and the other end of which is connected to the amplifier output. The sliding contact or arm terminates at the brown pin-jack and is fused with a $\frac{1}{32}$-amp MJB or 8AG fuse located under the chassis. To replace a fuse, the unit must be removed from the cabinet, and therefore to minimize this inconvenience, much care should be taken that the attenuator output is not accidentally patched into an amplifier output, a reference output, or ground.

The lowest item on the patch-board is a second 10-turn 30,000-ohm potentiometer labeled "attenuator," also fused, one end of which is grounded. The other end and the arm are available at the

2	GREEN	AMP IN X1	Amplifier input times (gain of) 1
2	RED	AMP OUTPUT	Amplifier output
1	YELLOW	AMP IN X10	Amplifier input times (gain of) 10
1	BROWN	AMP ATT OUT	Amplifier attenuator output
1	ORANGE	ATT IN	Coefficient attenuator input
1	PURPLE	ATT OUT	Coefficient attenuator output
1	WHITE	REF OUT	Reference voltage output
1	BLUE	INITIAL COND IN	Integrator initial condition input
2	BLACK	GROUND	Computer and chassis ground

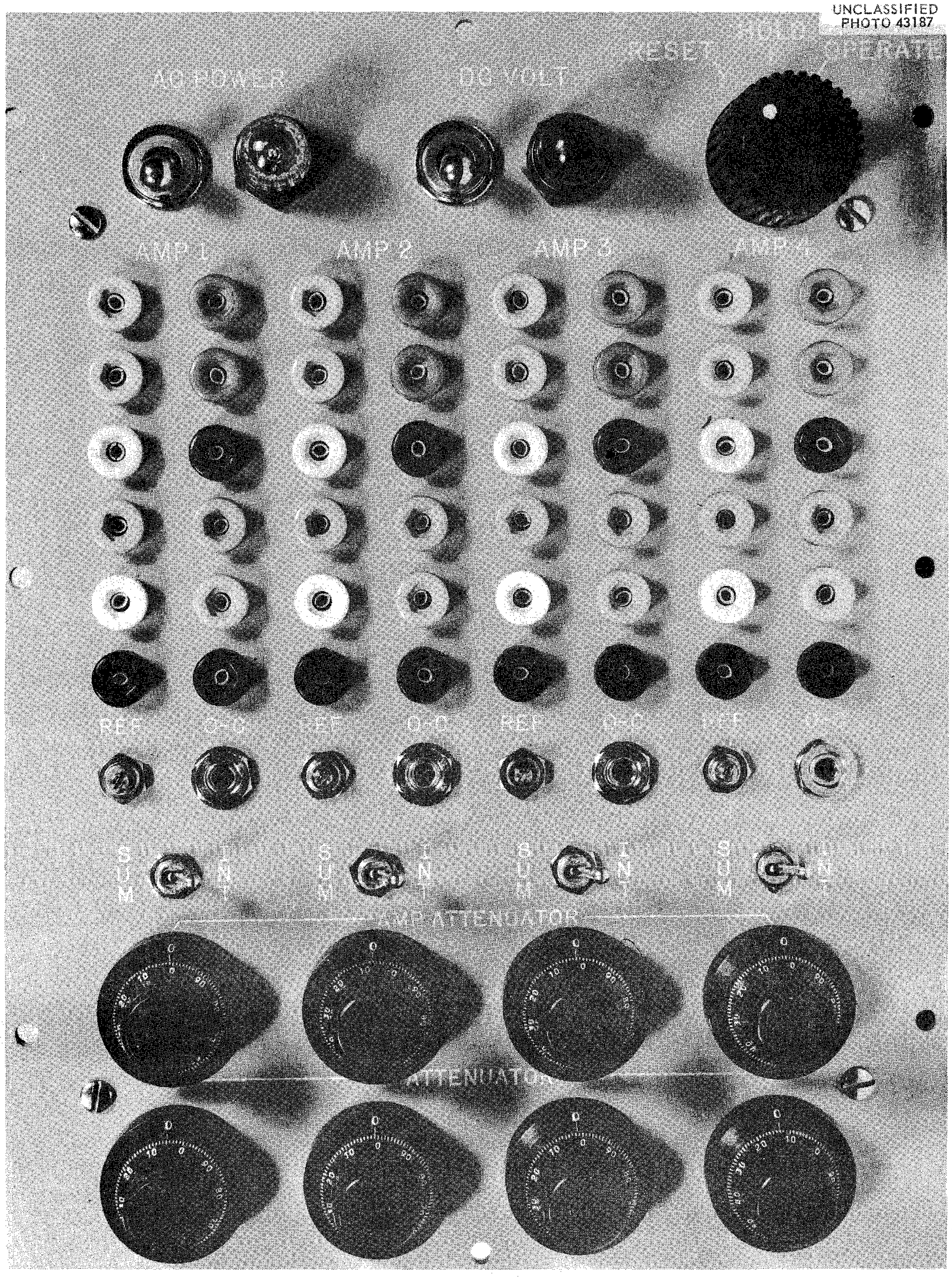

Fig. 10. Patch-Panel of FLEPAC.

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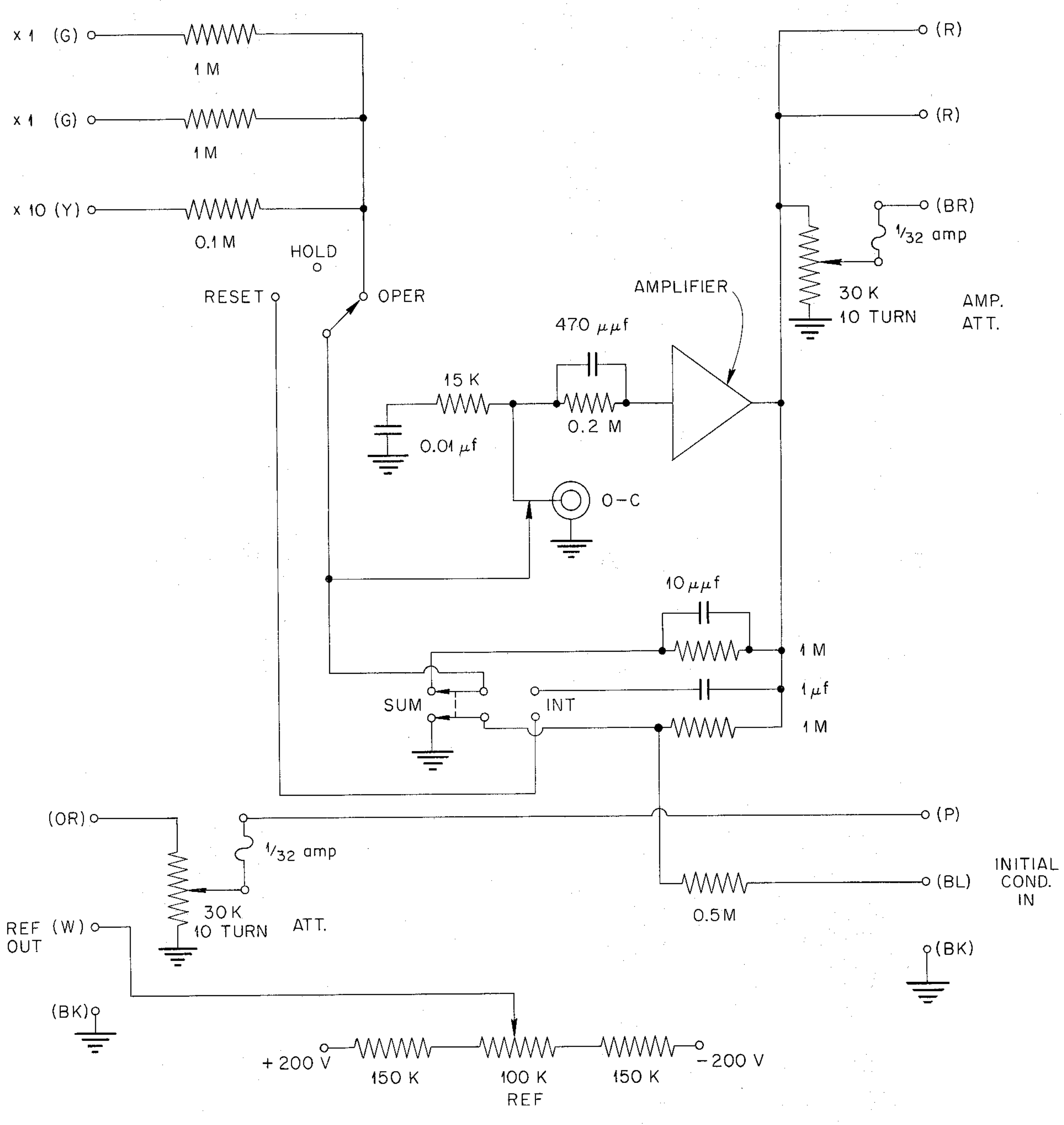

Fig. 11. Patch Panel Circuit for FLEPAC.

orange and purple pin-jacks. Each attenuator can be set precisely with the multiturn dial, by observing the first digit in the window above the reference line and two additional digits on the rotating knob below the reference line.

