

(Q4)

Given the curve $\sin(2x - y) + x^3y^2 + 1 = e^{y-x^2}$, we are required to find the equation of the tangent line at $(0, 0)$.

We can find the derivative by means of implicit differentiation:

$$\begin{aligned} & \frac{d}{dx} \sin(2x - y) + x^3y^2 + 1 - e^{y-x^2} \\ &= (\cos(2x - y) \cdot 2 - y') + (3x^2y^2 + x^3(2y \cdot y')) - (e^{y-x^2} \cdot (y' - 2x)) \end{aligned}$$

Considering the curve at $(0, 0)$:

$$\begin{aligned} & (\cos(0) \cdot 2 - y') + (3(0)^2(0)^2 + 0^3(0 \cdot y')) - (e^{0-0^2} \cdot (y' - 0)) \\ &= 2 - y' - y' = 0 \implies y' = 1 \end{aligned}$$

Therefore, the slope of the tangent is 1. Substituting the point $(0,0)$ and slope into the form $y = mx + b$:

$$\begin{aligned} 0 &= 1(0) + b \implies b = 0 \\ &\implies y = x \end{aligned}$$

Which is the equation of the tangent line.