(Q5)

Theorem 1. *Let $A,\ B \subseteq \mathbb{R}$. If $f\colon A \to B$ is strictly monotone, then there exists a subset $C \subseteq \mathbb{R}$ with the same cardinality as A.*

Proof. By Proposition 5.1.4, since f is strictly monotone, it is therefore injective on B.

In order to prove that $|C| = |A|$, we construct a bijection between them. Let $C = f(A)$. f is surjective on C by the definition of function image, and is also injective on C since $C \subseteq B$.

Therefore, since $f\colon A \to C$ is bijective, $|C| = |A|$. ■