

Bi-Directional Training in Interference Network

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0. System Model(2 Users)

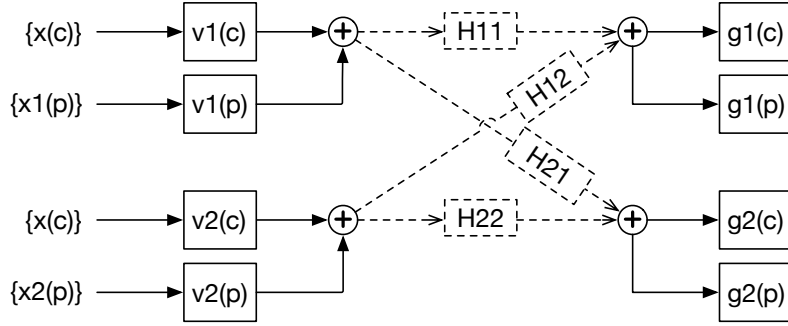


Figure 1: Forward Channel

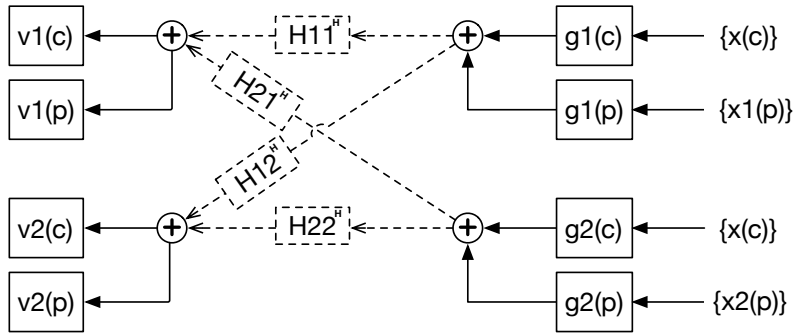


Figure 2: Backward Channel

1. Optimization Problem

$$\min_{\mathbf{v}_k, \mathbf{g}_k} \sum_k MSE_k^{(c)} + MSE_k^{(p)}$$

$$MSE_k^{(c)} = E[(x - \mathbf{g}_k^{H(c)} \mathbf{y}_k)(x - \mathbf{g}_k^{H(c)} \mathbf{y}_k)^H]$$

$$MSE_k^{(p)} = E[(x_k^{(p)} - \mathbf{g}_k^{H(p)} \mathbf{y}_k)(x_k^{(p)} - \mathbf{g}_k^{H(p)} \mathbf{y}_k)^H]$$

2. The received signal vector at k-th receiver

$$\mathbf{y}_k = \mathbf{H}_{kk}(\mathbf{v}_k^{(c)} x + \mathbf{v}_k^{(p)} x_k^{(p)}) + \sum_{j \neq k} \mathbf{H}_{kj}(\mathbf{v}_j^{(c)} x + \mathbf{v}_j^{(p)} x_j^{(p)}) + \mathbf{n}_k$$

3. SINR Derivation

$$s_k^{(c)} = \mathbf{g}_k^{H(c)} \left(\sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(c)} x \right)$$

$$s_k^{(p)} = \mathbf{g}_k^{H(p)} (\mathbf{H}_{kk} \mathbf{v}_k^{(p)} x_k^{(p)})$$

$$n_k^{(c)} = \mathbf{g}_k^{H(c)} \left(\sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(p)} x_i^{(p)} + \mathbf{n}_k \right)$$

$$n_k^{(p)} = \mathbf{g}_k^{H(p)} \left(\sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(c)} x + \sum_{j \neq k} \mathbf{H}_{kj} \mathbf{v}_j^{(p)} x_j^{(p)} + \mathbf{n}_k \right)$$

$$\frac{|s_k^{(c)}|^2}{|n_k^{(c)}|^2} = \frac{|\mathbf{g}_k^{H(c)} \sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(c)}|^2}{\sum_i |\mathbf{g}_k^{H(c)} \mathbf{H}_{ki} \mathbf{v}_i^{(p)}|^2 + |\mathbf{g}_k^{H(c)} \mathbf{R}_k \mathbf{g}_k^{(c)}|}$$

$$\frac{|s_k^{(p)}|^2}{|n_k^{(p)}|^2} = \frac{|\mathbf{g}_k^{H(p)} \mathbf{H}_{kk} \mathbf{v}_k^{(p)}|^2}{|\mathbf{g}_k^{H(p)} \sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(c)}|^2 + \sum_{j \neq k} |\mathbf{g}_k^{H(p)} \mathbf{H}_{kj} \mathbf{v}_j^{(p)}|^2 + |\mathbf{g}_k^{H(p)} \mathbf{R}_k \mathbf{g}_k^{(p)}|}$$

4. Max-SINR Algorithm[Gomadam,2011]

Forward Training(fix $\mathbf{v}_k^{(c)}, \mathbf{v}_k^{(p)}, \forall k$)

$$\begin{aligned} \mathbf{g}_k^{(c)} = & \left[\mathbf{H}_{kk} \mathbf{v}_k^{(c)} \mathbf{v}_k^{H(c)} \mathbf{H}_{kk}^H + \mathbf{H}_{kk} \mathbf{v}_k^{(p)} \mathbf{v}_k^{H(p)} \mathbf{H}_{kk}^H \right. \\ & + \left(\sum_{j \neq k} \mathbf{H}_{kj} \mathbf{v}_j^{(c)} \right) \left(\sum_{j \neq k} \mathbf{v}_j^{H(c)} \mathbf{H}_{kj}^H \right) + \left(\sum_{j \neq k} \mathbf{H}_{kj} \mathbf{v}_j^{(p)} \right) \mathbf{v}_j^{H(p)} \mathbf{H}_{kj}^H \\ & \left. + \mathbf{H}_{kk} \mathbf{v}_k^{(c)} \left(\sum_{j \neq k} \mathbf{v}_j^{H(c)} \mathbf{H}_{kj}^H \right) + \left(\sum_{j \neq k} \mathbf{H}_{kj} \mathbf{v}_j^{(c)} \right) \mathbf{v}_k^{H(c)} \mathbf{H}_{kk}^H + \sigma^2 \mathbf{I} \right]^{-1} \left(\sum_i \mathbf{H}_{ki} \mathbf{v}_i^{(c)} \right) \end{aligned}$$

$$\begin{aligned} \mathbf{g}_k^{(p)} = & \left[\mathbf{H}_{kk} \mathbf{v}_k^{(c)} \mathbf{v}_k^{H(c)} \mathbf{H}_{kk}^H + \mathbf{H}_{kk} \mathbf{v}_k^{(p)} \mathbf{v}_k^{H(p)} \mathbf{H}_{kk}^H \right. \\ & + \left(\sum_{j \neq k} \mathbf{H}_{kj} \mathbf{v}_j^{(c)} \right) \left(\sum_{j \neq k} \mathbf{v}_j^{H(c)} \mathbf{H}_{kj}^H \right) + \left(\sum_{j \neq k} \mathbf{H}_{kj} \mathbf{v}_j^{(p)} \right) \mathbf{v}_j^{H(p)} \mathbf{H}_{kj}^H \\ & \left. + \mathbf{H}_{kk} \mathbf{v}_k^{(c)} \left(\sum_{j \neq k} \mathbf{v}_j^{H(c)} \mathbf{H}_{kj}^H \right) + \left(\sum_{j \neq k} \mathbf{H}_{kj} \mathbf{v}_j^{(c)} \right) \mathbf{v}_k^{H(c)} \mathbf{H}_{kk}^H + \sigma^2 \mathbf{I} \right]^{-1} (\mathbf{H}_{kk} \mathbf{v}_k^{(p)}) \end{aligned}$$

