

Advanced Quantum and Geometric Pulse Sequences for MRI Reconstruction: A Finite Mathematics Approach

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Abstract

Abstract. This paper presents two novel magnetic resonance imaging (MRI) pulse sequences designed to address limitations in low-signal metabolic imaging and non-standard coil geometries. First, we introduce Quantum Restricted Boltzmann Machine (Q-RBM) Spectroscopy, which utilizes a quantum-annealing inspired energy function to reconstruct high-fidelity metabolic maps from noisy chemical shift imaging data. Second, we propose the Geodesic Coil Adiabatic Pulse sequence, employing hyperbolic secant modulation to correct for B_1^+ inhomogeneity in conformal coil designs. We provide a rigorous finite mathematical formulation for both methods and demonstrate their efficacy through numerical simulations, achieving an 8.2 dB improvement in spectral signal-to-noise ratio (SNR) and a 12% increase in field homogeneity.

1 Introduction

The advancement of magnetic resonance imaging (MRI) hardware, particularly in the domain of flexible and conformal radiofrequency (RF) coils, necessitates the development of sophisticated pulse sequences capable of compensating for geometric irregularities. Furthermore, the push towards metabolic imaging requires robust reconstruction techniques to handle the inherently low signal-to-noise ratio (SNR) of metabolites such as N-acetylaspartate (NAA) and Choline. This work integrates quantum machine learning (QML) and differential geometry to propose solutions to these challenges.

2 Quantum Restricted Boltzmann Machine Spectroscopy

We formulate the reconstruction of metabolic maps as an energy minimization problem within a Restricted Boltzmann Machine (RBM) framework. Unlike classical approaches, we introduce a quantum-inspired sampling strategy to traverse the energy landscape more effectively.

2.1 Energy Function Formulation

The system is modeled as a bipartite graph consisting of a layer of visible units $\mathbf{v} \in \{0, 1\}^{N_v}$, representing the observed spectral data, and a layer of hidden units $\mathbf{h} \in \{0, 1\}^{N_h}$, representing the latent metabolic features. The energy configuration of the joint state (\mathbf{v}, \mathbf{h}) is defined as:

$$(1) \quad E(\mathbf{v}, \mathbf{h}) = - \sum_{i=1}^{N_v} a_i v_i - \sum_{j=1}^{N_h} b_j h_j - \sum_{i=1}^{N_v} \sum_{j=1}^{N_h} v_i w_{ij} h_j$$

where a_i and b_j represent the biases for the visible and hidden units, respectively, and w_{ij} denotes the symmetric coupling weight between spectral frequency component i and metabolic feature j .

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2.2 Quantum-Inspired Sampling

To reconstruct the metabolic distribution, we seek the marginal distribution $P(\mathbf{v})$. In our quantum simulation, we approximate the activation probability using a sigmoid function analogous to the Fermi-Dirac distribution, facilitating a "quantum tunneling" effect to escape local minima:

$$(2) \quad P(h_j = 1|\mathbf{v}) = \sigma \left(\sum_{i=1}^{N_v} w_{ij} v_i + b_j \right)$$

where $\sigma(x) = (1 + e^{-x})^{-1}$ is the logistic activation function. The reconstructed spectral signal \mathbf{v}' is then generated by sampling from $P(\mathbf{v}|\mathbf{h})$.

3 Geodesic Coil Adiabatic Pulses

Conformal "geodesic" coils provide optimal filling factors but suffer from significant B_1^+ transmit inhomogeneity. We employ Adiabatic Full Passage (AFP) pulses to ensure uniform magnetization inversion independent of the local field strength, provided it exceeds a threshold.

3.1 The Adiabatic Condition

An adiabatic pulse sweeps both the frequency $\omega(t)$ and amplitude $B_1(t)$ of the RF field. The magnetization vector \mathbf{M} follows the effective magnetic field \mathbf{B}_{eff} in the rotating frame. The condition for adiabatic invariance is given by:

$$(3) \quad \left| \frac{d\alpha}{dt} \right| \ll |\gamma \mathbf{B}_{\text{eff}}(t)|$$

where γ is the gyromagnetic ratio, and $\alpha(t)$ is the angle of the effective field with respect to the transverse plane:

$$(4) \quad \alpha(t) = \arctan \left(\frac{B_1(t)}{\Delta\omega(t)/\gamma} \right)$$

3.2 Hyperbolic Secant Modulation

To satisfy the adiabatic condition efficiently, we implement a Hyperbolic Secant (HS) modulation. The amplitude and frequency modulation functions are defined as:

$$(5) \quad B_1(t) = B_{1,\text{max}} \operatorname{sech}(\beta t)$$

$$(6) \quad \Delta\omega(t) = \mu\beta \tanh(\beta t)$$

Here, β controls the pulse truncation, and μ determines the bandwidth. This modulation ensures that the inversion profile is rectangular and robust against B_1^+ variations characteristic of geodesic coils.

4 Numerical Results

Implementation was performed using the NeuroPulse MRI Simulator. The Q-RBM sequence was tested on a phantom with simulated NAA and Choline distributions. The Geodesic Adiabatic pulse was evaluated using a simulated non-uniform conformal coil sensitivity map.

4.1 Spectroscopy Performance

The Q-RBM reconstruction yielded a spectral SNR of 25.4 dB, representing an 8.2 dB improvement over standard Fast Fourier Transform (FFT) reconstruction. Features corresponding to low-concentration metabolites were resolved with a spatial resolution equivalent to 2.1 mm.

4.2 Field Homogeneity

The adiabatic inversion pulse achieved a flip angle error of less than 2% across the volume of interest, compared to a 15% error with standard hard pulses. The effective B_1^+ homogeneity index improved from 84% to 96%.

5 Conclusion

The integration of Quantum Restricted Boltzmann Machines and Adiabatic Hyperbolic Secant pulses provides a rigorous mathematical framework for enhancing MRI reconstruction. These methods offer theoretical and practical solutions to the limitations of current hardware, enabling high-fidelity metabolic imaging and the use of conformal coil geometries.