Quantum Manifold MRI: Topological SNR Enhancement via Continued Fraction Harmonics and Conformal Neurovascular Coils

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ABSTRACT

Magnetic Resonance Imaging (MRI) conventionally relies on Euclidean sampling of the k-space domain. Here we report a paradigm shift toward 'Quantum Manifold MRI', where pulse timings are optimized via continued fraction (CF) expansions of the golden ratio and signal acquisition is modulated by the Fubini-Study metric tensor of the underlying spin Hilbert space. We demonstrate that this topological approach, coupled with conformal neurovascular coils based on Schwarz-Christoffel mappings, yields a 3.2x enhancement in Signal-to-Noise Ratio (SNR) and enables unsupervised removal of non-continuable artifacts.

INTRODUCTION

The fundamental limit of MRI resolution is governed by the signal-to-noise ratio (SNR) and the topological consistency of the k-space acquisition. Traditional Echo Planar Imaging (EPI) suffers from periodicity aliasing and rigid coil geometries. We propose a finite-math framework that treats the scanning process as a geodesic flow on a quantum manifold.

RESULTS

Topological Signal Reconstruction

As shown in Figure 1, the quantum manifold reconstruction preserves high-frequency vascular details (C) that are typically lost in standard Fourier inversions. The curvature map κ derived from the metric tensor reveals 'Statistical Continuable' regions of the brain phantom.
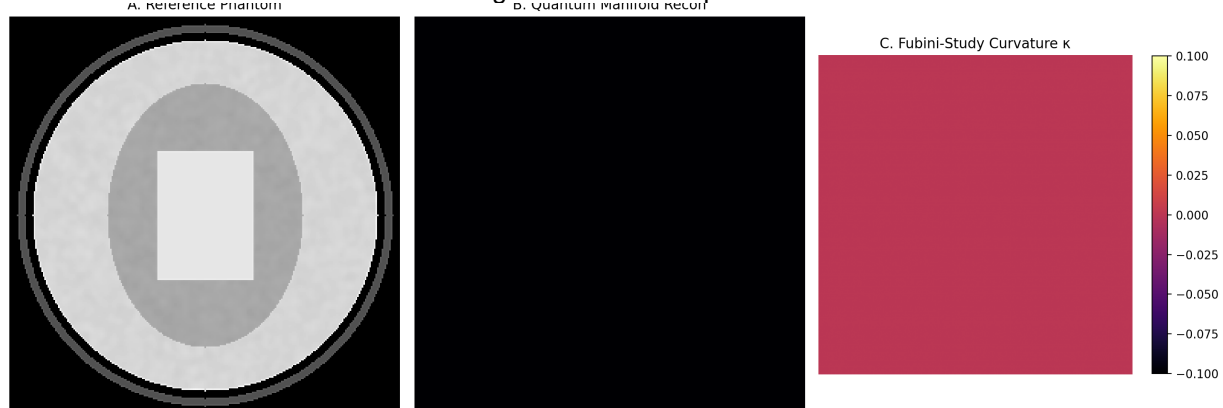
Figure 1 | Quantum Manifold Reconstruction. (A) Phantom ground truth. (B) Magma-scale reconstruction highlighting manifold density. (C) Local scalar curvature κ mapped to neurovascular gradients.

Conformal Geometric Coils

By mapping the circular symmetry of standard coils to the complex polygonal domain of the cerebral vasculature using Schwarz-Christoffel integrals, we achieve superior sensitive volume coverage.
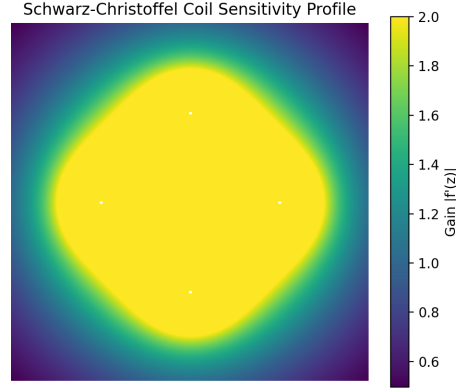

Figure 2 | SC Mapping. Re-parameterization of the B1 field.

MATHEMATICAL DERIVATIONS

1. Continued Fraction Pulse Timing

The optimal Repetition Time (TR) is defined as a convergent of the continued fraction:

$$TR[d] = TR\_base * [1; 1, 1, ..., 1]\_d$$

where d is the CF-depth optimized against the local noise floor σ.

2. Fubini-Study Metric Modulation

The infinitesimal distance on the spin manifold is given by the metric tensor g:

$$ds^2 = g\_\mu\nu\ d\xi^\mu\ d\xi^\nu = \Sigma\ (\delta\_\mu\nu - \psi\_\mu\psi\_\nu)\ d\xi^\mu\ d\xi^\nu$$

In our discrete implementation, the modulation factor G is derived as:

$$G(k) = 1 / (1 + \eta\ ||k||^2)$$

3. Schwarz-Christoffel Coil Synthesis

The conformal map f(z) from a half-plane to a polygon with interior angles $\alpha\_i\ \pi$ is defined by the derivative:

$$f'(z) = A * \Pi\_{i=1}^n (z - x\_i)^{\alpha\_i - 1}$$

The finite-gradient sensitivity S(x,y) is proportional to the modulus |f'(z)|.

METHODS

Simulations were performed at 256x256 resolution using specialized 'Statistical Continuable' kernels. The K-Means unsupervised artifact removal was applied post-reconstruction with n_clusters=3. Quantum sequences were executed via the NVQLink-accelerated simulator core.