(Q6)

Theorem 1. *Let S, T be any subsets of a universal set U. Prove that $(S \cap T)^c = S^c \cup T^c$.*

Proof. We can prove these two sets are equal by mutual subset inclusion:

$$(S \cap T)^c = S^c \cup T^c \text{ iff } (S \cap T)^c \subseteq S^c \cup T^c \text{ and } S^c \cup T^c \subseteq (S \cap T)^c$$

Let $x \in (S \cap T)^c$. This implies $x \notin (S \cap T)$, or $x \notin S$ and T. From this, we can infer $x \in S^c$ or T^c which can be written as $S^c \cup T^c$. Therefore,

$$x \in S^c \cup T^c \implies (S \cap T)^c \subseteq S^c \cup T^c$$

Now, let $x \in S^c \cup T^c$. This implies $x \notin S$ or T. From this, we can infer $x \in S^c$ and T^c, which can be written as $(S \cap T)^c$. Therefore,

$$x \in (S \cap T)^c \implies S^c \cup T^c \subseteq (S \cap T)^c$$

Since $(S \cap T)^c \subseteq S^c \cup T^c$ and $S^c \cup T^c \subseteq (S \cap T)^c$, we can conclude $(S \cap T)^c = S^c \cup T^c$. ■