

# Bi-Directional Training in Interference Network

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## 1. Optimization Problem

$$\min_{v_i, g_i} \sum_i MSE_i^{(c)} + MSE_i^{(p)}$$

## 2. The received signal vector at k-th receiver

$$\mathbf{y}_k = \mathbf{H}_{kk}(\mathbf{v}_k^{(c)} x + \mathbf{v}_k^{(p)} x_k^{(p)}) + \sum_{j \neq k} \mathbf{H}_{kj}(\mathbf{v}_j^{(c)} x + \mathbf{v}_j^{(p)} x_j^{(p)}) + \mathbf{n}_k$$

## 3. SINR Derivation

$$s_k^{(c)} = \mathbf{g}_k^{H(c)} \left( \sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(c)} x \right)$$

$$s_k^{(p)} = \mathbf{g}_k^{H(p)} (\mathbf{H}_{kk} \mathbf{v}_k^{(p)} x_k^{(p)})$$

$$n_k^{(c)} = \mathbf{g}_k^{H(c)} \left( \sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(p)} x_i^{(p)} + \mathbf{n}_k \right)$$

$$n_k^{(p)} = \mathbf{g}_k^{H(p)} \left( \sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(c)} x + \sum_{j \neq k} \mathbf{H}_{kj} \mathbf{v}_j^{(p)} x_j^{(p)} + \mathbf{n}_k \right)$$

$$\frac{|s_k^{(c)}|^2}{|n_k^{(c)}|^2} = \frac{|\mathbf{g}_k^{H(c)} \sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(c)}|^2}{|\mathbf{g}_k^{H(c)} \sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(p)}|^2 + |\mathbf{g}_k^{H(c)} \mathbf{R}_k \mathbf{g}_k^{(c)}|}$$

$$\frac{|s_k^{(p)}|^2}{|n_k^{(p)}|^2} = \frac{|\mathbf{g}_k^{H(p)} \mathbf{H}_{kk} \mathbf{v}_k^{(p)}|^2}{|\mathbf{g}_k^{H(p)} \sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(c)}|^2 + |\mathbf{g}_k^{H(p)} \sum_{j \neq k} \mathbf{H}_{kj} \mathbf{v}_j^{(p)}|^2 + |\mathbf{g}_k^{H(p)} \mathbf{R}_k \mathbf{g}_k^{(p)}|}$$

#### 4. Bi-Directional Training - Wiener Filter

*Forward Training*(fix  $\mathbf{v}_k^{(c)}, \mathbf{v}_k^{(p)}, \forall k$ )

$$\begin{aligned} \mathbf{g}_k^{H(c)} &= \sum_i \mathbf{v}_i^{H(c)} \mathbf{H}_{ki}^H [\mathbf{H}_{kk} \mathbf{v}_k^{(c)} \mathbf{v}_k^{H(c)} \mathbf{H}_{kk}^H + \mathbf{H}_{kk} \mathbf{v}_k^{(p)} \mathbf{v}_k^{H(p)} \mathbf{H}_{kk}^H \\ &\quad + (\sum_{j \neq k} \mathbf{H}_{kj} \mathbf{v}_j^{(c)}) (\sum_{j \neq k} \mathbf{v}_j^{H(c)} \mathbf{H}_{kj}^H) + (\sum_{j \neq k} \mathbf{H}_{kj} \mathbf{v}_j^{(p)}) (\sum_{j \neq k} \mathbf{v}_j^{H(p)} \mathbf{H}_{kj}^H) \\ &\quad + \mathbf{H}_{kk} \mathbf{v}_k^{(c)} (\sum_{j \neq k} \mathbf{v}_j^{H(c)} \mathbf{H}_{kj}^H) + (\sum_{j \neq k} \mathbf{H}_{kj} \mathbf{v}_j^{(c)}) \mathbf{v}_k^{H(c)} \mathbf{H}_{kk}^H + \sigma^2 \mathbf{I}]^{-1} \end{aligned}$$

$$\begin{aligned} \mathbf{g}_k^{H(p)} &= \mathbf{v}_k^{H(p)} \mathbf{H}_{kk}^H [\mathbf{H}_{kk} \mathbf{v}_k^{(c)} \mathbf{v}_k^{H(c)} \mathbf{H}_{kk}^H + \mathbf{H}_{kk} \mathbf{v}_k^{(p)} \mathbf{v}_k^{H(p)} \mathbf{H}_{kk}^H \\ &\quad + (\sum_{j \neq k} \mathbf{H}_{kj} \mathbf{v}_j^{(c)}) (\sum_{j \neq k} \mathbf{v}_j^{H(c)} \mathbf{H}_{kj}^H) + (\sum_{j \neq k} \mathbf{H}_{kj} \mathbf{v}_j^{(p)}) (\sum_{j \neq k} \mathbf{v}_j^{H(p)} \mathbf{H}_{kj}^H) \\ &\quad + \mathbf{H}_{kk} \mathbf{v}_k^{(c)} (\sum_{j \neq k} \mathbf{v}_j^{H(c)} \mathbf{H}_{kj}^H) + (\sum_{j \neq k} \mathbf{H}_{kj} \mathbf{v}_j^{(c)}) \mathbf{v}_k^{H(c)} \mathbf{H}_{kk}^H + \sigma^2 \mathbf{I}]^{-1} \end{aligned}$$