A set of 1 dozen patch-cords is supplied with the unit. Interconnection of FLEPAC with external equipment must be fabricated by the user, because adapter cords or cables are of types too numerous to predictably supply.

OPERATION

Startup. – Be sure the AC POWER and DC VOLT switches are off (down position), the control switch on RESET, and the SUM INT switches on SUM. Connect 60-cps, 115-v ac from a "clean" or Sola-regulated power source to the back of the cabinet. Throw the AC POWER switch on. After a minute or two, throw the DC VOLT switch on. The neon balance indicators on the amplifier panels will light up temporarily and then go out. Connect the pin plug of the special zero-check patch-cord, the one terminated with a "phone" plug,[3] into the amplifier output, and the "phone" plug into the O-C jack. Connect a voltmeter between ground and one amplifier output and set it on a sensitive d-c scale (1 to 3 v, full scale). By use of the numbered knob on the amplifier panel corresponding to the amplifier whose output is being measured, adjust the output to zero. The special cord contains a gain-of-1000 network, so that the meter scale indicates millivolts of grid offset voltage rather than volts; thus an indication of amplifier condition is observable. Noise fluctuations should be less than 0.1 mv or 100 μv. Remove the meter lead from the amplifier output first, transfer the zero-check cord to the second amplifier, and then reconnect the meter to the second amplifier output and repeat the zero adjustment. Repeat this operation for all four amplifiers. Zero adjustment should be repeated after 1 hr of warm-up and every 4 hr thereafter on those amplifiers in use. Remove the meter and zero-check cord before patching or operating.

Patching can be initiated at any time the control switch is in RESET or HOLD position. The amplifier panel balance indicators will light in any situation wherein (1) the amplifier output voltage exceeds the linear voltage range of the amplifier, or (2) the input and feedback networks provide a high-frequency, positive feedback signal permitting the amplifier to oscillate at a supersonic frequency. In the latter case, a 10,000-ohm series damping resistance added to a capacitance input usually eliminates the instability.

To leave the unit in standbv condition, return the control switch to RESET.

To shut down the unit, return the control switch to RESET and throw the DC VOLT switch off (down). If the FLEPAC is not to be used for two or more days, also throw the AC POWER switch off.

Malfunction. – Should a permanent overload or oscillatory indication appear on the amplifiers, even with simple-summation setting on the panel and with no voltage inputs, then amplifier or power supply malfunction is indicated. Substitution of the spare amplifier in one or the other of the amplifier cubicles may correct the difficulty. Return a disabled amplifier for maintenance if the difficulty seems to be caused by anything other than vacuum-tube failure. For other difficulties, check the vacuum tubes in the FLEPAC power supplies. If the difficulty is still unresolved, return the entire unit for replacement and servicing. If the unit gives no response to a-c or d-c voltage switching, check the fuses on the back and replace if necessary. All higher echelon maintenance should be performed by the Reactor Controls Department.

Elementary Patching Procedure

Any low-impedance voltage source of 100 v or less, capable of being measured by a volt-ohm-milliammeter (20,000 ohms/v), may be patched directly to an amplifier or attenuator input jack. A high-impedance voltage source or a low-current source may be plugged into an O-C jack. Observe that whenever special networks are connected to the O-C jack, all built-in networks of the associated amplifier are disconnected.

Attenuators. – An attenuator may be used as a variable-voltage source to simulate mathematical parameters being inserted as a voltage input to an amplifier; or it may be used as a coefficient potentiometer. Use the loading correction chart shown in Fig. 12 when the accuracy of solution desired will not tolerate the loading on the attenuator by the amplifier input resistor or succeeding

[3]"Tini Plug," ORNL Electronic Stores item No. 231-3420 (Switchcraft, Inc., part No. 780).

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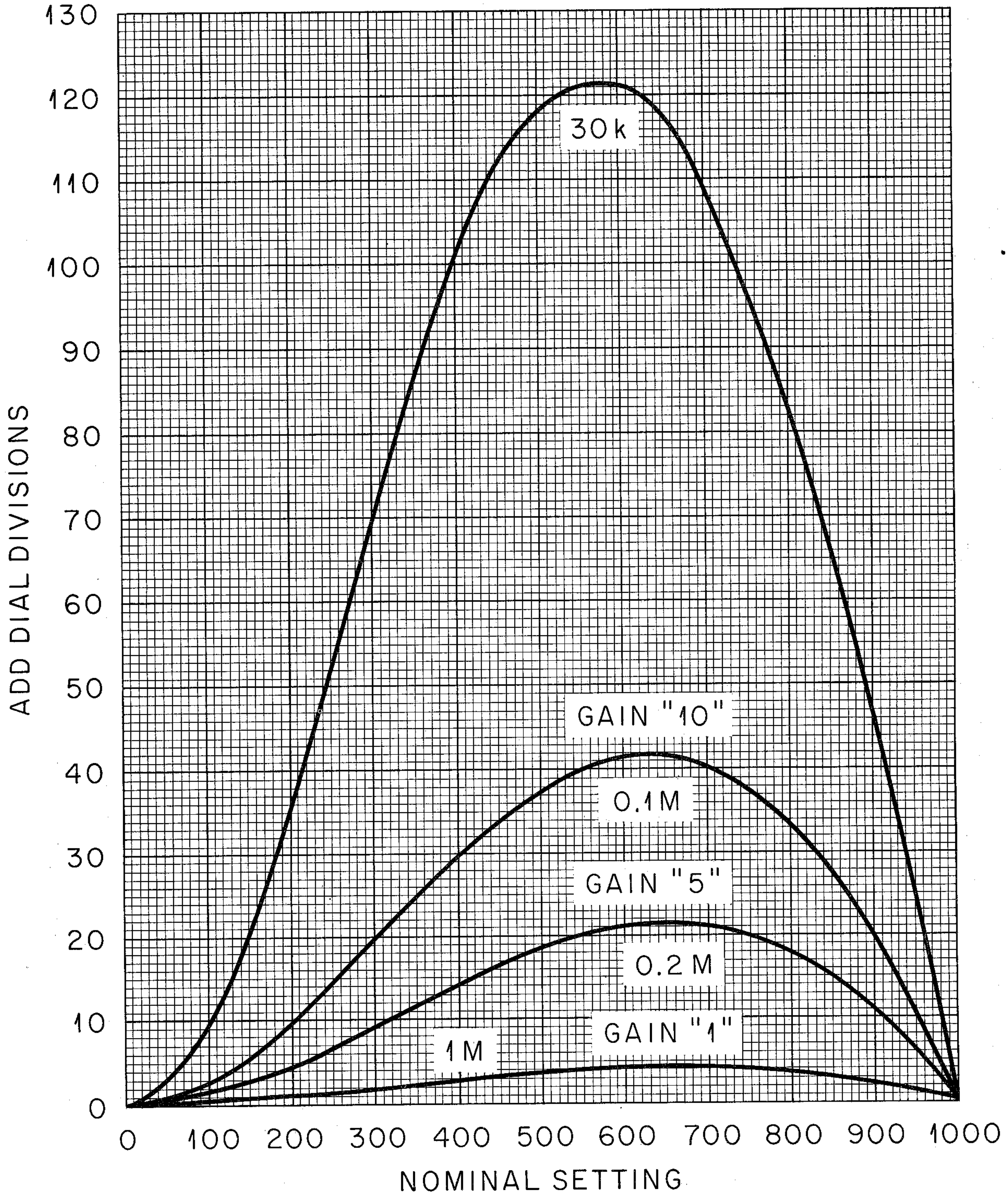

Fig. 12. Attenuator Loading Correction Chart (30-K Potentiometer).

attenuator. Curves shown are drawn for amplifier gains of 1 and 10, and for a tandem pair of attenuators. To obtain the corrected setting under loaded conditions, (1) locate the desired setting on the baseline, (2) pick the curve corresponding to the connection used, (3) note the intersection of the desired setting and the curve, (4) project this point horizontally until it intersects one of the "add dial divisions" lines, and (5) add the dial divisions noted to the attenuator setting.

Gain of 10 or Less. – As many as three voltage inputs may be patched into one amplifier, one of which can be amplified by a gain of 10, and any of which can be attenuated by as much as 1/1000. This applies to both summation and integration.

