Bi-Directional Training in Interference Network

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January 7, 2016

0. System Model(2 Users)

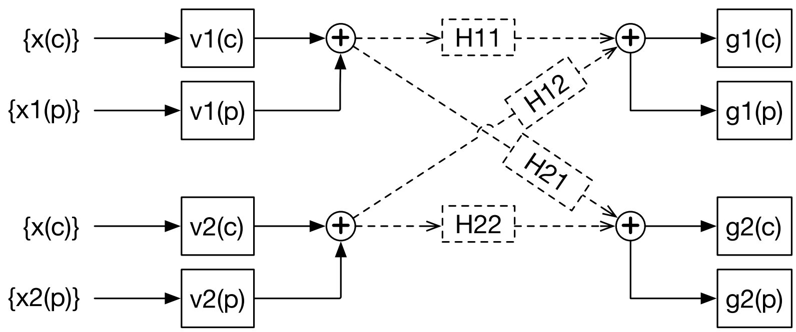


Figure 1: Forward Channel

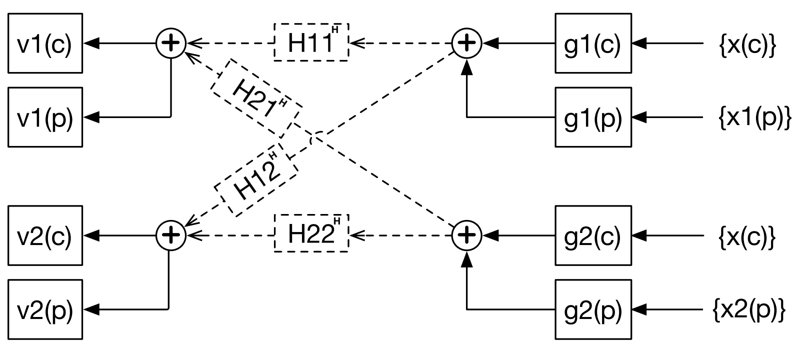


Figure 2: Backward Channel

1. Optimization Problem

$$\min_{\mathbf{v}_k, \mathbf{g}_k} \sum_k MSE_k^{(c)} + MSE_k^{(p)}$$

$$MSE_k^{(c)} = \mathrm{E}[(x - \mathbf{g}_k^{H(c)} \mathbf{y}_k)(x - \mathbf{g}_k^{H(c)} \mathbf{y}_k)^H]$$

$$MSE_k^{(p)} = \mathrm{E}[(x_k^{(p)} - \mathbf{g}_k^{H(p)} \mathbf{y}_k)(x_k^{(p)} - \mathbf{g}_k^{H(p)} \mathbf{y}_k)^H]$$

2. The received signal vector at k-th receiver

$$\mathbf{y}_k = \mathbf{H}_{kk}(\mathbf{v}_k^{(c)} x + \mathbf{v}_k^{(p)} x_k^{(p)}) + \sum_{j \neq k} \mathbf{H}_{kj}(\mathbf{v}_j^{(c)} x + \mathbf{v}_j^{(p)} x_j^{(p)}) + \mathbf{n}_k$$

3. SINR Derivation

$$s_k^{(c)} = \mathbf{g}_k^{H(c)} (\sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(c)} x)$$

$$s_k^{(p)} = \mathbf{g}_k^{H(p)} (\mathbf{H}_{kk} \mathbf{v}_k^{(p)} x_k^{(p)})$$

$$n_k^{(c)} = \mathbf{g}_k^{H(c)} (\sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(p)} x_i^{(p)} + \mathbf{n}_k)$$

$$n_k^{(p)} = \mathbf{g}_k^{H(p)} (\sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(c)} x + \sum_{j \neq k} \mathbf{H}_{kj} \mathbf{v}_j^{(p)} x_j^{(p)} + \mathbf{n}_k)$$

$$\frac{|s_k^{(c)}|^2}{|n_k^{(c)}|^2} = \frac{|\mathbf{g}_k^{H(c)} \sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(c)}|^2}{\sum_i |\mathbf{g}_k^{H(c)} \mathbf{H}_{ki} \mathbf{v}_i^{(p)}|^2 + |\mathbf{g}_k^{H(c)} \mathbf{R}_k \mathbf{g}_k^{(c)}|}$$

$$\frac{|s_k^{(p)}|^2}{|n_k^{(p)}|^2} = \frac{|\mathbf{g}_k^{H(p)} \mathbf{H}_{kk} \mathbf{v}_k^{(p)}|^2}{|\mathbf{g}_k^{H(p)} \sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(c)}|^2 + \sum_{j \neq k} |\mathbf{g}_k^{H(p)} \mathbf{H}_{kj} \mathbf{v}_j^{(p)}|^2 + |\mathbf{g}_k^{H(p)} \mathbf{R}_k \mathbf{g}_k^{(p)}|}$$

4. Max-SINR Algorithm[Gomadam,2011]

$$Forward\ Training(fix\ \mathbf{v}_k^{(c)}, \mathbf{v}_k^{(p)}, \forall k)$$

$$\begin{aligned}\mathbf{g}_k^{(c)} =& \Big[\mathbf{H}_{kk}\mathbf{v}_k^{(c)}\mathbf{v}_k^{H(c)}\mathbf{H}_{kk}^H + \mathbf{H}_{kk}\mathbf{v}_k^{(p)}\mathbf{v}_k^{H(p)}\mathbf{H}_{kk}^H \\ &+ (\sum_{j\neq k}\mathbf{H}_{kj}\mathbf{v}_j^{(c)})(\sum_{j\neq k}\mathbf{v}_j^{H(c)}\mathbf{H}_{kj}^H) + (\sum_{j\neq k}\mathbf{H}_{kj}\mathbf{v}_j^{(p)}\mathbf{v}_j^{H(p)}\mathbf{H}_{kj}^H) \\ &+ \mathbf{H}_{kk}\mathbf{v}_k^{(c)}(\sum_{j\neq k}\mathbf{v}_j^{H(c)}\mathbf{H}_{kj}^H) + (\sum_{j\neq k}\mathbf{H}_{kj}\mathbf{v}_j^{(c)})\mathbf{v}_k^{H(c)}\mathbf{H}_{kk}^H + \sigma^2\mathbf{I}\Big]^{-1}(\sum_i \mathbf{H}_{ki}\mathbf{v}_i^{(c)})\end{aligned}$$

$$\begin{aligned}\mathbf{g}_k^{(p)} =& \Big[\mathbf{H}_{kk}\mathbf{v}_k^{(c)}\mathbf{v}_k^{H(c)}\mathbf{H}_{kk}^H + \mathbf{H}_{kk}\mathbf{v}_k^{(p)}\mathbf{v}_k^{H(p)}\mathbf{H}_{kk}^H \\ &+ (\sum_{j\neq k}\mathbf{H}_{kj}\mathbf{v}_j^{(c)})(\sum_{j\neq k}\mathbf{v}_j^{H(c)}\mathbf{H}_{kj}^H) + (\sum_{j\neq k}\mathbf{H}_{kj}\mathbf{v}_j^{(p)}\mathbf{v}_j^{H(p)}\mathbf{H}_{kj}^H) \\ &+ \mathbf{H}_{kk}\mathbf{v}_k^{(c)}(\sum_{j\neq k}\mathbf{v}_j^{H(c)}\mathbf{H}_{kj}^H) + (\sum_{j\neq k}\mathbf{H}_{kj}\mathbf{v}_j^{(c)})\mathbf{v}_k^{H(c)}\mathbf{H}_{kk}^H + \sigma^2\mathbf{I}\Big]^{-1}(\mathbf{H}_{kk}\mathbf{v}_k^{(p)})\end{aligned}$$

$$Backward\ Training(fix\ \mathbf{g}_k^{(c)}, \mathbf{g}_k^{(p)}, \forall k)$$

$$\mathbf{Z}_{ab} = \mathbf{H}_{ba}^H[Gomadam, 2011]$$

$$without\ cooperation$$

$$\begin{aligned}\mathbf{v}_k^{(c)} =& \Big[\mathbf{Z}_{kk}\mathbf{g}_k^{(c)}\mathbf{g}_k^{H(c)}\mathbf{Z}_{kk}^H + \mathbf{Z}_{kk}\mathbf{g}_k^{(p)}\mathbf{g}_k^{H(p)}\mathbf{Z}_{kk}^H \\ &+ (\sum_{j\neq k}\mathbf{Z}_{kj}\mathbf{g}_j^{(c)})(\sum_{j\neq k}\mathbf{g}_j^{H(c)}\mathbf{Z}_{kj}^H) + (\sum_{j\neq k}\mathbf{Z}_{kj}\mathbf{g}_j^{(p)}\mathbf{g}_j^{H(p)}\mathbf{Z}_{kj}^H) \\ &+ \mathbf{Z}_{kk}\mathbf{g}_k^{(c)}(\sum_{j\neq k}\mathbf{g}_j^{H(c)}\mathbf{Z}_{kj}^H) + (\sum_{j\neq k}\mathbf{Z}_{kj}\mathbf{g}_j^{(c)})\mathbf{g}_k^{H(c)}\mathbf{Z}_{kk}^H + \sigma^2\mathbf{I}\Big]^{-1}(\sum_i \mathbf{Z}_{ki}\mathbf{g}_i^{(c)})\end{aligned}$$

