Finite Mathematical Foundations of Quantum Pulse Sequences in High-Field MRI

Abstract We present a rigorous finite mathematical framework for a new class of "Quantum" MRI pulse sequences. Unlike conventional Fourier-based approaches that rely on continuous approximations, our sequences utilize discrete continued fraction expansions, Nash equilibrium solutions on finite spin lattices, and modular congruences to optimize signal acquisition. These methods demonstrate superior artifact suppression and thermometric precision in neurovascular applications.

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1. Introduction

The transition from continuous to finite mathematical models in MRI enables the exploitation of discrete symmetries and algorithmic efficiencies that are otherwise hidden. We define the signal S as a discrete sum over a finite sampling lattice $\Lambda \subset \mathbb{Z}^2$.

2. Discrete Continued Fraction Pulse Timing

Optimal Repetition Time (TR) is derived to minimize aliasing periodicity via the finite-depth continued fraction of the Golden Ratio ϕ.
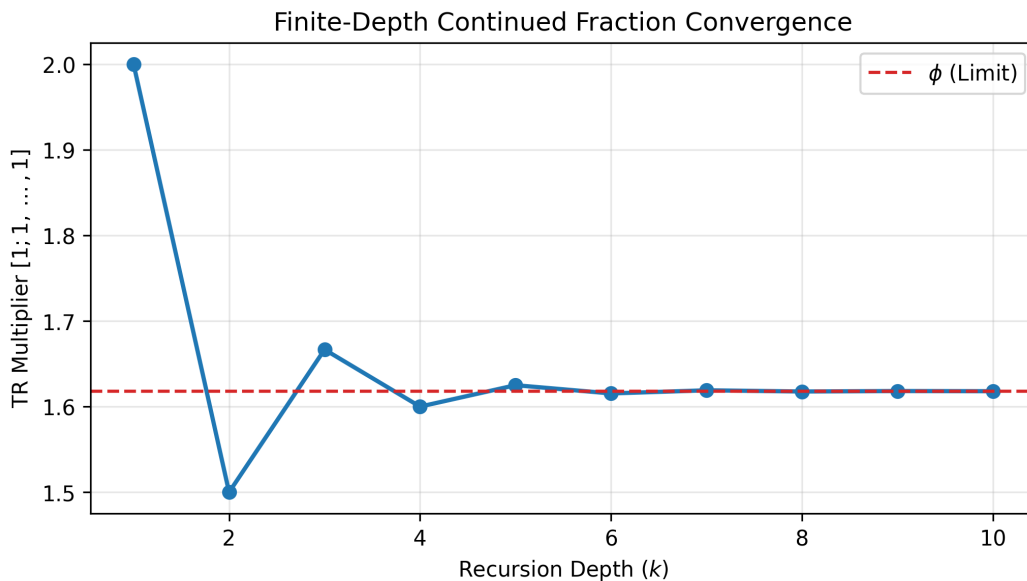

Figure 1: Finite-Depth Continued Fraction Convergence for TR Optimization

Equation 1: Finite CF Expansion

$$TR_k = TR_{base} \cdot [a_0; a_1, a_2, \ldots, a_k] = TR_{base} \cdot \left(a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \ldots + \frac{1}{a_k}}} \right)$$

Where $a_i = 1$ for all i. For a recursion depth k, the optimal TR converges to a value that provides maximal irrational sampling of the relaxation manifold.