Gain Greater than 10. – Several patching arrangements are available for obtaining calibrated amplifier gain greater than 10. Amplifier 4 circuitry has been modified slightly from the feedback connection shown in Fig. 11 to that in Fig. 13. The 1-megohm feedback resistor has been removed from the amplifier output and internally reconnected to the output pin jack (*BR*) of the amplifier attenuator. By reducing the setting of the potentiometer, α, from 1.00 to 0.020, gains, G, from 10 to 500 are available. Note: Setting α = 1.00 produces the same gain configuration for amplifier 4 as for amplifiers 1, 2, and 3. The potentiometer involved is identified by a red dot on the knob.

Either of two other circuit arrangements, each requiring two amplifiers, will provide a gain greater than 10. The first, two amplifiers in series, each patched for a gain of 10, provides a gain up to 100 with the output polarity identical to the input polarity. The second, two amplifiers in a closed loop, is illustrated in Fig. 14 and provides a gain up to several thousand with both

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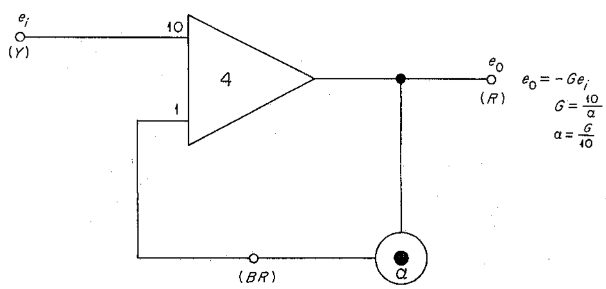

Fig. 13. Single Amplifier No. 4 Network for Gain Greater than Ten.

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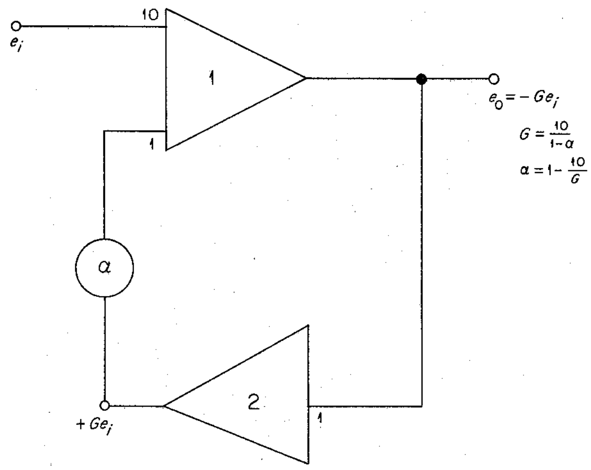

Fig. 14. Two-Amplifier Network for Gain Greater than Ten.

positive and negative output polarities available. The value of α is selected between 0 and $\lesssim$1.00.

Additional Exemplary Configurations

The form of the present ORNL FLEPAC package is not yet optimized for all laboratory applications. For unusual applications, extensive use can be made of the O-C jacks for special input and feedback networks and transducers. The applications suggested here are only a few of the hundreds of possible applications of FLEPAC. It will be appreciated if a description of any new applications developed by FLEPAC users be forwarded so that addenda to the manual may be issued.

Simple Analogs. – In all scientific and engineering practice, elementary variable relations exist which may be converted to electrical terms and simulated. A few such relations shown in Table 2 illustrate some electrical, mechanical, and thermal concepts.

Precision, Wide-Range Exponential Generator. – Suppose that it is desirable to generate a function of the form $f(t) = A\epsilon^{\pm\lambda t}$. For the circuit in Figs. 15*a* and 15*b*, the nodal equation for amplifier 1 without the amplifier 4 feedback is

$$e_1 = -\frac{R_2}{R_1} e - R_2 C_1 \frac{de_1}{dt} , \tag{41}$$

where the steady-state gain is R_2/R_1 and the time

Table 2. Simple Illustrative Analogs

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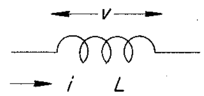
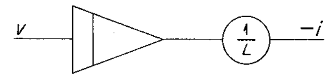
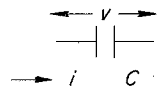
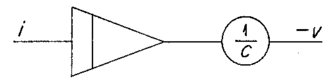
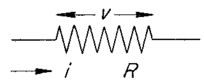
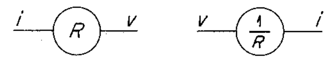
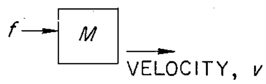
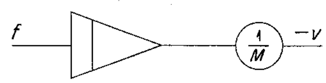
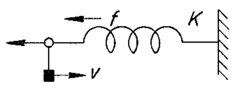
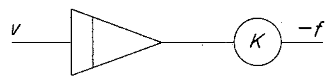
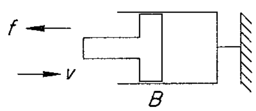
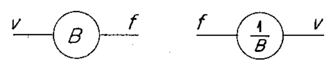
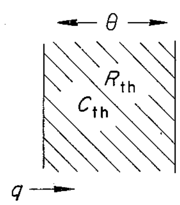
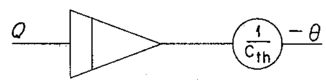
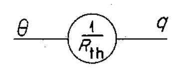
VARIABLE	SYMBOL	EQUATION	ANALOG
ELECTRICAL			
INDUCTANCE		$i = \frac{1}{L}\int v\,dt$	
CAPACITANCE		$v = \frac{1}{C}\int i\,dt$	
RESISTANCE		$v = iR$ $i = v/R$	
MECHANICAL			
MASS		$v = \frac{1}{M}\int f\,dt$	
SPRING		$f = K\int v\,dt$	
DAMPER		$f = Bv$ $v = f/B$	
THERMAL			
		$\theta = \frac{1}{C_{th}}\int Q\,dt$	
		$q = \theta/R_{th}$	

ELECTRICAL

i = CURRENT, amp

v = VOLTAGE, v

L = INDUCTANCE, henrys

C = CAPACITANCE, farads

R = RESISTANCE, ohms

t = TIME, sec

MECHANICAL

v = VELOCITY, fps

f = FORCE, poundals

M = MASS, lb

K = SPRING CONSTANT, poundals / ft

B = DAMPING COEFFICIENT, poundal − sec / ft

t = TIME, sec

THERMAL

q = HEAT FLOW, Btu / sec

θ = TEMPERATURE DIFFERENCE, °F

C_{th} = THERMAL HEAT CAPACITY, Btu / °F

Q = HEAT STORED, Btu

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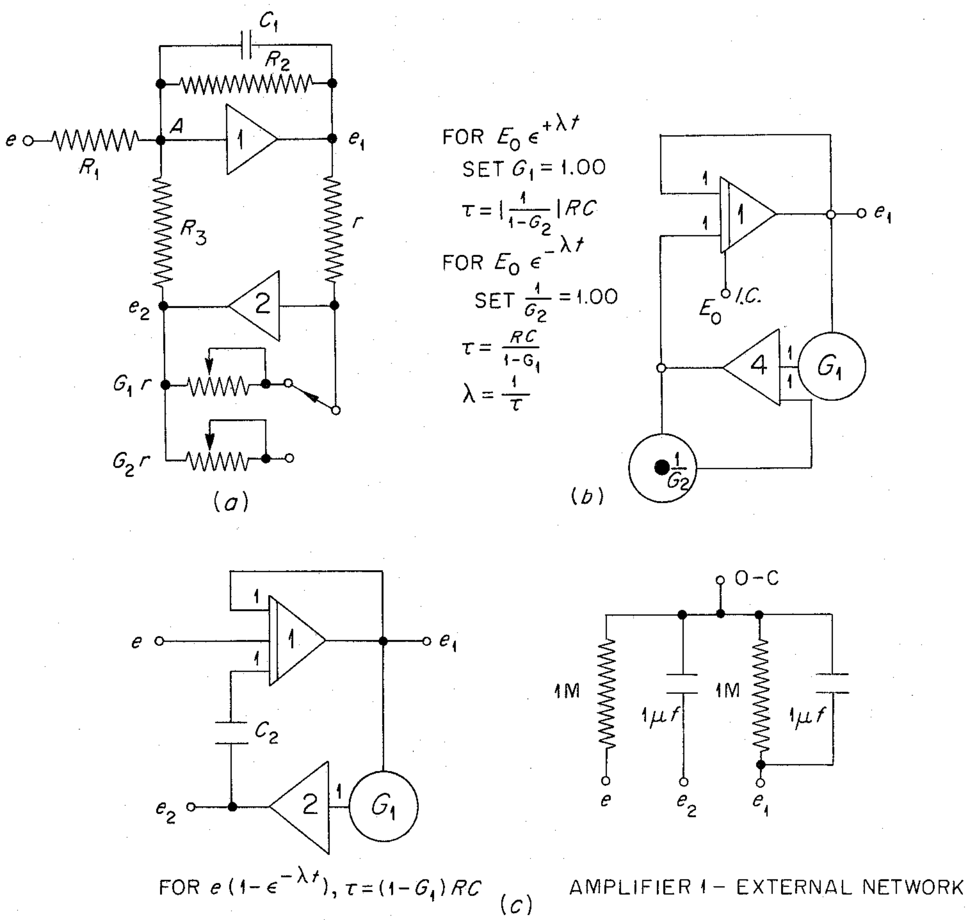

