

# Predictive Energy Management Strategies for Hybrid Vehicles

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**Abstract--** In this paper, predictive energy management strategies that utilize the previewed traffic pattern and terrain information are developed. A generalized predictive optimal control framework is used to find the conditions under which the predictive strategies, will give superior fuel economy to that of the instantaneous strategies. Mixed integer linear programming methodology, with no assumptions on the control structure, is used to find the predictive energy management strategies.

It is shown, by using theoretical work and simulation, that certain conditions are needed to make the predictive strategies, that utilize the previewed driving pattern and terrain information, give superior fuel economy to the instantaneous ones.

## I. INTRODUCTION

A hybrid-vehicle system, using a combination of an internal combustion engine (ICE) and electric motor(s), is an important concept to improve fuel economy and reduce emissions of vehicles. In the last two decades, the automotive industry has been actively working on several hybrid configurations, [1]. To improve the fuel economy and reduce the emissions of hybrid vehicles, it is important to optimize not only the architecture and components of the hybrid vehicles, but also the energy management strategy. The energy management strategy controls the energy flow among all components as well as the power generation and conversion in the individual components.

There are several approaches for the development of energy management strategies. One approach optimizes the engine operation, thus not using the full potential of hybrid technology. A second approach optimizes the instantaneous operation of the hybrid system. This can be done by the minimization of the current equivalent fuel flow and/or the instantaneous emissions. This optimization can be viewed as equivalent to local optimization with only instantaneous information available, see [2] for detail. In [3,4,5], instantaneous optimization of the vehicle operation is indirectly used as a basis for fuzzy logic control. In the third approach, predictive energy management strategies are used where the total fuel consumption and/or emissions of the vehicle in a predicted horizon of driving cycle and terrain profile are minimized. In [6] the dynamic optimal control methodology is used to find the optimal energy

management strategies for the Urban and the European cycles using numerical optimization techniques. In [7], the linear programming technique is used to find the ultimate fuel economy performance for a series hybrid vehicle. It was shown that using the driving cycle information in the energy management strategy might lead to only 2.4% fuel economy improvement.

The amount of improvement in fuel economy and/or emission that can be achieved by having future driving cycle and terrain information has not been adequately studied. The currently available results from the literature show surprisingly little or no improvement. In this paper, we assume that the future pattern information and terrain information are available. Actually, future driving pattern information can be predicted by using vehicle speeds transmitted from other surrounding vehicles. The future terrain information (going up hill or down hill) can also be predicted from GPS and map information.

In this paper, a theoretical framework, based on dynamic optimal control, is used to study the effect of previewed driving cycle/pattern and terrain information on vehicle fuel economy. Mixed integer linear programming method is conveniently used to find the predictive optimal solution and to assess its potential fuel economy benefits.

## II. HYBRID VEHICLE CONFIGURATION

A dual-axle drive parallel hybrid vehicle configuration is chosen in this study. In this configuration, the front wheels are driven by an electric motor through a gear reduction unit. The rear wheels are driven by another electric motor in parallel with a compression ignition engine, through a four speed automated-manual transmission. The dual axle drive of the vehicle increases the complexity of the control system but it adds a vast array of opportunities.

## III. VEHICLE AND COMPONENTS MODELS

To get a vehicle model suitable for predictive optimal control, the modeling of the vehicle and its components must be relatively simple from the dynamic point of view, but at the same time it has to describe the interesting and important effects with sufficient accuracy. For the purpose of fuel economy analysis, the important dynamics are fuel consumption and battery state of charge. The dynamics of

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the motors, the battery, and the engine power are relatively fast and can be neglected. The efficiency maps of the components are used to derive quasi-steady-state equations. These equations describe the relations between the input and the output power of the components. We assume that the previewed driving cycle is given as a trajectory of speed versus time. Using vehicle speed, and vehicle parameters (such as mass, drag coefficient, frontal area, and rolling resistance), the previewed power demanded at the wheel as a function of time can be calculated.

