(Q1)

Theorem 1. *Let* $f\colon A \to B$*. Then there exists a subset* $C \subseteq B$ *such that* $f\colon A \to C$ *is bijective.*

Proof. In order to prove that $f\colon A \to C$ is bijective, we prove that it is both surjective and injective.

Let $C = f(A)$, that is, C is the image of f. Thus, f is surjective on C by construction. Since f is injective on B and $C \subseteq B$ by definition of function image, f is also injective on C.

Since $f\colon A \to C$ is both surjective and injective, it is therefore bijective. ■