(Q2)
By limit laws, we know that:

$$\lim_{x\to a} f(x) = L \text{ and } \lim_{x\to a} g(x) = M \implies \lim_{x\to a} f(x) + g(x) = L + M$$
$$\lim_{x\to a} f(x) = L \text{ and } \lim_{x\to a} g(x) = M \implies \lim_{x\to a} f(x) \cdot g(x) = L \cdot M$$

Since $\lim_{x\to\infty} \frac{1}{x} = 0$, we can safely remove any operation involving adding a rational term.

(a)
First, some manipulations are in order:

$$\frac{2x^2 - 16x + 1}{x^3 + 137x + 2022} = \frac{x^2(2 - \frac{16}{x} + \frac{1}{x^2})}{x^2(x + \frac{137}{x} + \frac{2022}{x^2})}$$
$$= \frac{2 - \frac{16}{x} + \frac{1}{x^2}}{x + \frac{137}{x} + \frac{2022}{x^2}}$$

Removing the rational terms we get:

$$\lim_{x\to\infty} \frac{2 - \frac{16}{x} + \frac{1}{x^2}}{x + \frac{137}{x} + \frac{2022}{x^2}} = \lim_{x\to\infty} \frac{2}{x} = \lim_{x\to\infty} 2 \cdot \lim_{x\to\infty} \frac{1}{x}$$

By the limit laws,

$$\lim_{x\to\infty} 2 \cdot \lim_{x\to\infty} \frac{1}{x} = 2 \cdot 0 = 0$$

(b)
Similar manipulations are in order:

$$\frac{137x^5 - 10000x + 12}{2022x^5 + 20x^4 - x + 1} = \frac{x^5(137 - \frac{100000}{x^4} + \frac{12}{x^5})}{x^5(2022 + \frac{20}{x} - \frac{1}{x^4} + \frac{1}{x^5})}$$
$$= \frac{137 - \frac{100000}{x^4} + \frac{12}{x^5}}{2022 + \frac{20}{x} - \frac{1}{x^4} + \frac{1}{x^5}}$$

Removing the rational terms:

$$\lim_{x\to\infty} \frac{137 - \frac{100000}{x^4} + \frac{12}{x^5}}{2022 + \frac{20}{x} - \frac{1}{x^4} + \frac{1}{x^5}} = \frac{137}{2022}$$