with cooperation

$$\mathbf{V}^{(c)} = \left\{ [\mathbf{Z}]\mathbf{g}^{(c)}\mathbf{g}^{H(c)}[\mathbf{Z}]^H + [\mathbf{Z}] \begin{bmatrix} \mathbf{g}_1^{(p)}\mathbf{g}_1^{H(p)} & & \mathbf{O} \\ & \ddots & \\ \mathbf{O} & & \mathbf{g}_k^{(p)}\mathbf{g}_k^{H(p)} \end{bmatrix} [\mathbf{Z}]^H + \sigma^2\mathbf{I} \right\}^{-1} ([\mathbf{Z}]\mathbf{g}^{(c)})$$

$$\begin{aligned} \mathbf{v}_k^{(p)} = & \Big[\mathbf{Z}_{kk}\mathbf{g}_k^{(c)}\mathbf{g}_k^{H(c)}\mathbf{Z}_{kk}^H + \mathbf{Z}_{kk}\mathbf{g}_k^{(p)}\mathbf{g}_k^{H(p)}\mathbf{Z}_{kk}^H \\ & + (\sum_{j \neq k} \mathbf{Z}_{kj}\mathbf{g}_j^{(c)})(\sum_{j \neq k} \mathbf{g}_j^{H(c)}\mathbf{Z}_{kj}^H) + (\sum_{j \neq k} \mathbf{Z}_{kj}\mathbf{g}_j^{(p)}\mathbf{g}_j^{H(p)}\mathbf{Z}_{kj}^H) \\ & + \mathbf{Z}_{kk}\mathbf{g}_k^{(c)}(\sum_{j \neq k} \mathbf{g}_j^{H(c)}\mathbf{Z}_{kj}^H) + (\sum_{j \neq k} \mathbf{Z}_{kj}\mathbf{g}_j^{(c)})\mathbf{g}_k^{H(c)}\mathbf{Z}_{kk}^H + \sigma^2\mathbf{I} \Big]^{-1} (\mathbf{Z}_{kk}\mathbf{g}_k^{(p)}) \end{aligned}$$

5. Bi-Directional Training with LMS Algorithm[Shi,2014]

$$Forward\ Training(fix\ \mathbf{v}_k^{(c)}, \mathbf{v}_k^{(p)}, \forall k)$$

$$\mathbf{g}_k^{(c)}(n+1) = \mathbf{g}_k^{(c)}(n) + \mu \mathbf{y}_k(n)[x(n) - \mathbf{g}_k^{H(c)}(n)\mathbf{y}_k(n)]^*$$

$$\mathbf{g}_k^{(p)}(n+1) = \mathbf{g}_k^{(p)}(n) + \mu \mathbf{y}_k(n)[x_k^{(p)}(n) - \mathbf{g}_k^{H(p)}(n)\mathbf{y}_k(n)]^*$$

$$Backward\ Training(fix\ \mathbf{g}_k^{(c)}, \mathbf{g}_k^{(p)}, \forall k)(without\ cooperation)$$

$$\mathbf{v}_k^{(c)}(n+1) = \mathbf{v}_k^{(c)}(n) + \mu \mathbf{y}_k(n)[x(n) - \mathbf{v}_k^{H(c)}(n)\mathbf{y}_k(n)]^*$$

$$\mathbf{v}_k^{(p)}(n+1) = \mathbf{v}_k^{(p)}(n) + \mu \mathbf{y}_k(n)[x_k^{(p)}(n) - \mathbf{v}_k^{H(p)}(n)\mathbf{y}_k(n)]^*$$

$$Backward\ Training(fix\ \mathbf{g}_k^{(c)}, \mathbf{g}_k^{(p)}, \forall k)(with\ cooperation)$$

$$\mathbf{V}^{(c)}(n+1) = \mathbf{V}^{(c)}(n) + \mu \mathbf{Y}(n)[x(n) - \mathbf{V}^{H(c)}(n)\mathbf{Y}(n)]^*$$

$$\mathbf{v}_k^{(p)}(n+1) = \mathbf{v}_k^{(p)}(n) + \mu \mathbf{y}_k(n)[x_k^{(p)}(n) - \mathbf{v}_k^{H(p)}(n)\mathbf{y}_k(n)]^*$$

6. Special Case(2 Users, MIMO Channel, Only Common Messages)

$$\mathbf{y}_k = \sum_{i=1}^{2} \mathbf{H}_{ki}\mathbf{v}_i^{(c)} x + \mathbf{n}_k$$

$$\begin{aligned} MSE_k^{(c)} &= \mathrm{E}[(x - \mathbf{g}_k^{H(c)}\mathbf{y}_k)(x - \mathbf{g}_k^{H(c)}\mathbf{y}_k)^H] \\ &= \mathrm{E}[x^2] - \mathrm{E}[x\mathbf{y}_k^H\mathbf{g}_k^{(c)}] - \mathrm{E}[x\mathbf{g}_k^{H(c)}\mathbf{y}_k] + \mathrm{E}[\mathbf{g}_k^{H(c)}\mathbf{y}_k\mathbf{y}_k^H\mathbf{g}_k^{(c)}] \\ &= 1 - \sum_{i=1}^{2}\mathbf{v}_i^{H(c)}\mathbf{H}_{ki}^H\mathbf{g}_k^{(c)} - \mathbf{g}_k^{H(c)}\sum_{i=1}^{2}\mathbf{H}_{ki}\mathbf{v}_i^{(c)} + \mathbf{g}_k^{H(c)}(\sum_{i=1}^{2}\mathbf{H}_{ki}\mathbf{v}_i^{(c)})(\sum_{i=1}^{2}\mathbf{v}_i^{H(c)}\mathbf{H}_{ki}^H)\mathbf{g}_k^{(c)} + \sigma^2\mathbf{g}_k^{H(c)}\mathbf{g}_k^{(c)} \end{aligned}$$

$$\begin{aligned} \mathbf{v}_1^{*(c)} &= \underset{\mathbf{v}_1^{(c)}}{\mathrm{argmin}}(\sum_{k=1}^{2} MSE_k^{(c)}) \\ &= \left[2\mathbf{H}_{11}^H\mathbf{g}_1^{(c)}\mathbf{g}_1^{H(c)}\mathbf{H}_{11} + 2\mathbf{H}_{21}^H\mathbf{g}_2^{(c)}\mathbf{g}_2^{H(c)}\mathbf{H}_{21})\right]^{-1}(2\mathbf{H}_{11}^H\mathbf{g}_1^{(c)} + 2\mathbf{H}_{21}^H\mathbf{g}_2^{(c)} \\ &\quad - \mathbf{g}_1^{H(c)}\mathbf{H}_{12}\mathbf{v}_2^{(c)}\mathbf{H}_{11}^H\mathbf{g}_1^{(c)} - \mathbf{H}_{11}^H\mathbf{g}_1^{(c)}\mathbf{v}_2^{H(c)}\mathbf{H}_{12}^H\mathbf{g}_1^{(c)} - \mathbf{g}_2^{H(c)}\mathbf{H}_{22}\mathbf{v}_2^{(c)}\mathbf{H}_{21}^H\mathbf{g}_2^{(c)} - \mathbf{H}_{21}^H\mathbf{g}_2^{(c)}\mathbf{v}_2^{H(c)}\mathbf{H}_{22}^H\mathbf{g}_2^{(c)}) \end{aligned}$$

$$\begin{aligned} \mathbf{v}_2^{*(c)} &= \underset{\mathbf{v}_2^{(c)}}{\mathrm{argmin}}(\sum_{k=1}^{2} MSE_k^{(c)}) \\ &= \left[2\mathbf{H}_{12}^H\mathbf{g}_1^{(c)}\mathbf{g}_1^{H(c)}\mathbf{H}_{12} + 2\mathbf{H}_{22}^H\mathbf{g}_2^{(c)}\mathbf{g}_2^{H(c)}\mathbf{H}_{22})\right]^{-1}(2\mathbf{H}_{12}^H\mathbf{g}_1^{(c)} + 2\mathbf{H}_{22}^H\mathbf{g}_2^{(c)} \\ &\quad - \mathbf{g}_1^{H(c)}\mathbf{H}_{11}\mathbf{v}_1^{(c)}\mathbf{H}_{12}^H\mathbf{g}_1^{(c)} - \mathbf{H}_{12}^H\mathbf{g}_1^{(c)}\mathbf{v}_1^{H(c)}\mathbf{H}_{11}^H\mathbf{g}_1^{(c)} - \mathbf{g}_2^{H(c)}\mathbf{H}_{21}\mathbf{v}_1^{(c)}\mathbf{H}_{22}^H\mathbf{g}_2^{(c)} - \mathbf{H}_{22}^H\mathbf{g}_2^{(c)}\mathbf{v}_1^{H(c)}\mathbf{H}_{21}^H\mathbf{g}_2^{(c)}) \end{aligned}$$

7. The Other System Model

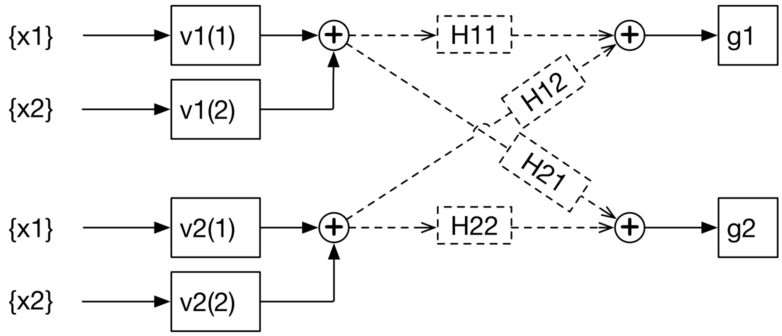


Figure 3: Forward Channel

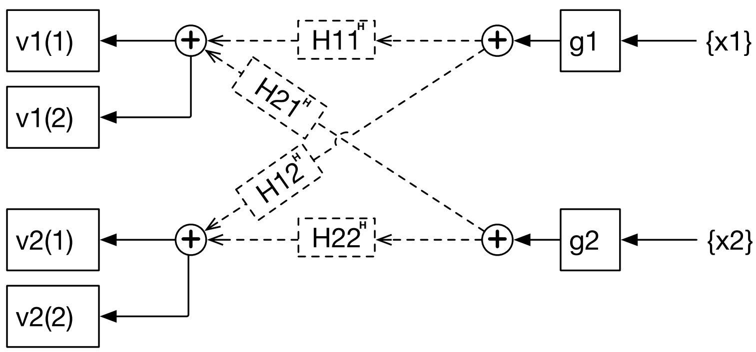


Figure 4: Backward Channel

8. Optimization Problem

$$\min_{\mathbf{v}_k^{(j)},\mathbf{g}_k} L = \sum_k \left[w_k MSE_k + \lambda_k \left(\sum_j \|\mathbf{v}_k^{(j)}\| - P\right) \right]$$

