

(Q8)

(a) True.

Proof. From the limit definition of the derivative:

$$\begin{aligned}\frac{d}{dx}(f(x) - g(x)) &= \lim_{x \rightarrow a} \frac{(f(x) - g(x)) - (f(a) - g(a))}{x - a} \\ &= \lim_{x \rightarrow a} \frac{f(x) - f(a) - g(x) + g(a)}{x - a} \\ &= \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} + \lim_{x \rightarrow a} \frac{g(x) - g(a)}{x - a}\end{aligned}$$

Therefore, since the limit of $(f - g)$ exists, these limits, which are the derivatives of f and g , have to exist, and thus f and g are differentiable. ■

(b) False.

As a counterexample, consider $f(x) = 1$, and $g(x) = |x|$. It follows:

$$\frac{f(x)}{g(x)} = \frac{1}{|x|}$$

Which is differentiable on its domain. Furthermore, f is differentiable on its domain, but g is not.

(c) False.

As a counterexample, consider the function:

$$f(x) = \begin{cases} 0 & \text{if } x \in \mathbb{Q} \\ 1 & \text{if } x \notin \mathbb{Q} \end{cases}$$

$f \circ f$ is differentiable, but f is not.

(d) True.

Proof. Since f is differentiable, this implies f is continuous. Since continuity is preserved across composition (provided both functions are continuous), $f \circ f$ is continuous. ■