

Hewlett Packard Correlator Model 3721A

PROBABILITY DENSITY FUNCTION

We shall discuss two ways of describing a signal — the power spectrum and, its equivalent in the time domain, the autocorrelation function. Neither of these, however, gives any indication of *waveshape*. Power spectrum and autocorrelation function describe a signal's frequency content, but do not characterize its waveshape.

No problem arises if the signal can be described by a simple expression (such as $y = a \sin \omega t$) but, unfortunately, this is not always the case.

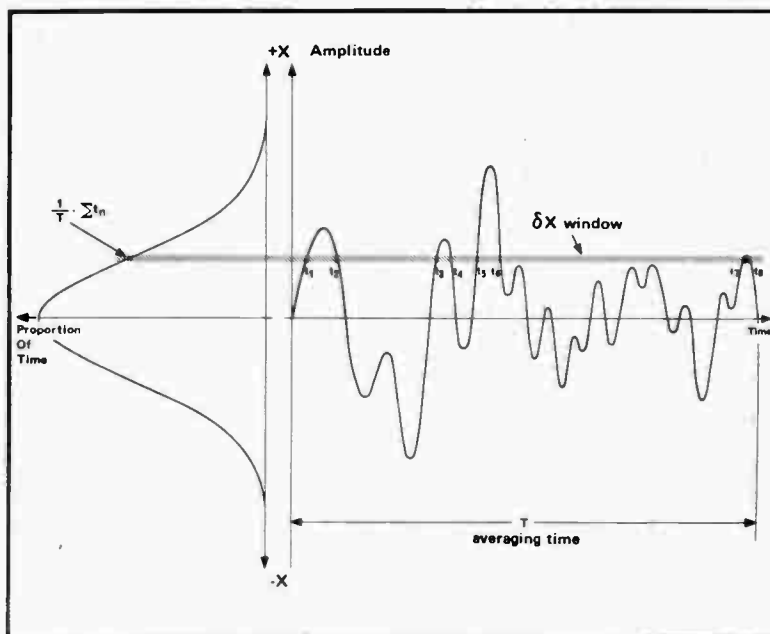
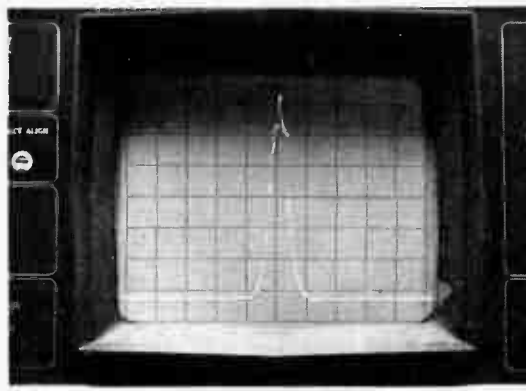
How can we describe a random signal (that is, noise) for which no expression can be found? Noise, by definition, is non-periodic and therefore cannot be specified by a simple time-varying function.

We can establish the power spectrum — and hence autocorrelation function — of a random signal just as we can that of a periodic signal. The spectrum alone may be all that we need to know about the signal in many instances but, for some purposes, we must have a means of characterising a random signal's amplitude behaviour.

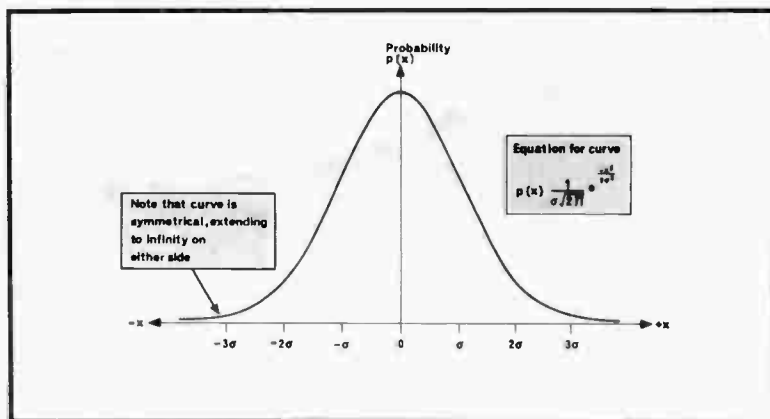
To do this, we determine the *proportion* of time spent by the signal at all possible amplitudes during a finite period of time. In practical terms, this means totalising the time spent by the signal in a selection of narrow (δx) amplitude windows, and then dividing the total for each window by the measurement, or averaging, time (T). The curve obtained by plotting the window totals against amplitude is known as the *probability density function* (pdf) of the signal.

The most commonly encountered pdf (for naturally occurring signals) is a curve which has a particular mathematical function known as the *Gaussian*, (that is, normal) distribution.

The amplitude (horizontal) scale of the pdf is calibrated in terms of sigma (δ), a symbol used in statistics to denote *standard deviation*; a measure of the spread of a set of values about their mean. In general δ is equal to the rms amplitude of the ac component of the signal. Power contained in the signal is therefore proportional to δ^2 . We see from the pdf in Figure 2 — 10 that a Gaussian-type noise signal spends most of the time between $\pm\delta$, and hardly ever exceeds $\pm 3\delta$ (in fact, it exceeds this value less than 0.1% of the time). Theoretically, however, it is quite possible for very large peaks to appear, although at very infrequent intervals.



Probability density function of a noise signal



Gaussian probability density function