Fig. 15. Wide-Range Linear, Time-Constant Modification.

constant is R_2C_1. With the feedback loop in place,

$$e_1 = \frac{-1}{R_1[(1/R_2) - (G/R_3)]}\, e - \frac{C_1}{[(1/R_2) - (G/R_3)]}\,\frac{de_1}{dt} . \quad (42)$$

In practice, set $R_3 = R_2$ so that

$$e_1 = \frac{-R_2}{R_1(1 - G)}\, e - \frac{R_2C_1}{(1 - G)}\,\frac{de_1}{dt} , \quad (43)$$

which shows that the effect of positive feedback is to divide both the time constant and the gain by $1 - G$. Where G is less than unity, for example, G_1, both the time constant and the gain are effectively increased, and time constants as large as 1000 sec are made possible without the need for impractically large values of R_2 or C_1. Where G exceeds unity, for example, G_2, negative time constants are created, corresponding to regenerative equations. The absolute value of the time constant may be larger or smaller than in the case without feedback, depending upon whether G_2 is less than or greater than 2.0.

Another case of interest is that in which R_3 is replaced by a capacitor, $C_2 = C_1$, assembled in the external network shown in Fig. 15c. The Kirchoff nodal equation at point A and the feedback are

$$\frac{e}{R_1} + \frac{e_1}{R_2} + C_1\frac{de_1}{dt} + C_1\frac{de_2}{dt} = 0 , \quad (44)$$

$$e_2 = -Ge_1 \ , \tag{45}$$

$$e_1 = -\frac{R_2}{R_1}e - R_2C_1(1 - G)\frac{de_1}{dt} \ . \tag{46}$$

Comparison of Eq. (46) with Eq. (41) demonstrates that in the case of capacitive positive feedback the sole effect is to multiply the time constant by $1 - G$. Where G_1 is less than unity, this effectively decreases R_2C_1 and gives access to a range of very short time constants.

The potentiometers may be calibrated and a linear plot of potentiometer setting vs λ (or τ) can be drawn for resistive or capacitive feedback. For positive time constants this is done for two or more settings of Gr by applying a constant potential, E, allowing the circuit to reach equilibrium, and then disconnecting the driving voltage and measuring the e folding time, τ, of the decaying function, e_1. For negative time constants this is done for two or more settings of Gr by applying a constant potential, E, differentiating the output, e_1, and measuring the rise-time constant, τ, of the expanding derivative. A typical plot of λ vs loop gain is shown in Fig. 16.

Low-Frequency Oscillator. – A sinusoid is a frequently required input and test function for system simulation. A low-frequency oscillator which can produce frequencies up to 1.59 cps is illustrated in Fig. 17. Its frequency is determined by

$$f = \frac{\omega}{2\pi RC} = \frac{\omega G}{2\pi} \ , \tag{47}$$

which, for the coefficient potentiometers, ω, set at 1.00 and for the integrator gain, $1/RC$, set at 1.00, is $1/2\pi$, or 0.159 cps. The amplitude is set with the initial-condition potentiometer, and if zero damping is desired, the output potentiometer of amplifier 4 is set empirically near mid-scale. Increasing the setting produces negative damping, and decreasing the setting produces positive damping.

Geometric Mean Generator. – The solution of a problem defined as one which requires determination of the geometric mean of two functions x and y is shown in Fig. 18. The variable x is the input of one Brown recorder. The variable y is fed through an inverting amplifier to the auxiliary slide-wire or potentiometer of the X_1 recorder, producing $-xy/100$ as the output of the slider. The analog voltage $-xy/100$ is then fed into amplifier 4, whose only feedback is through the auxiliary slide-wire of a second Brown recorder, labeled X_2. The servo amplifier of the Brown recorder is also fed from amplifier 4, so that the X_2 output equals $Z^2/100$. In order to keep the voltage at O near zero, this arrangement forces

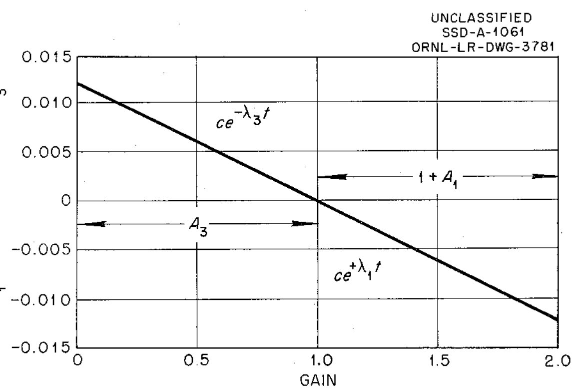

Fig. 16. Typical Plot of λ vs Loop Gain.

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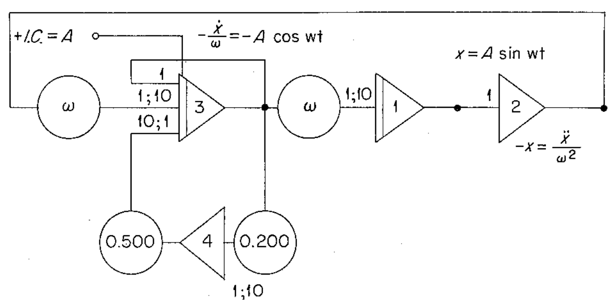

Fig. 17. Low-Frequency Oscillator.

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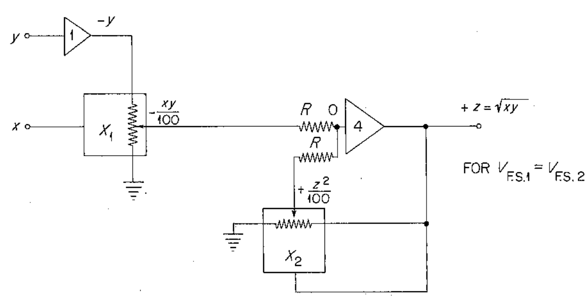

Fig. 18. Geometric Mean Generator.

the output of amplifier 4 to be such that

$$\frac{Z^2}{100} = \frac{xy}{100} \quad , \tag{48}$$

or at the amplifier output,

$$Z = \sqrt{xy} \quad . \tag{49}$$

The variable Z is recorded on the strip chart of the X_2 recorder.

Servo-controller. – The following analysis of a servo system illustrates the method of formulating a circuit. As a simplified model, it is assumed that all inertia and mass in the system may be combined as a single unit. This mass, perhaps a control rod, moves under the influence of an accelerating force composed of three parts. First, an error in the position or velocity of this mass is amplified and applied as a force; second, a frictional force which is proportional to the velocity of the mass retards its motion; third, a restoring force which is a function of its deviation from equilibrium may act upon the system. The total force acting upon the mass is the sum of these three components. The ratio of total force to mass equals acceleration, so that

$$A = \frac{F_t}{M} = \frac{F(E)}{M} - \frac{B_5 V}{M} - \frac{B_6}{M}(S - S_0) \quad , \tag{50}$$

where

A = acceleration of mass M,

F_t = total force acting upon mass M,

$F(E)$ = component of force derived from system error,

B_5 = coefficient of friction,

V = velocity of mass,

B_6 = stiffness coefficient,

S = position of mass,

S_0 = equilibrium position of mass.

As shown in Fig. 19, a circuit with three inputs, each associated with one of the terms on the right of Eq. (50), supplies a current to the input grid of amplifier 1 which is proportional to the total acceleration of the mass M. The reciprocal of the coefficient in Eq. (50) equals the resistance of the input circuit, and the variables are equal to the input voltages. Since

$$V = \int a \, dt \quad , \tag{51}$$

an integrator circuit, integrating the input current of the circuit of Fig. 18, produces an output proportional to velocity. Thus

$$-V = -\int a \, dt \quad . \tag{52}$$

Due to the polarity reversal of the amplifier, the output is proportional to the negative of the velocity. This is just the value of voltage required for one of the inputs; so, as shown by the dotted line in Fig. 19, the output may be coupled to the input.

