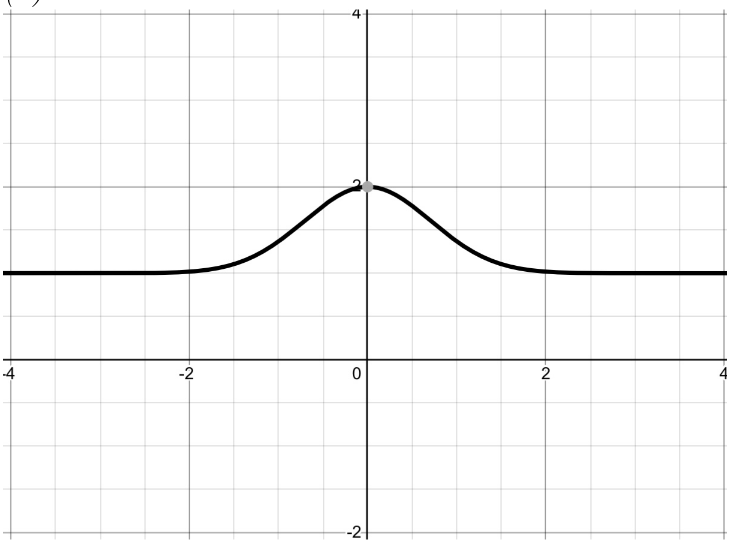
Q5

(a)



(b)

Proof. By properties of integrals, we observe

$$\begin{aligned}\int_0^x (e^{-t^2}+1)\,dt &= \int_0^x (e^{-t^2})\,dt + \int_0^x 1\,dt\\ &= \int_0^x (e^{-t^2}+1)\,dt + x\end{aligned}$$

Since $e^{-t^2} > 0$ for all t, $\int_0^x (e^{-t^2}+1)\,dt > x$ for $x \geq 0$.
Then, $e^{-t^2} \leq 1 \implies e^{-t^2}+1 \leq 2 \implies \int_0^x (e^{-t^2}+1)\,dt \leq 2x$, which leaves us with

$$x \leq F(x) \leq 2x$$

$\blacksquare$

(c)

Proof. By properties of integrals, we have

$$\int_0^x (e^{-t^2}+1)\,dt = -\int_x^0 (e^{-t^2}+1)\,dt$$

Thus for $x \leq 0$, this is simply the inverse of *(b)*. $\blacksquare$

(d)

Proof. By (b) and (c) and the Squeeze Theorem, for $x \geq 0$:

$$x \leq F(x) \leq 2x \implies \lim_{x\to\infty} F(x) = \infty$$

This also holds for the opposite case, $x \leq 0$. $\blacksquare$

(e) True.