(Q2)

Theorem 1. *Let A, B be sets. Prove that if $(A \cap B)^c = A^c$ then $A \setminus B = \phi$.*

Proof. We can prove this by contradiction.
We know that $A \setminus B = \phi$ iff $A \setminus B \subseteq \phi$ and $\phi \subseteq A \setminus B$.
Since $\phi \subseteq A \setminus B$ by its own definition, we only need to prove that $A \setminus B \subseteq \phi$.
For the sake of contradiction, we assume that $A \setminus B \nsubseteq \phi$. This results in the implication:

$$\exists x \text{ s.t. } x \in A \text{ and } x \notin B \tag{1}$$

This implies $x \notin A^c$. Assuming $(A \cap B)^c = A^c$ as given, we can rewrite this as:

$$x \notin (A \cap B)^c \implies x \in (A \cap B)$$

Which implies that $x \in A$ and $x \in B$. However, this is not possible as it contradicts the earlier statement $x \in A$ and $x \notin B$.
Therefore, $A \setminus B \subseteq \phi$, which combined with $\phi \subseteq A \setminus B$, allows us to conclude that $A \setminus B = \phi$.

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