#### IV. PREDICTIVE OPTIMAL CONTROL

In the formulation of the predictive optimal control problem, the system states, control variables, and their constraints should be identified. The states of the system are fuel mass and state of charge. The control variables are the engine power ( $P_{eng}$ ), rear motor regenerative power

( $P_{m1\_br}^-$ ), traction power ( $P_{m1}^+$ ) and charging power ( $P_{m1\_ch}^-$ ), as well as front motor regenerative power ( $P_{m2}^-$ ) and traction power ( $P_{m2}^+$ ). The systems equations can be put as:

$$\dot{x} = f(x, u, t)$$

$u(t)$  is the vector that represents the control variables. The predictive optimal control variables are found by minimizing the following objective function:

$$J = C_0 m(t+T) + \frac{1}{2} C_1 (SOC(t+T) - SOC(t))^2 + \frac{1}{2} \int_t^{t+T} (u^T R u) dt$$

The first term in the objective function represents the total fuel to be consumed in the previewed period, starting from the current time,  $t$  to the end of horizon time,  $t+T$ . The second term represents the square of the error between the current and final state of charge of the previewed

To simplify the solution, the state of charge is represented as a soft constraint. The third term represents the integral of the weighted sum of the square of the control variables mentioned earlier. This term is added to be able to have a closed-form solution. The non-dynamic constraints that include the equations of the components can be described by the following algebraic vector equation:

$$g(t, x, u) \leq 0$$

Using the minimum principle of Pontryagin, the following Hamiltonian equation has to be minimized:

$$H(x, u, \lambda, \mu) = \frac{1}{2} C_2 u^2 + \lambda(t)^T f(x, u, t)$$

$$+ \mu^T g(x, u, t)$$

The optimum solution should satisfy the following set of necessary conditions:

$$\begin{aligned} \frac{\partial H}{\partial u} &= 0 \\ \dot{x} &= f(x, u, t) \\ \dot{\lambda} &= -\frac{\partial H}{\partial x} = -\left(\frac{\partial f}{\partial x}\right)^T \lambda - \left(\frac{\partial g}{\partial x}\right)^T \mu \\ \mu^T g(x, u, t) &= 0 \end{aligned}$$

The above equation is equivalent to having  $\mu > 0$  for  $g(x, u, t) = 0$ , and  $\mu = 0$  for  $g(x, u, t) < 0$ .

The optimal control solution depends on the trajectories of the state and the adjoint variables ( $x, \lambda, \mu$ ). These variables can be found by solving coupled two-point boundary set of differential equations. The adjoint trajectory is solved backward in time by starting from the final conditions, while the state trajectory is solved forward in time by using the initial conditions. Therefore, the optimal solution depends on the availability of past, current, and future information about the system. However, it can be shown (by mathematically simplifying the necessary conditions), that if the constraints are strict inequalities (i.e., the SOC trajectory is within the limiting bounds), then the optimal control will be a function of only the current state of the system. The dependency of the solution on the past and future terrain and driving cycle information will vanish. This is a very important result because it implies that knowing the past or predicting the future trajectory of the states of the hybrid vehicle, driving cycle and terrain information will not improve the fuel economy as long as the battery SOC does not reach its bounds.

In this section, the mixed integer linear programming (MILP) technique will be used to find the predictive optimal control for the hybrid vehicle. The linear programming method is very popular because it has well-established numerical techniques that can handle problems with high dimensions. Its main disadvantage is that the problem is restricted to be linear. In this section, the control problem will be represented as a linear set of equations and a binary variable. The binary variable ( $X(k)$ ) is needed to account for the non-linear behavior of engine operation. This will make it possible to include the control flexibility of shutting the engine off when no power is requested from it. The presence of the binary variable makes it suitable to use the MILP methodology.