Backward Training(fix $\mathbf{g}_k^{(c)}, \mathbf{g}_k^{(p)}, \forall k$)

$$\mathbf{Z}_{ab} = \mathbf{H}_{ba}^H [\text{Gomadam}, 2011]$$

without cooperation

$$\begin{aligned} \mathbf{v}_k^{(c)} = & \left[\mathbf{Z}_{kk} \mathbf{g}_k^{(c)} \mathbf{g}_k^{H(c)} \mathbf{Z}_{kk}^H + \mathbf{Z}_{kk} \mathbf{g}_k^{(p)} \mathbf{g}_k^{H(p)} \mathbf{Z}_{kk}^H \right. \\ & + \left(\sum_{j \neq k} \mathbf{Z}_{kj} \mathbf{g}_j^{(c)} \right) \left(\sum_{j \neq k} \mathbf{g}_j^{H(c)} \mathbf{Z}_{kj}^H \right) + \left(\sum_{j \neq k} \mathbf{Z}_{kj} \mathbf{g}_j^{(p)} \right) \mathbf{g}_j^{H(p)} \mathbf{Z}_{kj}^H \\ & \left. + \mathbf{Z}_{kk} \mathbf{g}_k^{(c)} \left(\sum_{j \neq k} \mathbf{g}_j^{H(c)} \mathbf{Z}_{kj}^H \right) + \left(\sum_{j \neq k} \mathbf{Z}_{kj} \mathbf{g}_j^{(c)} \right) \mathbf{g}_k^{H(c)} \mathbf{Z}_{kk}^H + \sigma^2 \mathbf{I} \right]^{-1} \left(\sum_i \mathbf{Z}_{ki} \mathbf{g}_i^{(c)} \right) \end{aligned}$$

with cooperation

$$\mathbf{V}^{(c)} = \left\{ [\mathbf{Z}] \mathbf{g}^{(c)} \mathbf{g}^{H(c)} [\mathbf{Z}]^H + [\mathbf{Z}] \begin{bmatrix} \mathbf{g}_1^{(p)} \mathbf{g}_1^{H(p)} & & \mathbf{O} \\ & \ddots & \\ \mathbf{O} & & \mathbf{g}_k^{(p)} \mathbf{g}_k^{H(p)} \end{bmatrix} [\mathbf{Z}]^H + \sigma^2 \mathbf{I} \right\}^{-1} ([\mathbf{Z}] \mathbf{g}^{(c)})$$

$$\begin{aligned} \mathbf{v}_k^{(p)} = & \left[\mathbf{Z}_{kk} \mathbf{g}_k^{(c)} \mathbf{g}_k^{H(c)} \mathbf{Z}_{kk}^H + \mathbf{Z}_{kk} \mathbf{g}_k^{(p)} \mathbf{g}_k^{H(p)} \mathbf{Z}_{kk}^H \right. \\ & + \left(\sum_{j \neq k} \mathbf{Z}_{kj} \mathbf{g}_j^{(c)} \right) \left(\sum_{j \neq k} \mathbf{g}_j^{H(c)} \mathbf{Z}_{kj}^H \right) + \left(\sum_{j \neq k} \mathbf{Z}_{kj} \mathbf{g}_j^{(p)} \mathbf{g}_j^{H(p)} \mathbf{Z}_{kj}^H \right) \\ & \left. + \mathbf{Z}_{kk} \mathbf{g}_k^{(c)} \left(\sum_{j \neq k} \mathbf{g}_j^{H(c)} \mathbf{Z}_{kj}^H \right) + \left(\sum_{j \neq k} \mathbf{Z}_{kj} \mathbf{g}_j^{(c)} \right) \mathbf{g}_k^{H(c)} \mathbf{Z}_{kk}^H + \sigma^2 \mathbf{I} \right]^{-1} (\mathbf{Z}_{kk} \mathbf{g}_k^{(p)}) \end{aligned}$$

5. Bi-Directional Training with LMS Algorithm[Shi,2014]

Forward Training(fix $\mathbf{v}_k^{(c)}, \mathbf{v}_k^{(p)}, \forall k$)

$$\mathbf{g}_k^{(c)}(n+1) = \mathbf{g}_k^{(c)}(n) + \mu \mathbf{y}_k(n) [x(n) - \mathbf{g}_k^{H(c)}(n) \mathbf{y}_k(n)]^*$$

$$\mathbf{g}_k^{(p)}(n+1) = \mathbf{g}_k^{(p)}(n) + \mu \mathbf{y}_k(n) [x_k^{(p)}(n) - \mathbf{g}_k^{H(p)}(n) \mathbf{y}_k(n)]^*$$

Backward Training(fix $\mathbf{g}_k^{(c)}, \mathbf{g}_k^{(p)}, \forall k$)(without cooperation)

$$\mathbf{v}_k^{(c)}(n+1) = \mathbf{v}_k^{(c)}(n) + \mu \mathbf{y}_k(n) [x(n) - \mathbf{v}_k^{H(c)}(n) \mathbf{y}_k(n)]^*$$

$$\mathbf{v}_k^{(p)}(n+1) = \mathbf{v}_k^{(p)}(n) + \mu \mathbf{y}_k(n) [x_k^{(p)}(n) - \mathbf{v}_k^{H(p)}(n) \mathbf{y}_k(n)]^*$$

Backward Training(fix $\mathbf{g}_k^{(c)}, \mathbf{g}_k^{(p)}, \forall k$)(with cooperation)

$$\mathbf{V}^{(c)}(n+1) = \mathbf{V}^{(c)}(n) + \mu \mathbf{Y}(n)[x(n) - \mathbf{V}^{H(c)}(n)\mathbf{Y}(n)]^*$$

$$\mathbf{v}_k^{(p)}(n+1) = \mathbf{v}_k^{(p)}(n) + \mu \mathbf{y}_k(n)[x_k^{(p)}(n) - \mathbf{v}_k^{H(p)}(n)\mathbf{y}_k(n)]^*$$

6. Special Case(2 Users, MIMO Channel, Only Common Messages)

$$\mathbf{y}_k = \sum_{i=1}^2 \mathbf{H}_{ki} \mathbf{v}_i^{(c)} x + \mathbf{n}_k$$

$$\begin{aligned} MSE_k^{(c)} &= E[(x - \mathbf{g}_k^{H(c)} \mathbf{y}_k)(x - \mathbf{g}_k^{H(c)} \mathbf{y}_k)^H] \\ &= E[x^2] - E[x \mathbf{y}_k^H \mathbf{g}_k^{(c)}] - E[x \mathbf{g}_k^{H(c)} \mathbf{y}_k] + E[\mathbf{g}_k^{H(c)} \mathbf{y}_k \mathbf{y}_k^H \mathbf{g}_k^{(c)}] \\ &= 1 - \sum_{i=1}^2 \mathbf{v}_i^{H(c)} \mathbf{H}_{ki}^H \mathbf{g}_k^{(c)} - \mathbf{g}_k^{H(c)} \sum_{i=1}^2 \mathbf{H}_{ki} \mathbf{v}_i^{(c)} + \mathbf{g}_k^{H(c)} \left(\sum_{i=1}^2 \mathbf{H}_{ki} \mathbf{v}_i^{(c)} \right) \left(\sum_{i=1}^2 \mathbf{v}_i^{H(c)} \mathbf{H}_{ki}^H \right) \mathbf{g}_k^{(c)} + \sigma^2 \mathbf{g}_k^{H(c)} \mathbf{g}_k^{(c)} \end{aligned}$$

$$\begin{aligned} \mathbf{v}_1^{*(c)} &= \underset{\mathbf{v}_1^{(c)}}{\operatorname{argmin}} \left(\sum_{i=1}^2 MSE_k^{(c)} \right) \\ &= \left[2\mathbf{H}_{11}^H \mathbf{g}_1^{(c)} \mathbf{g}_1^{H(c)} \mathbf{H}_{11} + 2\mathbf{H}_{21}^H \mathbf{g}_2^{(c)} \mathbf{g}_2^{H(c)} \mathbf{H}_{21} \right]^{-1} (2\mathbf{H}_{11}^H \mathbf{g}_1^{(c)} + 2\mathbf{H}_{21}^H \mathbf{g}_2^{(c)} \\ &\quad - \mathbf{g}_1^{H(c)} \mathbf{H}_{12} \mathbf{v}_2^{(c)} \mathbf{H}_{11}^H \mathbf{g}_1^{(c)} - \mathbf{H}_{11}^H \mathbf{g}_1^{(c)} \mathbf{v}_2^{H(c)} \mathbf{H}_{12}^H \mathbf{g}_1^{(c)} - \mathbf{g}_2^{H(c)} \mathbf{H}_{22} \mathbf{v}_2^{(c)} \mathbf{H}_{21}^H \mathbf{g}_2^{(c)} - \mathbf{H}_{21}^H \mathbf{g}_2^{(c)} \mathbf{v}_2^{H(c)} \mathbf{H}_{22}^H \mathbf{g}_2^{(c)}) \end{aligned}$$

$$\begin{aligned} \mathbf{v}_2^{*(c)} &= \underset{\mathbf{v}_2^{(c)}}{\operatorname{argmin}} \left(\sum_{i=1}^2 MSE_k^{(c)} \right) \\ &= \left[2\mathbf{H}_{12}^H \mathbf{g}_1^{(c)} \mathbf{g}_1^{H(c)} \mathbf{H}_{12} + 2\mathbf{H}_{22}^H \mathbf{g}_2^{(c)} \mathbf{g}_2^{H(c)} \mathbf{H}_{22} \right]^{-1} (2\mathbf{H}_{12}^H \mathbf{g}_1^{(c)} + 2\mathbf{H}_{22}^H \mathbf{g}_2^{(c)} \\ &\quad - \mathbf{g}_1^{H(c)} \mathbf{H}_{11} \mathbf{v}_1^{(c)} \mathbf{H}_{12}^H \mathbf{g}_1^{(c)} - \mathbf{H}_{12}^H \mathbf{g}_1^{(c)} \mathbf{v}_1^{H(c)} \mathbf{H}_{11}^H \mathbf{g}_1^{(c)} - \mathbf{g}_2^{H(c)} \mathbf{H}_{21} \mathbf{v}_1^{(c)} \mathbf{H}_{22}^H \mathbf{g}_2^{(c)} - \mathbf{H}_{22}^H \mathbf{g}_2^{(c)} \mathbf{v}_1^{H(c)} \mathbf{H}_{21}^H \mathbf{g}_2^{(c)}) \end{aligned}$$