Since the output of the circuit of Fig. 19 equals the velocity of the mass M, integration of this output yields the displacement, S (Fig. 20). An adder follows the integrator for the purpose of obtaining the difference between S and some equilibrium value of position, S_0. A negative S_0 causes the difference between S and S_0 to appear at the output of amplifier 3. This difference equals the third input to amplifier 1 and may be so connected, as shown in Fig. 20, by a dotted line.

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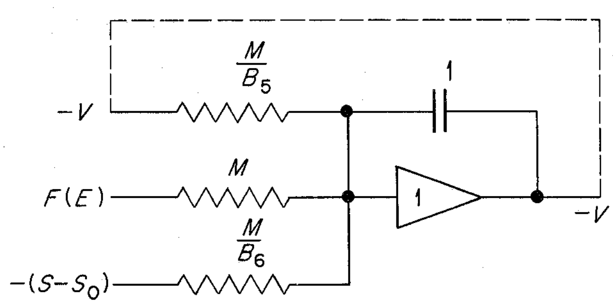

Fig. 19. Force Integrator.

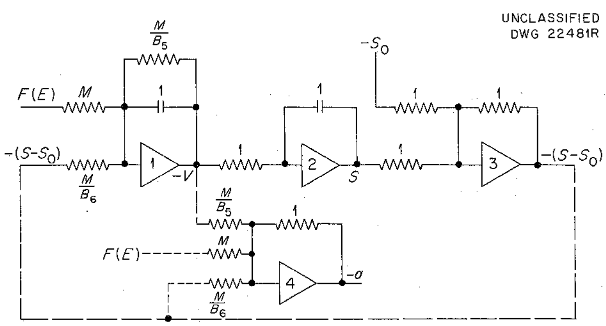

Fig. 20. Servo Simulator.

Amplifier 4 is a force-summing circuit added to obtain the acceleration of the body being simulated.

Although no equation of motion was employed directly in the description of the circuit in the preceding paragraphs, it follows from Eqs. (50) and (51) that

$$M\frac{d^2S}{dt^2} + B_5\frac{dS}{dt} + B_6(S - S_0) = F(E) \quad , \qquad (53)$$

and this is the equation which relates the model and the simulator. The same result could be obtained by starting with the model formulating Eq. (53) and designing an analog circuit for its solution.

Figure 21 shows the feedback circuits for an idealized servo system which the simulator provides. The circuit is the same as that shown in Fig. 19 with the addition of potentiometers to provide the means of altering the parameters.

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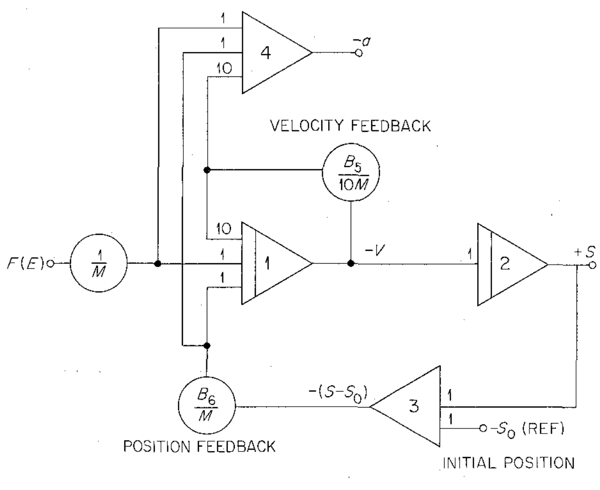

Fig. 21. Schematic Diagram of Servo Simulator.

A Proportional-Integral Servo-controller. – The operational amplifiers of FLEPAC can be used in a control device where proportional and integral control are desirable. The voltage to be controlled may be one derived from the sliding contact of a potentiometer in a Brown recorder for recording, for example, temperature, where a fixed potential is maintained across the potentiometer. In Fig. 22 the output of amplifier 2 can be used as the input to a power amplifier and an actuating device.

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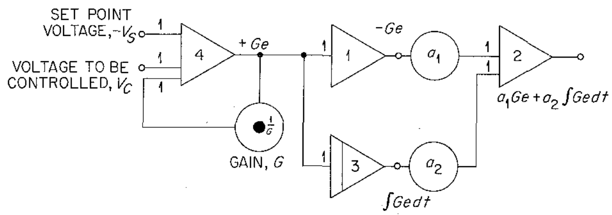

Fig. 22. Proportional-Integral Servo Controller.

Ballistic Galvanometer. – The operational amplifier used as an integrator can function as a ballistic galvanometer, as shown in Fig. 23. It is used to determine the magnetic lines threading a coil in the valuation of the hysteresis characteristics of a ferromagnetic material. Here the change in the number of lines threading the coil is

$$\frac{N\, d\phi/dt \cdot 10^{-8}}{R} = C\frac{dE}{dt} , \qquad (54)$$

where N is the number of turns of wire and $d\phi/dt$ is the rate of change of magnetic lines;

$$E = \frac{N \cdot 10^{-8}}{RC}(\phi - \phi_0) \quad , \qquad (55)$$

where E is in volts and $\phi - \phi_0$ is in maxwells.

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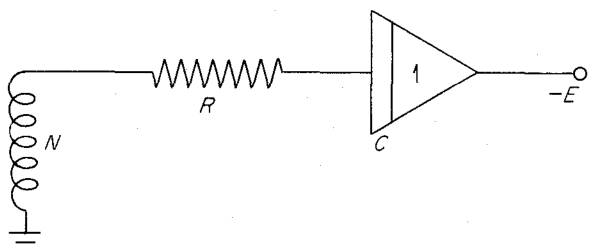

Fig. 23. Ballistic Galvanometer.

Count Rate Device. – In a case where a pulse shaper provides pulses of constant amplitude and width, a count rate device can be set up as shown in Fig. 24. For a fixed setting of the potentiometer, the output is proportional to the count rate over a range of three decades, that is, from 0.1 to 100 v.

One application of such a circuit might be with a pulse-type tachometer. The output E of the

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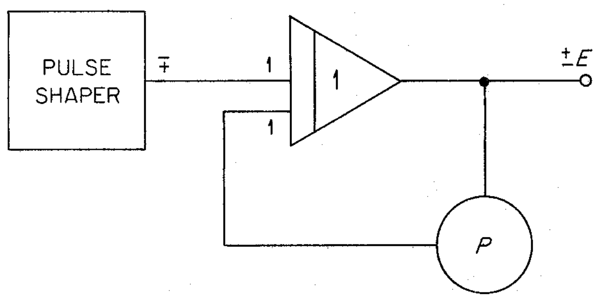

Fig. 24. Count-Rate Meter.

amplifier is then proportional to the speed of rotation for a fixed setting of potentiometer, P.

Nuclear Reactor Simulator. – The nuclear kinetic equations commonly employed to describe a reactor may be written as follows:

$$l^* \frac{dP}{dt} = (1 - \beta)\, kP - P + \sum_{i=1}^{i=6} \lambda_i X_i + S \quad , \qquad (56)$$

$$\frac{dX_i}{dt} = -\lambda_i X_i + \beta_i a_i kP \quad , \qquad (57)$$

where

l^* = mean lifetime of prompt neutrons from generation to absorption,

P = neutron concentration,

k = effective multiplication factor,

X_i = ith delayed neutron emitter concentration times l^*,

λ_i = decay rate of ith delayed neutron emitter,

S = neutron source in neutrons per l^* sec,

β_i = fraction of total neutron production which is delayed by the ith delay group,

β = total fraction of neutron production which is delayed by all delay groups,

a_i = factor employed to account for the reduction of the effective concentration of delayed neutrons due to fuel circulation.

Although the variables employed in Eqs. (56) and (57) were defined in terms of neutron concentration, little error results from the assumption that the power produced by the reactor is directly proportional to the neutron level. Thus P may be considered as the power production in Btu/sec, and X_i would be the potential power stored in the delayed-neutron emitter. The definition of units and terms depends upon the purpose of the reactor. For research reactors the neutron flux is important, and the terms would be as defined. For power-producing reactors, units of heat production are more conveniently employed. Since power reactors will be described in the following paragraphs, the units employed will be power units.

Since, for the circuit of Fig. 25,

$$i_1 + i_2 = 0 \quad , \qquad (58)$$

$$i_2 = -l^* \frac{dP}{dt} \quad , \qquad (59)$$

then it follows that

$$i_1 = l^* \frac{dP}{dt} \quad . \qquad (60)$$

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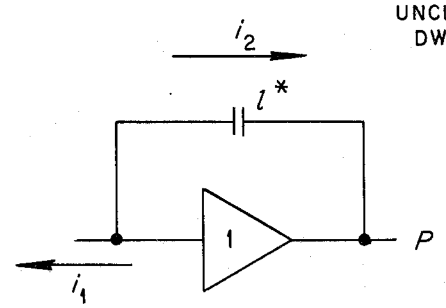

Fig. 25. Power Integrator Feedback Circuit.

