

(Q6)

Proof. We aim to prove the implication:

$$f(-x) = f(x) \implies f'(-x) = -f'(x)$$

Assuming that f is even (i.e. $f(x) = f(-x)$), it then follows:

$$\begin{aligned} \frac{d}{dx}f(x) &= \frac{d}{dx}f(-x) \implies f'(x) = -1 \cdot f'(-x) \text{ (by Chain Rule)} \\ &\implies f'(x) = -f'(-x) \\ &\implies -f'(x) = f'(-x) \end{aligned}$$

By the given definition of a odd function, we have proven that if f is even, then f' is odd, as required. ■