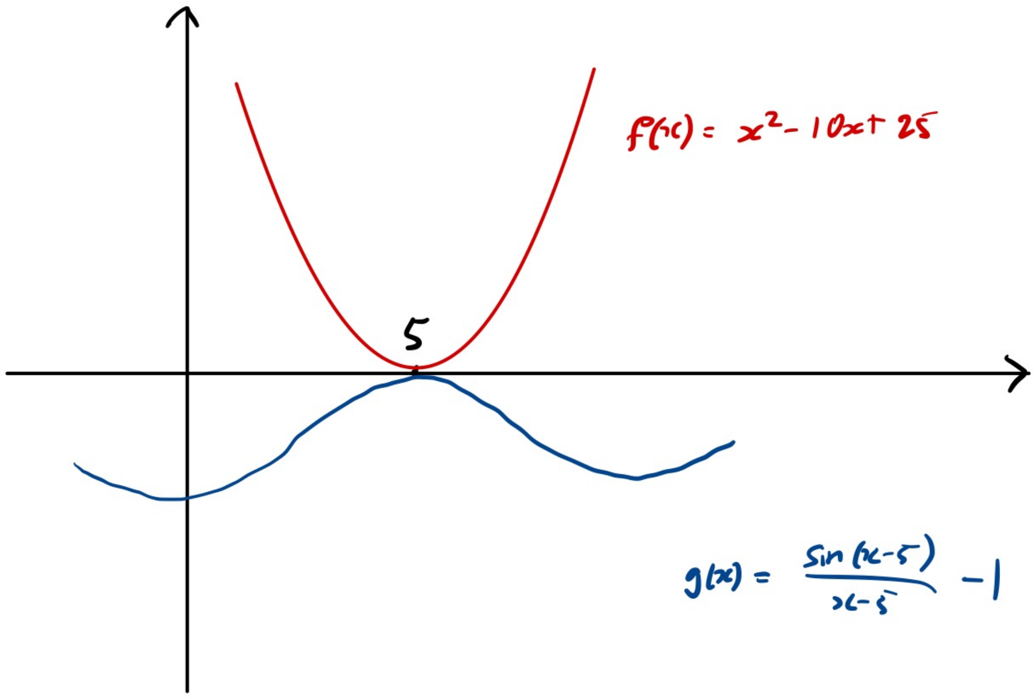
(Q3)

(a)



(b)

Proof. In order to prove that $f(x)$ kisses $g(x)$ at $x = 0$, we need to prove that

1. $\forall x \neq a,\ f(x) > g(x)$ or $f(x) < g(x)$
2. $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$.

By limit laws,

$$\lim_{x \to 0} x^2 + 1 = \lim_{x \to 0} x^2 + \lim_{x \to 0} 1 = 0 + 1 = 1$$

By earlier proof, we also know:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 = \lim_{x \to 0} x^2 + 1$$

Which satisfies Condition 2.
To prove Condition 1, we can use without proof that $\forall x \in \mathbb{R}\, |\sin x| \leq |x|$. Then $\forall x \neq 0$:

$$\begin{aligned} \frac{|\sin x|}{|x|} &\leq 1 \\ \implies \left|\frac{\sin x}{x}\right| &\leq 1 \\ \implies -1 \leq \frac{\sin x}{x} &\leq 1 \end{aligned}$$

Considering x^2:

$$x^2 \geq 0 \implies x^2 + 1 \geq 1$$

Since $g(0) = 1$, $g(x) > 1$ for all $x \neq 0$. Finally, for all $x \neq 0$,

$$\frac{\sin x}{x} \leq 1 < x^2 + 1 \implies \frac{\sin x}{x} < x^2 + 1$$

Which satisfies Condition 1.
Since both conditions are satisfied, we have proven that $f(x)$ kisses $g(x)$ at $x = 0$, as required. ■