

(Q2)

(a)(i)

Proof. Since $f \circ g$ is invertible, it must be bijective and thus injective and surjective. By the definition of surjectivity of $f \circ g$:

$$\forall y \in Z, \exists x \in X \text{ s.t. } f(g(x)) = y$$

Since by this definition, f maps from every member in $g(X)$ to its codomain Z , it must be surjective. ■

(a)(ii)

Proof. By earlier definition, $f \circ g$ is injective and surjective. Aiming for a contradiction, suppose that g is not injective. Thus:

$$\exists x_1, x_2 \in X \text{ s.t. } g(x_1) = g(x_2) \text{ and } x_1 \neq x_2$$

Then $g(x_1) = g(x_2) \implies f(g(x_1)) = f(g(x_2))$. However, simultaneously, $x_1 \neq x_2$. Thus, we have

$$f(g(x_1)) = f(g(x_2)) \text{ and } x_1 \neq x_2$$

Which suggests $f \circ g$ is not injective.

This is a contradiction, and thus g must be injective. ■

(b)

Let $g: \mathbb{R} \rightarrow \mathbb{R}$ be given by $g(x) = e^x$, and $f: \mathbb{R} \rightarrow (0, \infty)$ be given by $f(x) = x^2$. Thus, $f \circ g: \mathbb{R} \rightarrow (0, \infty)$ is given by $f(g(x)) = (e^x)^2 = e^{2x}$, which is bijective and is therefore invertible.