

(Q1)

Theorem. Let \mathbb{F} be a field. Then if $a \in \mathbb{F} \setminus \{0\}$, then a^{-1} is invertible and $(a^{-1})^{-1} = a$.

Proof. Since $a \neq 0_{\mathbb{F}}$, a^{-1} exists in \mathbb{F} . Then:

$$\begin{aligned} a &= 1 \cdot a \\ &= [(a^{-1})^{-1} \cdot a^{-1}] \cdot a \\ &= (a^{-1})^{-1} \cdot a^{-1} \cdot a \\ &= (a^{-1})^{-1} \cdot 1_{\mathbb{F}} \\ &= (a^{-1})^{-1} \end{aligned}$$

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