Q4

(a) Using integration by parts, we assign

$$u = t, \quad du = 1\,dx, \quad dv = e^{-t}\,dt, \quad v = -e^{-t}$$

$$\int te^{-t}\,dt = -te^{-t} + \int e^{-t}\,dx = -te^{-t} - e^{-t} + c$$

(b) We use a substitution, $u = \sqrt{x}, du = \frac{1}{2\sqrt{x}\,dx}$. Then

$$2\int -ue^{-u}\,du = -u^{-u} - e^{-u} + c$$

(c) By properties of absolute value, the function is even, so the two integrals are equal.
(d) Since the function is even, it suffices to determine whether one side of the doubly improper integral converges or diverges.

$$\int_0^\infty e^{-\sqrt{|x|}} = \lim_{x\to\infty} 2(-\sqrt{|x|}e^{-\sqrt{|x|}} - e^{-\sqrt{|x|}})\Big|_0^b$$

Since there is a defined limit, the integral converges.