7. The Other System Model

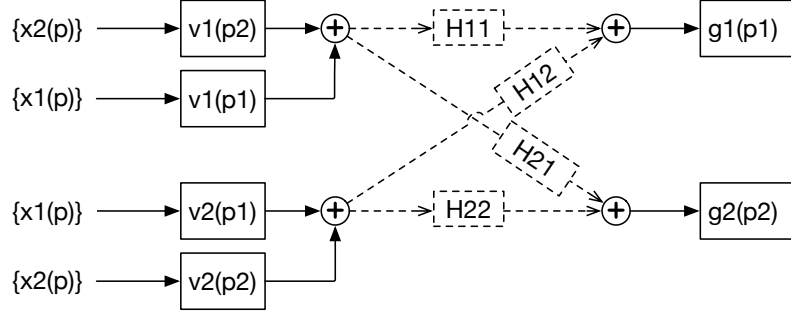


Figure 3: Forward Channel

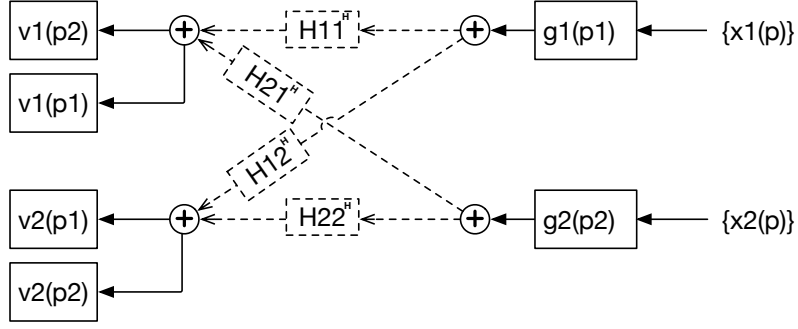


Figure 4: Backward Channel

8. Optimization Problem

$$\min_{\mathbf{v}_k(p_i), \mathbf{g}_k} \sum_k MSE_k^{(p)}$$

To be continued...

9. Numerical Simulation(2 Users, 2X2 MIMO Channel)

Rayleigh Fading Channel

*Cross Channel Gain = 0.8 * Direct Channel Gain*

$$SNR = \frac{1}{\sigma^2} = 10^3 = 30dB$$

Observation1 : If the training length is long enough, each LMS filter $(\mathbf{v}_k^{(c)}, \mathbf{v}_k^{(p)}, \mathbf{g}_k^{(c)}, \mathbf{g}_k^{(p)})$ will converge to Wiener filter

Observation2 : Use Wiener filters, and only send common messages. Sum rate $C = 11.6$ bit/channel

Observation3 : Use Wiener filters, and only send private messages. Sum rate $C = 2.63$ bit/channel

Observation4 : Use Wiener filters, and send both messages. Sum rate $C = 3.35$ bit/channel

Observation5 : Under the cooperation scheme, transmitters don't converge to Wiener filters

Observation5 : Under the cooperation scheme, $C = 3.15$ bit/channel

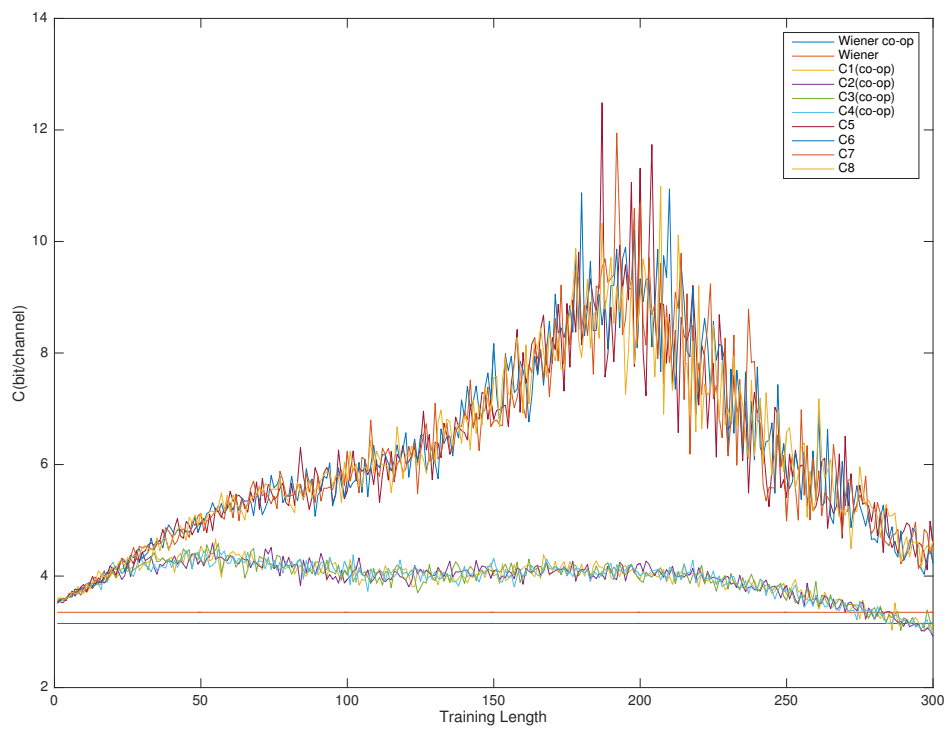


Figure 5: Insert caption

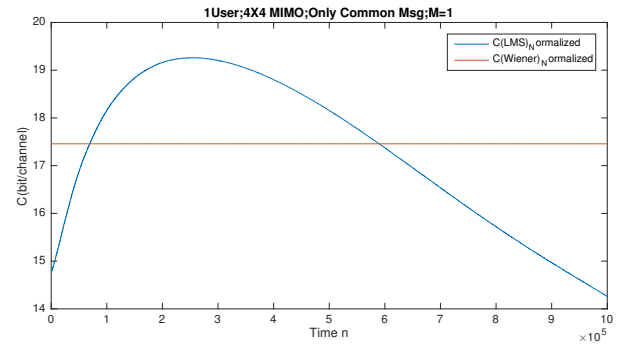
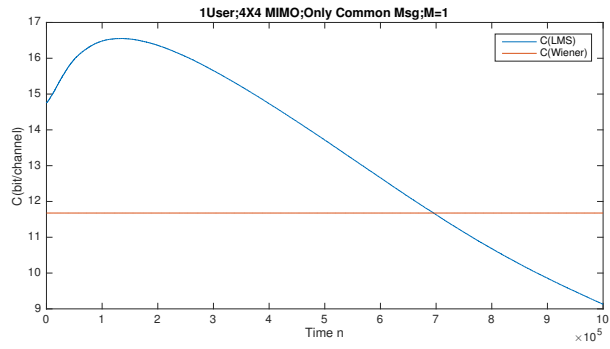
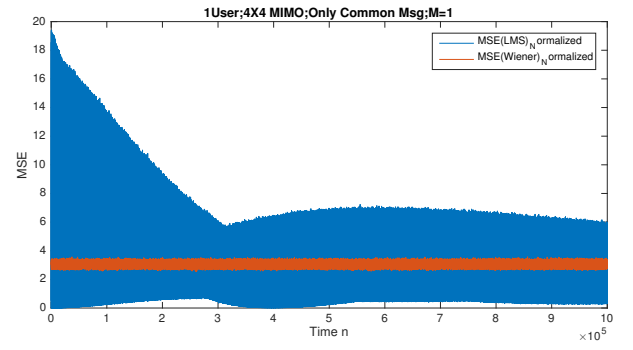
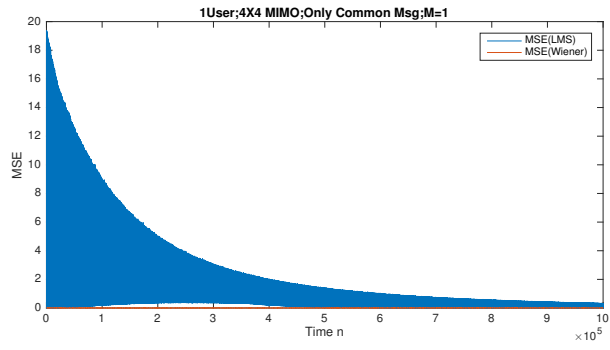


