(Q11)
(a) True.

Proof. Let $A \sim B$. Then $A = PBP^{-1}$, where $P \in \mathcal{M}_{n\times n}$. Then

$$A^k = (PBP^{-1})^k = \underbrace{(PBP^{-1})(PBP^{-1})\dots(PBP^{-1})}_{k \text{ times}}$$

Since between each occurrence of B, we have $P^{-1}P$, they cancel into I_n which then disappears, leaving us with

$$P\underbrace{BB\dots B}_{k \text{ times}}P^{-1} = PB^kP^{-1}$$

So $A^k = PB^kP^{-1} \iff A^k \sim B^k$. ■

(b) False.
As a counterexample, let

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$A^2 = B^2$, but A and B are clearly not similar.
(c) False.
As a counterexample, let

$$A = \begin{pmatrix} 5 & 7 \\ 2 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\det(AB) = 1$, but A and B are clearly not inverses of each other.