SSY280 Model Predictive Control

Assignment 2

Steady state targets and disturbance modeling

Due February 20 at 12:00

Division of Systems & Control

Department of Signals and Systems

Chalmers University of Technology

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Instructions

The assignments comprise an important part of the examination in this course. Each assignment is pursued in groups of two students and is reported both in written form and orally. It is important to comply with the following rules and instructions:

- Written report:
  - You should provide a written report with clear answers to the questions. These answers are expected to include your motivations, explanations, observations from simulations, and the Matlab program code if applicable.
  - Figures included in the report should have legends, and axes should be labelled.
  - The report should be uploaded *before the deadline* to your project's document area in PingPong.
  - Instructions concerning project group formation and naming conventions for files and email subjects, found at PingPong, should be followed!
- Oral examination:
  - During the oral examination, the assignment and the written report is discussed with the TA.
  - Both students are expected to be active during the examination.
- Grading:
  - The combination of *written report and oral exam* are graded EXCELLENT, GOOD, PASS or FAIL.
  - Bonus points are offered for the combination of written report and oral exam; GOOD gives 1 bonus point, and EXCELLENT 2 bonus points, to be added to the final exam score.
  - No bonus point is given if the report is handed in after the deadline, or after a FAIL in the first round.

1 Objectives

The objective of this assignment is to get some insight and experience in obtaining zero off-set when applying MPC control to a MIMO plant. First, the problem to assign steady-state values to a subset of system outputs is investigated in a couple of cases. Second, the code produced in assignment 1 is extended and used to control a chemical reactor model. The requirement here is to achieve zero off-set for the controlled outputs in the presence of constant but unknown disturbances.

2 Preliminaries

We start by briefly summarizing a few facts from the lectures for easy reference.

2.1 Steady-state targets

The condition for tracking a constant setpoint z_{sp} for the controlled outputs is that there is a steady-state solution satisfying

$$\begin{bmatrix} I-A & -B \\ C_z & 0 \end{bmatrix} \begin{bmatrix} x_s \\ u_s \end{bmatrix} = \begin{bmatrix} 0 \\ z_{sp} \end{bmatrix} \tag{1}$$

with $p_z \leq m$, where p_z is the number of controlled outputs and m is the number of manipulated variables (control inputs). If $p_z < m$, then the non-uniqueness of the solution of this equation can be circumvented by instead formulating the optimization problem

$$\begin{aligned} \underset{x_s, u_s}{\text{minimize}} \quad & \left(|u_s - u_{sp}|^2_{R_s} + |Cx_s - y_{sp}|^2_{Q_s} \right) \\ \text{subject to} \quad & \begin{bmatrix} I-A & -B \\ C_z & 0 \end{bmatrix} \begin{bmatrix} x_s \\ u_s \end{bmatrix} = \begin{bmatrix} 0 \\ z_{sp} \end{bmatrix} \end{aligned} \tag{2}$$

where setpoints or *desired values* for the inputs (u_{sp}) and possibly for the uncontrolled outputs (y_{sp}) have been included as well. We assume here that the controlled outputs are a selection of measured outputs, i.e. $C_z = HC$ for some "selection matrix" H.

2.2 Disturbance modeling and off-set free control

To account for constant but unknown disturbances, we augment the model used for state estimation with disturbance states d:

$$\begin{bmatrix} x \\ d \end{bmatrix}^+ = \begin{bmatrix} A & B_d \\ 0 & I \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} C & C_d \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix}$$

When this model is used for estimating the disturbance, the steady state target has to be modified accordingly. In part 2 of this assignment, dealing with a chemical reactor, we will for simplicity specialize to the case with as many controlled outputs as manipulated variables, and with no inequality constraints. We then avoid to solve an optimization problem to find the steady state target, which is instead obtained as the solution to the following equation:

$$\begin{bmatrix} I - A & -B \\ HC & 0 \end{bmatrix} \begin{bmatrix} x_s \\ u_s \end{bmatrix} = \begin{bmatrix} B_d \hat{d} \\ z_{sp} - HC_d \hat{d} \end{bmatrix} \tag{3}$$

2.3 A chemical reactor model

In the second task below, you will apply MPC to a linearized model of a chemical tank reactor, in which the species A undergoes a reaction and the species B is produced. The model states are

$$x = \begin{bmatrix} c \\ T \\ h \end{bmatrix}$$

where c is concentration of the substance A, T is temperature, and h is tank level. The manipulated variables u are the coolant temperature (the reaction is exothermic, i.e. it generates heat) and the outlet flowrate. The inlet flowrate acts as an unmeasured disturbance p. The linearized and discretized model (the sampling period is 1 min) at a desired operating point is given by

$$x^+ = Ax + Bu + B_p p$$

with

$$A = \begin{bmatrix} 0.2681 & -0.00338 & -0.00728 \\ 9.703 & 0.3279 & -25.44 \\ 0 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.00537 & 0.1655 \\ 1.297 & 97.91 \\ 0 & -6.637 \end{bmatrix} \quad B_p = \begin{bmatrix} -0.1175 \\ 69.74 \\ 6.637 \end{bmatrix}$$