Figure 6: Insert caption

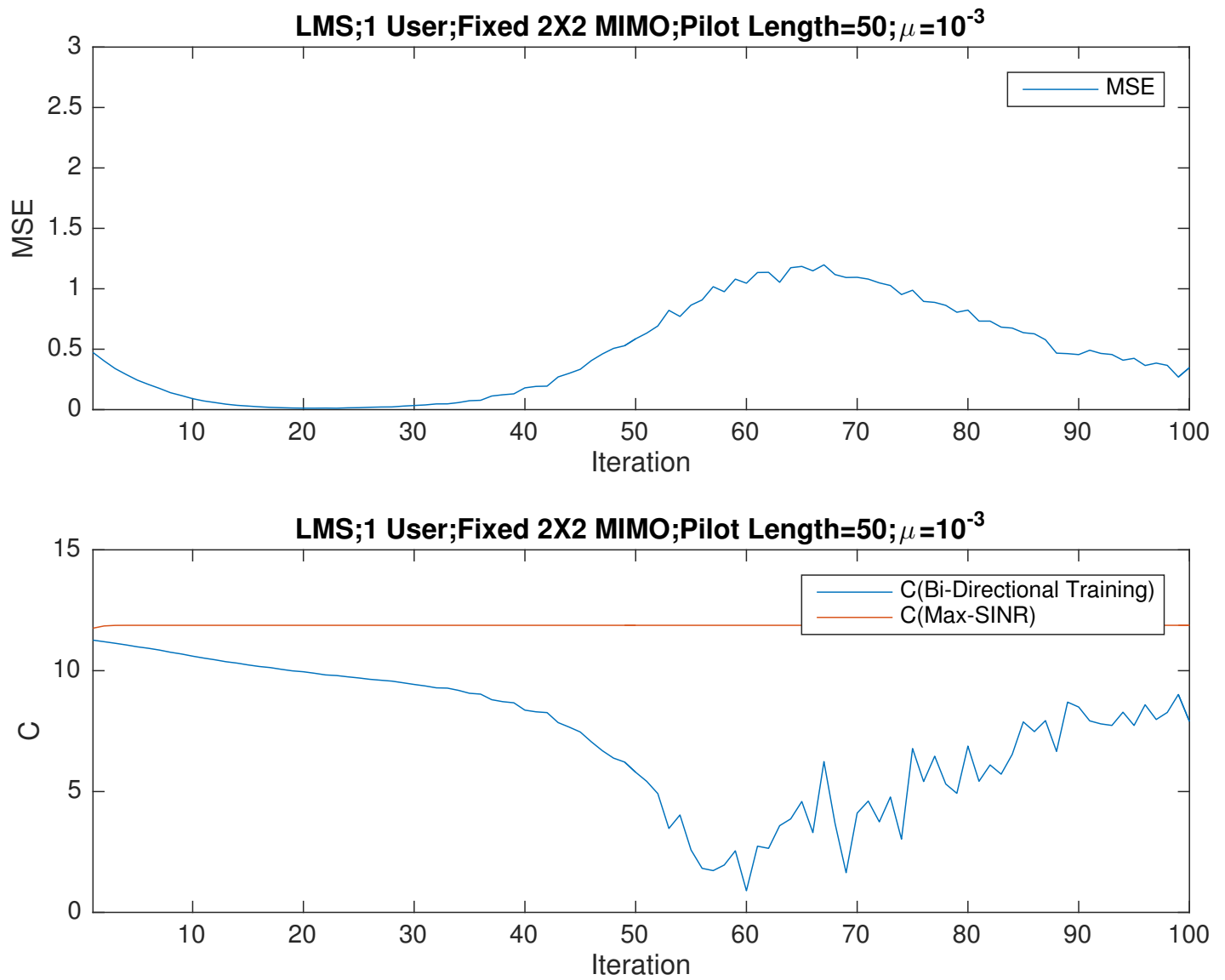


Figure 7: Insert caption

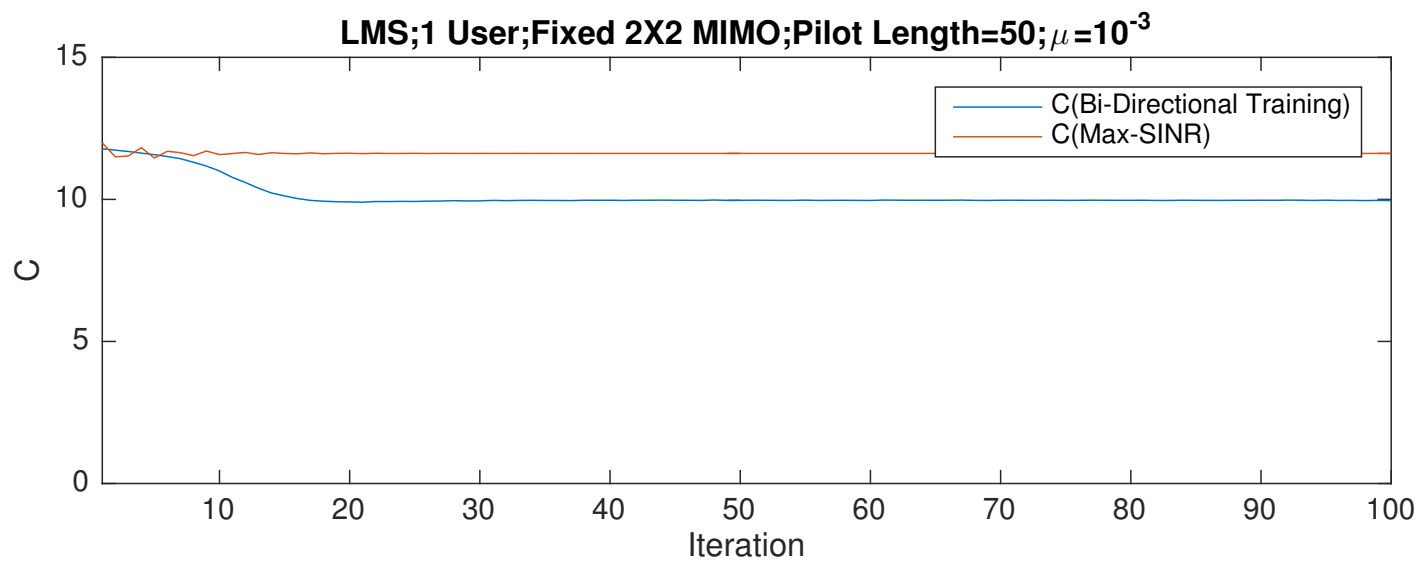
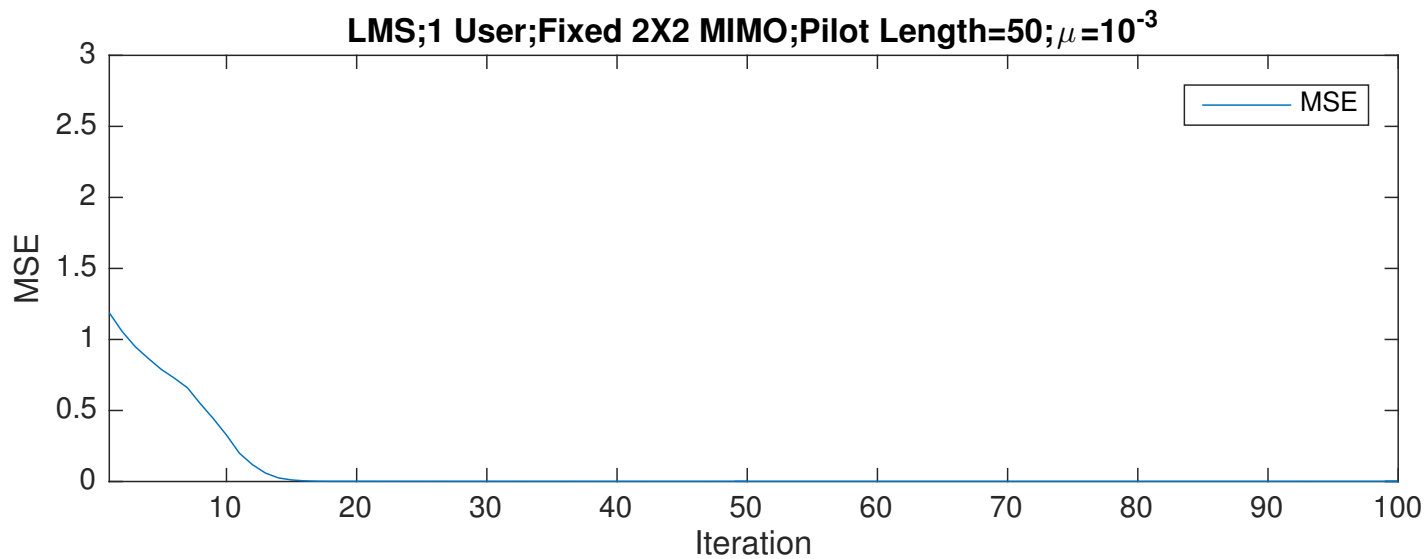


