

(Q5)

(a) Each vector \mathbf{a}_i in γ is given by $A\mathbf{e}_i$, so

$$A = [A\mathbf{e}_1, \dots, A\mathbf{e}_n] = [T_A(\mathbf{e}_1), \dots, T_A(\mathbf{e}_n)] = [I_{\mathbb{F}^n}]_{\gamma}^{\beta}$$

Since A is a change of basis matrix, it must be invertible.

(b) (i) Since the columns of A are given by the elements of γ :

$$A_i = \mathbf{a}_i = T_A(\beta_i)$$

any vector in $\text{im } T_A$ can be expressed as a linear combination of the columns of A , which is γ .