3 Tasks

3.1 Where is the steady state?

NB. This is an exercise that is *not* related to the next task on chemical reactor control!
Consider the following system with two inputs and two outputs:

$$A = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0.6 \end{bmatrix} \quad B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \\ 0.25 & 0 \\ 0 & 0.6 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

(a) In the first case, we assume that both manipulated inputs are used for control, and that we want to reach the output setpoint defined by $z_{sp} = \begin{bmatrix} 1 & -1 \end{bmatrix}^T$. Calculate the setpoint target (x_s, u_s, z_s). Is the output setpoint possible to attain, i.e. is it true that $C_z x_s = z_{sp}$? Motivate/explain your answer!

(b) In the second case, we assume that only the first manipulated input is available for control, giving less freedom in attaining setpoints. Is the output setpoint in (a) now attainable? What is the target setpoint in this case using $Q_s = I$ (and $R_s = 0$ since there is no setpoint defined for the input)? Motivate/explain your answer!

(c) Now assume that both manipulated inputs are available for control, but only the first output has a setpoint, $z_{1sp} = 1$. In this case, there are many ways to attain the output setpoint, and we can specify also setpoints (or rather desired values) for the control inputs: $u_{sp} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. What is the solution to the steady state target problem for $R_s = I$ (and $Q_s = 0$)? Motivate/explain your answer!

3.2 Chemical reactor control

Consider now the reactor model given above. The task is now to apply MPC to this model and in doing this, including a disturbance model to achieve off-set free control of the *controlled outputs*, namely the concentration and the tank level (the temperature is considered to be less important). You are supposed to use the MPC code that you prepared in assignment 1, but there are several extensions that need to be done:

1. An augmented model including disturbances should be created.
2. A state estimator should be designed to estimate both process and disturbance states.
3. The MPC algorithm needs to be extended with a steady state target calculation.
4. The MPC algorithm should be extended to handle different values for prediction and control horizon.

A Matlab skeleton script file for the assignment is provided at the course homepage. Each of the above items are commented below.

Augmented model. The skeleton script already includes three different cases to be investigated. They will be further described below. Note that the matrices B_d and C_d are given in the script, but you need to form the matrices for the augmented model yourself.

State estimator. A state estimator should be designed. The method used has less significance, but detectability of the system model is required. One possibility is to place the eigenvalues for the estimation error dynamics yourself using the command `place`; another option is to design a Kalman filter using e.g. the command `kalman`; a third option would be to design a moving horizon estimator based on a least squares criterion. Note that the code skeleton assumes that the state estimator updates the one-step ahead prediction $\hat{x}(k|k-1)$ — if you do it differently, the code should be modified accordingly!

Steady state target. The steady state target should be calculated using (3), where $\hat{d}$ is replaced by the current estimate $\hat{d}(k)$ and the setpoint z_{sp} is zero (since we consider deviations relative to a desired operating point). It is important to note how the result should be used: first, the *deviation* between the current state estimate and the state target x_s should be input to the QP computations; second, the QP algorithm delivers the control *deviation* relative to the target u_s, so in order to get the desired control signal, u_s should be added.

Control horizon. As can be seen in the code skeleton, you are asked to use a control horizon that is less than the prediction horizon. This means that the matrices used for the equality constraints need to be modified relative to assignment 1. Remember the usual assumption that the control stays constant beyond the control horizon.

We are now ready to formulate the tasks:

(d) Draw a block diagram of your model predictive controller, including all important variables with time arguments. Include the block diagram with your report.

(e) Since there are two inputs, we first try to remove steady-state offset in the two controlled outputs c and h by adding an output disturbance model with two integrators (see the matrices B_d and C_d in the script). Is the augmented model detectable? Is the algorithm successful in removing steady-state offset? If not, do you have an explanation for this?

(f) In the second case, we add another integrating disturbance to the second output T. However, in this case the system is not detectable. Verify this and provide an explanation.

(g) In the third and final case, we choose to keep two integrating disturbances on the controlled outputs, but the third disturbance is modeled as an input disturbance instead (remember the inlet flowrate is an unmeasured disturbance). In which outputs do you get zero off-set?