

(Q6)

Theorem 1. *Prove the following statement is true, by proving its contrapositive is true.*

"If $x + y \notin \mathbb{Q}$, then $x \notin \mathbb{Q}$ or $y \notin \mathbb{Q}$."

Proof. The given statement can be expressed mathematically as:

$$x + y \notin \mathbb{Q} \Rightarrow x \notin \mathbb{Q} \text{ or } y \notin \mathbb{Q}$$

The contrapositive of which is:

$$x \in \mathbb{Q} \text{ and } y \in \mathbb{Q} \Rightarrow x + y \in \mathbb{Q}$$

By the definition of rational numbers, we can assign values to x and y as such:

Let $x = \frac{a}{b}, y = \frac{c}{d}$, where $a, b, c, d \in \mathbb{Z}$ and $b, d \neq 0$.

Therefore,

$$\begin{aligned} x + y &= \frac{ad}{bd} + \frac{bc}{bd} \\ &= \frac{ad + bc}{bd} \end{aligned}$$

This holds because $b, d \neq 0$, thus $bd \neq 0$.

We assume that the set of integers is closed under addition and multiplication, thus ad , bc , and $ad + bc \in \mathbb{Z}$.

Therefore, $\frac{ad+bc}{bd}$ is a rational number, which means the contrapositive is true, and thus the original statement. ■