(Q2)

(a)

Proof. Using **Definition 1.3.1**, the absolute value of a real number x is:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

In order to prove the theorem, we need to consider cases where $b > 0$ and $b < 0$.

For cases where $b > 0$, $\frac{1}{b} \in (0, \infty)$.

$$\left|\frac{1}{b}\right| = \frac{1}{b} = \frac{1}{|b|}$$

For cases where $b < 0$, $\frac{1}{b} \in (-\infty, 0)$.

$$\left|\frac{1}{b}\right| = -\frac{1}{b} = \frac{1}{-b} = \frac{1}{|b|}$$

Since for both cases, $\left|\frac{1}{b}\right| = \frac{1}{|b|}$, this holds true for all $b \in \mathbb{R}$. ■

(b)

Proof. By **Proposition 1.3.2**, $|xy| = |x| \cdot |y|$.

Let $x = a$, $y = \frac{1}{b}$.

By *(a)* we know that $\frac{1}{|b|} = \left|\frac{1}{b}\right|$ for any non-zero real number b. Therefore:

$$\frac{|a|}{|b|} = |a| \cdot \frac{1}{|b|} = |a| \cdot \left|\frac{1}{b}\right| = \left|a \cdot \frac{1}{b}\right| = \left|\frac{a}{b}\right|$$

■