

(Q5)

(a)

These are the addition and multiplication tables for the given field \mathbb{F} .

Entries in **red** are entries that were pre-populated by the question.

The addition table is as follows:

+	0	1	x
0	0	1	x
1	1	x	0
x	x	0	1

The first row and first column were filled in as such to ensure Axiom 3 was fulfilled, specifically the existence of the additive identity. The remaining zeroes were filled in as such to ensure that Axiom 4 was fulfilled (existence of negatives).

The multiplication table is as follows:

\cdot	0	1	x
0	0	0	0
1	0	1	x
x	0	x	1

The first row and first column were filled in as such by proof of Claim 2.3.2.

The remaining two ones were filled in as such to ensure Axiom 4 was fulfilled (existence of reciprocals).

(b) Following the addition and multiplication tables:

$$x^2 = x \cdot x = 1$$

$$1 + x = 0$$

$$0 + 1 = 1$$