(Q7)
(a) True.

Proof. Formally speaking, what we are trying to prove is that:

$$\begin{gathered}\forall\varepsilon > 0,\ \exists\delta_1 > 0 \text{ s.t. } \forall x \in \mathbb{R}, \\ 0 < |x-a| < \delta_1 \implies |f(x)| < \varepsilon \quad (1) \\ \text{if and only if} \\ \forall\varepsilon > 0,\ \exists\delta_2 > 0 \text{ s.t. } \forall x \in \mathbb{R}, \\ 0 < |x-a| < \delta_2 \implies ||f(x)|| < \varepsilon \quad (2)\end{gathered}$$

In order to prove that (1) $\iff$ (2), we have to prove that (1) $\implies$ (2) and (2) $\implies$ (1). Assuming (1), let ε be arbitrary and fix $\delta_2 = \delta_1$. We observe that $||f(x)|| = |f(x)|$. Thus,

$$0 < |x-a| < \delta_1 \implies |f(x)| = ||f(x)||$$

Which proves (1) $\implies$ (2).
Assuming (2), let ε be arbitrary and fix $\delta_1 = \delta_2$. Using the same observation,

$$0 < |x-a| < \delta_2 \implies ||f(x)|| = |f(x)|$$

Which proves (2) $\implies$ (1).
Since we have proven that (1) $\implies$ (2) and (2) $\implies$ (1), we can conclude that (1) $\iff$ (2), as required.

■

(b) False.
As a counterexample, consider the function:

$$f(x) = \begin{cases} 1 \text{ if } x < a \\ -1 \text{ if } x > a \end{cases}$$

$f(x)$ has no limit at a, while $|f(x)|$ does. In other words,

$$\lim_{x\to a} |f(x)| = 1 \not\Rightarrow \lim_{x\to a} f(x) = 1$$