(Q3)

(a) $\dfrac{5}{6}$

(b) $\dfrac{n}{n+1}$

(c)

Proof. Let the theorem be $P(k)$ be $\sum_{i=1}^{k}(\frac{1}{i} - \frac{1}{i+1}) = \frac{k}{k+1}$.

We prove this by considering the base case $P(1)$ and then the induction hypothesis $P(k) \implies P(k+1)$ for some $k \in \mathbb{N}$:

$$\sum_{i=1}^{k}\left(\frac{1}{i} - \frac{1}{i+1}\right) = \frac{k}{k+1} \implies \sum_{i=1}^{k+1}\left(\frac{1}{i} - \frac{1}{i+1}\right) = \frac{k+1}{(k+1)+1}$$

Base Case.

$$P(1) = \sum_{i=1}^{1}\left(\frac{1}{1} - \frac{1}{1+1}\right) = \frac{1}{2} = \frac{1}{1+1}$$

Induction Step.

Assuming $P(k)$, it follows:

$$\begin{aligned}
\sum_{i=1}^{k+1}\left(\frac{1}{i} - \frac{1}{i+1}\right) &= \frac{k}{k+1} + \left(\frac{1}{k+1} - \frac{1}{(k+1)+1}\right) \\
&= \frac{k}{k+1} + \left(\frac{(k+1)+1}{(k+1)((k+1)+1)} - \frac{k+1}{(k+1)((k+1)+1)}\right) \\
&= \frac{k}{k+1} + \left(\frac{1}{(k+1)((k+1)+1)}\right) \\
&= \frac{k((k+1)+1)+1}{(k+1)((k+1)+1)} \\
&= \frac{k^2+2k+1}{(k+1)((k+1)+1)} \\
&= \frac{(k+1)^2}{(k+1)((k+1)+1)} \\
&= \frac{k+1}{(k+1)+1}
\end{aligned}$$

Which proves the induction hypothesis.

Since the base case and induction hypothesis both hold, the theorem is proved by induction, as required.

■