To be able to use the MILP, the cycle is divided into equal intervals. The length of each interval is reasonably chosen

to be one second. The value of the variables is available at every second. The differential equations are transformed into a discrete set of equations. The system variables are solved and put in terms of only the decision variables. The set of equations that describe the component behavior can be given at each of instant of time  $k$  as:

$$m(k) \geq A_1(gr(k) * \omega(k)) P_{eng}(k) + B_1(gr(k) * \omega(k)) X(k)$$

$$m(k) \geq A_2(gr(k) * \omega(k)) P_{eng}(k) + B_2(gr(k) * \omega(k)) X(k)$$

$$P_{e1}^+(k) = C(gr(k) * \omega(k)) P_{m1}^+ + D(gr(k) * \omega(k))$$

$$P_{e1}^-(k) = 1 / C(gr(k) * \omega(k)) (P_{m1}^-(k) - D(gr(k) * \omega(k)))$$

$$P_{e2}^+(k) = E(\omega(k)) P_{m2}^+(k) + F(\omega(k))$$

$$P_{e2}^-(k) = 1 / E(\omega(k)) (P_{m2}^-(k) - F(\omega(k)))$$

$$P_s^+(k) = H * (P_{e1}^+(k) + P_{e2}^+(k) + P_{acc}(k)) + I$$

$$P_s^-(k) = 1 / J * (P_{e1}^-(k) + P_{e2}^-(k) - P_{acc}(k)) - K / J$$

$$P_d^+(k) = (P_{eng}(k) + P_{m1}^+(k) - P_{m1ch}^-(k)) * dtr\_tract\_eff + P_{m2}^+(k)$$

$$P_d^-(k) = P_{m1br}^-(k) + P_{m2}^-(k)$$

$$P_{eng}(k), P_{m1}^+(k), P_{m1br}^-(k), P_{m1ch}^-(k), P_{m2}^+(k), P_{m2}^-(k)$$

is the sequence of control variables to be optimized, where  $k=1 \dots$ , length of the previewed cycle ( $K_f$ ). To reduce the number of variables to be optimized, we will assign the vehicle braking through the front motors because it is more efficient, till the maximum power is reached and then through the rear motor. The following equations compute the values of the regenerative braking sequence for the front and rear motors:

$$P_{m2}^-(k) = \min(P_{m2}^-_{max}, P_d^- * front\_regen\_eff)$$

$$P_{m1br}^-(k) = \max(0,$$

$$(P_d^-(k) - P_{m2}^-(k) / front\_regen\_eff) * dtr\_regen\_eff)$$

where  $front\_regen\_eff$  and

$dtr\_regen\_eff$  are the average regenerative braking efficiency for the front and rear wheels, respectively. The rear motor traction power ( $P_{m1}^+$ ) term can also be represented and then eliminated by using the following equation:

$$P_{m1}^+(k) = (-P_{eng}(k) + (P_d^+(k) - P_{m2}^+) / (dtr\_tract\_eff) + P_{m1ch}^-(k))$$

The objective of the optimization is to minimize the total fuel consumed. The following set of inequality constraints should be satisfied:

- 1- The difference between the initial and final state of charge is very small.
- 2- The battery charge power is limited
- 3- The battery discharge power is limited
- 4- The rear motor mechanical power output is non-negative
- 5- The battery state of charge at each instant should be less than the allowed maximum
- 6- The rear motor power output should be less than the maximum allowed value.
- 7- The equations that describe the engine performance
- 8- Decision variables Constraints are:

$X(k)$ : binary

$$0 \leq P_{eng}(k) \leq P_{eng\_max} X(k)$$

$$0 \leq P_{m1ch}^-(k) \leq P_{m1ch\_max}$$

$$0 \leq P_{m2}^+(k) \leq P_{m2\_max}$$

$$0 \leq m(k) \leq m_{max}$$

The binary variable  $X(k)$  is used as a switch. The switch will assure that the engine is off when the optimum engine power output ( $P_{eng}(k)$ ) is zero.

## V. SIMULATION RESULTS

Figures 1 and 2 show the MILP simulation results for a previewed Urban and Highway cycles. Figure 1 represents the state of charge trajectory for the previewed urban cycle. The optimization constraint assures that the SOC is the same at the beginning and at the end of the cycle. The SOC is clearly within the imposed limits of 0.3 and 0.6. The variation in SOC is very limited since it fluctuates from 0.443 to 0.463 (a range of about 2%). The SOC profile for the Highway cycle is shown in Figure 2. The variation in SOC is limited between 0.431 and 0.455 (less than 2.5% SOC fluctuation). From the profile of the SOC, it can be seen that the SOC is the lowest just before the final braking part of the cycle. The strategy is making use of the

previewed cycle information (knowing the predicted driving cycle information) to assure enough absorbing capacity in the battery to allow for the regenerative braking to be absorbed without much fluctuation in the SOC.

