

(Q4)

We row-reduce the matrix, checking how the determinant changes for each iteration:

$$\begin{aligned}
& \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & -1 & 2 & 1 \\ 2 & 2 & -1 & -1 \\ 2 & 2 & -1 & 2 \end{pmatrix} \xrightarrow{\substack{R2-R1 \\ R1-2R1 \\ R4-2R1}} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & -1 & 0 \end{pmatrix} \\
& \xrightarrow{R2 \cdot -\frac{1}{2}} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & -1 & 0 \end{pmatrix} \xrightarrow{\substack{R1-R2 \\ R3 \cdot -1 \\ R4 \cdot -1}} \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 0 \end{pmatrix} \\
& \xrightarrow{\substack{R1-R3 \\ R2+R3 \\ R4-R3}} \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & -3 \end{pmatrix} \xrightarrow{R4 \cdot -\frac{1}{3}} \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
& \xrightarrow{\substack{R1+2R4 \\ R2-3R4 \\ R4-3R4}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

We end up with the identity matrix, which has a determinant of 1.

In row-reducing this matrix A , we multiplied its determinant as follows:

$$\det A \cdot -\frac{1}{2} \cdot -1 \cdot -1 \cdot -\frac{1}{3} = \frac{1}{6} \det A = 1 \implies \det A = 6$$