(Q8)

(a) Let $x, y \in [a, b]$ and $x < y$. We observe the following:

$$\exists\, t \in [x, y] \;:\; t \in \mathbb{Q}$$
$$\exists\, s \in [x, y] \;:\; s \notin \mathbb{Q}$$

Therefore, $\inf f$ on any subinterval of $[a, b]$ on f is always 2022. In addition, $\sup f$ on any subinterval of $[a, b]$ on f is always 2023.
It follows that for any partition P of $[a, b]$:

$$L_P(f) = 2022(b - a), \quad U_P(f) = 2023(b - a)$$

(b)

Proof. Since $L_P(f) = 2022(b - a)$ and $U_P(f) = 2023(b - a)$ for any partition P of $[a, b]$ on f, we have:

$$\overline{I}\,{}_a^b(f) = \inf\{2023(b - a)\} = 2023(b - a)$$
$$\underline{I}\,{}_a^b(f) = \sup\{2022(b - a)\} = 2022(b - a)$$

Since $a < b$, $b - a > 0$ and thus $2023(b - a) > 2022(b - a)$.
Therefore, the upper and lower integrals are not equal and f is not integrable. ■