(Q4)

Theorem 1. *Prove that* $\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = 6$ *using the* $\delta - \varepsilon$ *definition of the limit.*

Proof. We observe that

$$\frac{x^2 - 9}{x - 3} = \frac{(x + 3)(x - 3)}{(x - 3)} = x + 3$$

Since we are working with limits, we can remove common factors with impunity as we do not risk dividing by 0.
The $\delta - \varepsilon$ definition of the limit for this function is

$$\forall \varepsilon > 0, \exists \delta > 0 \text{ s.t. } \forall x \in \mathbb{R},\ 0 < |x - 3| < \delta \implies |(x + 3) - 6| < \varepsilon$$

We can simplify the right hand side of this implication to $|x - 3| < \varepsilon$.
Let $\delta = \varepsilon$. It follows that:

$$0 < |x - 3| < \delta \implies 0 < |x - 3| < \varepsilon \implies |x - 3| < \varepsilon$$

$\blacksquare$