Bi-Directional Training in Interference Network

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1. Optimization Problem

$$\min_{v_i, g_i} \sum_i MSE_i^{(c)} + MSE_i^{(c)}$$

2. The received signal vector at k-th receiver

$$\mathbf{y}_k = \mathbf{H}_{kk}(\mathbf{v}_k^{(c)} x + \mathbf{v}_k^{(p)} x_k^{(p)}) + \sum_{j \neq k} \mathbf{H}_{kj}(\mathbf{v}_j^{(c)} x + \mathbf{v}_j^{(p)} x_j^{(p)}) + \mathbf{n}_k$$

3. SINR Derivation

$$s_k^{(c)} = \mathbf{g}_k^{H(c)}(\sum_i \mathbf{H}_{ki}\mathbf{v}_i^{(c)} x)$$

$$s_k^{(p)} = \mathbf{g}_k^{H(p)}(\mathbf{H}_{kk}\mathbf{v}_k^{(p)} x_k^{(p)})$$

$$n_k^{(c)} = \mathbf{g}_k^{H(c)}(\sum_i \mathbf{H}_{ki}\mathbf{v}_i^{(p)} x_i^{(p)} + \mathbf{n}_k)$$

$$n_k^{(p)} = \mathbf{g}_k^{H(p)}(\sum_i \mathbf{H}_{ki}\mathbf{v}_i^{(c)} x + \sum_{j \neq k} \mathbf{H}_{kj}\mathbf{v}_j^{(p)} x_j^{(p)} + \mathbf{n}_k)$$

$$\frac{|s_k^{(c)}|^2}{|n_k^{(c)}|^2} = \frac{|\mathbf{g}_k^{H(c)} \sum_i \mathbf{H}_{ki}\mathbf{v}_i^{(c)}|^2}{|\mathbf{g}_k^{H(c)} \sum_i \mathbf{H}_{ki}\mathbf{v}_i^{(p)}|^2 + |\mathbf{g}_k^{H(c)}\mathbf{R}_k\mathbf{g}_k^{(c)}|}$$

$$\frac{|s_k^{(p)}|^2}{|n_k^{(p)}|^2} = \frac{|\mathbf{g}_k^{H(p)}\mathbf{H}_{kk}\mathbf{v}_k^{(p)}|^2}{|\mathbf{g}_k^{H(p)}\sum_i \mathbf{H}_{ki}\mathbf{v}_i^{(c)}|^2 + |\mathbf{g}_k^{H(p)}\sum_{j\neq k}\mathbf{H}_{kj}\mathbf{v}_j^{(p)}|^2 + |\mathbf{g}_k^{H(p)}\mathbf{R}_k\mathbf{g}_k^{(p)}|}$$

4. Bi-Directional Training - Wiener Filter

$$Forward\ Training(fix\ \mathbf{v}_k^{(c)}, \mathbf{v}_k^{(p)}, \forall k)$$

$$\begin{aligned}
\mathbf{g}_k^{H(c)} =& \sum_i \mathbf{v}_i^{H(c)}\mathbf{H}_{ki}^H[\mathbf{H}_{kk}\mathbf{v}_k^{(c)}\mathbf{v}_k^{H(c)}\mathbf{H}_{kk}^H + \mathbf{H}_{kk}\mathbf{v}_k^{(p)}\mathbf{v}_k^{H(p)}\mathbf{H}_{kk}^H \\
&+ (\sum_{j\neq k}\mathbf{H}_{kj}\mathbf{v}_j^{(c)})(\sum_{j\neq k}\mathbf{v}_j^{H(c)}\mathbf{H}_{kj}^H) + (\sum_{j\neq k}\mathbf{H}_{kj}\mathbf{v}_j^{(p)})(\sum_{j\neq k}\mathbf{v}_j^{H(p)}\mathbf{H}_{kj}^H) \\
&+ \mathbf{H}_{kk}\mathbf{v}_k^{(c)}(\sum_{j\neq k}\mathbf{v}_j^{H(c)}\mathbf{H}_{kj}^H) + (\sum_{j\neq k}\mathbf{H}_{kj}\mathbf{v}_j^{(c)})\mathbf{v}_k^{H(c)}\mathbf{H}_{kk}^H + \sigma^2\mathbf{I}]^{-1}
\end{aligned}$$

