Advanced Quantum Mechanics		WiSe 2022/23
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Theoretical Physics 5		Instructor: Harvey Meyer

Interaction of a scalar field with a Dirac field

$$\mathcal{L}_{int} = -g\phi\bar{\psi}\psi$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\tilde{\kappa}^2\phi^2 + \bar{\psi}(i\gamma^\mu\partial_\mu - \kappa)\psi - g\phi\bar{\psi}\phi$$

$$\phi(\vec{x}) = \sqrt{\hbar}c\int\frac{d^3k}{(2\pi)^3\sqrt{2\Omega_{\vec{k}}}}(a(\vec{k})e^{i\vec{k}\cdot\vec{x}} + a(\vec{k})^\dagger e^{-i\vec{k}\cdot\vec{x}})$$

and

$$\psi(\vec{x}) = \sqrt{\hbar}c\int\frac{d^3\vec{q}}{(2\pi)^3\sqrt{2\omega_{\vec{q}}}}\sum_{s=1,2}(a^s(\vec{q})u^s(q)e^{i\vec{q}\cdot\vec{x}} + b^s(\vec{q})^\dagger v^s(q)e^{-i\vec{q}\cdot\vec{x}})$$

Note that the creation and annihilation operators in ϕ and ψ should not be confused. Those for ϕ hold the bosonic commutation relations Dirac ones hold fermionic anti-commutation relations.

The Higgs decays into a fermion anti-fermion pair $(f\bar{f})$. Probability per unit time (i.e. rate) of the transition $h \to \bar{f}f$:
Fermi's Golden Rule:

$$\Gamma = \frac{2\pi]}{\hbar}|\left\langle\bar{f}f\right| H_{int}\left|k\right\rangle|^2\delta(E_k - E_f - E_{\bar{f}})$$

$$H = \int d^3\vec{x}\left(\frac{1}{2}\pi(x)^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{1}{2}\tilde{\kappa}^2\phi^2 + \bar{\psi}(-i\vec{\gamma}\cdot\vec{\nabla} + \kappa)\psi + g\phi\bar{\psi}\psi\right)$$

$$H_\int = g\int d^3\vec{x}\phi\bar{\psi}\psi$$

$$H = \sum(p\cdot\dot{q}) - \mathcal{L}$$

Standard Model

$$\sqrt{\hbar c}g = \frac{m_f}{V} \simeq \frac{5}{250} = 0.02$$

V = Higgs vaccuum expectation value = 246 GeV. The mass $m_h \simeq 125$ GeV. The Higgs decays immediately. What are the products usually? The top quark is too heavy.

the bottom squark is the next lightest one $m_b \simeq 5$ GeV.

$$h_{\vec{k}} = c(\vec{k})^\dagger\left|0\right\rangle$$

(to avoid confusion we have started using a different symbol, c instead of a for the bosonic creation/annihilation operators).

Normalization:

$$\begin{aligned}\left\langle h'_{\vec{k}} \middle| h_{\vec{k}} \right\rangle &= \langle 0| c(\vec{k'})c(\vec{k})^\dagger |0\rangle \\ &= (2\pi)^3\delta^3(\vec{k}-\vec{k'}) \underbrace{\langle 0|0\rangle}_{1} \\ &= L^3\delta_{\vec{k},\vec{k'}}\end{aligned}$$

Since there are two particles

$$\left|f\bar{f}\right\rangle = \left|f,\vec{q},s;\bar{f},\vec{q}',s'\right\rangle$$

$$= a^s(\vec{q})^\dagger b^{s'}(\vec{q}')^\dagger |0\rangle$$

of states were unit-normalized ($(\left\langle h_{k'} \middle| h_{\vec{k}}\right\rangle = \delta_{\vec{k},\vec{k'}})$)

$$\Gamma = \sum_{\vec{q}} + \sum_{\vec{q}'}\sum_{r,s} |\left\langle f,\vec{q},s,\bar{f},\vec{q},s'\right| |H_{int}\left|h_{\vec{k}}\right\rangle|^2$$

Note that we'd prefer these states here to be unit-normalized, but while dealing with the original states $\left|f\bar{f}\right\rangle$ for instance, it's convenient to work without normalization

$$\xrightarrow{L\to\infty} = (L^3)^2 \int \frac{d^3q}{(2\pi^3)\frac{d^3q'}{(2\pi)^3}\sum_{r,s}}(\ \dots\ )$$

$$= \int \frac{d^3q}{(2\pi)^3}\int\sum_{r,s}\frac{2\pi}{\hbar}|\left\langle f,\bar{f}\right| H_{int}\left|h\right\rangle|^2\delta(E_{\vec{k}}-E_{\vec{q}}-E_{\vec{q}'})$$