Current i_1 may be produced by multiple inputs to amplifier 1. Each input equals the negative of one term on the right side of Eq. (56). Three input currents may be produced directly and are shown in Fig. 26. Since P is both an input and an output, the dotted connection may be used to couple them.

The currents proportional to the delayed emitters require further consideration. It is required that currents be produced at the input to amplifier 1 which equal $\Sigma \lambda_i X_i$. This involves six inputs, since six classes of delayed emitters are utilized in the simulator. The current, or contribution, from each group is described by Eq. (57). A steady-state solution of these equations yields

$$\lambda_i X_{i0} = \beta_i a_i k P_0 \quad . \qquad (61)$$

These are proportional to kP_0. During transient

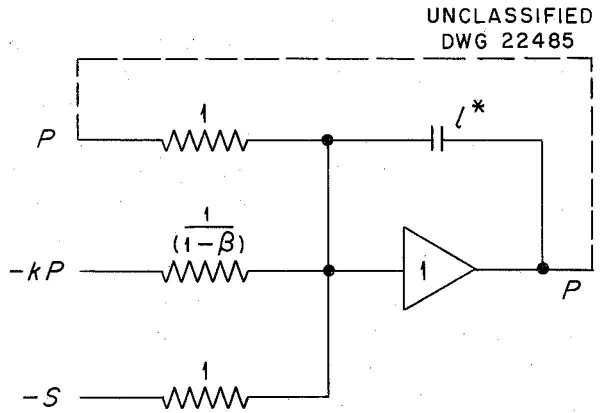

Fig. 26. Power Integrator with Prompt Inputs.

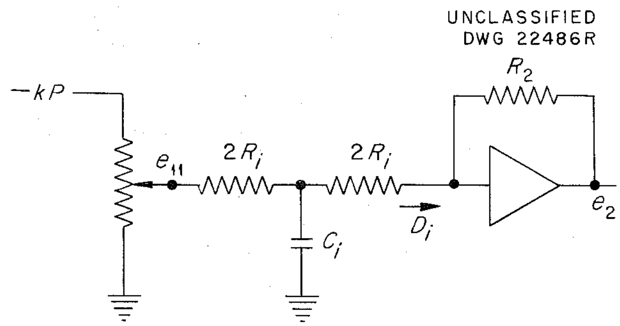

Fig. 27. First-Order Delay in Input Circuit.

conditions, however, each current differs from that of Eq. (61) and is delayed by a time constant equal to $1/\lambda_i$. The source term, or driving function, in Eq. (57) is proportional to kP.

The input circuit shown in Fig. 27 was described previously. The input current, here denoted by D_i, may be described by the following:

$$C_i \frac{d(2R_i D_i)}{dt} + D_i = \frac{e_{11} - 2R_i D_i}{2R_i} , \tag{62}$$

or, by rearranging,

$$\dot{D}_i = -\frac{1}{R_i C_i} D_i + \frac{e_{11}}{4R_i^2 C_i} . \tag{63}$$

From Fig. 27,

$$e_{11} = -\gamma_i kP , \tag{64}$$

where γ_i = gain of input potentiometer. Then Eq. (63) becomes

$$\dot{D}_i = -\frac{1}{R_i C_i} D_i - \frac{\gamma_i}{4R_i^2 C_i} kP . \tag{65}$$

The necessary input current to amplifier 1 equals $-\lambda_i X_i$. Therefore let

$$D_i = -\lambda_i X_i . \tag{66}$$

Substituting this in Eq. (57) yields

$$\dot{D}_i = -\lambda_i D_i - \beta_i a_i \lambda_i kP , \tag{67}$$

which equals Eq. (65) if

$$\lambda_i = \frac{1}{R_i C_i} , \tag{68}$$

$$\beta_i a_i = \frac{\gamma_i}{4R_i} . \tag{69}$$

Figure 28 shows the circuit of Fig. 26 after six networks were added to furnish the necessary delayed contribution. Amplifier 2 performs as a scalar to produce $-kP$ and closes the loop.

Figure 29 illustrates in a more complete manner the reactor nuclear loop. The feedback circuits associated with amplifier 1 of Fig. 28 are represented as a single block. The circuit shows the means of generating $-kP$ from P; k is the effective multiplication factor and is variable. It equals the effective scalar product of P when comparing the output of amplifier 2 with that of amplifier 1. Four inputs furnish current to amplifier 2. Each current is proportional to P, and the output of amplifier 2 is proportional to the sum of all input currents. Consider, therefore, each input separately.

The upper three inputs furnish currents which are

$$i_{11} = 0.95P , \tag{70}$$

$$i_{12} = 0.10P\beta_1 , \tag{71}$$

$$i_{13} = 0.05P\beta_2 \int . \tag{72}$$

The currents i_{12} and i_{13} may be controlled manually by potentiometers. The "unit function" symbol in Eq. (72) indicates that closure of switch 1 permits application of i_{13} as a "step."

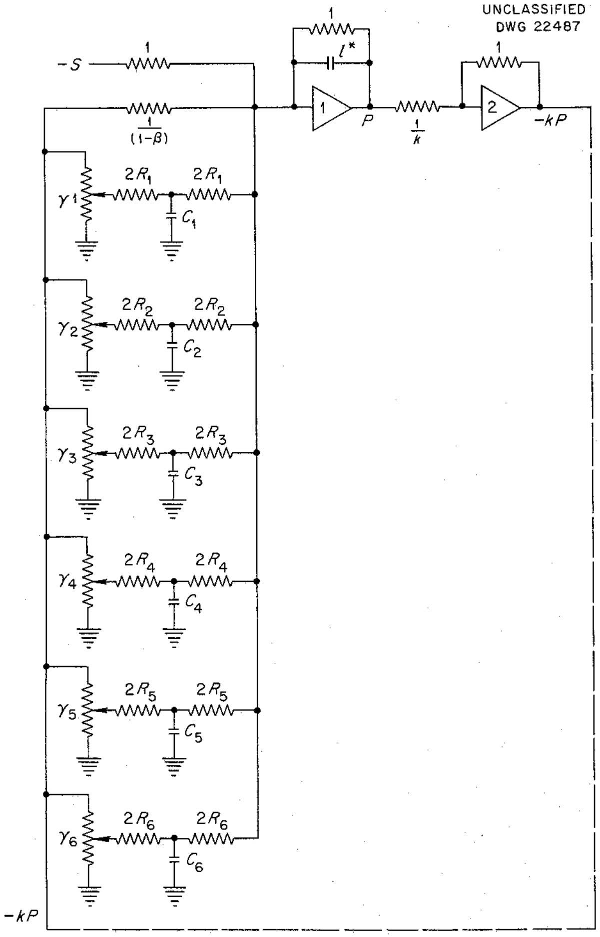

Fig. 28. Nuclear Kinetic Simulator.

The input, θ, to the multiplier includes all other continuously variable factors which affect the value of k. Thus

$$i_{14} = -\frac{P\theta\beta_3}{1000} . \quad (73)$$

The sum of all input currents is, therefore,

$$i_{\text{total}} = 0.95P + 0.10P\beta_1 + + 0.05P\beta_2 - 0.001P\theta\beta_3 . \quad (74)$$

Factoring out P from the terms on the right side of Eq. (74) yields

$$k = 0.95 + 0.10\beta_1 + 0.05\beta_2 - 0.001\theta\beta_3 . \quad (75)$$

In the most elementary form, the production of power in a reactor will raise the temperature of the fuel. Some means in the form of a fluid coolant is provided to remove this heat from the critical lattice. The exchange of heat between point A and point B of a stationary system requires a difference of temperature between A and B, provided that no change of state of the material at either point is involved. Heat flows from the higher temperature point to the lower one. The rate of flow of heat, or power, depends upon the thermal conductivity and the difference of temperature between the points. An interface modifies the thermal conductivity between two points. Within and between the mediums employed for heat conduction or heat transfer, temperature variations exist. A complete solution of the thermodynamic properties, therefore, involves both spatial and time variations. This computation is not suited for FLEPAC solution. The mean temperature of each medium is employed in the simulator, and the average differences in temperature between two mediums are used.

As a typical reactor, consider one in which the fuel is stationary and a coolant circulates past the fuel, carrying the heat to an external heat exchanger. The inlet temperature of the gas is fixed by the load and is not affected by the reactor.

