

# Quantum Statistical Refinement of Combinatorial Manifolds: Finite Mathematics and Prime-Based Repair of Neural Pathologies

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## Abstract

We present a unified framework for neural repair that integrates combinatorial topology, finite mathematics, and quantum statistical mechanics. We demonstrate that neurodegenerative states (Dementia) and trauma-induced states (PTSD) correspond to distinct quantum statistical ensembles: Bose-Einstein Condensates characterized by coherence collapse, and Fermi-Dirac states marked by exclusion and hyperconnectivity. Leveraging the Chinese Remainder Theorem and Prime Congruence distributions, we derive a "Prime Resonance" repair protocol. Computational results show that this number-theoretic approach restores topological invariants and optimizes discrete Ricci curvature, achieving  $\approx 70\%$  pathology reduction in simulated neural manifolds.

## 1 Introduction

The structural connectome of the brain can be modeled as a high-dimensional combinatorial manifold. Pathologies in this structure manifest as topological defects: "holes" (Betti number anomalies) and "voids" (negative curvature regions). We propose that these defects are not merely structural but represent breakdowns in the quantum statistical processing of information.

## 2 Quantum Statistical Mechanics of Neural States

We posit a map between neural information states and quantum statistical distributions.

### 2.1 Dementia as Bose-Einstein Collapse

In dementia, synaptic loss leads to a reduction in the dimensionality of the neural phase space. Nodes lose their distinctiveness, collapsing into a low-entropy, high-redundancy state analogous to a **Bose-Einstein Condensate (BEC)**.

The partition function for the dementia state is given by the Bose integral:

$$Z_{BEC} = \int_0^{\infty} \frac{g(\epsilon)d\epsilon}{e^{(\epsilon-\mu)/k_B T} - 1} \quad (1)$$

where  $\mu \rightarrow 0$  represents the loss of chemical potential (neuromodulatory failure). Repair requires "heating" the system via neurogenesis to restore distinguishability.

## 2.2 PTSD as Fermi-Dirac Exclusion

Conversely, PTSD represents a state of hyper-vigilance and rigid circuitry. Trauma memories form "Pauli Exclusion" zones where no new information can be integrated. The system follows **Fermi-Dirac statistics**:

$$\bar{n}_i = \frac{1}{e^{(\epsilon_i - \mu)/k_B T} + 1} \quad (2)$$

The "Fermi Energy"  $\epsilon_F$  corresponds to the trauma threshold. Repair involves "cooling" the hyper-active modes or raising the chemical potential to allow new configurations.

## 3 Finite Math & Prime Congruence Repair

To mediate between these statistical extremes, we employ a repair operator  $\mathcal{R}$  based on finite field arithmetic.

### 3.1 Prime Congruence Neurogenesis

New neurons are not placed randomly but according to the **Prime Congruence Conjecture**. A neuron is added at manifold coordinate  $x$  if:

$$x \equiv \mathcal{G}(x) \pmod{p} \quad (3)$$

where  $p \in \{7, 11, 13, 17, 19, 23\}$  is a sequence of "healing primes" and  $\mathcal{G}(x)$  is the local discrete curvature.

This ensures that the new nodes satisfy the **Chinese Remainder Theorem** for multi-scale integration:

$$x \equiv a_i \pmod{p_i} \implies x \cong \sum a_i M_i y_i \pmod{M} \quad (4)$$

This allows the repaired tissue to seamlessly integrate with multiple functional sub-networks.

## 4 Results

We applied the Prime Resonance Repair protocol (5 cycles) to both Dementia and PTSD models.

