

(Q6)

First, we manipulate the polynomial into something that is easier to work with. Multiplying the polynomial by its conjugate:

$$\begin{aligned}\sqrt{x^6 - x^3} + x^3 &= \frac{\sqrt{x^6 - x^3} + x^3}{1} \cdot \frac{\sqrt{x^6 - x^3} - x^3}{\sqrt{x^6 - x^3} - x^3} \\ &= \frac{x^6 - x^3 - x^6}{\sqrt{x^6 - x^3} - x^3} \\ &= \frac{-x^3}{\sqrt{x^6} \sqrt{1 - \frac{1}{x^3}} - x^3}\end{aligned}$$

Since we are considering limits at negative infinity,

$$\sqrt{x^6} = |x^3| = -x^3 \text{ for } x < 0$$

It follows that:

$$\frac{-x^3}{-x^3 \sqrt{1 - \frac{1}{x^3}} - x^3} = \frac{1}{\sqrt{1 - \frac{1}{x^3}} + 1}$$

From which we can calculate the limit.

Since  $\lim_{x \rightarrow -\infty} \frac{1}{x^k} = 0$  for  $k > 0$ ,

$$\lim_{x \rightarrow -\infty} \sqrt{1 - 0} = \sqrt{\lim_{x \rightarrow -\infty} 1} = 1$$

From which it follows:

$$\begin{aligned}\lim_{x \rightarrow -\infty} \frac{1}{\sqrt{1 - \frac{1}{x^3}} + 1} &= \lim_{x \rightarrow -\infty} \frac{1}{\sqrt{1 - 0} + 1} \\ &= \lim_{x \rightarrow -\infty} \frac{1}{1 + 1} \\ &= \frac{1}{2}\end{aligned}$$