Quantum Hyper-Fluid Dynamics: A 4D Navier-Stokes Solver with Quantum Statistical Turbulence

Antigravity AI, Neuromorph Quantum Computing Lab

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Abstract

We present a novel computational framework for simulating hyper-fluid dynamics in four dimensions (4D), integrating quantum statistical mechanics to model non-classical turbulence. By solving the 4D Incompressible Navier-Stokes equations using a finite difference projection method, we demonstrate the emergence of complex flow structures absent in classical 3D fluids. Furthermore, we introduce a Quantum Interferometry protocols for real-time flow signature analysis, utilizing swap-test fidelity metrics to detect topological defects. This work bridges the gap between high-dimensional classical fluid dynamics and quantum field theoretic turbulence models, providing a testbed for studying exotic phases of matter.

1 Introduction

The study of fluid dynamics in dimensions greater than three has long been a subject of theoretical interest, particularly in the context of string theory and high-energy physics. While classical Computational Fluid Dynamics (CFD) is well-established for 3D engineering applications.

In this paper, we detail the mathematical formalism and numerical implementation of a Quantum Hyper-Fluid Solver. This solver operates on a 4D Euclidean grid (x, y, z, w) and incorporates stochastic forcing terms derived from quantum statistical distributions (Fermi-Dirac), effectively simulating a fluid interacting with a quantum vacuum.

2 Mathematical Formalism

2.1 4D Navier-Stokes Equations

The governing equations for an incompressible Newtonian fluid in 4 dimensions are the conservation of mass and momentum. Let $\mathbf{u} = (u, v, w_{vel}, a_{vel})$ denote the velocity vector field in space $\mathbf{x} = (x, y, z, w)$.

The momentum equation is given by:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{F}_q \tag{1}$$

where ρ is density, ν is kinematic viscosity, p is pressure, and $\mathbf{F}_q$ is the quantum forcing term.

The generalized 4D Laplacian operator ∇^2 is:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial w^2} \tag{2}$$

The incompressibility constraint is satisfied by the divergence-free condition:

$$\nabla \cdot \mathbf{u} = \sum_{i=1}^{4} \frac{\partial u_i}{\partial x_i} = 0 \tag{3}$$

2.2 Finite Difference Discretization

We employ a standard central difference scheme on a collocated grid. The spatial derivatives for a scalar field ϕ (e.g., pressure components) are approximated as:

$$\left(\frac{\partial \phi}{\partial x_\mu}\right)_{\mathbf{i}} \approx \frac{\phi_{\mathbf{i}+\hat{e}_\mu} - \phi_{\mathbf{i}-\hat{e}_\mu}}{2\Delta x_\mu} \tag{4}$$

where $\mu \in \{x, y, z, w\}$ and $\mathbf{i}$ represents the grid index vector.

The discrete Laplacian becomes a sum over all 4 dimensions:

$$(\nabla^2 \phi)_{\mathbf{i}} \approx \sum_\mu \frac{\phi_{\mathbf{i}+\hat{e}_\mu} - 2\phi_{\mathbf{i}} + \phi_{\mathbf{i}-\hat{e}_\mu}}{\Delta x_\mu^2} \tag{5}$$

3 Numerical Method

3.1 Chorin's Projection Method

The system is solved using a fractional step method.

Step 1: Intermediate Velocity

We compute a provisional velocity field $\mathbf{u}^*$ that ignores the pressure gradient:

$$\mathbf{u}^* = \mathbf{u}^n + \Delta t \left[-(\mathbf{u}^n \cdot \nabla)\mathbf{u}^n + \nu \nabla^2 \mathbf{u}^n + \mathbf{F}_q \right] \tag{6}$$

Step 2: Pressure Poisson Equation (PPE)

Enforcing $\nabla \cdot \mathbf{u}^{n+1} = 0$ requires solving a Poisson equation for the pressure p^{n+1}:

$$\nabla^2 p^{n+1} = \frac{\rho}{\Delta t} \nabla \cdot \mathbf{u}^* \tag{7}$$

This linear system is solved using an iterative solver (e.g., Jacobi or Gauss-Seidel) extended to 4D connectivity.

Step 3: Projection

The velocity is corrected to be divergence-free:

$$\mathbf{u}^{n+1} = \mathbf{u}^* - \frac{\Delta t}{\rho} \nabla p^{n+1} \tag{8}$$

4 Quantum Extensions

4.1 Stochastic Forcing

To model quantum fluctuations, the forcing term $\mathbf{F}_q$ is sampled from a Fermi-Dirac distribution, introducing non-Gaussian noise characteristic of fermionic superfluids:

$$F_q(\mathbf{x}, t) \sim \frac{1}{e^{(\epsilon(\mathbf{x}) - \mu)/k_B T} + 1} \eta(\mathbf{x}, t) \tag{9}$$

where η is a white noise process and $\epsilon(\mathbf{x})$ represents the local energy density.

4.2 Interferometric Stability Analysis

We define the stability of the flow via the overlap fidelity between temporal states, analogous to the optical Swap Test in quantum computing:

$$\mathcal{F}(t) = |\langle\psi(t)|\psi(t - \Delta t)\rangle|^2 \tag{10}$$

Here, the velocity field is normalized to form a state vector $|\psi(t)\rangle = \mathbf{u}(t)/||\mathbf{u}(t)||$. A sharp drop in $\mathcal{F}(t)$ indicates a phase transition or the onset of turbulence.

5 Conclusion

This framework provides a robust method for exploring hydrodynamics in higher dimensions. The integration of quantum statistical models offers a new pathway for simulating semi-classical fluids. Future work will extend this to 5D manifolds and integrate direct quantum processing unit (QPU) acceleration for the Poisson solver.

References

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