(Q3)

Proof. We know by the Intermediate Value Theorem (IVT) that if a function f:

- is continuous, and
- there exists $a, b, y \in \mathbb{R}$ where $f(a) < f(b)$ and $y \in (f(a), f(b))$,

then there exists $c \in \mathbb{R}$ where $f(c) = y$.
Consdering the given equation $x^3 - x\cos x = 10$, we define a function $g(x) = x^3 - x\cos x - 10$, and we aim to show that $\exists c \in \mathbb{R}$ such that $g(c) = 0$.
Since x^3, $x\cos x$ and -10 are all continuous, $g(x)$ is continuous by limit laws.
Considering $g(2)$ and $g(2.5)$:

$$\begin{aligned} g(2) &= 8 - 2\cos 2 - 10 \\ &= -2 - 2\cos 2 \\ g(2.5) &= 15.625 - 2.5\cos 2.5 - 10 \\ &= 5.625 - 2.5\cos 2.5 \end{aligned}$$

Since cos is bounded within $[-1, 1]$, $2\cos 2 \in (-2, 2)$, and thus $-2 - 2\cos 2 < 0$. On the other hand, $2.5\cos 2.5 < 0$ since $2.5 > \frac{\pi}{2}$, so $5.625 - 2.5\cos 2.5 > 0$.
Since $g(x)$ is continuous and $g(2) < 0$ while $g(2.5) > 0$, by IVT we can conclude that $\exists c \in (2, 2.5) \subseteq \mathbb{R}$ such that $g(c) = 0$; in other words, that $g(x)$ has a solution. ■

Since the solution is within the interval $(2, 2.5)$, the closest integer is 2.