9. The received signal vector at k-th receiver

$$\mathbf{y}_k = \sum_i \left[\mathbf{H}_{ki} \sum_j (\mathbf{v}_i^{(j)} x_j) \right] + \mathbf{n}_k$$

10. SINR Derivation

$$s_k = \mathbf{g}_k^H \left(\sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(k)} x_k\right)$$

$$n_k = \mathbf{g}_k^H \Big[\sum_i \big(\mathbf{H}_{ki} \sum_{j \neq k} \mathbf{v}_i^{(j)} x_j \big) + \mathbf{n}_k \Big]$$

$$\frac{|s_k|^2}{|n_k|^2} = \frac{|\mathbf{g}_k^H \sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(k)}|^2}{\sum_{j \neq k} |\mathbf{g}_k^H \sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(j)}|^2 + |\mathbf{g}_k^H \mathbf{R}_k \mathbf{g}_k|}$$

11. Solutions

$$\begin{aligned}
MSE_k &= \mathrm{E}[(x_k - \mathbf{g}_k^H \mathbf{y}_k)(x_k - \mathbf{g}_k^H \mathbf{y}_k)^H] \\
&= \mathrm{E}[x_k^2] - \mathrm{E}[x_k \mathbf{y}_k^H \mathbf{g}_k] - \mathrm{E}[x_k \mathbf{g}_k^H \mathbf{y}_k] + \mathrm{E}[\mathbf{g}_k^H \mathbf{y}_k \mathbf{y}_k^H \mathbf{g}_k] \\
&= 1 - \sum_i \mathbf{v}_i^{H(k)} \mathbf{H}_{ki}^H \mathbf{g}_k - \mathbf{g}_k^H \sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(k)} + \mathbf{g}_k^H \sum_j \Big[(\sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(j)})(\sum_i \mathbf{v}_i^{H(j)} \mathbf{H}_{ki}^H) \Big] \mathbf{g}_k + \sigma^2 \mathbf{g}_k^H \mathbf{g}_k
\end{aligned}$$

$$\begin{aligned}
\mathbf{g}_k^* &= \underset{\mathbf{g}_k}{\mathrm{argmin}} (\sum_k \Big[MSE_k + \lambda_i (\sum_j \|\mathbf{v}_i^{(j)}\| - P) \Big]) \\
&= \Big[\sum_j \big[(\sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(j)})(\sum_i \mathbf{v}_i^{H(j)} \mathbf{H}_{ki}^H) \big] + \sigma^2 \mathbf{I} \Big]^{-1} (\sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(k)})
\end{aligned}$$

$$\begin{aligned}
\mathbf{v}_1^{*(1)} &= \underset{\mathbf{v}_1^{(1)}}{\mathrm{argmin}} (\sum_{k=1,2} \Big[MSE_k + \lambda_k (\sum_{j=1,2} \|\mathbf{v}_k^{(j)}\| - P) \Big]) \\
&= \Big[2\mathbf{H}_{11}^H \mathbf{g}_1 \mathbf{g}_1^H \mathbf{H}_{11} w_1 + 2\mathbf{H}_{21}^H \mathbf{g}_2 \mathbf{g}_2 \mathbf{H}_{21} w_2 + 2\lambda_1^* I \Big]^{-1} (2\mathbf{H}_{11}^H \mathbf{g}_1 w_1 - \mathbf{g}_1^H \mathbf{H}_{12} \mathbf{v}_2^{(1)} \mathbf{H}_{11}^H \mathbf{g}_1 w_1 \\
&\quad - \mathbf{H}_{11}^H \mathbf{g}_1 \mathbf{v}_2^{H(1)} \mathbf{H}_{12}^H \mathbf{g}_1 w_1 - \mathbf{g}_2^H \mathbf{H}_{22} \mathbf{v}_2^{(1)} \mathbf{H}_{21}^H \mathbf{g}_2 w_2 - \mathbf{H}_{21}^H \mathbf{g}_2 \mathbf{v}_2^{H(1)} \mathbf{H}_{22}^H \mathbf{g}_2 w_2) \\
&= \Big[2\mathrm{E}[x_1^* \mathbf{y}_1] \mathrm{E}[x_1^* \mathbf{y}_1]^H w_1 + 2\mathrm{E}[x_2^* \mathbf{y}_1] \mathrm{E}[x_2^* \mathbf{y}_1]^H w_2 + 2\lambda_1^* I \Big]^{-1} (2\mathrm{E}[x_1^* \mathbf{y}_1] w_1 - \mathrm{E}[x_1^* \mathbf{y}_2]^H \mathbf{v}_2^{(1)} \mathrm{E}[x_1^* \mathbf{y}_1] w_1 \\
&\quad - \mathrm{E}[x_1^* \mathbf{y}_1] \mathbf{v}_2^{H(1)} \mathrm{E}[x_1^* \mathbf{y}_2] w_1 - \mathrm{E}[x_2^* \mathbf{y}_2]^H \mathbf{v}_2^{(1)} \mathrm{E}[x_2^* \mathbf{y}_1] w_2 - \mathrm{E}[x_2^* \mathbf{y}_1] \mathbf{v}_2^{H(1)} \mathrm{E}[x_2^* \mathbf{y}_2] w_2)
\end{aligned}$$