$$\begin{aligned}
\mathbf{g}_k^{H(p)} =& \mathbf{v}_k^{H(p)}\mathbf{H}_{kk}^H[\mathbf{H}_{kk}\mathbf{v}_k^{(c)}\mathbf{v}_k^{H(c)}\mathbf{H}_{kk}^H + \mathbf{H}_{kk}\mathbf{v}_k^{(p)}\mathbf{v}_k^{H(p)}\mathbf{H}_{kk}^H \\
&+ (\sum_{j\neq k}\mathbf{H}_{kj}\mathbf{v}_j^{(c)})(\sum_{j\neq k}\mathbf{v}_j^{H(c)}\mathbf{H}_{kj}^H) + (\sum_{j\neq k}\mathbf{H}_{kj}\mathbf{v}_j^{(p)})(\sum_{j\neq k}\mathbf{v}_j^{H(p)}\mathbf{H}_{kj}^H) \\
&+ \mathbf{H}_{kk}\mathbf{v}_k^{(c)}(\sum_{j\neq k}\mathbf{v}_j^{H(c)}\mathbf{H}_{kj}^H) + (\sum_{j\neq k}\mathbf{H}_{kj}\mathbf{v}_j^{(c)})\mathbf{v}_k^{H(c)}\mathbf{H}_{kk}^H + \sigma^2\mathbf{I}]^{-1}
\end{aligned}$$

$$Backward\ Training(fix\ \mathbf{g}_k^{(c)}, \mathbf{g}_k^{(p)}, \forall k)$$

$$\mathbf{Z}_{ab} = \mathbf{H}_{ba}^T$$

$$\begin{aligned}
\mathbf{v}_k^{H(c)} =& \sum_i \mathbf{g}_i^{H(c)}\mathbf{Z}_{ki}^H[\mathbf{Z}_{kk}\mathbf{g}_k^{(c)}\mathbf{g}_k^{H(c)}\mathbf{Z}_{kk}^H + \mathbf{Z}_{kk}\mathbf{g}_k^{(p)}\mathbf{g}_k^{H(p)}\mathbf{Z}_{kk}^H \\
&+ (\sum_{j\neq k}\mathbf{Z}_{kj}\mathbf{g}_j^{(c)})(\sum_{j\neq k}\mathbf{g}_j^{H(c)}\mathbf{Z}_{kj}^H) + (\sum_{j\neq k}\mathbf{Z}_{kj}\mathbf{g}_j^{(p)})(\sum_{j\neq k}\mathbf{g}_j^{H(p)}\mathbf{Z}_{kj}^H) \\
&+ \mathbf{Z}_{kk}\mathbf{g}_k^{(c)}(\sum_{j\neq k}\mathbf{g}_j^{H(c)}\mathbf{Z}_{kj}^H) + (\sum_{j\neq k}\mathbf{Z}_{kj}\mathbf{g}_j^{(c)})\mathbf{g}_k^{H(c)}\mathbf{Z}_{kk}^H + \sigma^2\mathbf{I}]^{-1}
\end{aligned}$$

$$
\begin{aligned}
\mathbf{v}_k^{H(p)} =& \mathbf{v}_k^{H(p)}\mathbf{Z}_{kk}^H[\mathbf{Z}_{kk}\mathbf{g}_k^{(c)}\mathbf{g}_k^{H(c)}\mathbf{Z}_{kk}^H + \mathbf{Z}_{kk}\mathbf{g}_k^{(p)}\mathbf{g}_k^{H(p)}\mathbf{Z}_{kk}^H \\
&+ (\sum_{j\neq k}\mathbf{Z}_{kj}\mathbf{g}_j^{(c)})(\sum_{j\neq k}\mathbf{g}_j^{H(c)}\mathbf{Z}_{kj}^H) + (\sum_{j\neq k}\mathbf{Z}_{kj}\mathbf{g}_j^{(p)})(\sum_{j\neq k}\mathbf{g}_j^{H(p)}\mathbf{Z}_{kj}^H) \\
&+ \mathbf{Z}_{kk}\mathbf{g}_k^{(c)}(\sum_{j\neq k}\mathbf{g}_j^{H(c)}\mathbf{Z}_{kj}^H) + (\sum_{j\neq k}\mathbf{Z}_{kj}\mathbf{g}_j^{(c)})\mathbf{g}_k^{H(c)}\mathbf{Z}_{kk}^H + \sigma^2\mathbf{I}]^{-1}
\end{aligned}
$$

5. Bi-Directional Training - Least Mean Square Algorithm

$$Forward\ Training(fix\ \mathbf{v}_k^{(c)}, \mathbf{v}_k^{(p)}, \forall k)$$

$$\mathbf{g}_k^{(c)}(n+1) = \mathbf{g}_k^{(c)}(n) + \mu\mathbf{y}_k(n)[x(n) - \mathbf{g}_k^{H(c)}(n)\mathbf{y}_k(n)]^*$$

$$\mathbf{g}_k^{(p)}(n+1) = \mathbf{g}_k^{(p)}(n) + \mu\mathbf{y}_k(n)[x_k^{(p)}(n) - \mathbf{g}_k^{H(p)}(n)\mathbf{y}_k(n)]^*$$

$$Backward\ Training(fix\ \mathbf{g}_k^{(c)}, \mathbf{g}_k^{(p)}, \forall k)(without\ cooperation)$$

$$\mathbf{v}_k^{(c)}(n+1) = \mathbf{v}_k^{(c)}(n) + \mu\mathbf{y}_k(n)[x(n) - \mathbf{v}_k^{H(c)}(n)\mathbf{y}_k(n)]^*$$

