

**(Q10)**

(a) False.

As a counterexample, let  $a = -a$  and  $b = -b$ . Then  $a + a = 0 = b + b$ , while  $a$  and  $b$  are not necessarily equal.

(b) False.

Let  $\mathbb{F}$  be a field with the following addition table:

+	0	1	a	b
0	0	1	a	b
1	1	0	b	a
a	a	b	0	1
b	b	a	1	0

In this field,  $\text{char}(\mathbb{F}) = 2$ , which is prime, but  $\mathbb{F}$  is not  $\mathbb{Z}_2$ .

(c) False.

Let  $p(x) = (x^2 + 1)^2 = x^4 + 2x^2 + 1$  in  $\mathbb{R}$ .  $p$  is reducible, but has no solutions in  $\mathbb{R}$ .

(d) True.

*Proof.* Suppose for the sake of contradiction that  $p$  is irreducible and  $p$  has solutions.

Then  $\exists a : p(x) = (x - a)q(x)$ , where  $q$  is another polynomial.

Then  $\deg p > 1 \implies \deg q \geq 1$ , which implies that  $p$  is in fact reducible, thus forming a contradiction. ■