

(Q5)

Theorem 1. *Let \mathbb{F} be a field. Suppose that $1 + 1 + 1 = 0$.*

Prove that for all $x \in \mathbb{F}$, we have $x + x + x = 0$.

Proof. We can express $1 + 1 + 1$ as $1 + (1 + 1)$.

By Axiom 5,

$$x(1 + (1 + 1)) = x + x(1 + 1) = x + (x + x) = x + x + x = 0$$

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