(Q5)

(a)

Since f is defined on a closed interval, this follows from the Extreme Value Theorem.

(b)

We know that maximum and minimum values occur when $f'(x) = 0$. Therefore:

$$f'(x) = \frac{7}{3}x^{\frac{4}{3}} - \frac{7}{3}x^{-\frac{2}{3}}$$

$$\begin{aligned} f'(x) = 0 &\implies \frac{7}{3}x^{\frac{4}{3}} = \frac{7}{3}x^{-\frac{2}{3}} \\ &\implies x^{\frac{4}{3}} = x^{-\frac{2}{3}} \\ &\implies x^{\frac{4}{3}} = \frac{1}{x^{\frac{2}{3}}} \\ &\implies x^{\frac{6}{3}} = x^2 = 1 \\ &\implies x = \pm 1 \end{aligned}$$

Therefore, f has maximum and/or minimum points at -1 and 1. Then,

$$f''(x) = \frac{29}{9}x^{\frac{1}{3}} - \frac{14}{9}x^{-\frac{5}{3}}$$

$f''(-1) < 0$, so f is concave down at that point and thus $x = -1$ is a maximum point, and $f''(1) > 0$, so f is concave up at that point, and thus $x = 1$ is a minimum point.