

(Q3)

We can express $\left(1 + \frac{M}{x}\right)^{Nx}$ as $e^{Nx \cdot \ln(1 + \frac{M}{x})}$.

Then:

$$\begin{aligned}\lim_{x \rightarrow \infty} Nx \cdot \ln\left(1 + \frac{M}{x}\right) &= N \cdot \frac{\ln\left(1 + \frac{M}{x}\right)}{\frac{1}{x}} \\ &\stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} N \cdot \frac{\frac{\frac{1}{1 + \frac{M}{x}}} \cdot \frac{-M}{x^2}}{-\frac{1}{x^2}} \\ &= \lim_{x \rightarrow \infty} N \cdot \frac{M}{\frac{M+x}{x}} \\ &= \lim_{x \rightarrow \infty} N \cdot M \cdot \frac{x}{x+M} \\ &\stackrel{\text{L'H}}{=} \lim_{x \rightarrow \infty} N \cdot M \cdot \frac{1}{1} \\ &= \lim_{x \rightarrow \infty} M \cdot N\end{aligned}$$

$$\text{Thus, } \lim_{x \rightarrow \infty} \left(1 + \frac{M}{x}\right)^{Nx} = e^{MN}.$$