(Q2)

(a) $\dfrac{1}{36}$

(b) $\dfrac{1}{(n+1)^2}$

(c)

Proof. Let the theorem $P(k)$ be $\prod_{i=1}^{k} \frac{i^2}{(i+1)^2} = \frac{1}{(k+1)^2}$.

We prove this theorem by considering the base case $P(1)$ and then the induction hypothesis $P(k) \implies P(k+1)$ for some $k \in \mathbb{N}$:

$$\prod_{i=1}^{k} \frac{i^2}{(i+1)^2} = \frac{1}{(k+1)^2} \implies \prod_{i=1}^{k+1} \frac{i^2}{(i+1)^2} = \frac{1}{((k+1)+1)^2}$$

Base Case.

$$P(1) = \prod_{i=1}^{1} \frac{i^2}{(i+1)^2} = \frac{1}{4} = \frac{1}{(1+1)^2}$$

Which shows the base case holds.

Induction Step.

Assuming $P(k)$, it follows:

$$\begin{aligned}\prod_{i=1}^{k+1} &= \frac{1}{(k+1)^2} \cdot \frac{(k+1)^2}{((k+1)+1)^2} \\ &= \frac{(k+1)^2}{(k+1)^2((k+1)+1)^2} \\ &= \frac{1}{((k+1)+1)^2}\end{aligned}$$

By PMI, since both the base case and induction step hold, we have proven the theorem, as required.

$\blacksquare$