

Q2

(a) Using the substitution $u = t^2$, $du = 2t dt$:

$$\begin{aligned}\int t \arctan(t^2) dt &= \frac{1}{2} \int \arctan(u) du \\ &= \frac{1}{2} \left(u \arctan(u) - \frac{1}{2} \ln |u^2 + 1| \right) + c \\ &= \frac{1}{2} \left(t^2 \arctan(t^2) - \frac{1}{2} \ln |t^4 + 1| \right) + c\end{aligned}$$

The antiderivative of $\arctan t$ is given by earlier proof.

(b) Using integration by parts:

$$u = u, \quad du = 1, \quad dv = f''(u), \quad v = f'(u)$$

Then

$$\int_0^1 u f''(u) = u f'(u) - f(u) \Big|_0^1 = -6$$