DRIVING SIMULATION OF A PARALLEL HYBRID ELECTRIC VEHICLE USING RECEDING HORIZON CONTROL

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ABSTRACT

Fuel-consumption and catalyst-out emissions of a parallel HEV(hybrid electric vehicle) are affected by operating region of an engine. It is generally known that it is profitable in fuel- consumption to operate engine in OOL(optimal operating line). We established the mathematical model of a parallel HEV, which is linear time-invariant. To operate an engine in OOL, we applied RHC(receding horizon control), because it has advantage such as good tracking performance under the state and control constraints. This RHC is obtained by using the semi-definite programming in the LMI(linear matrix inequality) form.

In this paper, there are two main topics. First, under state and control constraints by the engine and motor performances, the optimal tracking of OOL was simulated. Second, we combined the RHC with the iterative simulation to extract the optimal gear ratio. In this simulation, the vehicle is commanded to track the reference vehicle trajectory, and the engine is operated in the optimal operating region which is made by the state constraints.

1. INTRODUCTION

In the driving simulation of a parallel HEV, there are two types. One is the simulation using the sliding mode control or Pontryagin's minimum principal in the basis of the mathematical models of engine, motor, transmission, fuel-consumption of engine, and state of charge of battery, etc [1]. The other is the simulation using the map data of its components such as engine and motor without other specific mathematical models except the equation of motion [2].

In this paper, we use RHC [3] in the driving simulation of a parallel HEV, because it has advantage of good tracking performance under the state and control constraints when the finite future tracking commands are given. For this purpose, used are the equation of motion of this system, and the equations related to the dynamic characteristics of the engine and motor, assuming that this system is linear time-invariant.

This paper provides the simplified mathematical model of a parallel HEV in Section 2. In Section 3, the optimal tracking of OOL under the state and control constraints is simulated. In Section 4, the optimal gear ratio is extracted by combining RHC with the iterative simulation.

2. SYSTEM MODELING

In this study, we consider a parallel HEV whose required power from driving load is supplied by a 38 kw gasoline engine and a 30kw AC induction motor(Fig.1). Figure 2 shows the schematic diagram of this system. The simple mathematical model of this system is as follows.

Equation (1) shows the equation of motion of this system, and the dynamic characteristics of an engine and a motor are modeled as the first-order systems with time constants, τ_e and τ_m like Eq. (2) and (3).

The control variables are the engine torque command, Te_{set} and the motor toruqe command, Tm_{set}. If the gear ratio in included in the control variables, the total system of Eq. (1) becomes the nonlinear time-varying system. So, to simplify the system equation, the gear ratio is assumed to be a constant value like Table 1. This assumption means that the transcient state during shifting is not modeled.

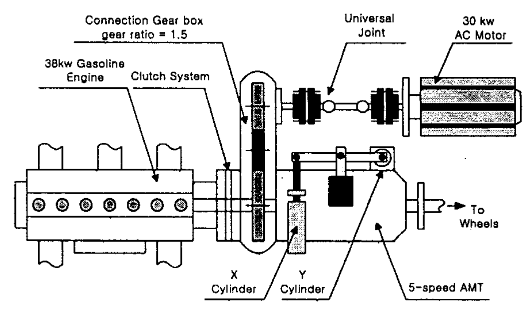


Fig. 1 Configuration of a Parallel HEV

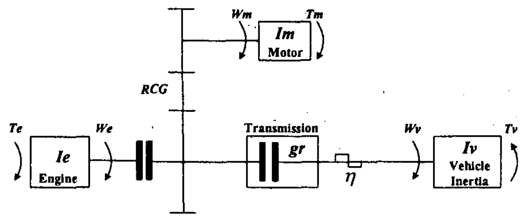


Fig. 2 Schematic Diagram of a Parallel HEV

$$\left[Ie + RCG^2 \cdot \mathrm{Im} + \frac{Iv}{(gr \cdot \eta)^2}\right] \cdot \dot{We} = Te + RCG \cdot Tm - \frac{Tv}{gr \cdot \eta} \tag{1}$$

$$Wm = RCG \cdot We, \quad Wv = \frac{We}{gr \cdot \eta}$$

$$\dot{Te} = \frac{Te_{set} - Te}{\tau_e} \tag{2}$$

$$\dot{Tm} = \frac{Tm_{set} - Tm}{\tau_m} \tag{3}$$

Equation (4) and (5) show the control and state variables(Te :Engine torque[Nm], Tm :Motor torque[Nm], We :Engine velocity[rad/s]). The discretized state and output equations are as Eq. (6) and (7), and this system is controllable and observable.

