Q(4)

Theorem. Let V be a finite dimensional vector space over a field $\mathbb{F}$ and $W \subseteq V$ a supspace. Then $\dim W = \dim V \iff W = V$.

Proof. Suppose $V = W$. Then $\dim V = \dim W$.
Now suppose $\dim V = \dim W = n$. Let the set $A = \{x_1, x_2, \ldots, x_n\}$ be a basis for W. Since A is linearly independent in W, it is also linearly independent in V. Then since $\dim V = \dim W$, A is also a basis for V.
Since V and W have the same basis, $V = W$. ■