$$\mathbf{v}_k^{(p)}(n+1) = \mathbf{v}_k^{(p)}(n) + \mu\mathbf{y}_k(n)[x_k^{(p)}(n) - \mathbf{v}_k^{H(p)}(n)\mathbf{y}_k(n)]^*$$

$$Backward\ Training(fix\ \mathbf{g}_k^{(c)}, \mathbf{g}_k^{(p)}, \forall k)(with\ cooperation)$$

$$\mathbf{v}_k^{(c)}(n+1) = \mathbf{v}_k^{(c)}(n) + \mu\mathbf{y}_k(n)[x(n) - \frac{\sum_i \mathbf{v}_i^{H(c)}(n)\mathbf{y}_i(n)}{Cardinality\ of\ i}]^*$$

$$\mathbf{v}_k^{(p)}(n+1) = \mathbf{v}_k^{(p)}(n) + \mu\mathbf{y}_k(n)[x_k^{(p)}(n) - \mathbf{v}_k^{H(p)}(n)\mathbf{y}_k(n)]^*$$

6. Numerical Simulation(2 Users, 2X2 MIMO Channel)

$$Rayleigh\ Fading\ Channel$$

$$Cross\ Channel\ Gain = 0.8 * Direct\ Channel\ Gain$$

$$SNR = \frac{1}{\sigma^2} = 10^3 = 30dB$$

$$\begin{aligned} Observation1:& If\ the\ training\ length\ is\ long\ enough,\ each\ LMS\ filter\ (\mathbf{v}_k^{(c)}, \mathbf{v}_k^{(p)}, \mathbf{g}_k^{(c)}, \mathbf{g}_k^{(p)},)\\ & will\ converge\ to\ Wiener\ filter \end{aligned}$$

$$Observation2:\ Use\ Wiener\ filters,\ and\ only\ send\ common\ messages.\ Sum\ rate\ C\ =\ 11.6\ bit/channel$$

$$Observation3:\ Use\ Wiener\ filters,\ and\ only\ send\ private\ messages.\ Sum\ rate\ C\ =\ 2.63\ bit/channel$$

$$Observation4:\ Use\ Wiener\ filters,\ and\ send\ both\ messages.\ Sum\ rate\ C\ =\ 3.35\ bit/channel$$

$$Observation5:\ Under\ the\ cooperation\ scheme,\ transmitters\ don't\ converge\ to\ Wiener\ filters$$

$$Observation5:\ Under\ the\ cooperation\ scheme,\ C = 3.15\ bit/channel$$

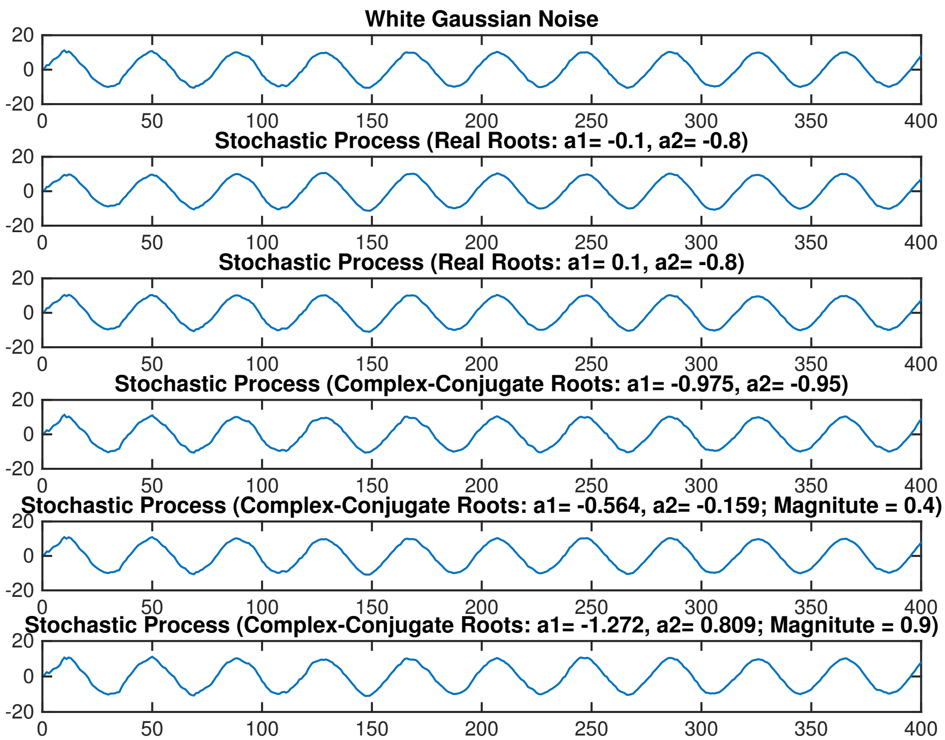

Figure 1: Insert caption