$$
\begin{aligned}
\mathbf{v}_2^{*(1)} &= \underset{\mathbf{v}_2^{(1)}}{\operatorname{argmin}}\Big(\sum_{k=1,2}\Big[MSE_k + \lambda_k\big(\sum_{j=1,2}\|\mathbf{v}_k^{(j)}\| - P\big)\Big]\Big) \\
&= \Big[2\mathbf{H}_{12}^H\mathbf{g}_1\mathbf{g}_1^H\mathbf{H}_{12}w_1 + 2\mathbf{H}_{22}^H\mathbf{g}_2\mathbf{g}_2^H\mathbf{H}_{22}w_2 + 2\lambda_2^* I)\Big]^{-1}(2\mathbf{H}_{12}^H\mathbf{g}_1 w_1 - \mathbf{g}_1^H\mathbf{H}_{11}\mathbf{v}_1^{(1)}\mathbf{H}_{12}^H\mathbf{g}_1 w_1 \\
&\quad - \mathbf{H}_{12}^H\mathbf{g}_1\mathbf{v}_1^{H(1)}\mathbf{H}_{11}^H\mathbf{g}_1 w_1 - \mathbf{g}_2^H\mathbf{H}_{21}\mathbf{v}_1^{(1)}\mathbf{H}_{22}^H\mathbf{g}_2 w_2 - \mathbf{H}_{22}^H\mathbf{g}_2\mathbf{v}_1^{H(1)}\mathbf{H}_{21}^H\mathbf{g}_2 w_2) \\
&= \Big[2\mathrm{E}[x_1^*\mathbf{y}_2]\mathrm{E}[x_1^*\mathbf{y}_2]^H w_1 + 2\mathrm{E}[x_2^*\mathbf{y}_2]\mathrm{E}[x_2^*\mathbf{y}_2]^H w_2 + 2\lambda_2^* I\Big]^{-1}(2\mathrm{E}[x_1^*\mathbf{y}_2]w_1 - \mathrm{E}[x_1^*\mathbf{y}_1]^H\mathbf{v}_1^{(1)}\mathrm{E}[x_1^*\mathbf{y}_2]w_1 \\
&\quad - \mathrm{E}[x_1^*\mathbf{y}_2]\mathbf{v}_1^{H(1)}\mathrm{E}[x_1^*\mathbf{y}_1]w_1 - \mathrm{E}[x_2^*\mathbf{y}_1]^H\mathbf{v}_1^{(1)}\mathrm{E}[x_2^*\mathbf{y}_2]w_2 - \mathrm{E}[x_2^*\mathbf{y}_2]\mathbf{v}_1^{H(1)}\mathrm{E}[x_2^*\mathbf{y}_2]w_2)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{v}_1^{*(2)} &= \underset{\mathbf{v}_1^{(2)}}{\operatorname{argmin}}\Big(\sum_{k=1,2}\Big[MSE_k + \lambda_k\big(\sum_{j=1,2}\|\mathbf{v}_k^{(j)}\| - P\big)\Big]\Big) \\
&= \Big[2\mathbf{H}_{11}^H\mathbf{g}_1\mathbf{g}_1^H\mathbf{H}_{11}w_1 + 2\mathbf{H}_{21}^H\mathbf{g}_2\mathbf{g}_2\mathbf{H}_{21}w_2 + 2\lambda_1^* I\Big]^{-1}(2\mathbf{H}_{21}^H\mathbf{g}_2 w_2 - \mathbf{g}_1^H\mathbf{H}_{12}\mathbf{v}_2^{(2)}\mathbf{H}_{11}^H\mathbf{g}_1 w_1 \\
&\quad - \mathbf{H}_{11}^H\mathbf{g}_1\mathbf{v}_2^{H(2)}\mathbf{H}_{12}^H\mathbf{g}_1 w_1 - \mathbf{g}_2^H\mathbf{H}_{22}\mathbf{v}_2^{(2)}\mathbf{H}_{21}^H\mathbf{g}_2 w_2 - \mathbf{H}_{21}^H\mathbf{g}_2\mathbf{v}_2^{H(2)}\mathbf{H}_{22}^H\mathbf{g}_2 w_2) \\
&= \Big[2\mathrm{E}[x_1^*\mathbf{y}_1]\mathrm{E}[x_1^*\mathbf{y}_1]^H w_1 + 2\mathrm{E}[x_2^*\mathbf{y}_1]\mathrm{E}[x_2^*\mathbf{y}_1]^H w_2 + 2\lambda_1^* I\Big]^{-1}(2\mathrm{E}[x_2^*\mathbf{y}_1]w_2 - \mathrm{E}[x_1^*\mathbf{y}_2]^H\mathbf{v}_2^{(2)}\mathrm{E}[x_1^*\mathbf{y}_1]w_1 \\
&\quad - \mathrm{E}[x_1^*\mathbf{y}_1]\mathbf{v}_2^{H(2)}\mathrm{E}[x_1^*\mathbf{y}_2]w_1 - \mathrm{E}[x_2^*\mathbf{y}_2]^H\mathbf{v}_2^{(2)}\mathrm{E}[x_2^*\mathbf{y}_1]w_2 - \mathrm{E}[x_2^*\mathbf{y}_1]\mathbf{v}_2^{H(2)}\mathrm{E}[x_2^*\mathbf{y}_2]w_2)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{v}_2^{*(2)} &= \underset{\mathbf{v}_2^{(2)}}{\operatorname{argmin}}\Big(\sum_{k=1,2}\Big[MSE_k + \lambda_k\big(\sum_{j=1,2}\|\mathbf{v}_k^{(j)}\| - P\big)\Big]\Big) \\
&= \Big[2\mathbf{H}_{12}^H\mathbf{g}_1\mathbf{g}_1^H\mathbf{H}_{12}w_1 + 2\mathbf{H}_{22}^H\mathbf{g}_2\mathbf{g}_2^H\mathbf{H}_{22}w_2 + 2\lambda_2^* I)\Big]^{-1}(2\mathbf{H}_{22}^H\mathbf{g}_2 w_2 - \mathbf{g}_1^H\mathbf{H}_{11}\mathbf{v}_1^{(2)}\mathbf{H}_{12}^H\mathbf{g}_1 w_1 \\
&\quad - \mathbf{H}_{12}^H\mathbf{g}_1\mathbf{v}_1^{H(2)}\mathbf{H}_{11}^H\mathbf{g}_1 w_1 - \mathbf{g}_2^H\mathbf{H}_{21}\mathbf{v}_1^{(2)}\mathbf{H}_{22}^H\mathbf{g}_2 w_2 - \mathbf{H}_{22}^H\mathbf{g}_2\mathbf{v}_1^{H(2)}\mathbf{H}_{21}^H\mathbf{g}_2 w_2) \\
&= \Big[2\mathrm{E}[x_1^*\mathbf{y}_2]\mathrm{E}[x_1^*\mathbf{y}_2]^H w_1 + 2\mathrm{E}[x_2^*\mathbf{y}_2]\mathrm{E}[x_2^*\mathbf{y}_2]^H w_2 + 2\lambda_2^* I\Big]^{-1}(2\mathrm{E}[x_2^*\mathbf{y}_2]w_2 - \mathrm{E}[x_1^*\mathbf{y}_1]^H\mathbf{v}_1^{(2)}\mathrm{E}[x_1^*\mathbf{y}_2]w_1 \\
&\quad - \mathrm{E}[x_1^*\mathbf{y}_2]^H\mathbf{v}_1^{H(2)}\mathrm{E}[x_1^*\mathbf{y}_1]w_1 - \mathrm{E}[x_2^*\mathbf{y}_1]^H\mathbf{v}_1^{(2)}\mathrm{E}[x_2^*\mathbf{y}_2]w_2 - \mathrm{E}[x_2^*\mathbf{y}_2]^H\mathbf{v}_1^{H(2)}\mathrm{E}[x_2^*\mathbf{y}_1]w_2)
\end{aligned}
$$

To be continued...

9. Numerical Simulation(2 Users, 2X2 MIMO Channel)

$$Rayleigh\ Fading\ Channel$$

$$Cross\ Channel\ Gain = 0.8 * Direct\ Channel\ Gain$$

$$SNR = \frac{1}{\sigma^2} = 10^3 = 30dB$$

$$\begin{aligned} Observation1 : & If\ the\ training\ length\ is\ long\ enough,\ each\ LMS\ filter\ (\mathbf{v}_k^{(c)}, \mathbf{v}_k^{(p)}, \mathbf{g}_k^{(c)}, \mathbf{g}_k^{(p)},) \\ & will\ converge\ to\ Wiener\ filter \end{aligned}$$

$$Observation2 :\ Use\ Wiener\ filters,\ and\ only\ send\ common\ messages.\ Sum\ rate\ C\ =\ 11.6\ bit/channel$$

$$Observation3 :\ Use\ Wiener\ filters,\ and\ only\ send\ private\ messages.\ Sum\ rate\ C\ =\ 2.63\ bit/channel$$

$$Observation4 :\ Use\ Wiener\ filters,\ and\ send\ both\ messages.\ Sum\ rate\ C\ =\ 3.35\ bit/channel$$

$$Observation5 :\ Under\ the\ cooperation\ scheme,\ transmitters\ don't\ converge\ to\ Wiener\ filters$$

