(Q1)
(a)

Proof. We define the two multiplicands as follows:

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ a_{m1} & \dots & \dots & a_{mn} \end{pmatrix} \begin{pmatrix} 0_1 \\ 0_2 \\ \vdots \\ 1_j \\ 0_{j+1} \\ \vdots \\ 0_n \end{pmatrix}$$

By definition of matrix multiplication, this would evaluate to:

$$\begin{pmatrix} 0+0+\ldots+a_{1j}+\ldots+0 \\ 0+0+\ldots+a_{2j}+\ldots+0 \\ 0+0+\ldots+a_{3j}+\ldots+0 \\ \vdots 0+0+\ldots+a_{nj}+\ldots+0 \end{pmatrix} = \begin{pmatrix} a_{1j} \\ a_{2j} \\ a_{3j} \\ \vdots \\ a_{nj} \end{pmatrix}$$

■

(b)

Proof. By definition of matrix multiplication, the ij-th entry of the product AB of two matrices $A \in \mathcal{M}_{m\times n}(\mathbb{F})$ and $B \in \mathcal{M}_{n\times k}(\mathbb{F})$ is

$$(AB)_{ij} = \sum_{l=1}^{n} A_{il}B_{lj}$$

Where B_{lj} is the l-th entry of the j-th column of B.
Then, treating the j-th column of B as a matrix and applying matrix multiplication to $A\mathbf{b}_j$, we have:

$$A\mathbf{b}_j = \sum_{l=1}^{n} A_{il}(\mathbf{b}_j)_l$$

Where $(\mathbf{b}_j)_l$ is the l-th entry of $\mathbf{b}_j$, the j-th column of B, the same as what was defined for the product of AB above.
Since j is arbitrary, this applies to all columns $1\ldots k$ of matrix B. ■