

(Q2)

In order to find and classify all critical points, we require the first and second derivatives of f :

$$\begin{aligned}f'(x) &= x^{2/5} - x^{-4/5} \\f''(x) &= \frac{2}{5}x^{-3/5} + \frac{4}{5}x^{-9/5}\end{aligned}$$

Solving f and f' for 0:

$$\begin{aligned}f(x) &= \frac{5}{7}x^{7/5} - 5x^{1/5} = 0 \\5x\left(\frac{1}{7}x^{2/5} - x^{-4/5}\right) &= 0 \implies x = 0 \\ \frac{1}{7}x^{2/5} &= x^{-4/5} \\x^{6/5} &= (x^{3/5})^2 = 7 \\x^{3/5} &= \pm\sqrt{7} \\x &= \pm(\sqrt{7})^{5/3}\end{aligned}$$

$$\begin{aligned}f'(x) &= x^{2/5} - x^{-4/5} = 0 \\x^{2/5} &= x^{-4/5} \\x^{6/5} &= (x^{3/5})^2 = 1 \\x^{3/5} &= \pm 1 \\x &= \pm 1\end{aligned}$$

Thus, f has roots at $\pm(\sqrt{7})^{5/3}$ and 0, as well as max/min points at ± 1 .

Another thing to note is that $f'(x)$ is undefined at $x = 0$, so f has a vertical tangent line at $x = 0$.

Using f'' to determine concavity, we have:

$$\begin{aligned}f''(1) &= \frac{2}{5}(1)^{-3/5} + \frac{4}{5}(1)^{-9/5} = \frac{6}{5} > 0 \\f''(-1) &= \frac{2}{5}(-1)^{-3/5} + \frac{4}{5}(-1)^{-9/5} = -\frac{6}{5} < 0\end{aligned}$$

So f is concave up at $x = 1$ and concave down at $x = -1$, so its maximum point is at $x = -1$ and its minimum point is at $x = 1$.

With all this information, we can then sketch the graph of f :