$$u = \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} Te_{set} \\ Tm_{set} \end{Bmatrix} \tag{4}$$

$$x = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} Te \\ Tm \\ We \end{Bmatrix} \tag{5}$$

$$x(i+1) = Ax(i) + Bu(i) \tag{6}$$

$$y(i) = x(i) \tag{7}$$

$$A = \begin{Bmatrix} 1 - \Delta t / \tau_e & 0 & 0 \\ 0 & 1 - \Delta t / \tau_m & 0 \\ \Delta t / I & \Delta t \cdot RCG / I & 1 - \Delta t \cdot c / \left(I \cdot (gr \cdot \eta)^2\right) \end{Bmatrix}$$

$$B = \begin{Bmatrix} \Delta t / \tau_e & 0 \\ 0 & \Delta t / \tau_m \\ 0 & 0 \end{Bmatrix}$$

$I = Ie + RCG^2 \cdot \mathrm{Im} + \frac{Iv}{(gr \cdot \eta)^2}$	Equivalent inertia of total system
$\mathrm{Im} = 0.02\,[kgm^2]$	Motor inertia
$Ie = 0.1\,[kgm^2]$	Engine inertia
$Iv = 100.2\,[kgm^2]$	Vehicle inertia
$Wm\,[rad/s]$	Motor velocity
$Wv\,[rad/s]$	Vehicle velocity
$c = 30.456\,[Nm/(rad/s)]$	Proportional constant related to driving load(Tv)
$gr = 4.0$	Gear ratio
$RCG = 1.5$	Ratio of connection gear
$\eta = 2.837$	Final gear ratio

Table 1 System Data

3. CONSTRAINED OPTIMAL TRACKING

To improve the fuel-economy of a parallel HEV, it is desirable to operate an engine around OOL[4]. Figure 3 shows the BSFC(brake specific fuel-consumption [*g/kwh*]) map of an engine, and the OOL is defined as the set of the minimum points of BSFC on the iso-power lines.

In this section, simulated is the optimal tracking of OOL under the constraints of state and control variables like Eq. (8) and (9).

The performance measure is as Eq. (10), and this is partitioned into two parts. One(Eq.(11)) is the summation of tracking error and control effort from $i = 0$ to $N-1$, here, N is the prediction horizon length. The other(Eq.(12)) is related to the tracking error at the final horizon. The OOL is expressed in $x_r(t+i\,|\,t)$ and $x_r(t+N\,|\,t)$, which are assumed to be available over the future horizon [$i = 0, N$].

The state and control weighting matrix, Q and R are selected like Eq. (13). It means that the tracking performance is considered to be more important than the control effort. And, in matrix Q, the weight of engine torque is set to be largest.

We use the semi-definite programming as the optimization algorithm, which solves the optimization problem in a linear performance measure that is subject to the LMI forms. This optimization problem with performance measure of Eq. (10)~(13) and constraints of Eq. (8) and (9) can be formulated into the LMI forms like Eq. (14)~(21) [3].

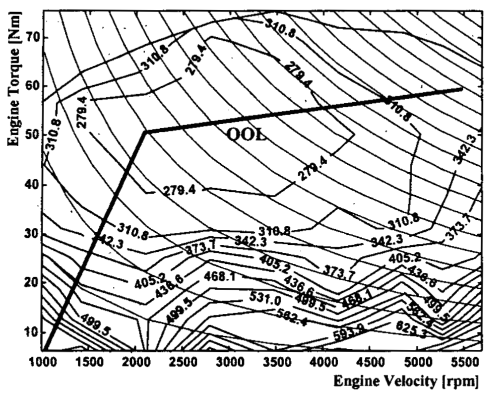


Fig. 3 OOL of 38kw Gasoline Engine

The overall procedure is as follows. First, at current time, the control U is calculated, which is the solutions that minimize $\gamma_1+\gamma_2$ of Eq. (14) on a fixed horizon from the current time to the current time plus a prediction horizon N, satisfying Eq. (15)~(21). Second, among the solutions of the fixed horizon, only the first one is implemented as the current control law. This procedure is repeated at the next time.