*Backward Training*(fix  $\mathbf{g}_k^{(c)}, \mathbf{g}_k^{(p)}, \forall k$ )

$$\mathbf{Z}_{ab} = \mathbf{H}_{ba}^T$$

$$\begin{aligned} \mathbf{v}_k^{H(c)} &= \sum_i \mathbf{g}_i^{H(c)} \mathbf{Z}_{ki}^H [\mathbf{Z}_{kk} \mathbf{g}_k^{(c)} \mathbf{g}_k^{H(c)} \mathbf{Z}_{kk}^H + \mathbf{Z}_{kk} \mathbf{g}_k^{(p)} \mathbf{g}_k^{H(p)} \mathbf{Z}_{kk}^H \\ &\quad + (\sum_{j \neq k} \mathbf{Z}_{kj} \mathbf{g}_j^{(c)}) (\sum_{j \neq k} \mathbf{g}_j^{H(c)} \mathbf{Z}_{kj}^H) + (\sum_{j \neq k} \mathbf{Z}_{kj} \mathbf{g}_j^{(p)}) (\sum_{j \neq k} \mathbf{g}_j^{H(p)} \mathbf{Z}_{kj}^H) \\ &\quad + \mathbf{Z}_{kk} \mathbf{g}_k^{(c)} (\sum_{j \neq k} \mathbf{g}_j^{H(c)} \mathbf{Z}_{kj}^H) + (\sum_{j \neq k} \mathbf{Z}_{kj} \mathbf{g}_j^{(c)}) \mathbf{g}_k^{H(c)} \mathbf{Z}_{kk}^H + \sigma^2 \mathbf{I}]^{-1} \end{aligned}$$

$$\begin{aligned}
\mathbf{v}_k^{H(p)} = & \mathbf{v}_k^{H(p)} \mathbf{Z}_{kk}^H [\mathbf{Z}_{kk} \mathbf{g}_k^{(c)} \mathbf{g}_k^{H(c)} \mathbf{Z}_{kk}^H + \mathbf{Z}_{kk} \mathbf{g}_k^{(p)} \mathbf{g}_k^{H(p)} \mathbf{Z}_{kk}^H \\
& + (\sum_{j \neq k} \mathbf{Z}_{kj} \mathbf{g}_j^{(c)}) (\sum_{j \neq k} \mathbf{g}_j^{H(c)} \mathbf{Z}_{kj}^H) + (\sum_{j \neq k} \mathbf{Z}_{kj} \mathbf{g}_j^{(p)}) (\sum_{j \neq k} \mathbf{g}_j^{H(p)} \mathbf{Z}_{kj}^H) \\
& + \mathbf{Z}_{kk} \mathbf{g}_k^{(c)} (\sum_{j \neq k} \mathbf{g}_j^{H(c)} \mathbf{Z}_{kj}^H) + (\sum_{j \neq k} \mathbf{Z}_{kj} \mathbf{g}_j^{(c)}) \mathbf{g}_k^{H(c)} \mathbf{Z}_{kk}^H + \sigma^2 \mathbf{I}]^{-1}
\end{aligned}$$

## 5. Bi-Directional Training - Least Mean Square Algorithm

$$Forward\ Training(fix\ \mathbf{v}_k^{(c)}, \mathbf{v}_k^{(p)}, \forall k)$$

$$\mathbf{g}_k^{(c)}(n+1) = \mathbf{g}_k^{(c)}(n) + \mu \mathbf{y}_k(n) [x(n) - \mathbf{g}_k^{H(c)}(n) \mathbf{y}_k(n)]^*$$

$$\mathbf{g}_k^{(p)}(n+1) = \mathbf{g}_k^{(p)}(n) + \mu \mathbf{y}_k(n) [x_k^{(p)}(n) - \mathbf{g}_k^{H(p)}(n) \mathbf{y}_k(n)]^*$$

$$Backward\ Training(fix\ \mathbf{g}_k^{(c)}, \mathbf{g}_k^{(p)}, \forall k)(without\ cooperation)$$