In the final state, there's no scalar particle ($\langle 0|$), so we are essentially interested in the matrix element

$$\langle 0| |g\int d^3\vec{x}\phi(\bar{\psi}\psi)(c(\vec{k})^\dagger|0\rangle)$$

In $\phi(\vec{x})$ the only term of interest to us here is the one with $c(\vec{k})e^{i\vec{k}\cdot\vec{x}}$.
Let's focus on how ϕ acts on $c(\vec{k})^\dagger|0\rangle$ which is

$$\langle 0|\phi(\vec{x})c(\vec{k})^\dagger|0\rangle = \frac{\sqrt{\hbar}c}{\sqrt{2\Omega_k}}e^{i\vec{k}\cdot\vec{x}}$$

And then

$$\underbrace{\left\langle f\bar{f}\right|\bar{\psi}(\vec{x})}\psi(\vec{x})|0\rangle$$

The $\left|f\bar{f}\right\rangle = \langle 0| b^{s'}(\vec{q}')a^s(\vec{q})$. In the $\bar{\psi}\psi$, $\bar{\psi}$ provides an $a^\dagger$ and the ψ will a $b^\dagger$.
so we have

$$= \frac{(\sqrt{\hbar}c)^2}{\sqrt{2\omega_q 2\omega_{q'}}}\bar{u}^s(\vec{q})e^{-i\vec{q}\cdot\vec{x}}V^{s'}(\vec{q}')$$

In the end, this matrix element

$$\langle \bar{f}f|\, H_{int}\, |h_{\vec{k}}\rangle = g\frac{(\sqrt{\hbar}c)^3}{\sqrt{2\Omega_{\vec{k}}2\omega_{q'}2\omega_q}}\bar{u}^s(\vec{q})v^{s'}(\vec{q}\,')\underbrace{\int_d^3 xe^{i(\vec{k}-\vec{q}-\vec{q}\,')\cdot\vec{x}}}_{(2\pi)^3\delta^3(\vec{k}-\vec{q}-\vec{q}\,')}$$

Where we see that the last part is inforcing momentum conservation.

$$\langle \bar{f}f|\, H_{int}\, |h_{\vec{k}}\rangle = g^2(\sqrt{\hbar}c)\ |\bar{u}^s(\vec{q})v^{s'}(\vec{q}\,')|^2\ \frac{1}{2\Omega_{\vec{k}}2\omega_{\vec{q}}2\omega_{\vec{q}\,'}}(2\pi)^3\delta^3(\vec{k}-\vec{q}-\vec{q}\,')\underbrace{(2\pi)^3\delta^3(\vec{k}-\vec{q}-\vec{q}\,')}_{L^3\delta_{\vec{k},\vec{q},\vec{q}\,'}}$$

$$\Gamma = \int\frac{d^3q}{(2\pi)^3}\int\frac{d^3q'}{(2\pi)^3}\sum_{s,s'}\frac{2\pi}{\hbar}\frac{g^2(\sqrt{\hbar}c)^6}{2\Omega_{\vec{k}}2\omega_{\vec{q}}2\omega_{\vec{q}\,'}}\bar{u}^s...$$

One thing we have to do is to select the rest frame of our scalar particle. If we pick the rest frame ($\vec{k}=0$). That means the δ function of this energy of the particles is just mc^2.
That gives $\delta(m_hc^2-2E_q)$, with $E_q=\sqrt{(\hbar q)^2+m_f^2c^2}$

and therefore, the δ function takes up a certain value, one that conserves energy

$$\delta(mc^2-2E_q) = \frac{E_{\vec{q}^*}}{2|\vec{q}^*|^2\hbar^2c^2}\delta(|\vec{q}-|\vec{q}^*)$$

(where $\vec{q}^*$ is the value that conserves energy)

so finally

$$\Gamma = \frac{g^2\hbar c^2}{16\pi\tilde{\kappa}}\frac{2|\vec{q}^*|}{\tilde{\kappa}}\underbrace{\sum_{s,s'}|\bar{u}^s(\vec{q})v^{s'}(\vec{q})|^2}_{8\ \vec{q}^{*2}}$$

$$\hbar\Gamma = (g^2\hbar c)\frac{m_hc^2}{8\pi}$$

$\hbar\Gamma$ is also apparently the natural width (actually, partial width) of the Lorentzian resonance peak of the particle signature.

Here, it turns out to be $\hbar\Gamma = 2$ MeV.