

(Q6)

Theorem 1. *Prove that if $\lim_{x \rightarrow 0} x^2 f(x) = 2022$, then $\lim_{x \rightarrow 0} f(x)$ does not exist.*

Proof. We assume that $\lim_{x \rightarrow 0} x^2 = 0$.

For the sake of contradiction, assume that $\lim_{x \rightarrow 0} f(x)$ exists. Since both functions have limits that exist, we can make use of Limit Laws.

By the Limit Law of Products, $\lim_{x \rightarrow 0} x^2 f(x) = \lim_{x \rightarrow 0} x^2 \cdot \lim_{x \rightarrow 0} f(x)$. Thus,

$$\lim_{x \rightarrow 0} x^2 f(x) = 0 \cdot \lim_{x \rightarrow 0} f(x) = 0$$

Which is not possible as $\lim_{x \rightarrow 0} x^2 f(x) = 2022$. Thus, $\lim_{x \rightarrow 0} f(x)$ does not exist. ■