$$\mathbf{v}_k^{(c)}(n+1) = \mathbf{v}_k^{(c)}(n) + \mu \mathbf{y}_k(n) [x(n) - \mathbf{v}_k^{H(c)}(n) \mathbf{y}_k(n)]^*$$

$$\mathbf{v}_k^{(p)}(n+1) = \mathbf{v}_k^{(p)}(n) + \mu \mathbf{y}_k(n) [x_k^{(p)}(n) - \mathbf{v}_k^{H(p)}(n) \mathbf{y}_k(n)]^*$$

$$Backward\ Training(fix\ \mathbf{g}_k^{(c)}, \mathbf{g}_k^{(p)}, \forall k)(with\ cooperation)$$

$$\mathbf{v}_k^{(c)}(n+1) = \mathbf{v}_k^{(c)}(n) + \mu \mathbf{y}_k(n) [x(n) - \frac{\sum_i \mathbf{v}_i^{H(c)}(n) \mathbf{y}_i(n)}{Cardinality\ of\ i}]^*$$

$$\mathbf{v}_k^{(p)}(n+1) = \mathbf{v}_k^{(p)}(n) + \mu \mathbf{y}_k(n) [x_k^{(p)}(n) - \mathbf{v}_k^{H(p)}(n) \mathbf{y}_k(n)]^*$$

6. Numerical Simulation(2 Users, 2X2 MIMO Channel)

*Rayleigh Fading Channel*

*Cross Channel Gain = 1 \* Direct Channel Gain*

$$SNR = \frac{1}{\sigma^2} = 10^3 = 30dB$$

*Observation1 : If the training length is long enough, each LMS filter  $(\mathbf{v}_k^{(c)}, \mathbf{v}_k^{(p)}, \mathbf{g}_k^{(c)}, \mathbf{g}_k^{(p)})$  will converge to Wiener filter*

*Observation2 : Use Wiener filters, and only send common messages. Sum rate  $C = 11.6$  bit/channel*

*Observation3 : Use Wiener filters, and only send private messages. Sum rate  $C = 2.63$  bit/channel*

*Observation4 : Use Wiener filters, and send both messages. Sum rate  $C = 3.35$  bit/channel*

*Observation5 : Under the cooperation scheme, transmitters don't converge to Wiener filters*

*Observation5 : Under the cooperation scheme,  $C = 3.15$  bit/channel*

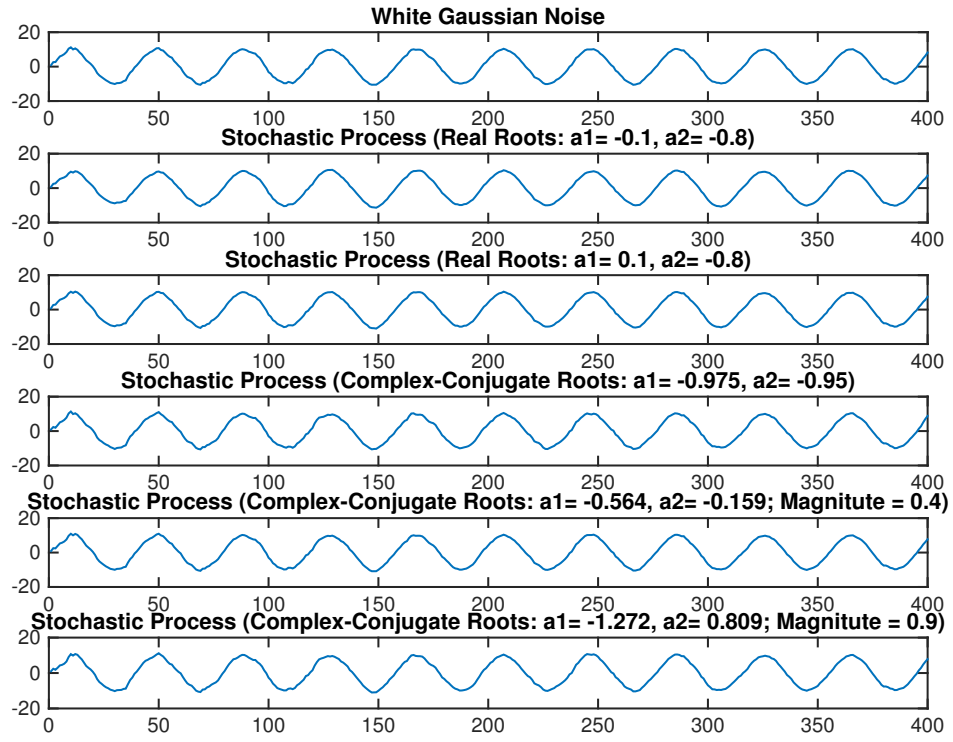


Figure 1: Insert caption