Figures 3, and 4 show the predictive MILP simulation results for the urban cycle with 6 degrees negative grade terrain. In Figure 4, it can be seen that SOC variation is between 0.43 and 0.53 (about 10% SOC fluctuation). This SOC fluctuation is much more than the previous cases. Moreover, with the availability of predictive cycle information and terrain, the algorithm discharges the battery ahead of the negative grade to allow enough space for the regenerative braking. The limits of the SOC did not reach the bounds of 0.3 and 0.6. Figure 5 shows the SOC profile with 6 degrees negative grade with a controller that does not have previewed information. This controller is based on fuel instantaneous optimization. The charge neutral condition occurs with an initial SOC of 0.53. SOC variation is between 0.46 and 0.6 (SOC fluctuation of 14%). The upper bound of SOC is reached very fast. This puts a limit on the amount of the regeneration that can be captured during braking on a negative grade. The fuel economy improvement of the predictive MILP algorithm over the default one is about 75%. The improvement is due to the fact that the MILP algorithm makes use of the future cycle and terrain information by discharging the battery through the FETS action and allowing enough space to capture all available regeneration power. The controller without the previewed information did not discharge the battery enough and was bounded by SOC upper limit. The large percentage improvement in fuel economy is due to the fact the higher grade leads to reduction in the fuel required for the cycle. The energy saving, due to the predictive energy management strategy, results in a high percentage improvement

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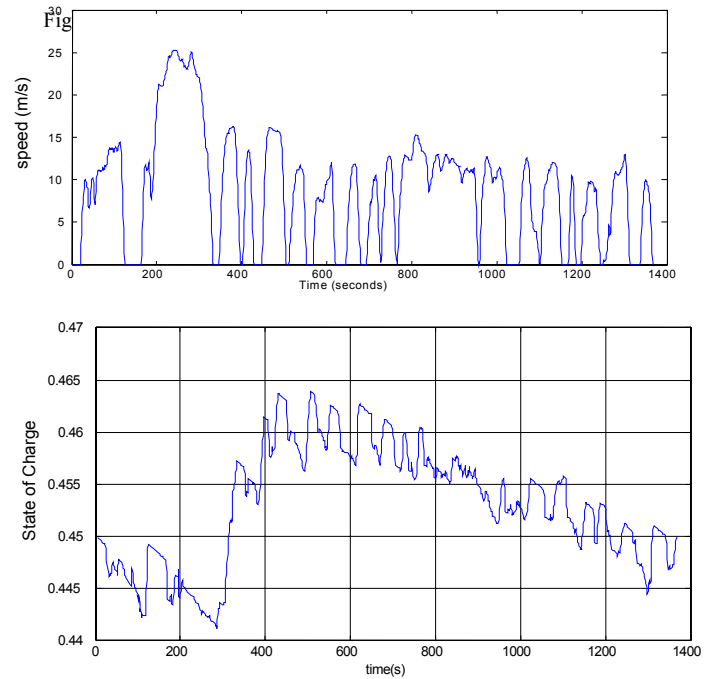


Figure 1: Predictive optimal SOC trajectory for urban cycle (zero- grade)

