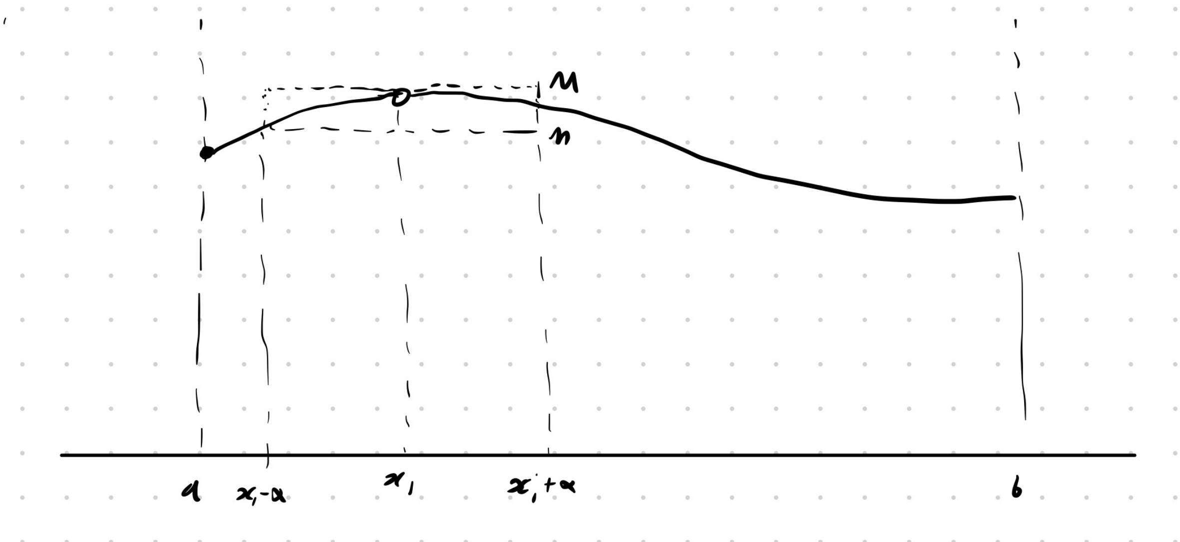
(Q3)

Proof. *(a)(i)*
We can sketch such a partition as shown:



(a)(ii) We observe that $\Delta x = 2\alpha$. Then we want to show:

$$M\Delta x - m\Delta x = 2\alpha(M - m) \leq \frac{\varepsilon}{3}$$

Since M and m are dependent on α, set α such that $\alpha = \min\{M - m \leq \frac{1}{3}, \frac{\varepsilon}{2}\}$.
Then we have

$$2\alpha(M - m) \leq \frac{1}{3} \cdot \varepsilon = \frac{\varepsilon}{3}$$

(a)(iii) Since f is continuous on $[a, x_1 - \alpha]$ and $[x_1 + \alpha, b]$, it is integrable and thus fulfils the ε-characterization of integrability:

$$\exists P_1 \,:\, U_{P_1} - L_{P_1} < \frac{\varepsilon}{3}$$
$$\exists P_2 \,:\, U_{P_2} - L_{P_2} < \frac{\varepsilon}{3}$$

(a)(iv) Hence, we have:

$$2\alpha(M - m) + U_{P_1} - L_{P_1} + U_{P_2} - L_{P_2} < \varepsilon$$

Which fulfils the ε-characterization of integrability and thus satisfies the base case.
(b)(i) Again, since f is continuous on $[a, x_N - \alpha]$ and $[x_N + \alpha, b]$, it fulfills the ε-characterization of integrability.
(b)(ii) We refer to $(a)(ii)$ to confirm that this holds.
(b)(iii) Thus, for arbitrary N, we have:

$$2\alpha(M - m) + U_{P_1} - L_{P_1} + U_{P_2} - L_{P_2} < \varepsilon$$

And so the induction hypothesis holds.

■