Equation (15) and (16) are the LMI forms related to Eq. (11) and (12). And Eq. (17) shows the condition of the terminal matrix Q_f for stabilizing the performance measure. The constraints of Eq. (8) and (9) are converted into LMI forms of Eq. (18) and (19). Equation (20) and (21) are related to the conditions for checking the feasibility of stabilizing RHC [3].

$$Te_{\min} \le x_1 \le Te_{\max}\,, Tm_{\min} \le x_2 \le Tm_{\max} \qquad (8)$$

$$We_{\min} \le x_3 \le We_{\max}$$

$$Te_{\min} \le u_1 \le Te_{\max}\,,\ \ Tm_{\min} \le u_2 \le Tm_{\max} \qquad (9)$$

$$(Te_{\min}=0.0\,[Nm]\,, Te_{\max}=70.0\,[Nm]\,,$$
$$Tm_{\min}=-30.0\,[Nm]\,, Tm_{\max}=30.0\,[Nm]$$
$$We_{\min}=62.8\,[rad/s]\,, We_{\max}=576.0\,[rad/s])$$

$$J(x(t),t)=J_1(x(t),t)+J_2(x(t),t) \qquad (10)$$

$$J_1(x(t),t) = \sum_{i=0}^{N-1}\begin{pmatrix}(x(t+i\,|\,t)-x_r(t+i\,|\,t))^T Q\,(x(t+i\,|\,t)-x_r(t+i\,|\,t))\\ +u^T(t+i\,|\,t)\,R\,u(t+i\,|\,t)\end{pmatrix} \qquad (11)$$

$$J_2(x(t+N\,|\,t),t) = (x(t+N\,|\,t)-x_r(t+N\,|\,t))^T Q_f\,(x(t+N\,|\,t)-x_r(t+N\,|\,t)) \qquad (12)$$

$$Q=\begin{bmatrix}20.0 & 0 & 0\\ 0 & 0.001 & 0\\ 0 & 0 & 1.0\end{bmatrix},\ R=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix} \qquad (13)$$

$$\underset{\Lambda}{Minimize}\ \ \gamma_1+\gamma_2 \qquad (14)$$

$$\Lambda=\{\gamma_1,\gamma_2,X,Y,Z,V,U\}\,,\ \ Y=HX\,, X=\gamma_2 Q_f^{-1}$$

$$\begin{bmatrix}\gamma_1-V^TU(t)-V_0 & (W^{1/2}U(t))^T\\ W^{1/2}U(t) & I\end{bmatrix}\ge 0 \qquad (15)$$

$$\begin{bmatrix}1 & (A^N x(t)+\bar{B}U(t))^T\\ A^N x(t)+\bar{B}U(t) & X\end{bmatrix}\ge 0 \qquad (16)$$

$$\begin{bmatrix}X & (AX+BY)^T & (Q^{1/2}X)^T & (R^{1/2}Y)^T\\ AX+BY & X & 0 & 0\\ Q^{1/2}X & 0 & \gamma_2 I & 0\\ R^{1/2}Y & 0 & 0 & \gamma_2 I\end{bmatrix}\ge 0 \qquad (17)$$

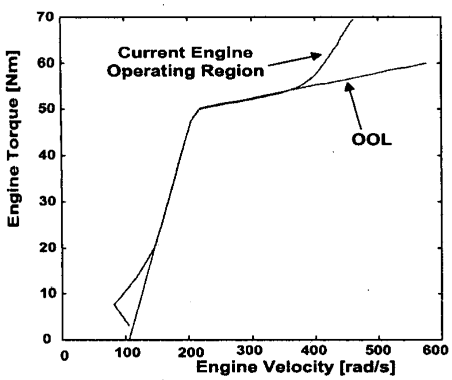
$$\hat{u}_{\min}\le U(t)\le \hat{u}_{\max} \qquad (18)$$
$$\hat{u}_{\min}=[Te_{\min}, Tm_{\min},\cdots\cdots]^T$$
$$\hat{u}_{\max}=[Te_{\max}, Tm_{\max},\cdots\cdots]^T$$

$$\hat{g}_{\min}\le \hat{G}U(t)+\hat{g}_0\le \hat{g}_{\max} \qquad (19)$$
$$\hat{g}_{\min}=[Te_{\min}, Tm_{\min}, We_{\min},\cdots\cdots]^T$$
$$\hat{g}_{\max}=[Te_{\max}, Tm_{\max}, We_{\max},\cdots\cdots]^T$$