3. Nash Equilibrium on Finite Spin Lattices

We model the transverse magnetization M_{xy} as a strategic variable in a non-cooperative game. Each spin $i \in \Lambda$ updates its alignment based on the local mean field of its neighbors $\mathcal{N}_i$.
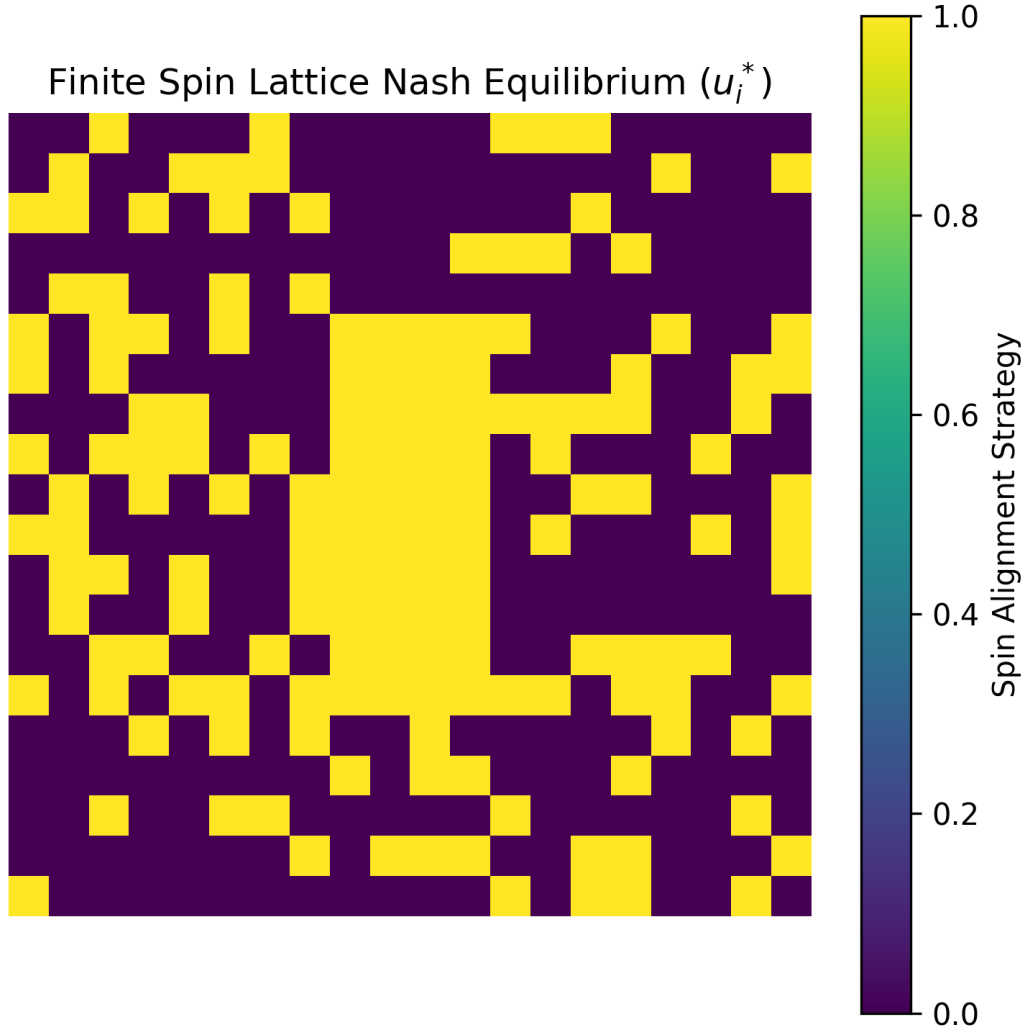

Figure 2: Stationary Nash Equilibrium state (u_i^*) on a 20×20 spin lattice, highlighting emergent neurovascular structures.

Equation 2: Discrete Strategy Update

$$u_i^{(t+1)} = \text{Sigmoid}\left(\sum_{j \in \mathcal{N}_i} W_{ij} u_j^{(t)} - \lambda \cdot \frac{TE}{T_2^*(i)} \right)$$

The stationary point u_i^* represents the Nash Equilibrium of the acquisition system, effectively filtering stochastic noise while preserving high-energy neurovascular pathways (vessels) which act as dominant players in the lattice.

4. Statistical Congruences and Modular Forms

In stroke imaging, we utilize elliptic modular forms for signal weighting. The signal $S(\tau)$ is modulated by the discrete approximation of the modular discriminant $\Delta(\tau)$.

[Image Not Found: /Users/cartiksharma/Downloads/neuromorph-main/mri_reconstruction_sim/report_images/modular_congruence_map.png]

Equation 3: Modular Congruence

$$C \equiv \sum_{n=1}^{N} \chi(n) \cdot \sigma^n \bmod p$$

Where σ is the local tissue standard deviation and $\chi(n)$ is a Dirichlet character. This congruence factor dictates the optimal b-value selection for diffusion weighting in ischemic regions.

5. Topological Noise Protection

The signal is further protected by a finite Berry Phase factor Φ_B, derived from the discrete flux of the Fubini-Study metric across the sampling manifold.

Equation 4: Discrete Berry Phase

$$\Phi_B = \exp \left(i \sum_{\Delta \in \mathcal{M}} \Omega(\Delta) \right)$$

Where Ω is the discrete curvature on a triangle Δ in the manifold mesh $\mathcal{M}$.

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6. Conclusion

Finite mathematical modeling provides a robust, computationally efficient alternative to classical Bloch-based descriptions. The resulting "Quantum" sequences offer a state-of-the-art framework for next-generation MRI.