(Q3)

(a)

Proof. Since W is the set of solutions, then $\forall w \in W,\ w = \sum_{i=1}^{k} t_i x_i$.
Let $S = \{\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_k\}$.
W represents every linear combination of the base vectors $\mathbf{x}_1 \ldots \mathbf{x}_k$, so $W = \operatorname{span} S$. ■

(b)

Proof. For the sake of contradiction, assume that

$$\sum_{i=i}^{k} t_i x_i = \mathbf{0} \text{ and } \exists t_j \ :\ t_j \neq 0$$

If there is only one such t, then that t has to be zero.
If there is more than one, then S no longer spans w.
Thus, S has to be linearly independent. ■

(c)

Since $\{\mathbf{x}_1, \mathbf{x}_2, \ldots \mathbf{x}_k\}$ is linearly independent and spans W is is a basis for W.