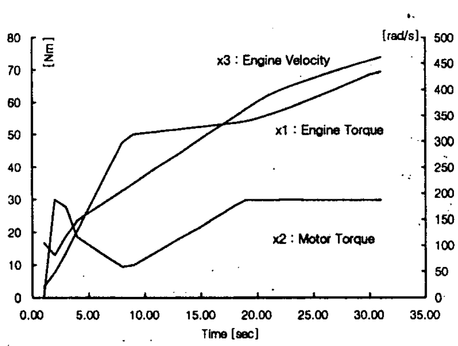
$$\begin{bmatrix}Z & Y\\ Y^T & X\end{bmatrix}\ge 0\,, Z\le\begin{bmatrix}(Te_{\max})^2 & 0\\ 0 & (Tm_{\max})^2\end{bmatrix} \qquad (20)$$

$$GXG^T\le V\,,\ \ V\le\begin{bmatrix}(Te_{\max})^2 & 0 & 0\\ 0 & (Tm_{\max})^2 & 0\\ 0 & 0 & (We_{\max})^2\end{bmatrix} \qquad (21)$$

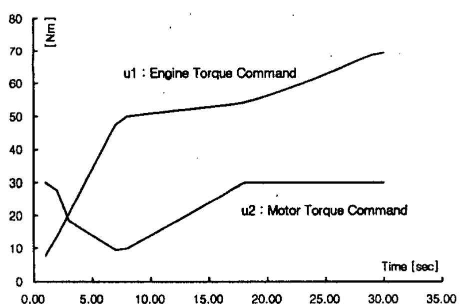
The simulation result is as Fig. 4. The OOL tracking performance is good until 19 second except the initial transient situation. During this time, the motor torque is assisting the engine torque to follow OOL reasonably. But, after 19 second, the motor torque is limited by the upper constraint, so, at the cost of tracking error, the engine torque is increased to meet the high driving load.



(a) Engine Operating Region



(b) State Trajectory



(c) Control History

Fig. 4 Simulation Result of Constrained Optimal Tracking

4. STUDY ON OPTIMAL GEAR RATIO

In section 2, the gear ratio is assumed as a constant value. But, this assumption gives rise to the following tracking error like Fig. 5. In the tracking problem of the reference vehicle trajectory, the parallel HEV of the gear ratio such as 4.0 or 5.0 does not track the reference trajectory reliably, because the engine velocity is limited by its upper constraint.

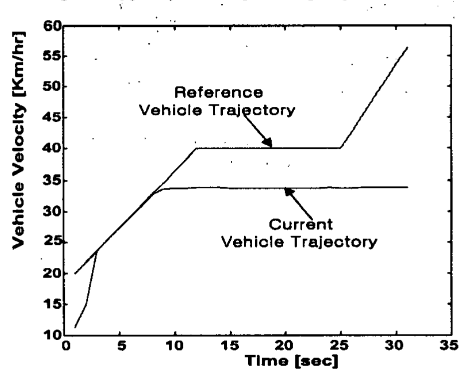
In this section, to solve the above problem, that is to say, to extract the optimal gear ratio, we combine the RHC with the iterative simulation. The conditions of simulation are as follows.

- The parallel HEV tracks the reference vehicle trajectory like Fig. 5.
- At the same time, the engine is operated in the optimal operating region such as Fig. 8 (b), which is made by the state constraints. In this region, the BSFC is low, and the carbon monoxide is emitted relatively a little.
- In above conditions, the optimal gear ratio is obtained by sweeping the allowable range [Gear_min, Gear_max] of gear ratio.

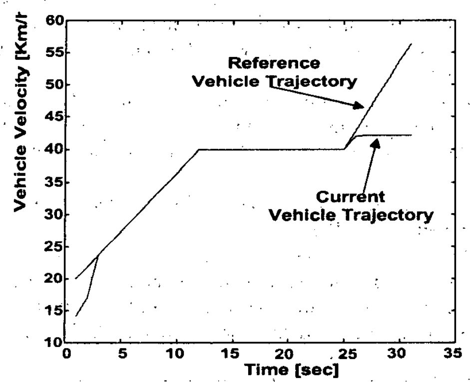
Figure 6 shows the flow of this simulation. After setting the current gear ratio to Gear_max, solve the RHC. Then, if the tracking error is smaller than 0.5 km/h, calculate cost. This cost is not the performance measure of Eq. (10) but the cost related to the fuel-consumption of engine and the state of charge of battery. Namely, the cost is defined as the weighted summation of the engine and motor power.

This gear ratio sweeping sequence is repeated until Gear_min. Then the gear ratio that minimizes the cost is determined as the optimal gear ratio at current time. Finally, by using this optimized gear ratio, solve the RHC once more. The above procedure is repeated until Time_end.

