

(Q4)

Proof. By earlier proof, $f(x)$ is differentiable. Thus,

$$f'(x) = 12x^3 + 12x^2 + 12x = 12x(x^2 + x + 1)$$

Since the discriminant of $x^2 + x + 1$ is less than 0, this expression has no real roots, thus the only root of $f'(x)$ is $x = 0$.

By Rolle's Theorem, this means that $f(x)$ has at most 2 roots.

We also have:

$$f(-2) = 3(2)^4 + 4(2)^3 + 6(2)^2 - 10 = 94$$

$$f(0) = -10$$

$$f(1) = 3$$

By the IVT, $\exists c \in (-2, 0)$ s.t. $f(c) = 0$, and the same for $(0, 1)$.

Thus, f has two zeroes. ■