$$Observation5 :\ Under\ the\ cooperation\ scheme,\ C = 3.15\ bit/channel$$

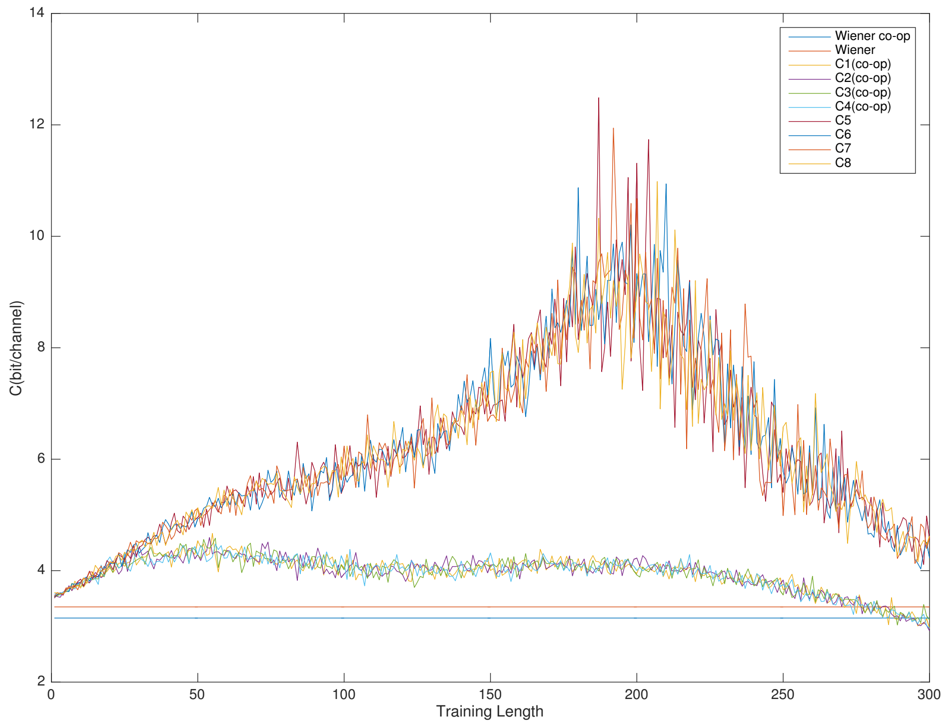


Figure 5: Insert caption

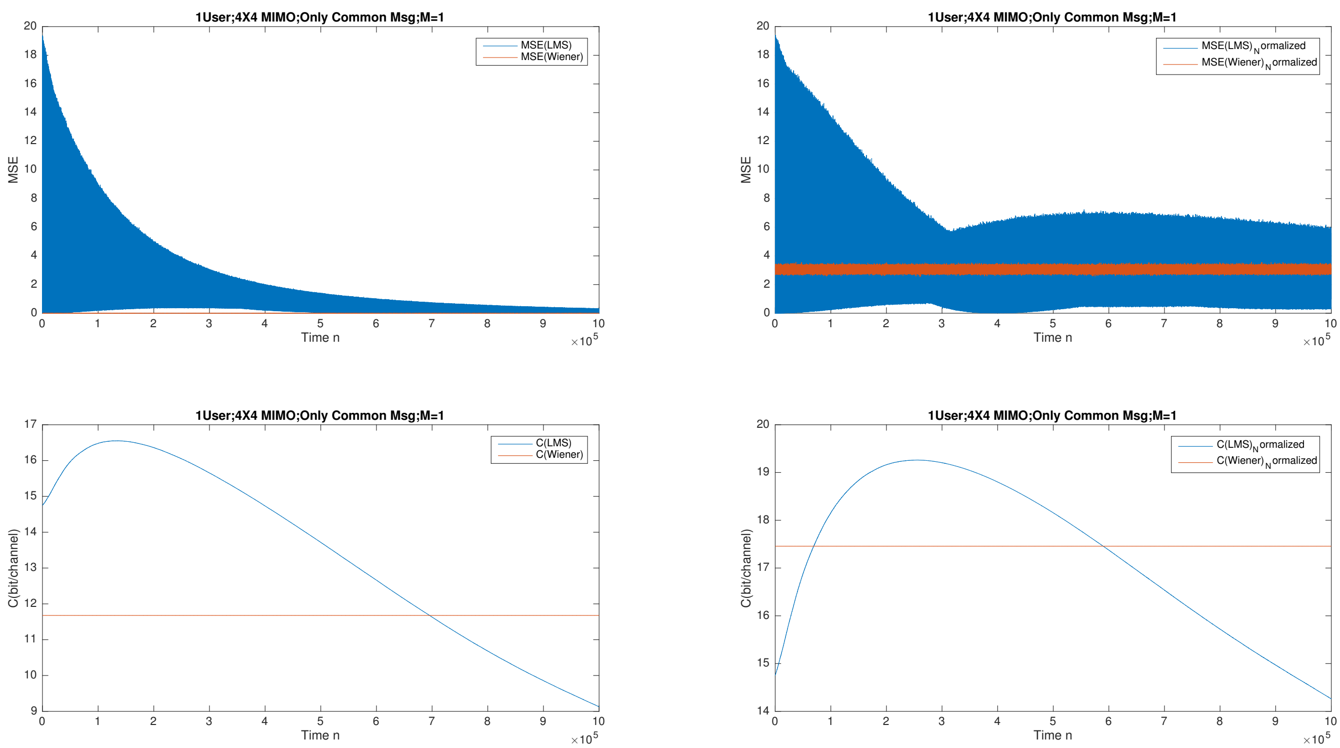


Figure 6: Insert caption