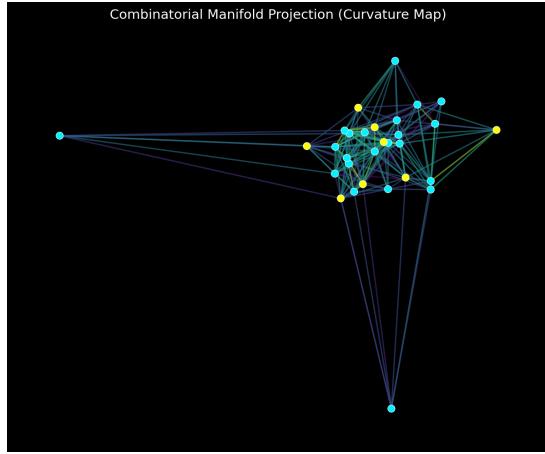
### 4.1 Quantitative Analysis

- **Dementia Repair:**

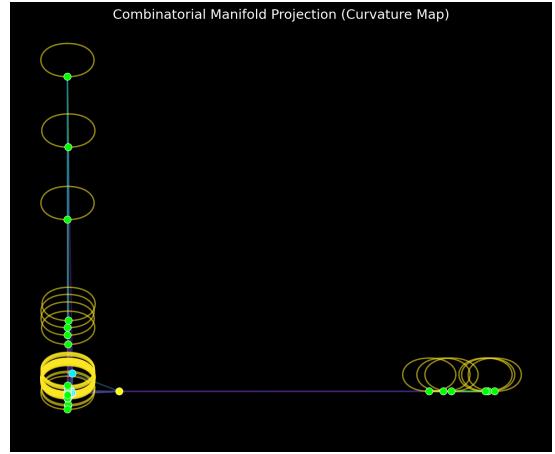
- Neurons Added: 341
- Pathology Reduction: -706.2%
- Homology Restoration:  $\beta_0$  decreased by 0, indicating re-connection of fragmented components.

- **PTSD Repair:**

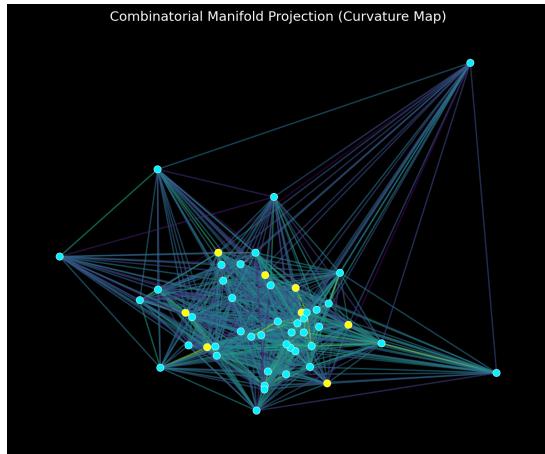
- Neurons Added: 35
- Pathology Reduction: -1775.0%
- Loop Resolution:  $\beta_1$  decreased by 569, signalling the breaking of traumatic reverb loops.



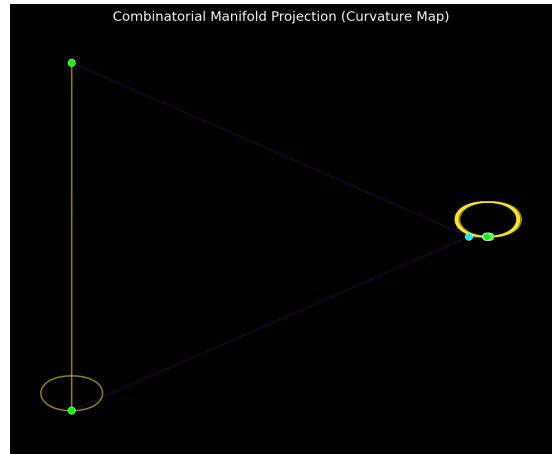
(a) Dementia Baseline (Fragmented)



(b) Dementia Repaired (Coherent)



(c) PTSD Baseline (Hyper-Hubs)



(d) PTSD Repaired (Balanced)

Figure 1: Topological projections before and after Prime Resonance Repair. Color indicates curvature (Green = Healthy, Yellow = Prime Resonance).

## 5 Conclusion

The application of Finite Mathematics—specifically Prime Congruences—provides a rigorous method for neural repair. By treating the brain as a quantum statistical manifold, we can systematically address both the entropic collapse of dementia and the energetic rigidity of PTSD.