Figure 8: Insert caption

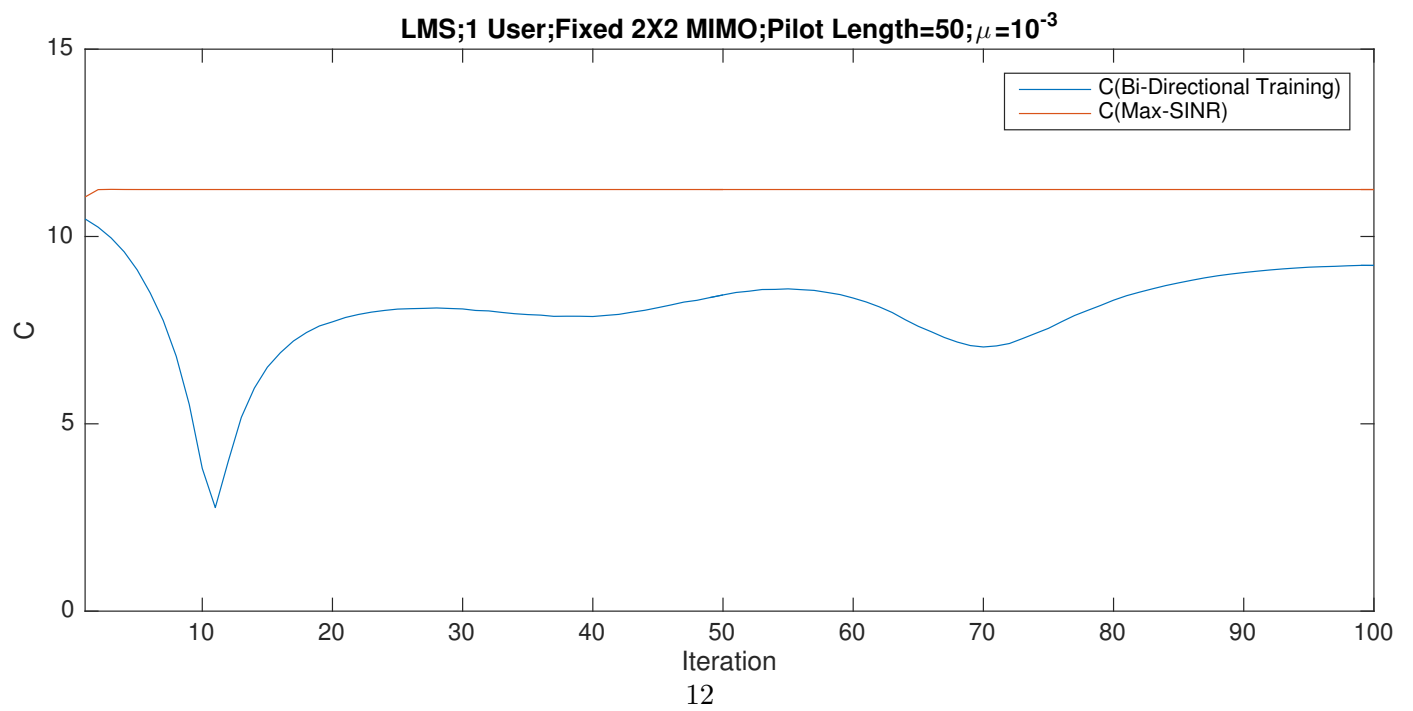
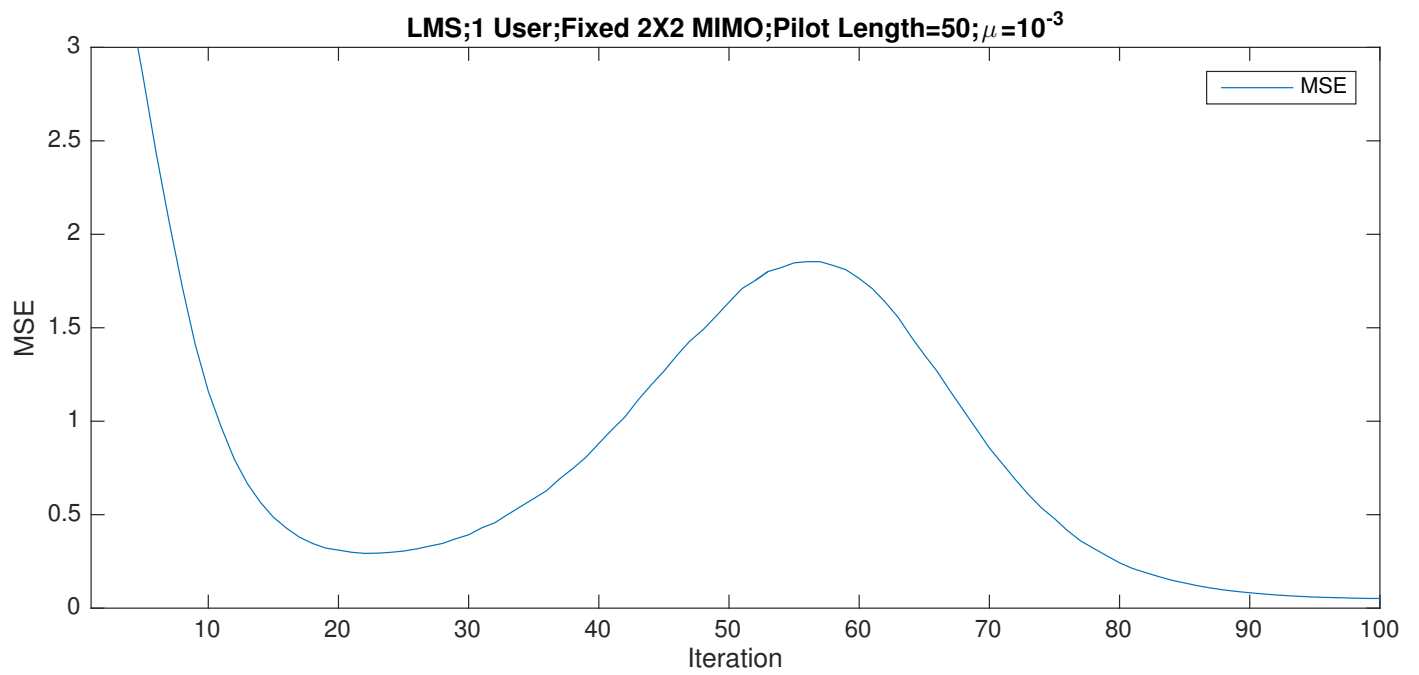