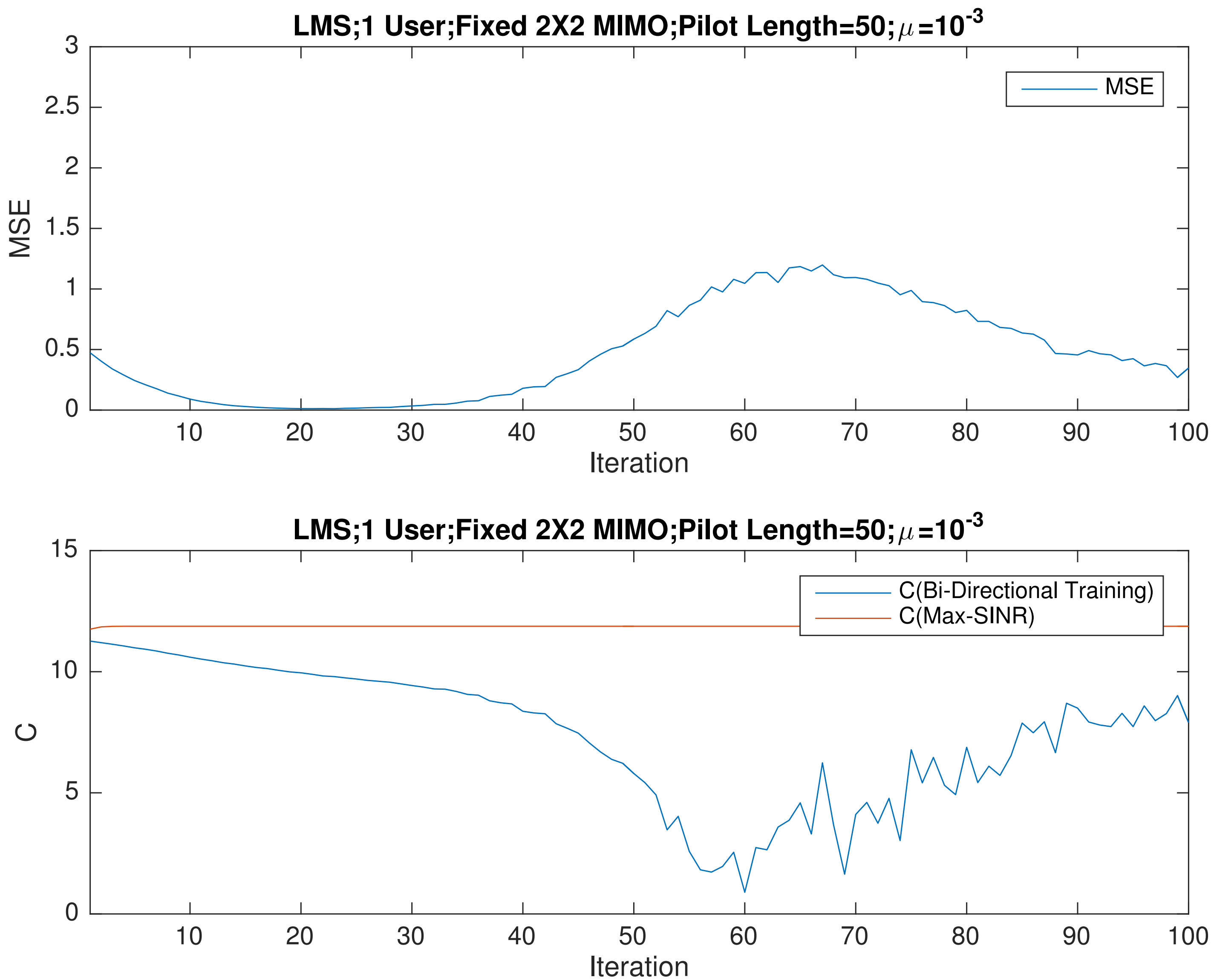


Figure 7: Insert caption

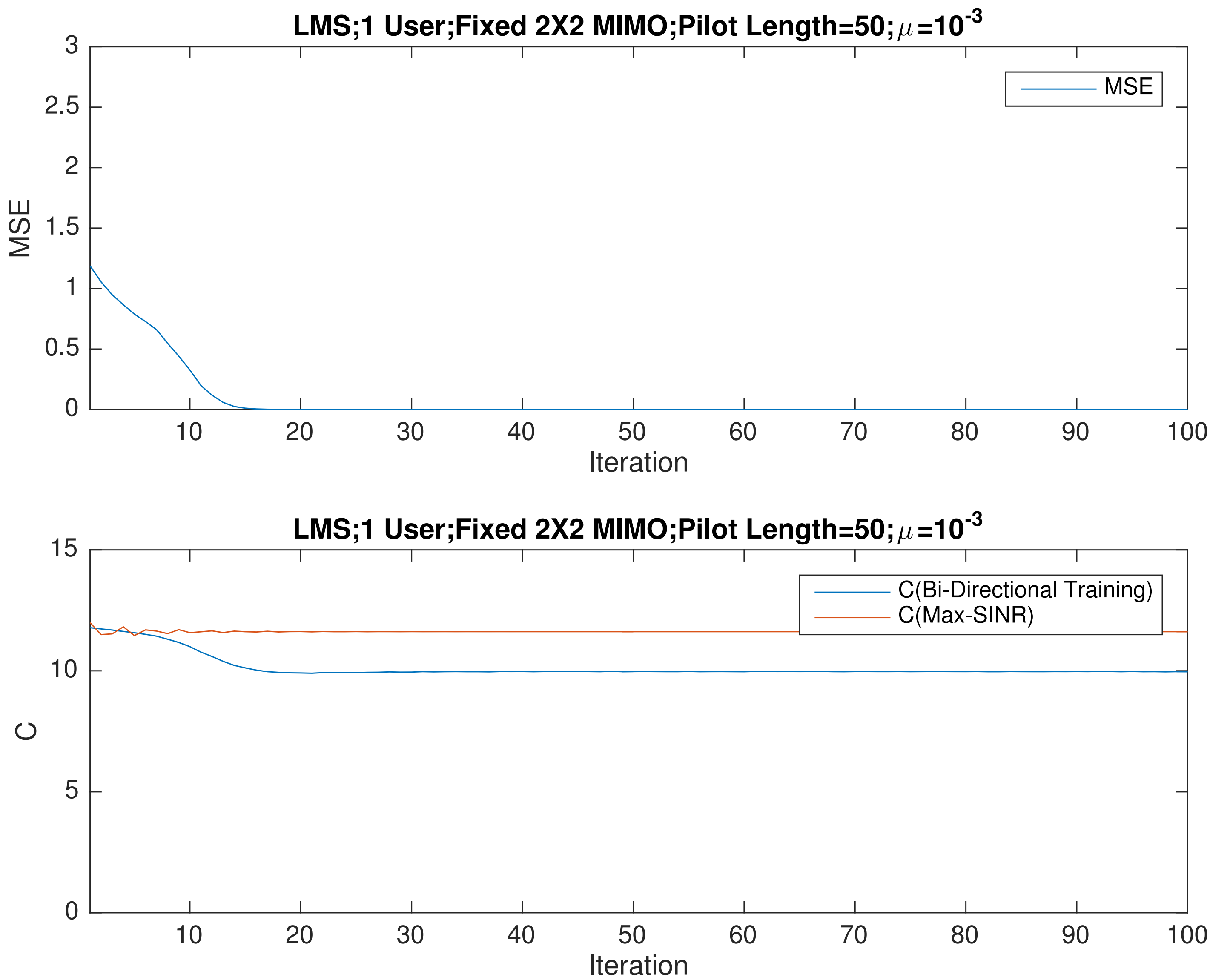


Figure 8: Insert caption

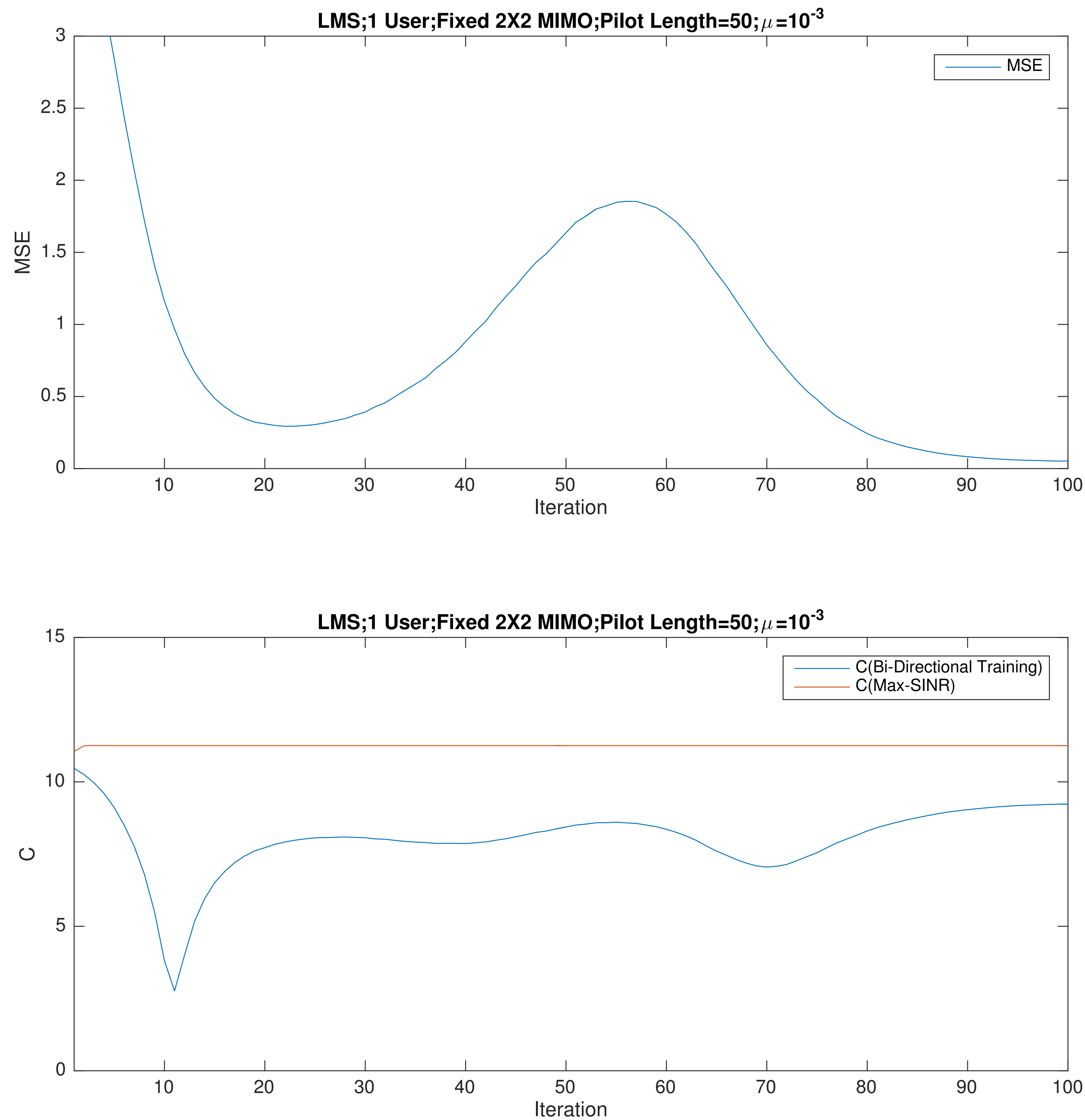
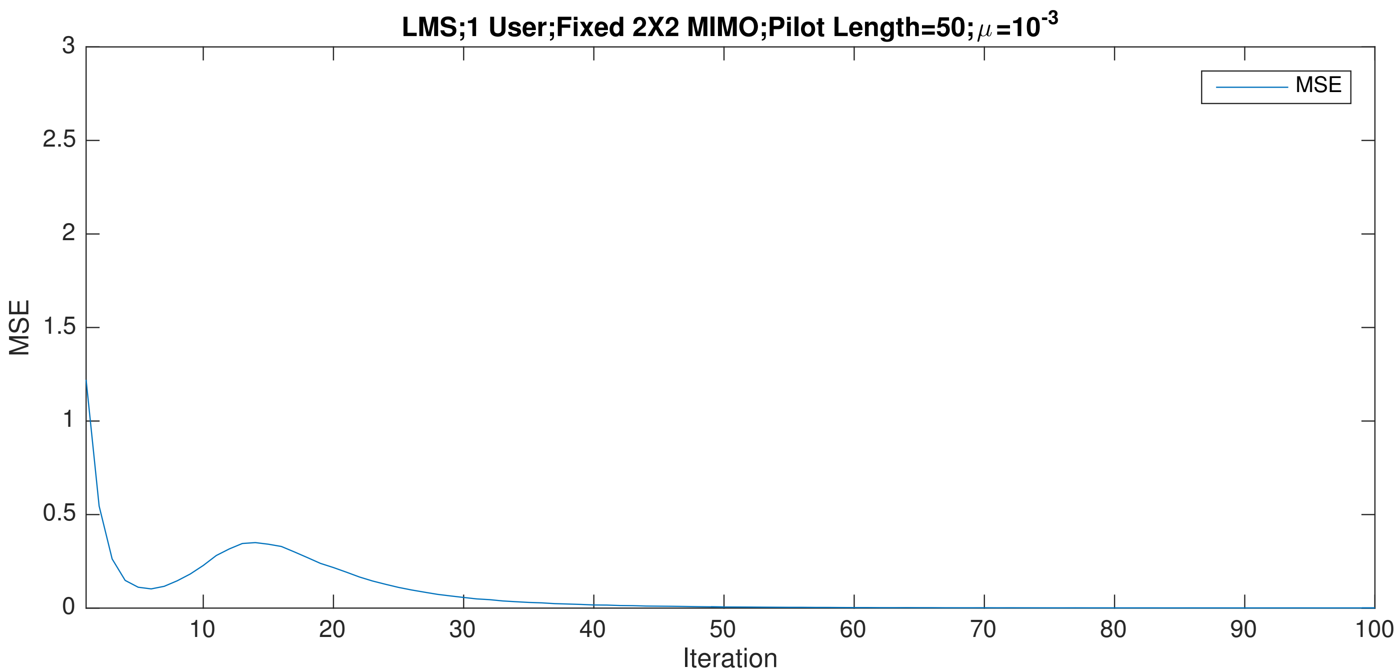


