(Q2)

Theorem 1. *Prove that if* $|2x+2| < 4$*, then* $|x^3+1| < 26$*.*

Proof. We can factorize the inequality as follows:

$$|2x+2| < 4 \implies 2|x+1| < 4 \implies |x+1| < 2$$

Thus, it follows that:

$$\begin{aligned} |x+1| < 2 &\implies -2 < x+1 < 2 \\ &\implies -3 < x < 1 \\ &\implies -27 < x^3 < 1 \\ &\implies -26 < x^2+1 < 2 \\ &\implies |x^3+1| < 26 \end{aligned}$$

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