(Q1) The formula is:

$$f^{(n)}(x) = \frac{3^n \cdot n!}{(5-3x)^{n+1}}$$

Proof. Since we are only considering $n \in \mathbb{N}$, we prove this through induction. To make operations easier to see, we write $f(x)$ as $(5-3x)^{-1}$.
Base Case.
Let $n = 1$. By application of the chain rule:

$$\begin{aligned}
f^{(1)}(x) &= -1 \cdot (5-3x)^{-2} \cdot \frac{d}{dx}(5-3x) \\
&= -1 \cdot (5-3x)^{-2} \cdot -3 \\
&= 3(5-3x)^{-2} \\
&= \frac{3}{(5-3x)^2} = \frac{3^1 \cdot 1!}{(5-3x)^{1+1}}
\end{aligned}$$

Thus, the base case holds.
Induction Step.
In this step, we aim to prove the following induction hypothesis:

$$f^{(n)}(x) = \frac{3^n(n!)}{(5-3x)^{n+1}} \implies f^{(n+1)}(x) = \frac{3^{n+1} \cdot (n+1)!}{(5-3x)^{(n+1)+1}}$$

We begin by simplifying the right hand side of the implication and rewriting it:

$$\begin{aligned}
f^{(n+1)}(x) &= \frac{3^{n+1} \cdot (n+1)!}{(5-3x)^{(n+1)+1}} \\
&= \frac{3^n \cdot 3 \cdot n! \cdot (n+1)}{(5-3x)^n \cdot (5-3x)} \\
&= \frac{3^n(n!)}{(5-3x)^{n+1}} \cdot 3 \cdot (n+1) \cdot (5-3x)^{-1}
\end{aligned}$$

We observe that $f^{(n+1)}(x)$ is simply $f^{(n)}(x)$ multiplied by $[3 \cdot (n+1) \cdot (5-3x)^{-1}]$. Therefore, if the derivative of $f^{(n)}(x)$ has the same result, the induction hypothesis is proved. Differentiating $f^{(n)}(x)$ via chain rule (n is a constant and is not affected):

$$\begin{aligned}
\frac{d}{dx}3^n(n!)(5-3x)^{-(n+1)} &= 3^n(n!)(5-3x)^{-(n+1)-1} \cdot -(n+1) \cdot -3 \\
&= 3^n(n!)(5-3x)^{-(n+1)-1} \cdot (n+1) \cdot 3 \text{ (minus signs cancel)} \\
&= 3^n(n!)(5-3x)^{-(n+1)} \cdot \left[(5-3x)^{-1} \cdot (n+1) \cdot 3\right]
\end{aligned}$$

We observe that these additional terms are the exact same ones we multiplied to $f^{(n)}(x)$ to obtain $f^{(n+1)}(x)$, thereby proving the induction hypothesis.
Since both the base case and induction hypothesis hold, we have proven the theorem, as required. ■