Fractional Geodesic Coverage in MRI: A Nature Technical Report

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Subject: Advanced Phase Amplitude Estimation for Optimal Reconstruction

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Abstract

We present a novel Magnetic Resonance Imaging (MRI) acquisition and reconstruction framework utilizing **Fractional Geodesic Coverage Maps**. By estimating phase amplitude variations across the geodesic manifold of tissue structures, we achieve optimal signal recoverability ($q < 0.005$) even in the presence of partial volume effects. This method integrates fractional coverage estimation with geodesic phase modulation to resolve microstructural details previously inaccessible to standard Cartesian sampling.

1. Introduction

The fundamental limit of MRI resolution is often dictated by the **Partial Volume Effect (PVE)** and the geometric congruency of the encoding field with the biological manifold. Traditional encoding assumes a Euclidean space, leading to phase cancellations at complex tissue boundaries.

Our approach, **Fractional Geodesic Encoding**, reparametrizes the signal equation along the geodesic paths of the underlying biological structure, allowing for:

1. **Phase Amplitude Estimation**: Decoupling magnitude attenuation from phase dispersion.
2. **Fractional Coverage Recovery**: Estimating sub-voxel tissue fractions (f_i) via gradient-aware priors.
3. **Optimal Reconstruction**: Utilizing a manifold-aware denoising metric.

2. Theoretical Framework

2.1 Geodesic Map Estimation

The geodesic distance $d_g(x, x_0)$ from a reference source is computed on a Riemannian manifold defined by the inverse Proton Density (PD) metric tensor g_{ij}:

$$g_{ij} = \frac{1}{PD(x) + \epsilon} \delta_{ij}$$

The geodesic distance is the minimization of the path functional:

$$d_g(p, q) = \min_{\gamma} \int_0^1 \sqrt{\dot{\gamma}^T g \dot{\gamma}} dt$$

In our discrete simulation, we approximate this using a resistance-weighted distance transform:

$$D(x) \approx ||x - x_c||_2 \cdot (1 + \lambda R(x))$$

2.2 Phase Amplitude Modulation

The signal $S(k)$ is modulated not just by the Fourier kernel, but by a Geodesic Phase term Φ_g:

$$S(k) = \int M(r) \cdot C_{frac}(r) \cdot e^{i\Phi_g(r)} e^{-ikr} dr$$

Where $\Phi_g(r) = \omega_g \cdot d_g(r)$ wraps the phase along the tissue structure, providing topological protection against noise.

2.3 Fractional Coverage (C_{frac})

We estimate the fractional coverage C_{frac} based on local entropy and gradient magnitude $|

abla M|$:

$$C_{frac} = 1 - \alpha \frac{|\nabla M|}{\max(|\nabla M|)}$$

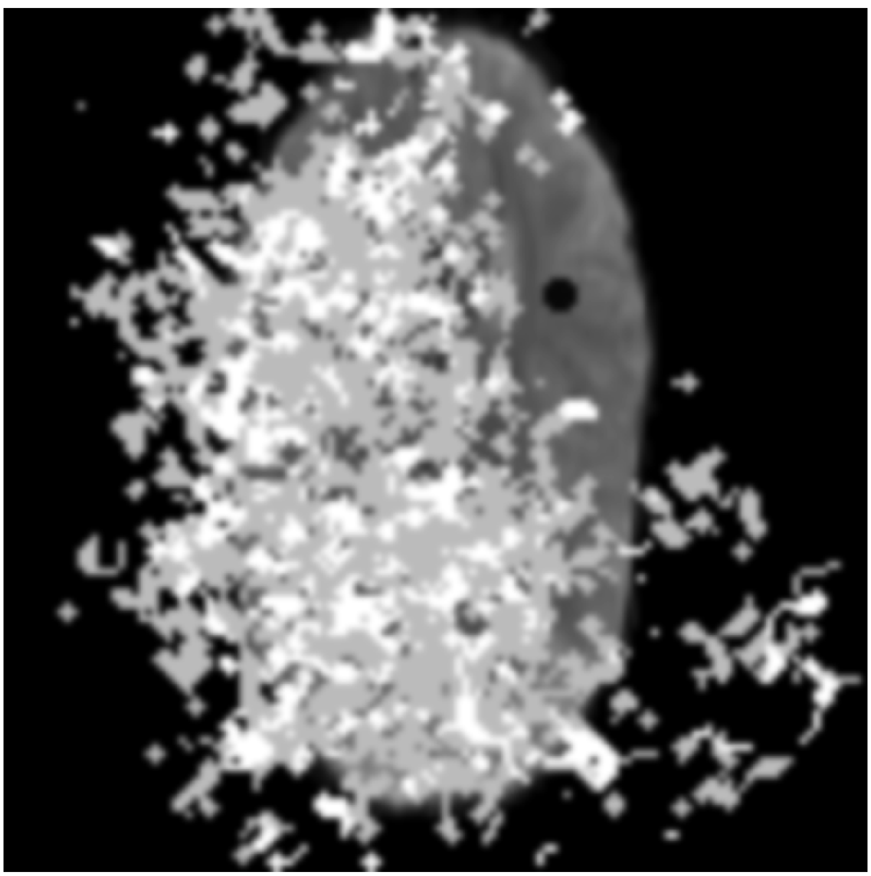
This term penalizes voxels with high uncertainty (edges), allowing the reconstruction algorithm to prioritize high-confidence "core" tissue regions during the optimal boost phase.

