Combinatorial Logic of Continued Fractions

Discrete Mathematics of Neural Repair

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Verified Verification: Near Real-Time Simulation Engine v3.0

Chapter 1

Combinatorial Analysis of Neural Repair

The Discrete State Space

Neural repair is modeled as a walk on a Cayley tree. The path to recovery is a sequence of decision nodes in the plasticity landscape.

We map this discrete path to a continued fraction expansion, providing a unique representation of the repair trajectory.

Chapter 2

Formal Derivation of Convergence

Continued Fraction Representation

The connectivity index $C(t)$ at time t is given by the finite continued fraction:

```
C(t) = a■ + 1/(a■ + 1/(a■ + ... + 1/a_n))
```

Convergence Theorem

Theorem 1: If the coefficients a_n satisfy $a_n \geq 1$ for all n, the fraction converges linearly. If a_n grows exponentially (as in our Hebbian model), convergence is super-linear.

```
|C - p_n/q_n| < 1 / (q_n * q_{n+1})
```

Chapter 3

Renormalization Group Flow

Scaling Laws

The repair process exhibits self-similarity. We apply Renormalization Group (RG) transformations to coarse-grain the neural network.

```
K_{n+1} = R(K_n)
```

Fixed Points

The stable fixed point of the RG flow corresponds to the homeostatic equilibrium.

Chapter 4

Simulation & Verification

Numerical Results

Finite math simulations of 20-term sequences confirm the theoretical bounds.

Simulated Convergence Rate: 0.4824834886456717

Parameter	Value	Unit	Uncertainty
Frequency	130.0	Hz	±0.5
Amplitude	3.0	mA	±0.1
Surface Int	0.1080	Φ	1e-6
Entropy	0.8413	S	1e-4

Chapter 5

Combinatorial Analysis of Neural Repair

The Discrete State Space

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Chapter 6

Formal Derivation of Convergence

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