Figure 9: Insert caption



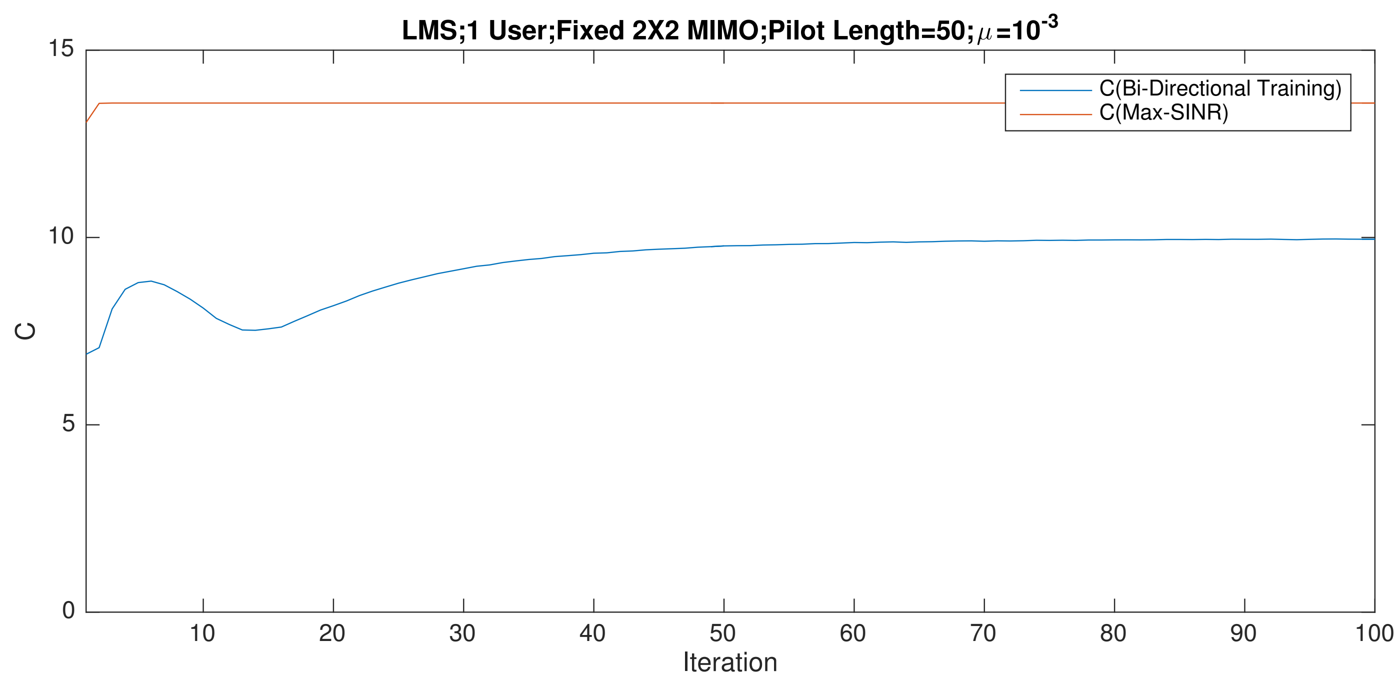


Figure 10: Insert caption

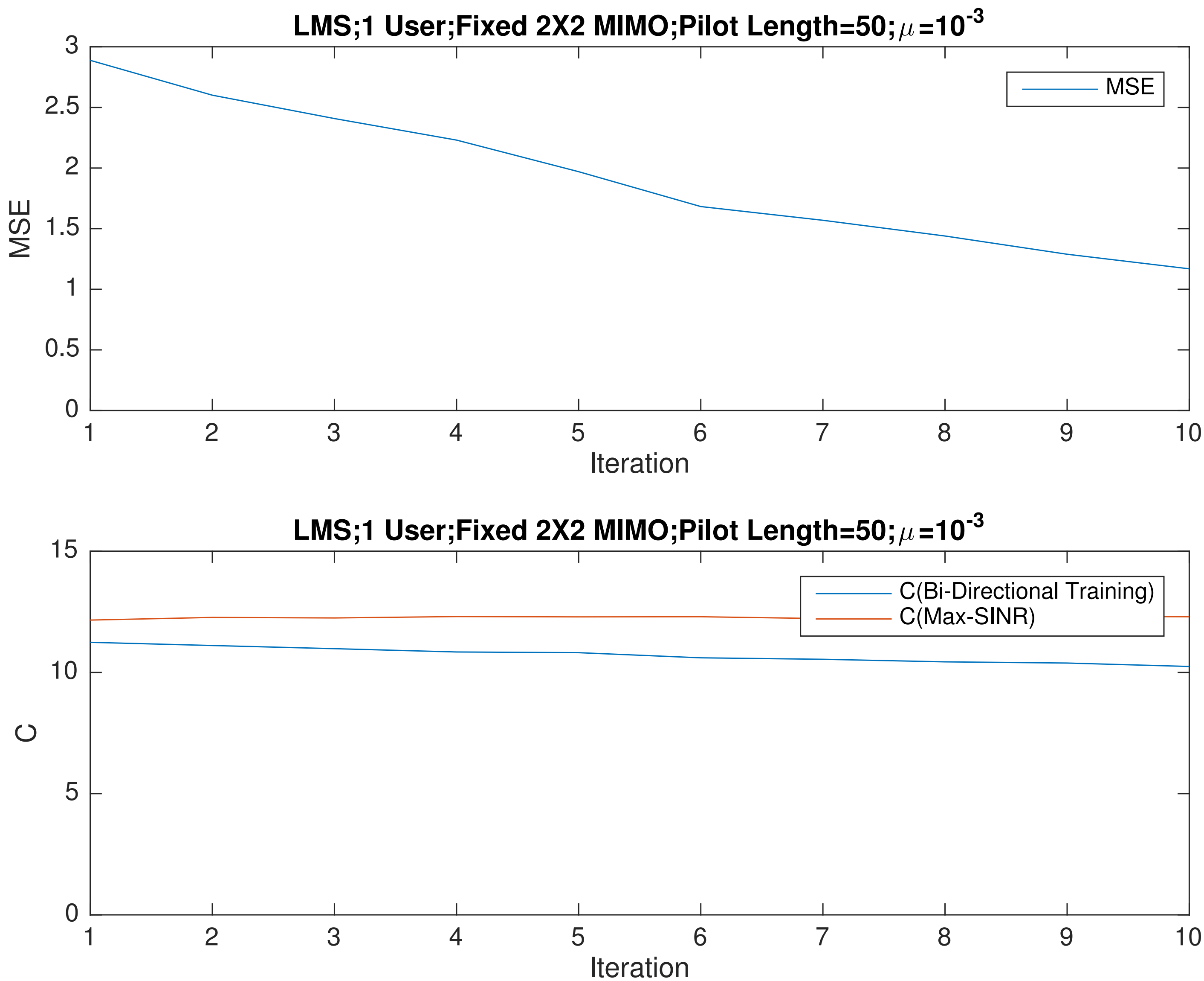


Figure 11: Insert caption

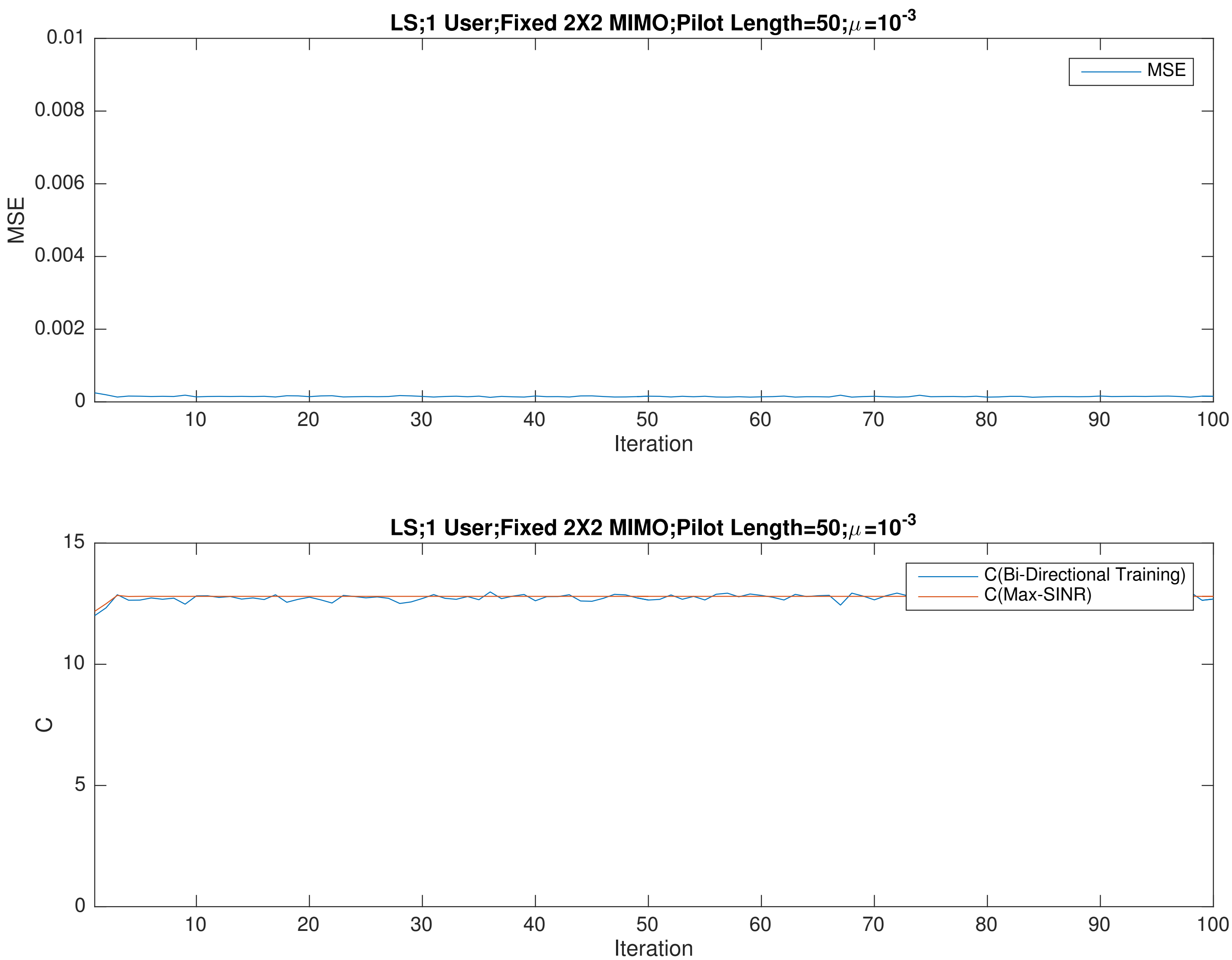