Figure 9: Insert caption

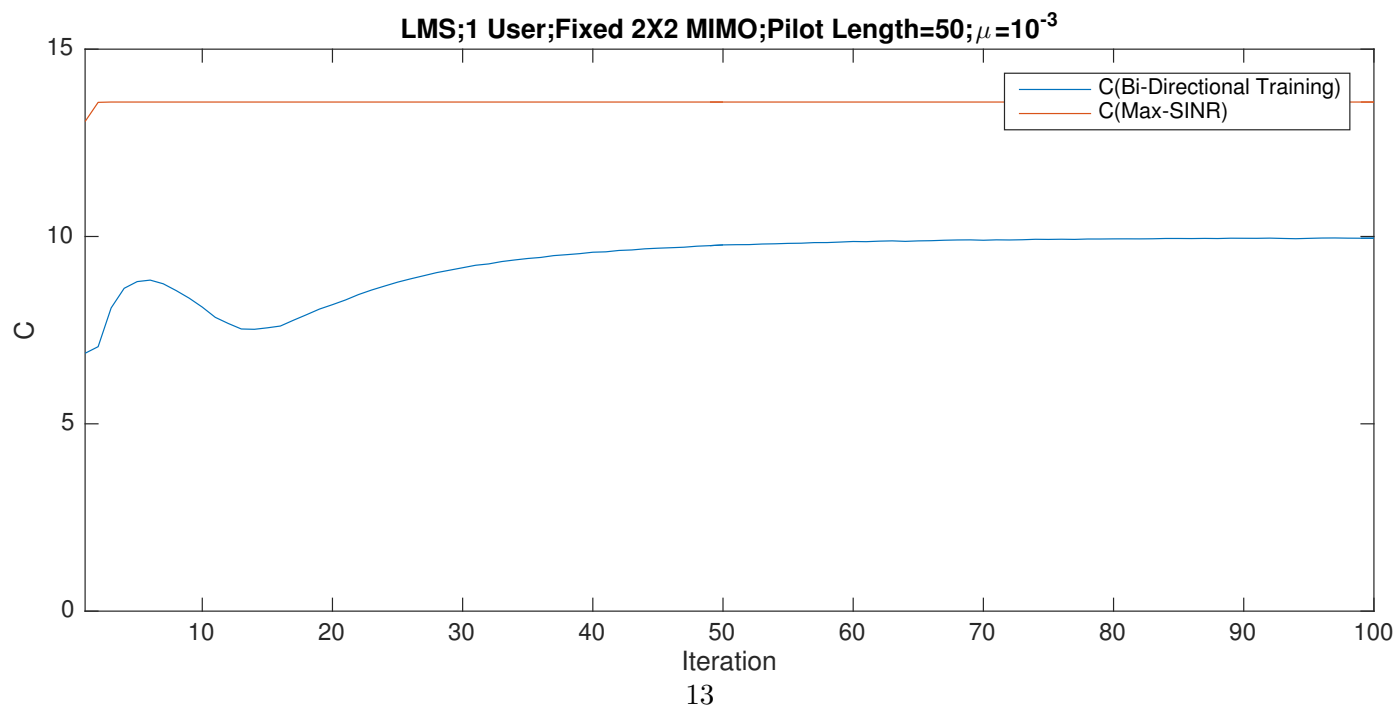
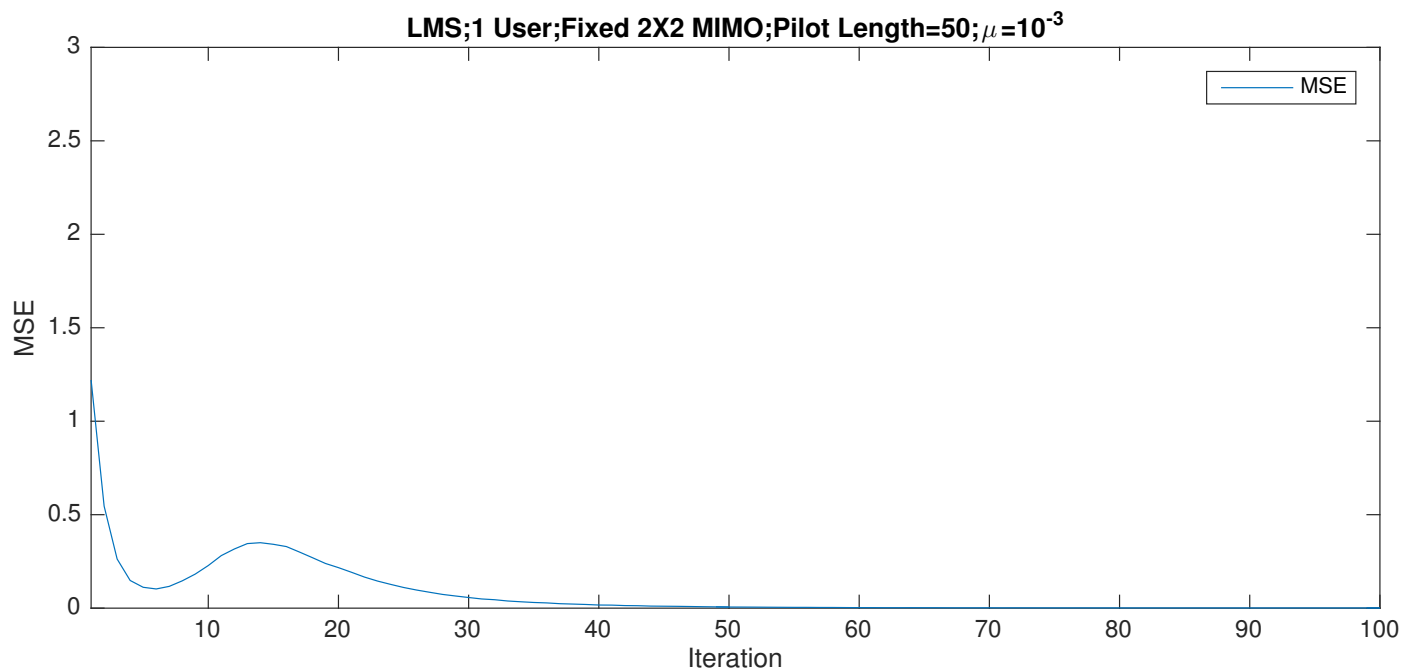


Figure 10: Insert caption

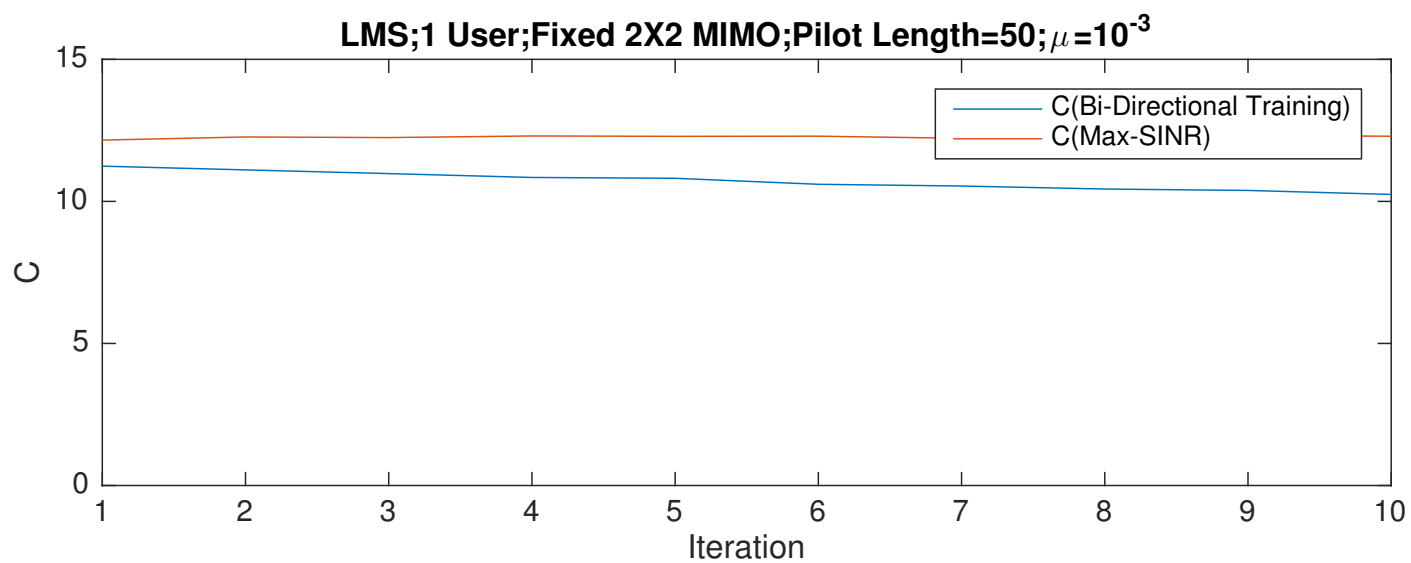
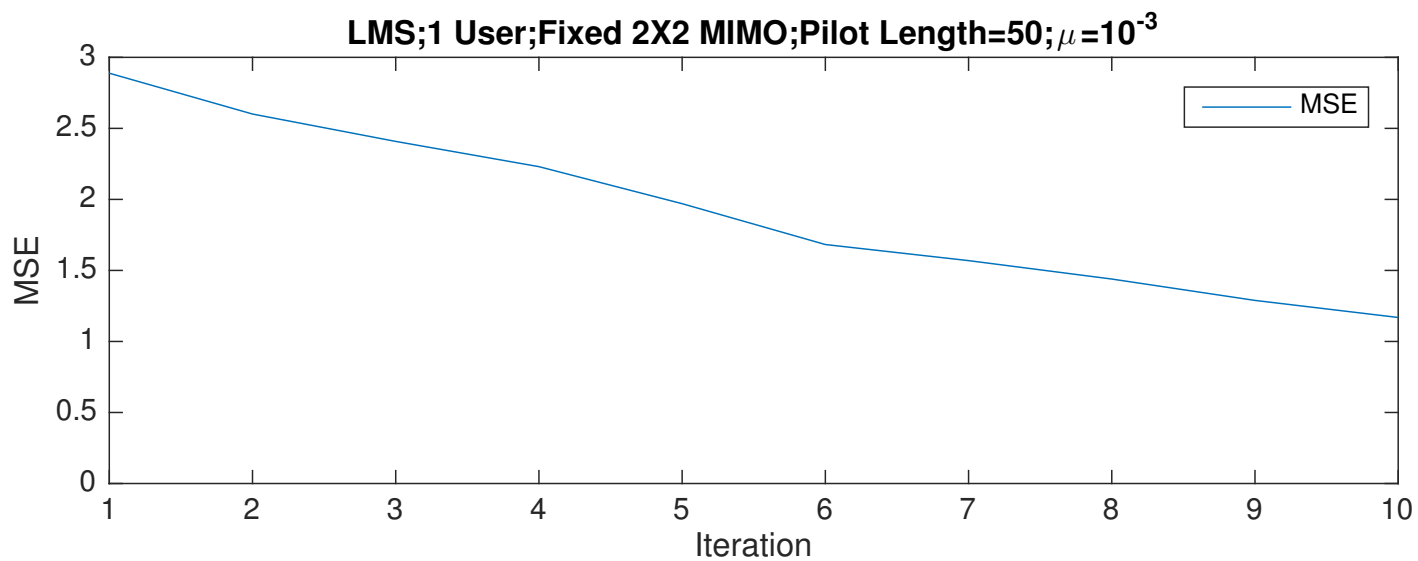


