(Q4)

Theorem 1. *Let $\mathbb{F}$ be a field. Let $a, b \in \mathbb{F}$.*
Prove that if $a \cdot b = 0$, then either a or $b = 0$.

Proof. In order to prove this, we can consider a few cases:
$a = 0$: This case is trivial.
$a \neq 0$: By Axiom 4, $a \neq 0 \implies a^{-1} \in \mathbb{F}$. It follows that:

$$\begin{aligned} & a^{-1}(ab) = 0 \text{ by Claim 2.3.2} \\ \implies & (a^{-1} \cdot a) \cdot b = b = 0 \end{aligned}$$

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