Consider the fuel. If no coolant is supplied, the temperature of the fuel, θ_f, at design-point power rises at a mean rate of θ_f deg/sec. This is obtained by dividing the design-point power, P_0, by the total heat capacity, C_f, of the fuel. Thus

$$\theta_f = \frac{P_0}{C_f} . \quad (76)$$

By use of an integrator, such as that shown in Fig. 30, P_0 determines θ_f according to Eq. (76). The negative sign is due to the polarity reversal of amplifier 1. Without cooling, however, the fuel would melt. Heat is removed from the fuel at a rate proportional to the difference in temperature between the fuel and the coolant and is a function of the geometry of the physical system. At design point, however, Z_1 degrees exists between the two means, and this, by definition, is the difference required to transfer heat from the fuel to the coolant at the rate at which it is produced. Coolant flow removes heat from the reactor at this rate, so that at design point the coolant mean temperature, θ_c, is constant.

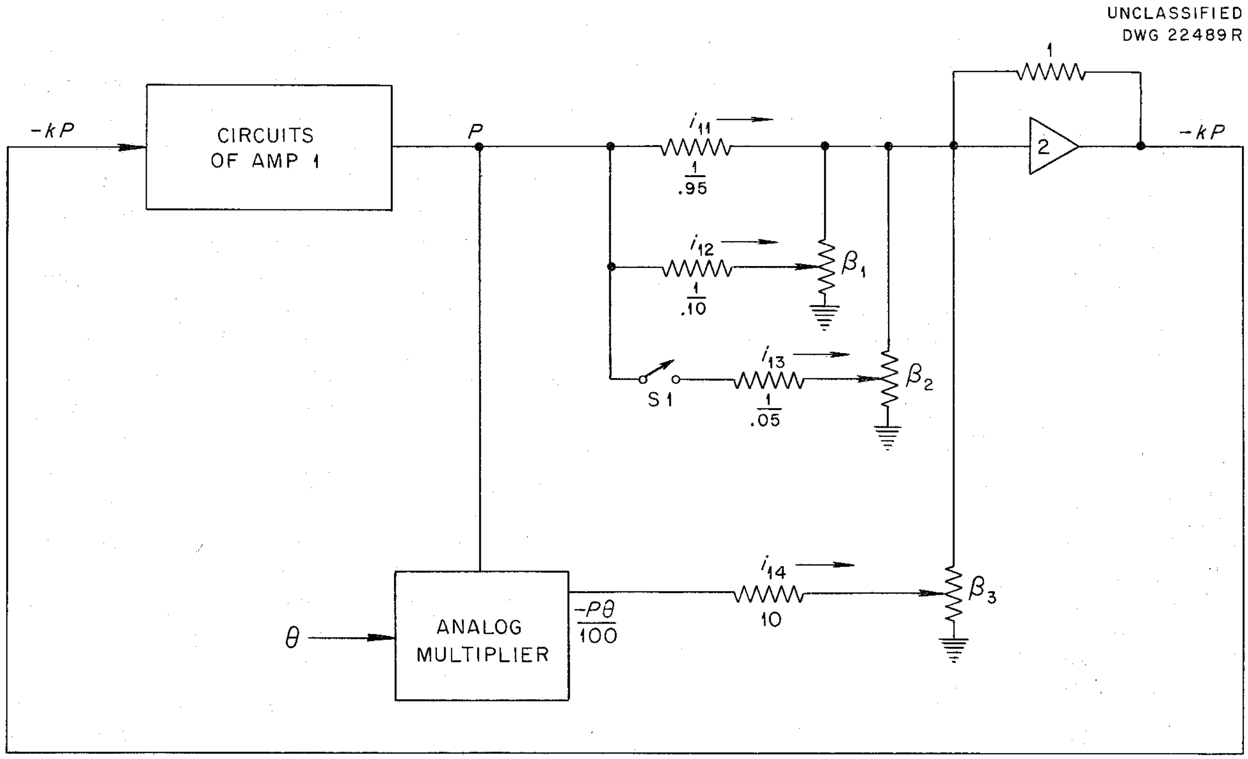

Fig. 29. Nuclear Kinetic Simulator Circuit with Means of Controlling k.

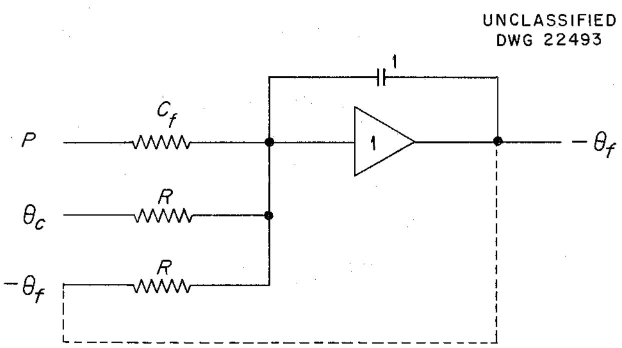

Fig. 30. Fuel Temperature Integrator with Heat Extraction.

Since the mean fuel temperature is constant at design-point power, P_0, the sum of all input currents to amplifier 1 must be zero at this power. Therefore

$$\frac{P_0}{C_f} + \frac{\theta_c}{R} - \frac{\theta_f}{R} = 0 \quad , \tag{77}$$

but

$$\theta_f - \theta_c = Z_1 \quad , \tag{78}$$

so that

$$\frac{P_0}{C_f} - \frac{Z_1}{R} = 0 \quad ; \tag{79}$$

therefore

$$R = \frac{Z_1 C_f}{P_0} \quad . \tag{80}$$

The difference between the power introduced and the power removed from the coolant, within the reactor, determines the time behavior of the mean coolant temperature. In this simplified case, θ_c is considered to be a load-demand parameter and would simulate an open-loop cooling system. The mean fuel temperature, θ_f, is the feedback parameter supplied to the θ input in Fig. 29.

The complete circuit for such a reactor simulation is shown in Fig. 31. The necessary networks of input and feedback passive elements have been assembled in a separate small package, the FLEPAC Nuclear Reactor Simulator.

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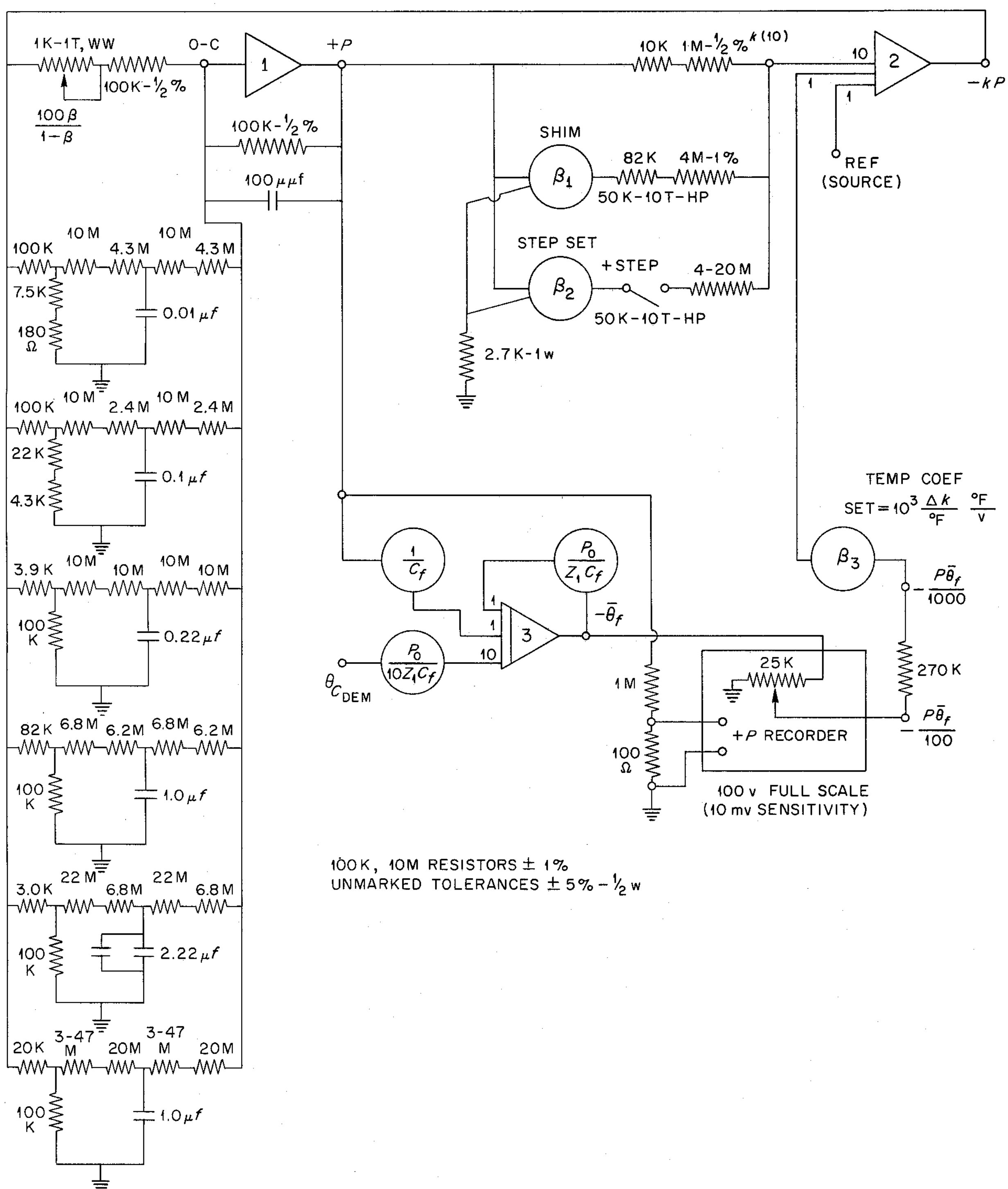

Fig. 31. Nuclear Kinetic Simulator Including Fuel Temperature Coefficient of Reactivity.