Figure 11: Insert caption

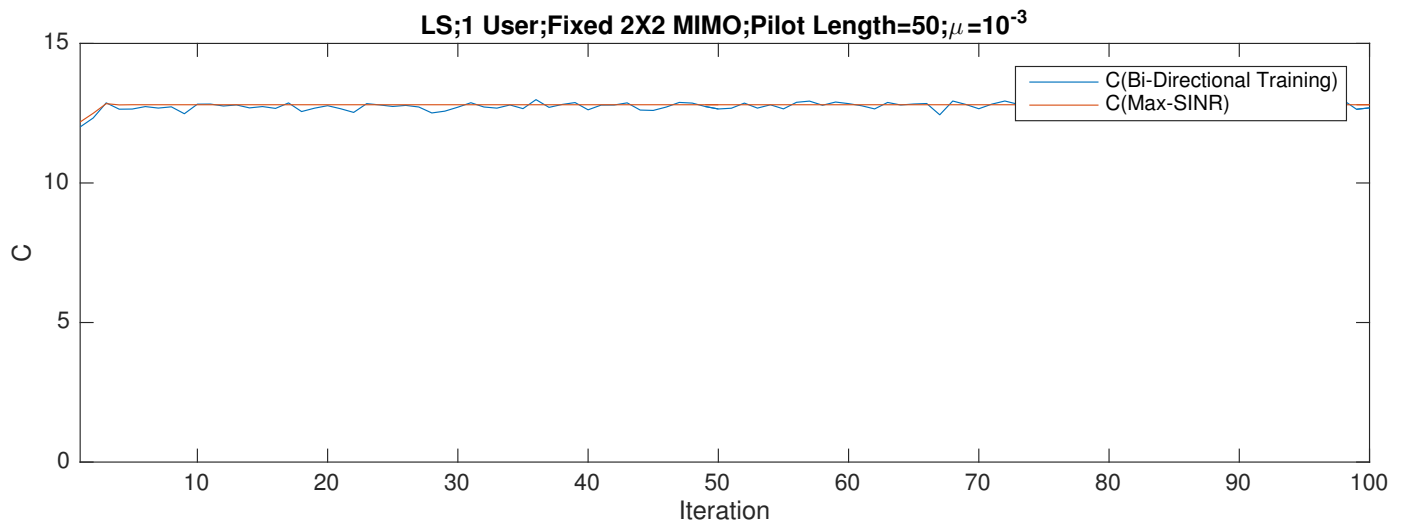
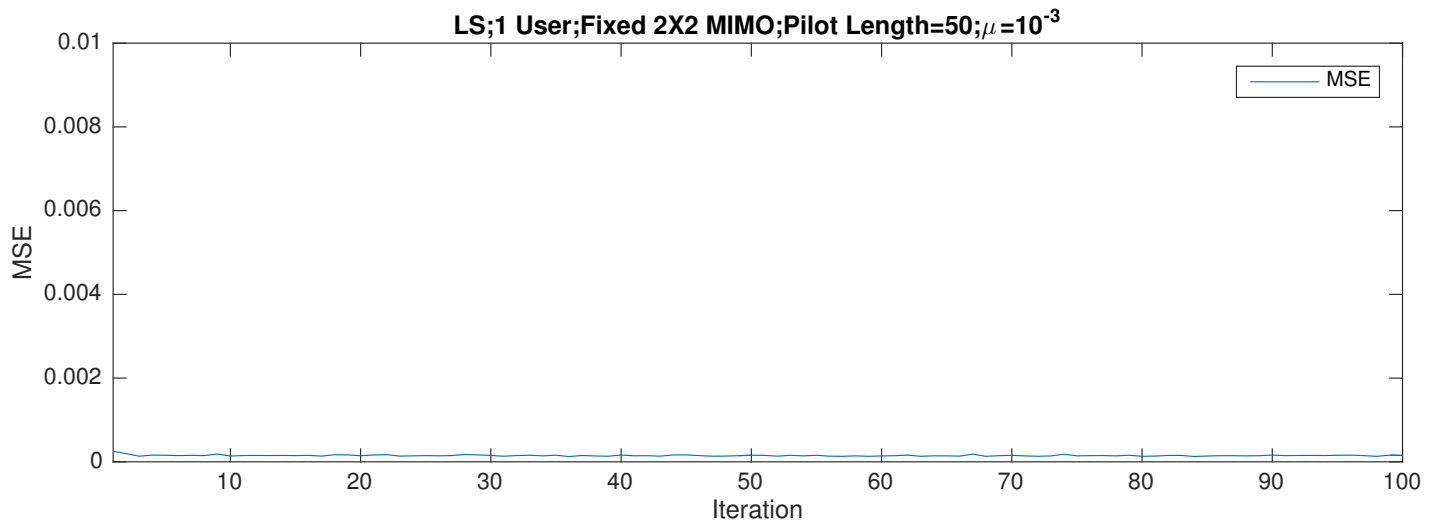


