

**(Q2)**

(a)

$$x + 0y - 2z + w = 0$$

$$x + y - z + 0w = 0$$

$$2x + y - 3z + w = 0$$

(b)

$$\left( \begin{array}{cccc|c} 1 & 0 & 2 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 2 & 1 & -3 & 1 & 0 \end{array} \right)$$

$\downarrow$

$$\left( \begin{array}{cccc|c} 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 & 0 \end{array} \right)$$

$\downarrow$

$$\left( \begin{array}{cccc|c} 1 & 0 & -2 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

We then have

$$x = 2s - t$$

$$y = -s + t$$

$$z = s$$

$$w = t$$

And thus

$$\begin{aligned} & (2s - t, -s + t, s, t) \\ &= (2s, -s, s, 0) + (-t, t, 0, t) \\ &= s(2, -1, 1, 0) + t(-1, 1, 0, 1) \end{aligned}$$

(c)

This follows from the fact that  $(2, -1, 1, 0)$  and  $(-1, 1, 0, 1)$  are linearly independent, and thus is a basis for  $W$ .

(d)

$W$  is closed under scaling and vector addition, so it is a subspace.