(Q5)

(a)

$$\forall \varepsilon > 0,\ \exists M \in \mathbb{R} \text{ s.t. } \forall x \in \mathbb{R},\ x < M \implies |f(x) - L| < \varepsilon$$

(b)

Proof. Formally, the implication we are trying to prove is:

$$\forall \varepsilon > 0,\ \exists M \in \mathbb{R} \text{ s.t. } \forall x \in \mathbb{R},\ x < M \implies |e^x| < \varepsilon$$

Fix ε as arbitrary. Let $M = \ln \varepsilon$. It follows:

$$\begin{aligned} x < \ln \varepsilon &\implies e^x < e^{\ln \varepsilon} \text{ (since } a < b \implies e^a < e^b\text{)} \\ &\implies e^x < \varepsilon \\ &\implies |e^x| < \varepsilon \text{ (since } e^x > 0 \text{ for all } x \in \mathbb{R}\text{)} \end{aligned}$$

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