(Q1)

Theorem 1. *Using induction, prove that for all even natural numbers* n*, the integer* $n^3 + 2n^2 - 32$ *is divisible by 8.*

Proof. Let the theorem be $P(n) = n^3 + 2n^2 - 32$. We first consider the base case $P(2)$ and then the induction hypothesis where for a given even natural number k:

$$P(k) \implies P(k+2)$$

Base Case. Considering $P(2)$:

$$P(2) = 2^3 + 2(2)^2 - 32 = 8 + 8 - 32 = -16$$

Which is divisible by 8, so the base case holds.
Induction Step. We consider the induction hypothesis:

$$P(k) \implies P(k+2) \text{ where } k \text{ is an even natural number}$$

It follows:

$$\begin{aligned}(k+2)^3 + 2(k+2)^2 - 32 &= (k^3 + 2k^2 + 4k^2 + 8k + 4k + 8) + (2k^2 + 8k + 8) - 32 \\ &= (k^3 + 2k^2 - 32) + 6k^2 + 20k + 16\end{aligned}$$

Assuming that $(k^3 + 2k^2 - 32)$ is divisible by 8, it suffices to prove that $6k^2 + 20k + 16$ is divisible by 8.

Since k is even, we can express k as $2m$, where m is a natural number. We can thus express $6k^2$ and $20k$ as $6(2m)^2$ and $20(2m)$ respectively.

Then

$$6k^2 + 20k + 16 = 6(2m)^2 + 20(2m) + 16 = 24m^2 + 40m + 16 = 8(3m^2 + 5m + 2)$$

Since $\mathbb{N}$ is closed under multiplication, $3m^2 + 5m + 2$ is a natural, thus $8(3m^2 + 5m + 2)$ is divisible by 8, proving the induction hypothesis.

By PMI, since the base case and induction hypothesis hold, we have proved the theorem, as required. ■