(Q5)
(a)

Proof. By definition of symmetry, an entry in row i column j should equal the entry in row j column i.
This is a result of the Axiom of Commutativity. ■

(b)

Proof. Given the layout of the multiplication table, this means that

$$\forall a \in \mathbb{F}, (a \cdot 0) = (0 \cdot a) = 0$$

We can prove this as follows:

$$\begin{aligned}
\text{(Add. Iden.)} \quad a \cdot 0 &= (a \cdot 0) + 0 \\
\text{(Add. Inv.)} \quad &= (a \cdot 0) + (a \cdot 0 + (-a) \cdot 0) \\
\text{(Assoc.)} \quad &= (a \cdot 0 + a \cdot 0) + ((-a) \cdot 0) \\
\text{(Dist.)} \quad &= a \cdot (0 + 0) + (-a) \cdot 0 \\
\text{(Add. Iden.)} \quad &= (0 \cdot a) + (0 \cdot (-a)) \\
\text{(Add. Inv.)} \quad &= 0
\end{aligned}$$

■

(c) This is a result of the uniqueness of each field element:

$$a + b = c + b \implies a = c$$

Proof. For some $a, b, c \in \mathbb{F}$, we have:

$$\begin{aligned}
a + b = c + b &\implies a + b + (-b) = c + b + (-b) \\
&\implies a + 0 = c + 0 \\
&\implies a = c
\end{aligned}$$

■

(d) This is also a result of the uniqueness of each field element:

$$a \cdot c = b \cdot c \implies a = b$$

Proof. For some $a, b, c \in \mathbb{F} \setminus \{0\}$, we have:

$$\begin{aligned}
a \cdot b = c \cdot b &\implies a \cdot b \cdot b^{-1} = c \cdot b \cdot b^{-1} \\
&\implies a \cdot 1 = c \cdot 1 \\
&\implies a = c
\end{aligned}$$

■

(e) This set is not a field, as the properties addressed in *(a)*, *(c)* and *(d)* are violated.