Q1

(a)

We first manipulate the rational into something easier to work with:

$$\begin{aligned}\frac{x^2}{x^2+9} &= \frac{A}{x^2+9} + \frac{x^2+9}{x^2+9} \\ &= \frac{-9}{x^2+9}\end{aligned}$$

Using substitution, let $u = \frac{x}{3}, du = \frac{1}{3}dx$. Then

$$\begin{aligned}-3\int \frac{1}{\frac{x^2}{9}+1} \cdot \frac{1}{3} dx &= -3\int \frac{1}{u^2+1} du \\ &= -3\arctan(u) \\ &= -3\arctan(\frac{x}{3}) + c\end{aligned}$$

(b)

We manipulate the rational into something easier to work with:

$$\begin{aligned}\frac{x^2}{x^2+2x+2} &= \frac{A}{x^2+2x+2} + 1 \\ &= \frac{-2x-2}{x^2+2x+2} + 1\end{aligned}$$

Then

$$\begin{aligned}\int \frac{x^2}{x^2+2x+2} dx &= \int \left(\frac{-2x-2}{x^2+2xc+2} + 1 \right) dx \\ &= -\int \left((2x+2) \cdot \frac{1}{x^2+2xc+2} + 1 \right) dx\end{aligned}$$

Using the substitution $u = x^2+2x+2, du = (2x+2)dx$, then

$$\begin{aligned}-\int \frac{1}{u} du + \int 1 dx &= \ln|u| + x \\ &= -\ln|x^2+2x+2| + x + c\end{aligned}$$