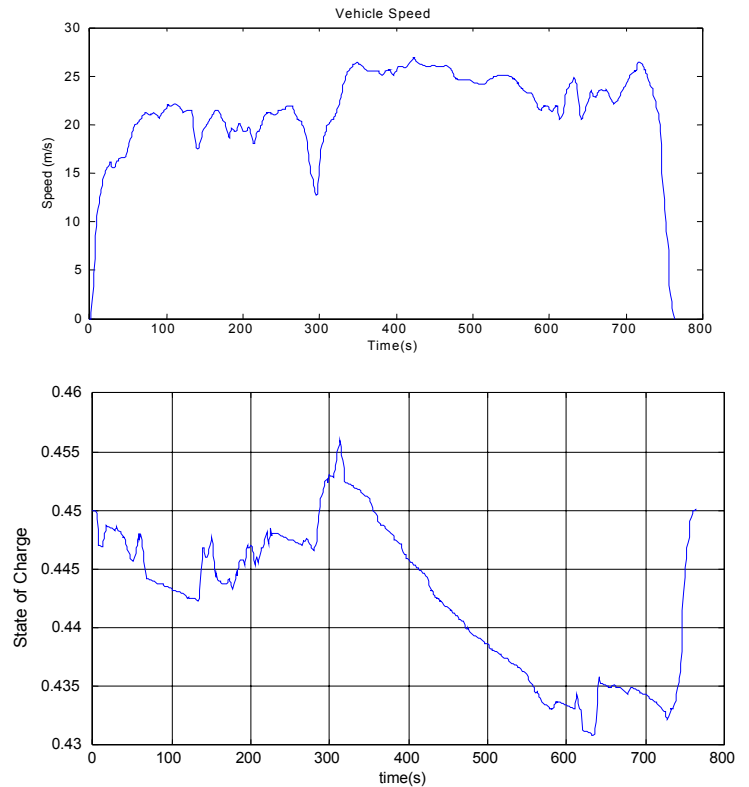


Figure 2: Predictive optimal SOC for the highway cycle with zero grade

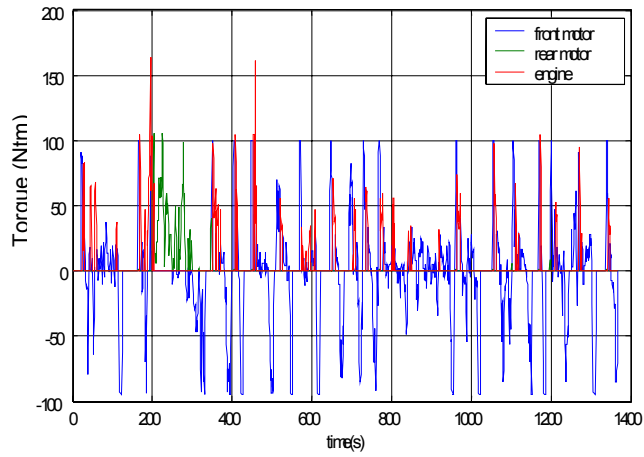


Figure 3: Predictive optimal torque trajectories us for the urban cycle with 6 degrees negative grade

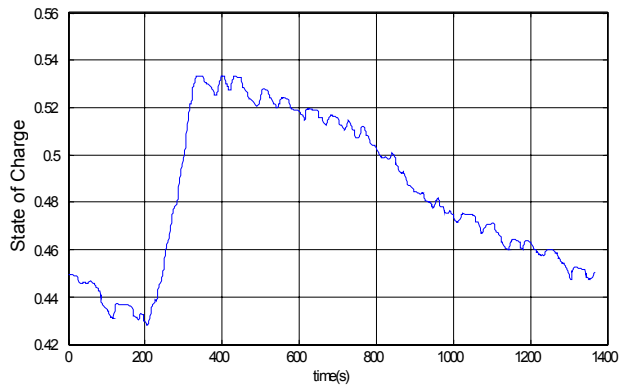


Figure 4: Predictive optimal SOC for the urban cycle with 6 degrees negative grade

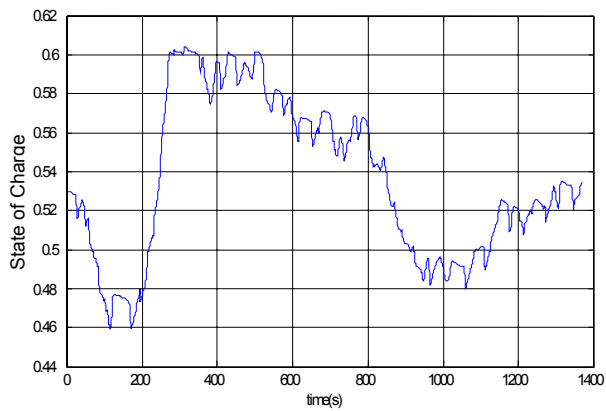


Figure 5: SOC trajectory (with no preview) for the urban cycle with 6 degrees negative grade (0.3 and 0.6 are SOC limits)