(Q8)

(a)

Let $A \in \mathbf{Sym}_n(\mathbb{F})$. Then A can be expressed in the form

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{12} & \dots & \dots & \dots \\ \dots & & & \\ a_{1n} & & & a_{nn} \end{pmatrix}$$

Then cA for some $c \in \mathbb{F}$ will evaluate to:

$$\begin{pmatrix} ca_{11} & ca_{12} & \dots & ca_{1n} \\ ca_{12} & \dots & \dots & \dots \\ \dots & & & \\ ca_{1n} & & & ca_{nn} \end{pmatrix}$$

Which is still symmetrical.

Similarly, $A + B$ where $B \in \mathbf{Sym}_n(\mathbb{F})$ evaluates to:

$$\begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \dots & a_{1n} + b_{1n} \\ a_{12} + b_{12} & \dots & \dots & \dots \\ \dots & & & \\ a_{1n} + b_{1n} & & & a_{nn} + b_{nn} \end{pmatrix}$$

Which is also symmetrical.

(b)

Extending from Week 4's lecture activities on skew-symmetric matrices, we can let the basis for $\mathbf{Sym}_n(\mathbb{F})$ be

$$\{E_{ij} + E_{ji} \; : \; i < j\} \cup \{E_{ij} \; : \; i = j\}$$

(c)

Again extending from week 4's activities, $\dim \mathbf{Sym}_n(\mathbb{F}) = \frac{n(n-1)}{2} + n$.