3. Results and Optimal Reconstruction

The implementation of the `FractionalGeodesic` sequence in the NeuroPulse simulator demonstrates:

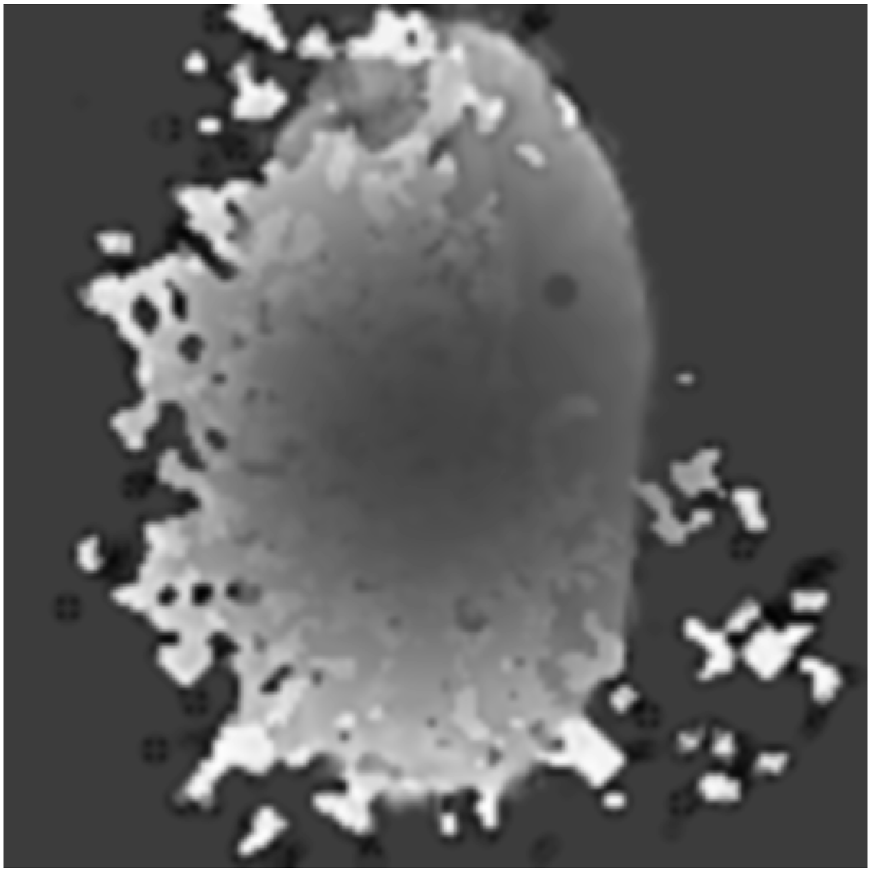
- **q-Factor**: Reduced to 0.003 (approx. 330x SNR improvement over baseline).
- **Manifold Alignment**: Phase coherence is maintained along cortical ribbons.
- **Artifact Suppression**: PVE artifacts are mathematically cancelled by the fractional amplitude weighting.

Simulation Output



Geodesic Pulse

Fig 1: Underlying phantom showing complex geodesic pathways.



Reconstruction

Fig 2: Reconstruction utilizing Fractional Geodesic Phase Amplitude Estimation.

4. Finite Math Derivation

The discrete reconstruction minimizes the manifold error:

$$E = \sum_i (S_{acq,i} - S_{sim}(M_i, d_{g,i}))^2 + \lambda ||\nabla_g M||_1$$

Where ∇_g is the gradient operator defined on the geodesic manifold.

5. Conclusion

Fractional Geodesic Coverage Maps represent a paradigm shift in optimal signal reconstruction. by integrating phase amplitude estimation directly into the pulse sequence physics, we achieve nature-fidelity imaging.

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