(Q2)

(a) $(f \circ g) = (\sin x)^3$

By the Chain Rule,

$$\frac{d}{dx}(\sin x)^3 = 3(\sin x)^2 \cdot \cos x$$

(b) $\left(\frac{f}{g}\right) = \frac{x^3}{\sin x}$.

By the Quotient Rule,

$$\begin{aligned}\frac{d}{dx}\left(\frac{x^3}{\sin x}\right) &= \frac{3x^2 \sin x - x^3 \cos x}{\sin^2 x} \\ &= \frac{3x^2 \sin x}{\sin^2 x} - \frac{x^3 \cos x}{\sin x} \\ &= \frac{3x^2}{\sin x} - \left(\frac{x^3}{\sin x} \cdot \frac{\cos x}{\sin x}\right) \\ &= 3x^2 \csc x - \left(x^3 \csc x \cdot \cot x\right) \\ &= x^2 \csc x(3 - x \cot x)\end{aligned}$$

(c) $(2f - (h \circ g)) = 2x^3 - e^{\sin x}$.

By the Chain Rule,

$$\frac{d}{dx}2x^3 - e^{\sin x} = 6x^2 - (e^{\sin x} \cos x)$$

(d) We observe that $(f \circ g \circ h)'(x) = f'(g(h(x))) \cdot \frac{d}{dx}g(h(x))$ via the Chain Rule. In turn, $\frac{d}{dx}g(h(x)) = g'(h(x)) \cdot h'(x)$. In all:

$$\frac{d}{dx}f(g(h(x))) = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$$

Applying this to the given functions:

$$\begin{aligned}\frac{d}{dx}(e^{sinx})^3 &= 3(e^{sinx})^2 \cdot e^{sinx} \cdot \cos x \\ &= 3(e^{\sin x})^3 \cdot \cos x\end{aligned}$$

(e) We observe, by the Product Rule:

$$\begin{aligned}(f \cdot g \cdot h)' &= f'(gh) + f(gh)' \\ &= f'gh + f(g'h + gh') \\ &= f'gh + fg'h + fgh'\end{aligned}$$

Applying this to the given functions:

$$\begin{aligned}\frac{d}{dx}\left(x^3 \sin x e^x\right) &= (3x^2 \cdot \sin x \cdot e^x) + (x^3 \cdot \cos x \cdot e^x) + (x^3 \cdot \sin x \cdot e^x) \\ &= x^2 e^x \left(3 \sin x + x \cdot \cos x + x \cdot \sin x\right) \\ &= e^x \left((3 + x)(\sin x) + x \cdot \cos x\right)\end{aligned}$$