

1. The Quantum Neural Hamiltonian

1. The Hamiltonian Formulation

We model the neural system as a graph $G(V, E)$ where each node (neuron/column) is a qubit state $|\psi\rangle$. The total system Hamiltonian is:

$$H_{sys} = H_{local} + H_{int}$$

1.1 Local Term:

Fluctuations in local cognitive potential are modeled as Pauli-Z rotations:

$$H_{local} = \sum_i \hbar \omega_i \sigma_z^{(i)}$$

1.2 Interaction Term (Synaptic Coupling):

The edges E represent entangled states. We use an XY-interaction model for Hebbian exchange:

$$H_{int} = - \sum_{\langle i, j \rangle} J_{ij}(t) (\sigma_+^{(i)} \sigma_-^{(j)} + \text{h.c.})$$

Step 1 Derivation:

Information transfer probability is proportional to the square of the transition amplitude:

$$P_{i \rightarrow j}(t) \approx \sin^2 (J_{ij}t/\hbar)$$

Thus, maximizing J_{ij} directly maximizes the coherent information flux.

2. Hebbian Plasticity First Principles

2. Deriving the Hebbian Operator

Hebbian plasticity states: 'Neurons that fire together, wire together.' In Quantum terms: 'Qubits that phase-lock maximize mutual inductance.'

We define a Plasticity Operator K that acts on the couplings J :

$$\frac{dJ_{ij}}{dt} = \eta \langle \Psi | \hat{K}_{ij} | \Psi \rangle$$

Step-by-Step Derivation:

1. Let the system seek the ground state of an 'Optimization Hamiltonian' where aligned spins have lower energy:

$$H_{opt} = - \sum \sigma_x^{(i)} \sigma_x^{(j)}$$

2. The 'force' driving plasticity is the gradient of the energy expectation:

$$F_{ij} = - \nabla_{J_{ij}} \langle H_{opt} \rangle$$

3. Substituting the correlations ($\langle \sigma_x \sigma_x \rangle \sim \cos \Delta \phi$):

$$J_{ij}(t+1) = J_{ij}(t) + \alpha \cos(\phi_i - \phi_j)$$

3. Prime Resonance Regularization

3. Prime Resonance Field Theory

We postulate that critical stability in neural networks follows the Montgomery-Odlyzko Law, relating quantum energy levels to the zeros of the Riemann Zeta function.

3.1 The Prime Potential:

We define a scalar potential V_p on the discretized manifold:

$$V_p(x) = \sum_k \delta(x - x_k) (\ln p_k)^{-1}$$

3.2 Modified Hebbian Update:

To prevent runaway excitation, we damp the Hebbian term with the Prime Potential density. This acts as a regularization term R :

$$\mathcal{R}_{ij} = \frac{1}{\ln p_i \ln p_j}$$

3.3 Final Update Equation:

$$\Delta J_{ij} = \alpha \cos(\Delta \phi_{ij}) \cdot (\ln p_i \ln p_j)^{-1}$$

4. Quantum Surface Integral Flux

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To categorize the global state, we calculate the flux of the wavefunction across the topology.

Definition:

Let L be the Graph Laplacian. We define the 'Neuro-Flux' Φ as:

$$\Phi = \oint_{\partial\mathcal{G}} (\psi^\dagger \nabla \psi) \cdot d\mathbf{S}$$

Discretization:

1. The gradient maps to $L|\psi\rangle$.
2. The surface element $d\mathbf{S}$ is weighted by the Prime Metric g_{kk} .

$$\Phi \approx \psi^\dagger \cdot (\mathcal{L} \cdot \mathbf{g}) \cdot \psi$$

Interpretation:

- High Flux: Critical State (Healthy)
- Low Flux: Disconnected (Dementia)

Our repair algorithm injects edges specifically to maximize this integral.