Figure 12: Insert caption

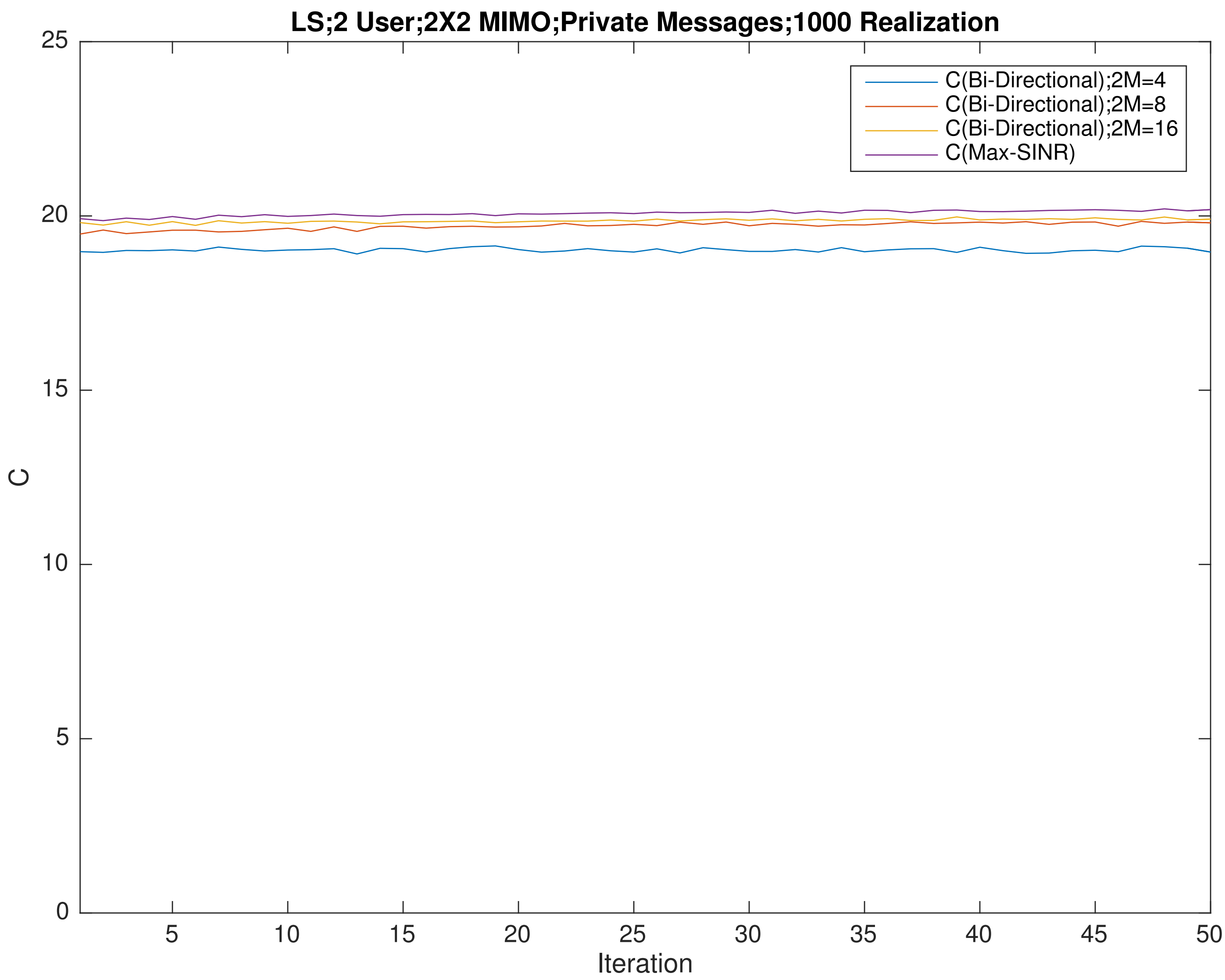


Figure 13: Insert caption

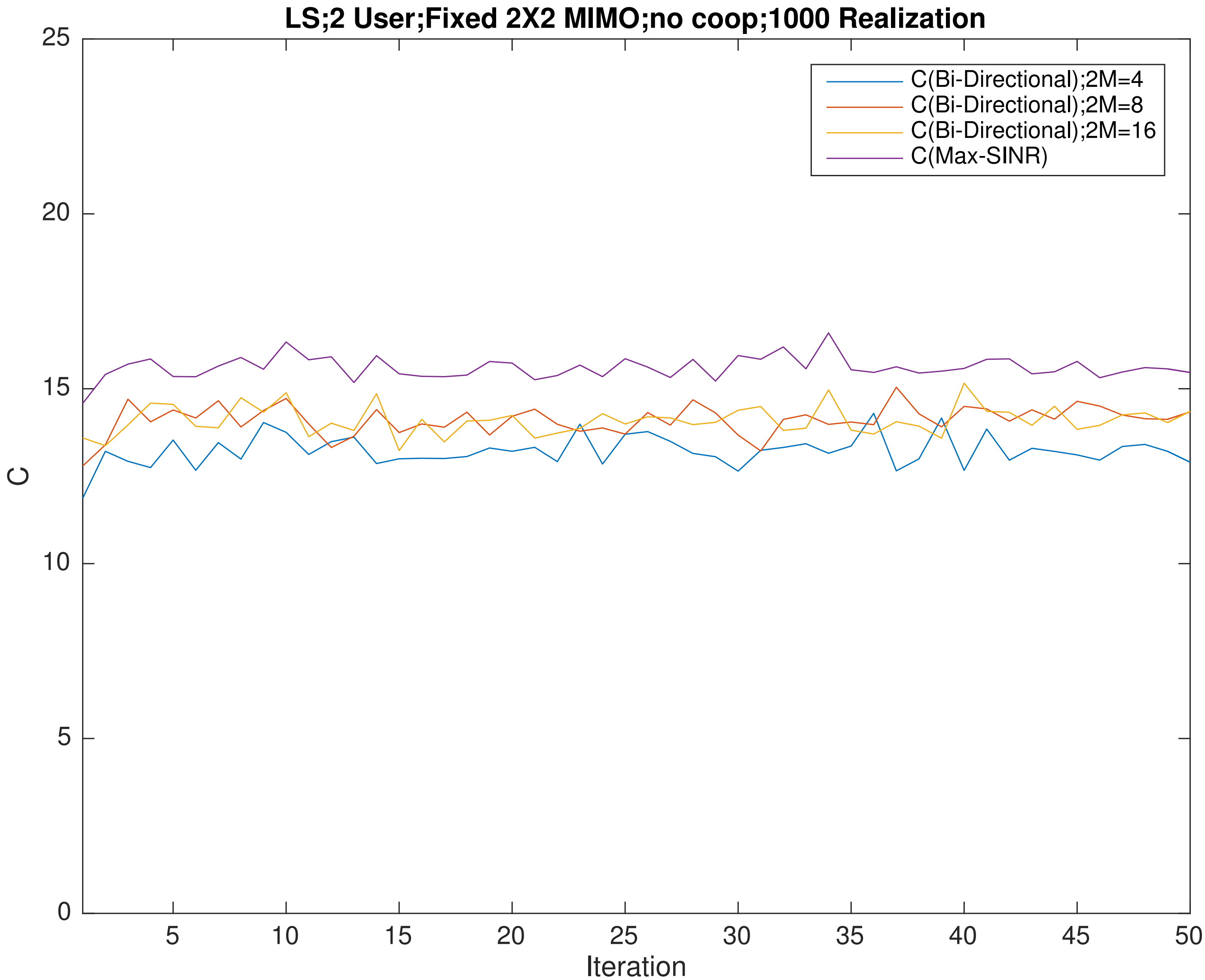


Figure 14: Insert caption

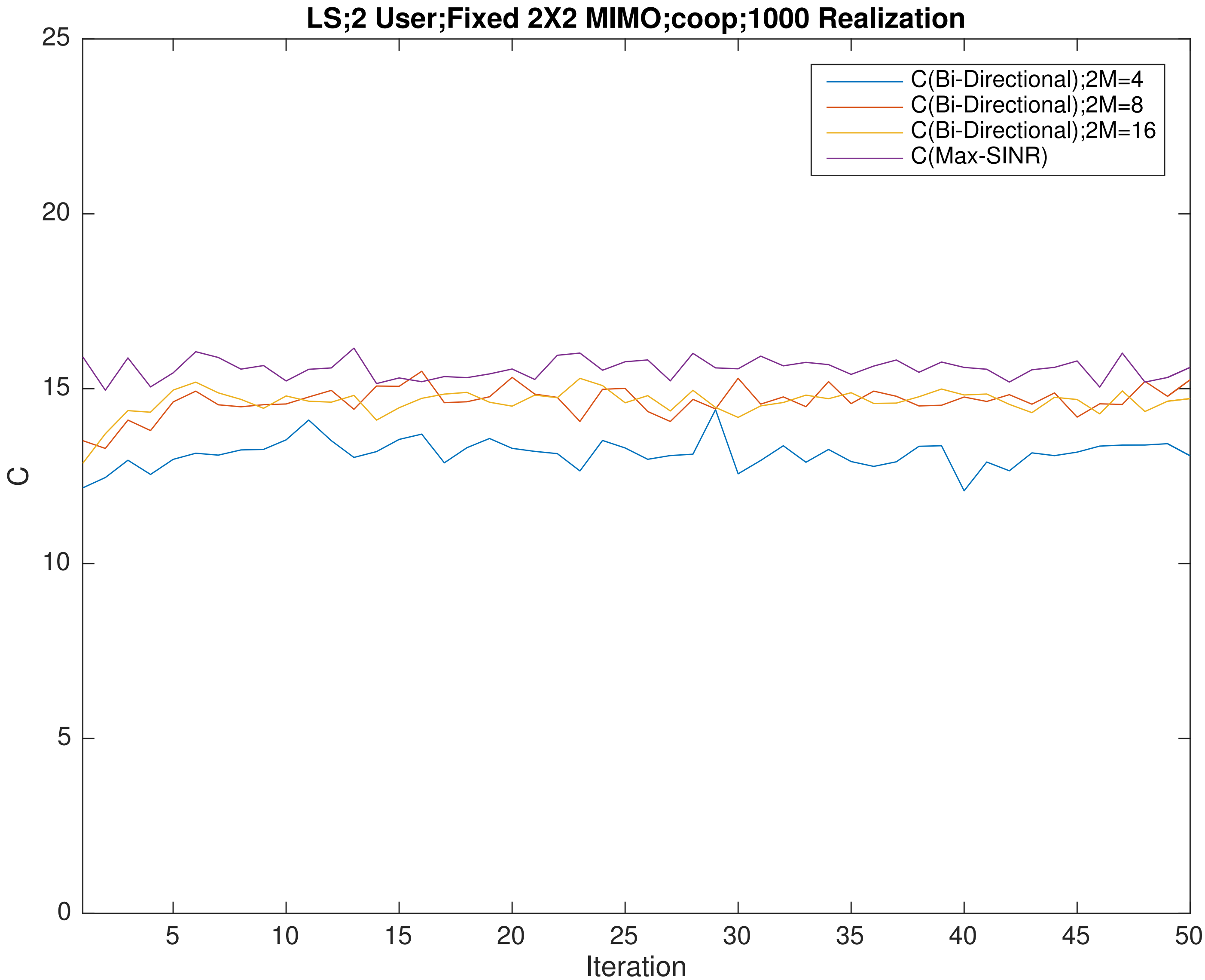


Figure 15: Insert caption