

(Q1)

Theorem 1. *Prove that if $|x| < 10$, then $|2x^3 - 2x + 2| < 2022$.*

Proof. We know the triangle inequality to be:

For any $x, y \in \mathbb{R}$, we have

$$|x + y| \leq |x| + |y|$$

Knowing that $|x| < 10$, we can do the following:

$$\begin{aligned} |x| &< 10 \\ 2|x|^3 &= |2x^3| < 2000 \\ |2x^3| + |-2x| &< 2000 + 20 = 2020 \\ |2x^3| + |-2x| + |2| &< 2000 + 20 + 2 = 2022 \end{aligned}$$

By the triangle inequality,

$$\begin{aligned} |2x^3 - 2x + 2| &\leq |2x^3| + |-2x| + |2| < 2022 \\ \implies |2x^3 - 2x + 2| &< 2022 \end{aligned}$$

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