(Q5)

Proof. We prove this with induction on n.
For $n = 1$, there is nothing to prove.
For $n = 2$, let $A = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$. $\det A$ is then $ac - 0b = ac$.
For the inductive step, we assume this holds for an upper triangular matrix $A \in \mathcal{M}_{n \times n}$ and prove for $n + 1$.
Let $A \in \mathcal{M}_{(n+1)\times(n+1)}$. We expand $\det A$ with "row-i" expansion on row $n + 1$, the last row.
This gives us

$$\det A = \sum_{j=1}^{n+1} (-1)^{(n+1)+j} \det(\tilde{A}_{nj})$$

Since every element in the last row is 0 except for element $A_{(n+1)(n+1)}$, we can simplify this to:

$$(-1)^{2(n+1)} A_{(n+1)(n+1)} \det(\tilde{A}_{nn}) \qquad (1)$$

Since $(-1)^{2n}$ is always positive, it disappears. We also observe that $\tilde{A}_{nn}$ is an upper triangular matrix of $n \times n$, so we can assume $\det(\tilde{A}_{nn})$ is a product of its diagonal entries. Since $A_{(n+1)(n+1)}$ is the last diagonal entry of A, by (1), $\det A$ is also the product of its diagonal entries. ∎