(Q3)
In this question, we are asked to prove the statement

For all $x, y \in \mathbb{R}$ we have

$$\big||x| - |y|\big| \leq |x + y|$$

is true, and if false, to provide a counterexample.
This statement is true, and the proof is as follows.

Proof. We know from **Proposition 1.3.2** that $x \leq |x|$.

Let $x = xy$. Therefore, $xy \leq |xy|$.

We can then do the following:

$$\begin{aligned} |xy| &\geq xy \\ -2|xy| &\leq 2xy \\ |x|^2 - 2|xy| + |y|^2 &\leq |x|^2 + 2xy + |y|^2 \end{aligned}$$

By **Proposition 1.3.2**:

$$\sqrt{x^2} = |x|$$

We can then factorise and take the square root of both sides:

$$\begin{aligned} \big||x| - |y|\big|^2 &\leq |x + y|^2 \\ \big||x| - |y|\big| &\leq |x + y| \end{aligned}$$

$\blacksquare$