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SOCP formulation for autonomous overtaking sampling in the spatial domain

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I. Translation

If we disregard the constraint on the lateral velocity (because it is not necessary and is not used in the code) and refrain from using any convexity schemes we have from Nikolces paper the problem:

$$\min_{\hat{\mathbf{u}}_{\mathrm{E}}(\tilde{x})} \tilde{J}(\hat{\mathbf{x}}_{\mathrm{E}}(\tilde{x}), \hat{\mathbf{u}}_{\mathrm{E}}(\tilde{x}), \hat{\mathbf{u}}'_{\mathrm{E}}(\tilde{x})) \tag{1a}$$

subject to

$$\hat{\mathbf{x}}'_{\mathrm{E}}(\tilde{x}) = \left[a_{\mathrm{E}x}(\tilde{x}), v_{\mathrm{E}y}(\tilde{x}), \frac{1}{\tilde{v}_{\mathrm{E}x}(\tilde{x})}\right]^T \tag{1b}$$

$$\hat{\mathbf{x}}_{\mathrm{E}}(\tilde{x}) \in [\hat{\mathbf{x}}_{\mathrm{min}}(\tilde{x}), \hat{\mathbf{x}}_{\mathrm{max}}(\tilde{x})] \tag{1c}$$

$$\tilde{a}_{\mathrm{Ex}}(\tilde{x}) \in [a_{\mathrm{xmin}}(\tilde{x}), a_{\mathrm{xmax}}(\tilde{x})]/\tilde{v}_{\mathrm{Ex}}(\tilde{x}) \tag{1d}$$

$$v_{\mathrm{E}y}(\tilde{x}) \in [s_{\mathrm{min}}, s_{\mathrm{max}}]\left(1 + \frac{v_{\mathrm{L}}(\tilde{x})}{\tilde{v}_{\mathrm{E}x}(\tilde{x})}\right) \tag{1e}$$

$$\frac{\tilde{x} - x_{\mathrm{A0}}(\tilde{x}) - (v_{\mathrm{A}} - v_{\mathrm{L}})\tilde{t}_{\mathrm{E}}(\tilde{x})}{l_{\mathrm{Af}}} - \frac{y_{\mathrm{E}}(\tilde{x}) - y_{\mathrm{A}}}{\omega_l} \geq \quad 1 \quad \forall i \in \mathcal{A}_{\mathrm{FCC}}. \tag{1f}$$

$$\hat{\mathbf{x}}_{\mathrm{E}}(0) = \hat{\mathbf{x}}_{\mathrm{E}\mathbf{0}} \tag{1g}$$

where

$$\hat{\mathbf{x}}_{\mathrm{E}}(\tilde{x}) = [\tilde{v}_{\mathrm{E}x}(\tilde{x}), y_{\mathrm{E}}(\tilde{x}), \tilde{t}_{\mathrm{E}}(\tilde{x})],$$
$$\hat{\mathbf{u}}_{\mathrm{E}}(\tilde{x}) = [\tilde{a}_{\mathrm{E}x}(\tilde{x}), v_{\mathrm{E}y}(\tilde{x})],$$

Studying this problem we can see that the constraints (12b), (12d) and (12e) are all non-constant due to the fact that we divide by the velocity. To get rid of this we introduce the variable change:

$$z_{\mathrm{E}}(\tilde{x}) = \frac{1}{\tilde{v}_{\mathrm{E}x}(\tilde{x})},$$
$$z'_{\mathrm{E}}(\tilde{x}) = (\frac{1}{\tilde{v}_{\mathrm{E}x}(\tilde{x})})' = -\frac{a_{\mathrm{Ex}}(\tilde{x})}{\tilde{v}_{\mathrm{E}x}(\tilde{x})^2} = -a_{\mathrm{Ex}}(\tilde{x})z_{\mathrm{E}}^2(\tilde{x}) = -\frac{F_{\mathrm{E}}(\tilde{x})}{m}z_{\mathrm{E}}^3(\tilde{x}) = -u(\tilde{x}).$$

If we introduce the new state and control vectors

$$\hat{\mathbf{x}}_{\mathrm{E}}(\tilde{x}) = [z_{\mathrm{E}}(\tilde{x}), y_{\mathrm{E}}(\tilde{x}), \tilde{t}_{\mathrm{E}}(\tilde{x})]^T,$$
$$\hat{\mathbf{u}}_{\mathrm{E}}(\tilde{x}) = [u(\tilde{x}), v_{\mathrm{E}y}(\tilde{x})]^T,$$

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the full problem formulation reads

$$\min_{\hat{\mathbf{u}}_{\mathrm{E}}(\tilde{x})} \tilde{J}(\hat{\mathbf{x}}_{\mathrm{E}}(\tilde{x}), \hat{\mathbf{u}}_{\mathrm{E}}(\tilde{x}), \hat{\mathbf{u}}'_{\mathrm{E}}(\tilde{x})) \tag{2a}$$

subject to

$$\hat{\mathbf{x}}'_{\mathrm{E}}(\tilde{x}) = [-u(\tilde{x}), v_{\mathrm{E}y}(\tilde{x}), z_{\mathrm{E}}(\tilde{x})]^T \tag{2b}$$

$$\hat{\mathbf{x}}_{\mathrm{E}}(\tilde{x}) \in [\hat{\mathbf{x}}_{\min}(\tilde{x}), \hat{\mathbf{x}}_{\max}(\tilde{x})] \tag{2c}$$

$$u(\tilde{x}) \in [a_{\min}(\tilde{x}), a_{\max}(\tilde{x})] z_{\mathrm{E}}^3(\tilde{x}) \tag{2d}$$

$$v_{\mathrm{E}y}(\tilde{x}) \in [s_{\min}, s_{\max}] (1 + v_{\mathrm{L}}(\tilde{x}) z_{\mathrm{E}}(\tilde{x})) \tag{2e}$$

$$\frac{\tilde{x} - x_{\mathrm{A}0}(\tilde{x}) - (v_{\mathrm{A}} - v_{\mathrm{L}})\tilde{t}_{\mathrm{E}}(\tilde{x})}{l_{\mathrm{Af}}} - \frac{y_{\mathrm{E}}(\tilde{x}) - y_{\mathrm{A}}}{\omega_l} \geq \quad 1 \quad \forall i \in \mathcal{A}_{\mathrm{FCC}} \tag{2f}$$

$$\hat{\mathbf{x}}_{\mathrm{E}}(0) = \hat{\mathbf{x}}_{\mathrm{E}\mathbf{0}} \tag{2g}$$