

(Q8)

(a) $\{(x, mx) | x \in \mathbb{R}\}$ and $\{(x, -\frac{1}{m}x) | x \in \mathbb{R}\}$.

$y = -\frac{1}{m}x$ is orthogonal to $y = mx$, so any reflection across the latter of any element in $y = -\frac{1}{m}x$ would still lie on the same line.

(b)

Proof. By earlier proof, T is an isomorphism, so $N(T) = \{0_{\mathbb{R}^3}\}$. Then $T(N(T)) = N(T)$ by linearity, so $N(T)$ is T -invariant.

Since T is isomorphic,

$$\text{rank } T = \dim T = \dim(\text{im } T) = n \implies \text{im } T = \mathbb{R}^3 \subseteq \mathbb{R}^3$$

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