Detailed Derivations: Prime Resonance Hebbian Amplification & Quantum Surface Integral Flux

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1. The Quantum Hebbian Hamiltonian

We begin by defining the interaction Hamiltonian describing synaptic plasticity. In classical neuroscience, Hebb's postulate allows synaptic weight w_{ij} to grow if firing is correlated. In our Quantum model, 'firing' relates to phase alignment on the Bloch sphere.

The Hamiltonian H_{Hebb} governing the evolution of entanglement weights J_{ij} is:

$ H_{Hebb} = - \sum_{\langle i,j \rangle} J_{ij}(t) \left(\sigma_+^{(i)} \sigma_-^{(j)} + \text{h.c.} \right) $

To induce **Hebbian Amplification**, we introduce a time-dependent driving force $U_{drive}(\tau)$ that minimizes the system energy when phases align $(\Delta\phi \to 0)$.

$ \frac{dJ_{ij}}{dt} = \eta \langle \psi | \left(\sigma_x^{(i)} \sigma_x^{(j)} + \sigma_y^{(i)} \sigma_y^{(j)} \right) | \psi \rangle $

2. Prime Resonance Modulation

Standard Hebbian learning can lead to runaway excitation (epileptic states). We modulate this using the **Prime Number Theorem**. We postulate that stable biological networks follow the distribution of Prime Gaps $g_n = p_{n+1} - p_n$.

We impose a **Prime Potential** scalar field $V(k)$ on each node k:

$$V(k) = \frac{1}{\ln(p_k)}$$

Where p_k is the k-th prime number mapping to node k. This potential acts as a 'damping factor' derived from the asymptotic density of primes.

The Modified Hebbian Update Rule becomes:

$ J_{ij}^{new} = J_{ij}^{old} + \alpha \cdot \cos(\phi_i - \phi_j) \cdot \left(\frac{1}{\ln(p_i)\ln(p_j)} \right) $

3. Quantum Surface Integral Flux

To quantify the global health of the connectome, we treat the graph as a discrete manifold $\mathcal{M}$. We calculate the 'Surface Flux' Φ_Σ of the wavefunction ψ across this manifold.

The divergence theorem states:

$ \int_{\mathcal{V}} (\nabla \cdot \mathbf{F}) \, dV = \oint_{\partial \mathcal{V}} (\mathbf{F} \cdot \mathbf{n}) \, dA $

In our graph theoretic limit, the Laplacian $\mathcal{L}$ acts as the divergence operator. The Surface Flux is thus the projection of the Laplacian-transformed state vector onto the Prime Potential field:

$$\Phi_\Sigma = \left\langle \psi | \mathcal{L}^\dagger \hat{P} | \psi \right\rangle$$

Where $\hat{P} = \mathrm{diag}(1/\ln p_1, \dots, 1/\ln p_N)$.

4. Derivation of the Prime Vortex

A 'Vortex' in this field corresponds to a topological defect (broken connection). The winding number W around a cycle C is:

$$W = \frac{1}{2\pi} \oint_C \nabla\theta \, dl$$

Neurodegeneration (dementia) is characterized by $W \to 0$ (loss of phase coherence).

Our repair algorithm injects 'Prime Gliders'—edges with weight equal to normalized Prime Gaps—to restore non-trivial topology ($W \neq 0$). This forces the system back into a Quantum Critical Regime (GUE Statistics).

$$J_{repair} \propto \frac{p_{n+1} - p_n}{\ln p_n}$$

This ensures that the energy spectrum E_n of the repaired graph mirrors the zeros of the Riemann Zeta Function.