

(Q4)

We observe that f is symmetrical about $x = 0$, this we can take advantage of this symmetry to simplify our expressions for upper bounds.

However, we need to consider separate cases for even n and odd n . As such, for even n :

$$U_{P_n} = \frac{2}{n} \left[2 \cdot \sum_{i=1}^{\frac{n}{2}} f\left(\frac{2i}{n}\right) \right]$$

$$L_{P_n} = \frac{2}{n} \left[2 \cdot \sum_{i=0}^{\frac{n}{2}} f\left(\frac{2i}{n}\right) \right]$$

And for odd n :

$$U_{P_n} = \frac{2}{n} \left[f\left(\frac{1}{n}\right) + 2 \cdot \sum_{i=1}^{\frac{n-1}{2}} f\left(\frac{2i+1}{n}\right) \right]$$

$$L_{P_n} = \frac{2}{n} \left[f\left(\frac{1}{n}\right) + 2 \cdot \sum_{i=1}^{\frac{n-1}{2}} f\left(\frac{2i-1}{n}\right) \right]$$