Mechanical interpretation of the KKT conditions

Note the different notation used in these slides compared to the Lecture notes:

	Lecture notes	**These slides**
Optimization variables	x	w
Objective function	$f(x)$	$\Phi(w)$
Inequality constraint function	$g(x)$	$h(w)$
Equality constraint function	$h(x)$	$g(w)$

Some intuitions on the KKT conditions

$$\min_{\mathbf{w}} \ \Phi(\mathbf{x})$$
$$\text{s.t.} \ \mathbf{h}(\mathbf{w}) \leq \mathbf{0}$$

Mechanical analogy

Ball rolling down a valley blocked by a fence

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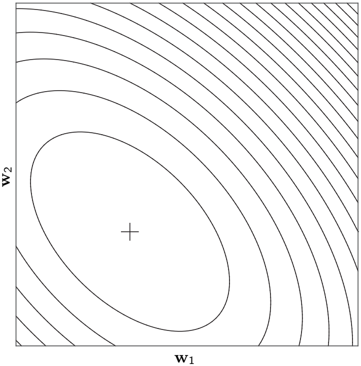

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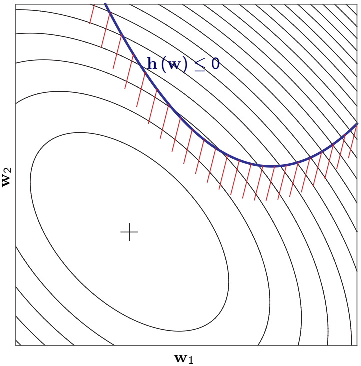

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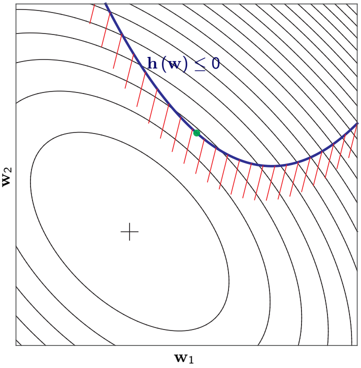

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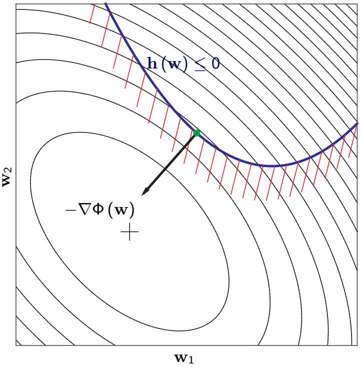
- $-\nabla\Phi$ is the gravity



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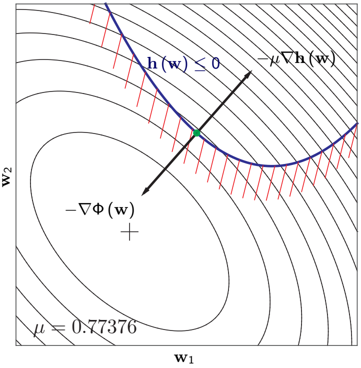

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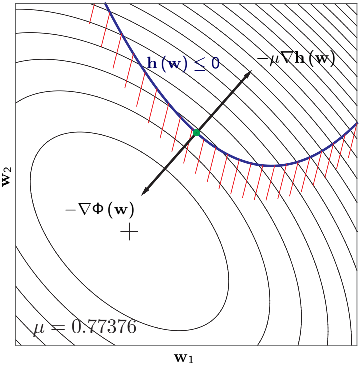

Balance of the forces:
$$\nabla\mathcal{L} = \nabla\Phi(\mathbf{w}) + \mu\nabla h(\mathbf{w}) = 0$$

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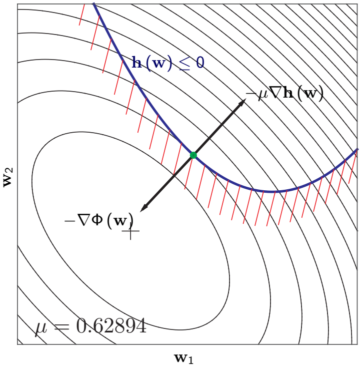