The extracted optimal gear ratio(Fig. 7) is decreasing until 7 second, after this time, it becomes the constant value of 2.50. Figure 8 shows the results of simulation using the optimal gear ratio. The parallel HEV is tracking the reference vehicle trajectory in the error bound of 0.5 km/h, and simultaneously, the engine is being operated in the rectangular-type optimal operating region.



(a) Gear Ratio(gr) = 5.0



(b) Gear Ratio(gr) = 4.0

Fig. 5 Tracking Error as to Gear Ratio(gr)

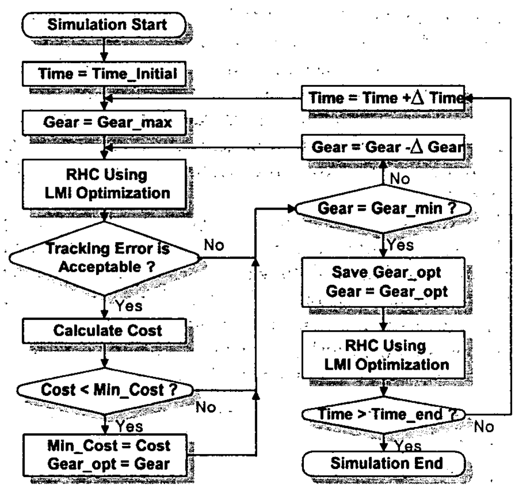


Fig. 6 Flowchart of the Simulation for Extracting Optimal Gear Ratio

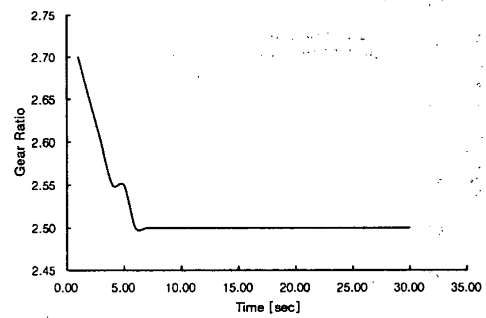
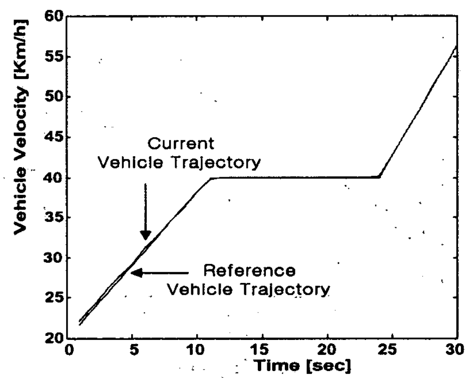
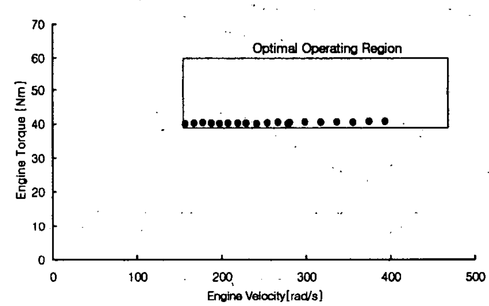


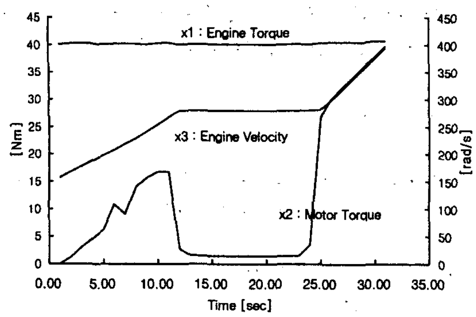
Fig. 7 Optimal Gear Ratio



(a) Vehicle Trajectory



(b) Engine Operating Region



(c) State Trajectory

Fig. 8 Results of Simulations using Optimal Gear Ratio

5. CONCLUSION

In this paper, we applied the RHC to the driving control of a parallel HEV, and the results are as follows.

1. The linear time-invariant model of a parallel HEV was established, that is to say, the transient situation during shifting was excluded. And the RHC is obtained by using the semi-definite programming in the LMI form.

2. Under the state and control constraints by the engine and motor performance, the tracking of OOL was simulated. The tracking error can be increased by the high driving load, so, the power matching of an engine and a motor is very important to prevent this error.

3. To extract the optimal gear ratio every time step, we combined the RHC with the iterative simulation for sweeping the allowable range of gear ratio. In the simulation using the optimal gear ratio, the tracking performance of the reference vehicle trajectory was improved, and the engine was operated in the optimal operating region which is made by the state constraints.

6. ACKNOWLEDGEMENT

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7. REFERENCES

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