Figure 12: Insert caption

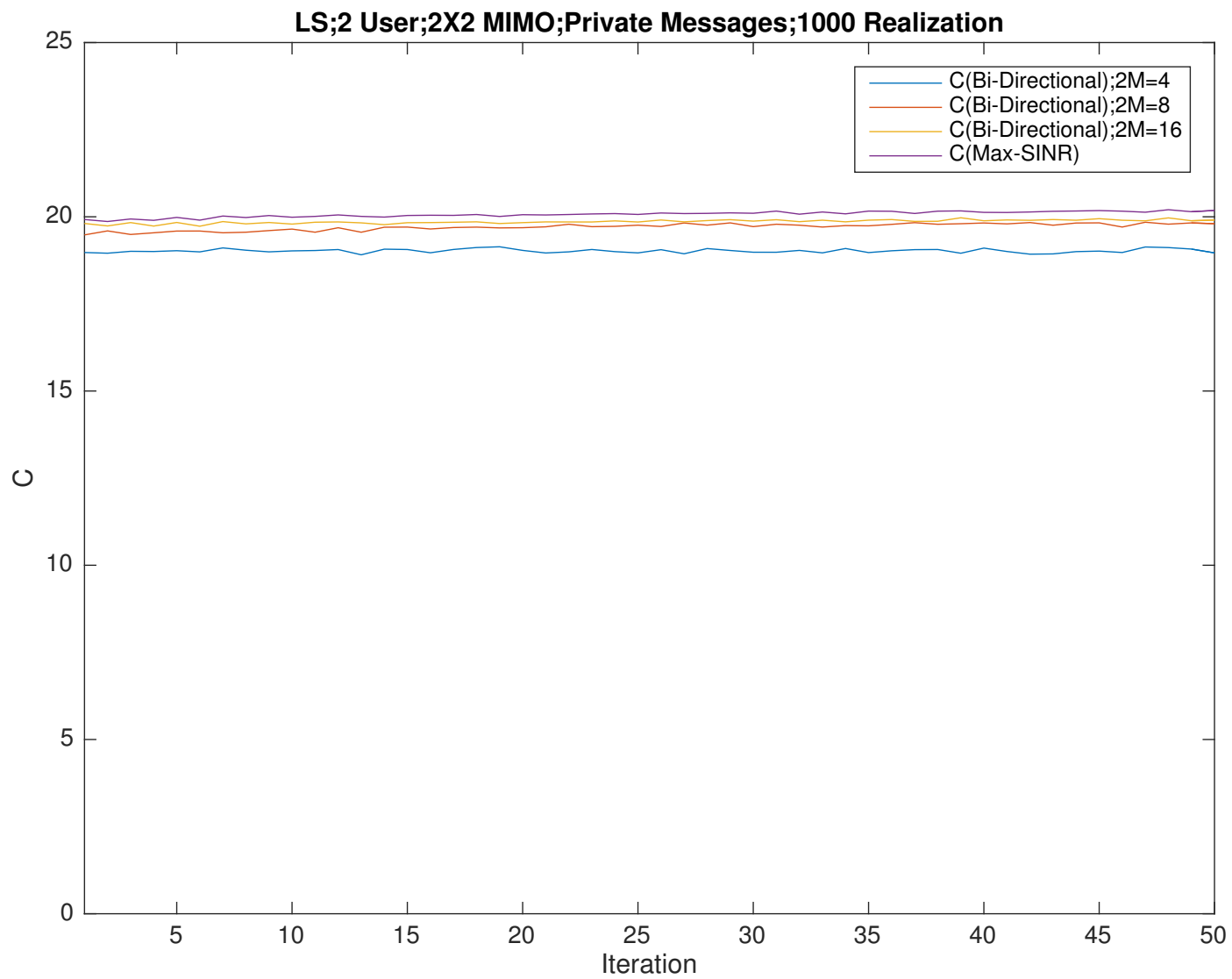


Figure 13: Insert caption

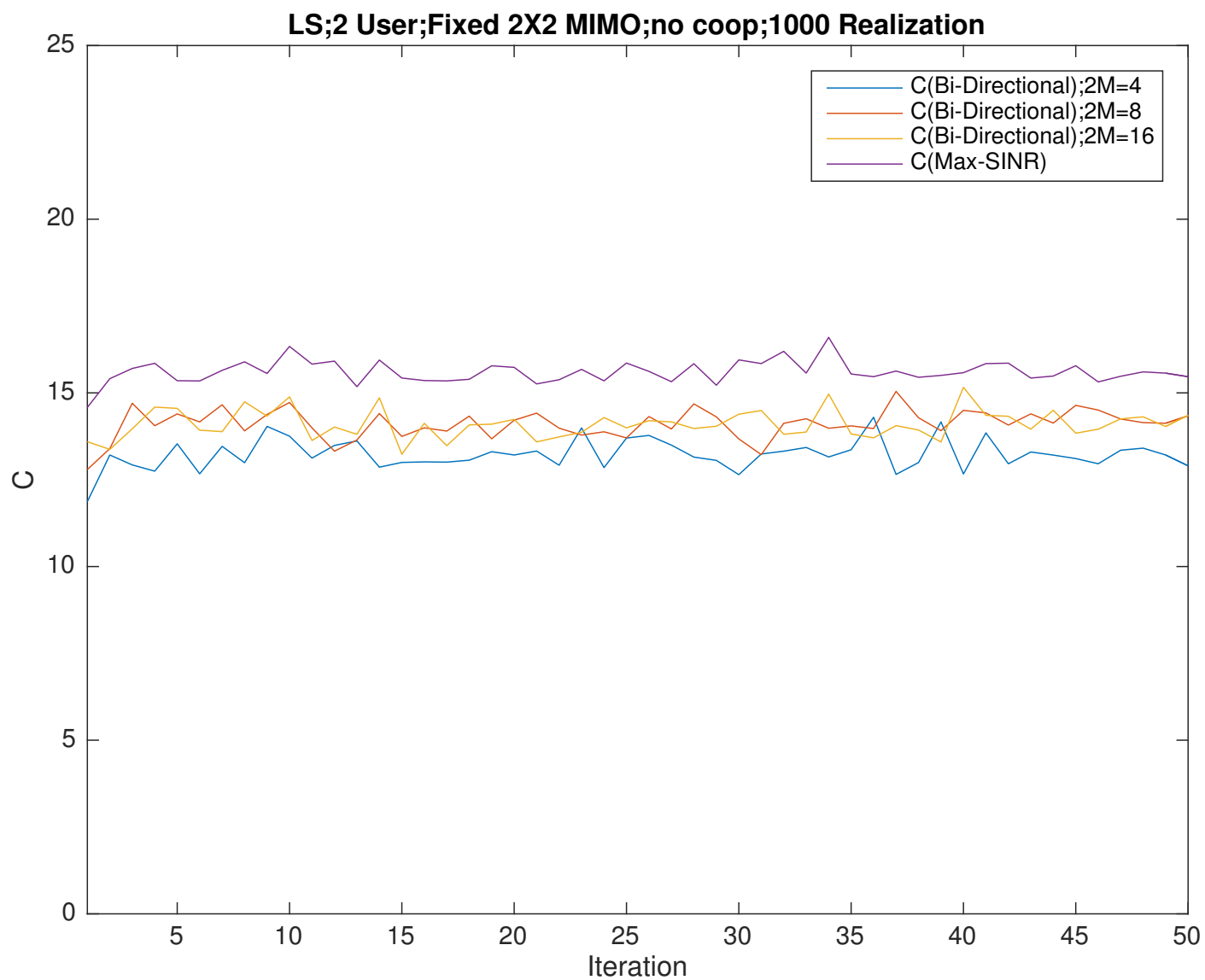


Figure 14: Insert caption

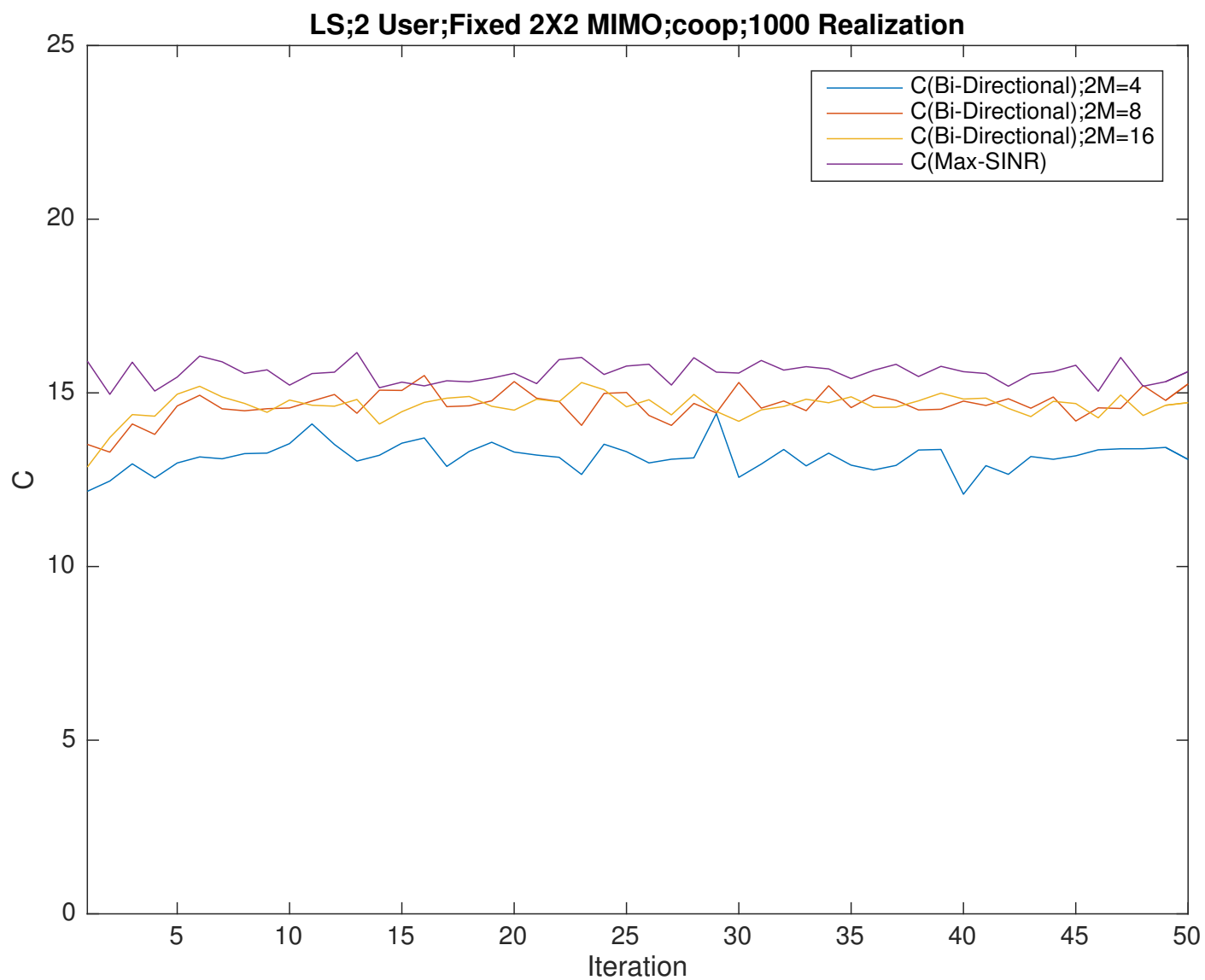


Figure 15: Insert caption