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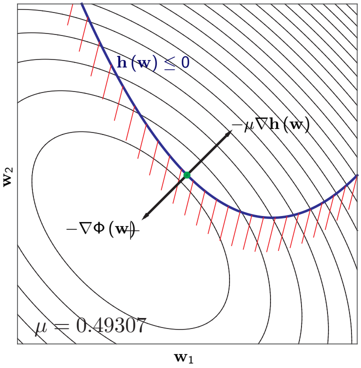

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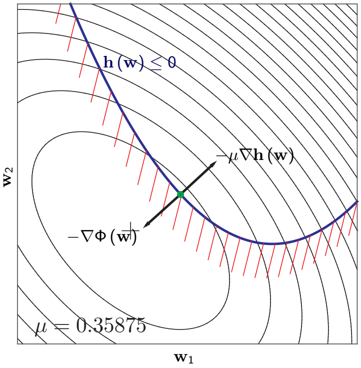

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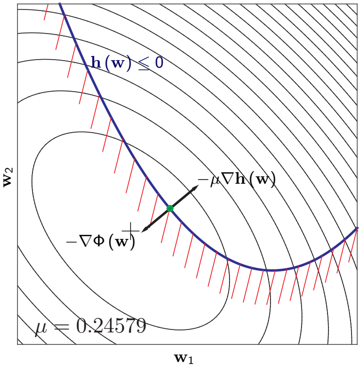

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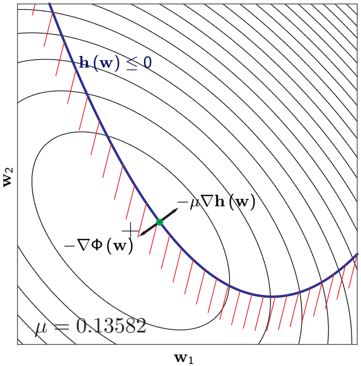

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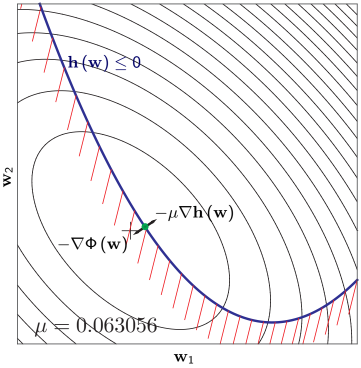

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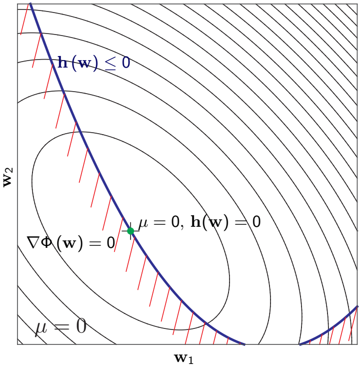
$$h(\mathbf{w}) = 0, \quad \mu = 0$$

the ball touches the fence but no force is needed.



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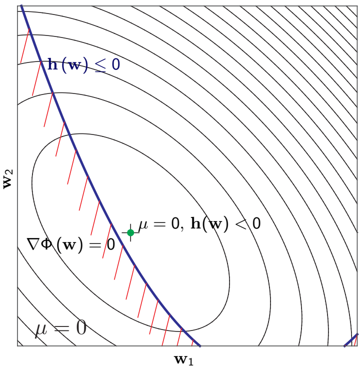

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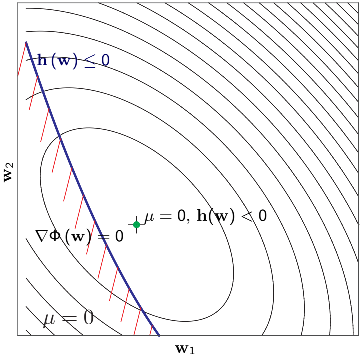

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