APPENDIX

FLEPAC TANDEM OPERATION ACCESSORY

NOTE: WHEN CONNECTED, SELECTION OF REMOTE OR LOCAL OPERATION MADE BY SETTING UNUSED CONTROL SWITCHES ON **HOLD.**

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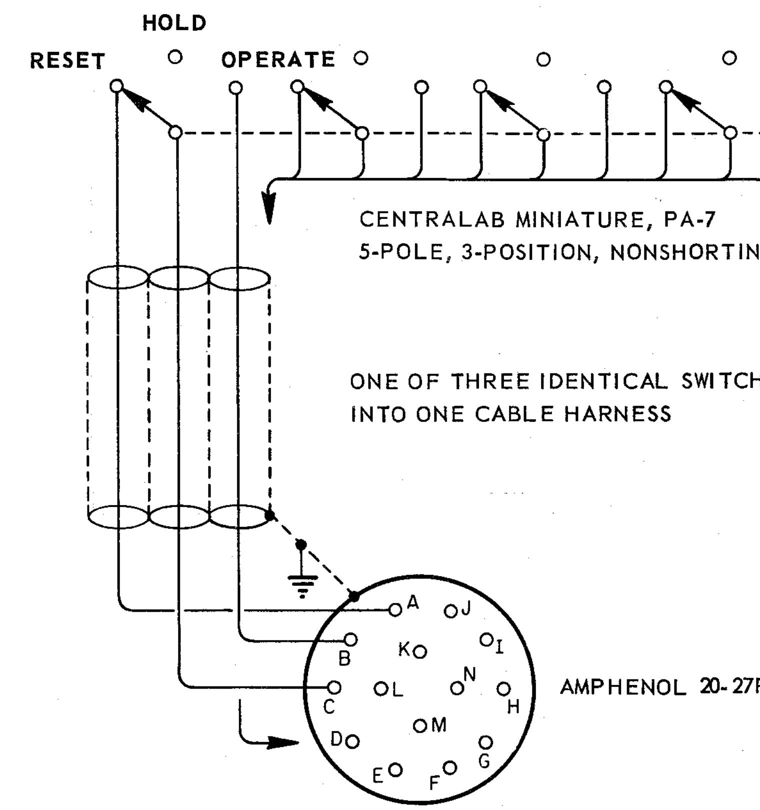
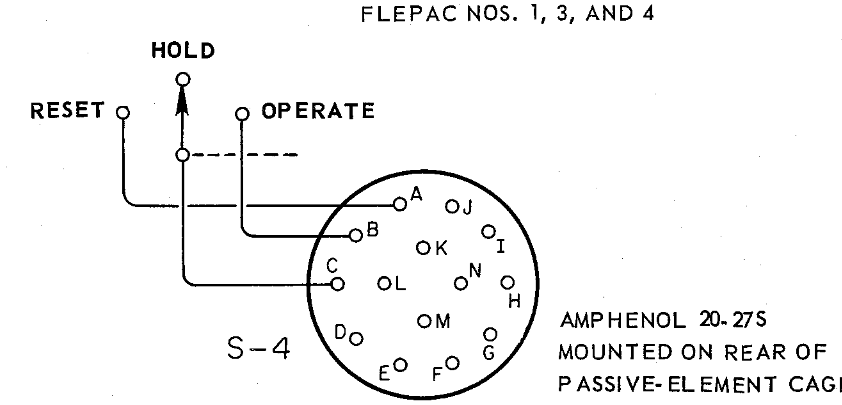
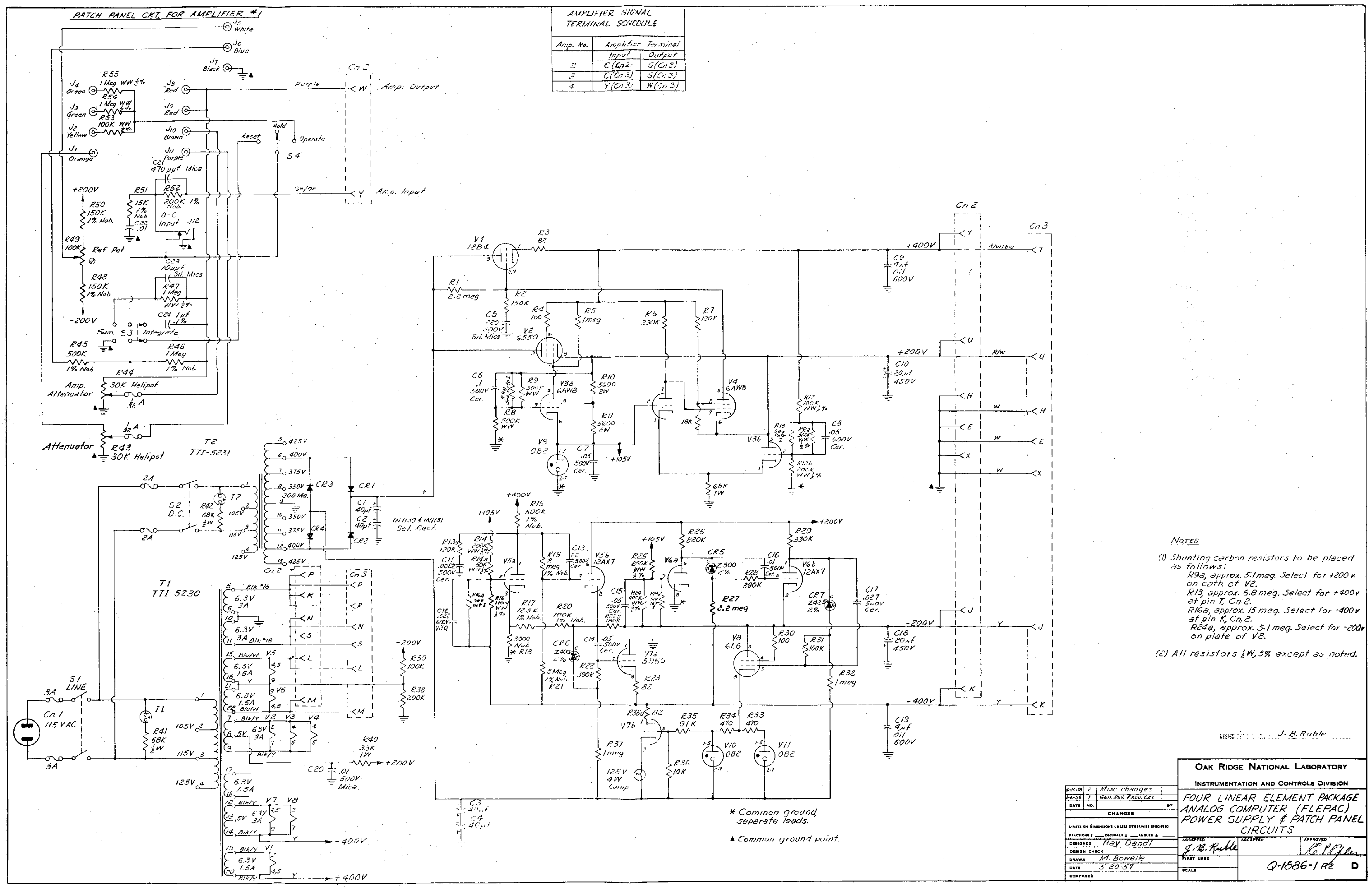


FLEPAC NOS. 1, 3, AND 4



AMPLIFIER 1: ABC
AMPLIFIER 2: DEF
AMPLIFIER 3: GHI
AMPLIFIER 4: JKL



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