(Q1)
(a) Since $\mathcal{F}$ is the function space mapping into a vector space, these properties arise from the scalar and additive properties of V.
(b)

Proof. Set $s \in S$ as arbitrary, and $f, g, h \in \mathcal{F}$. Then

$$\begin{aligned}(f+g)(s) &= f(s) + g(s)\\ (g+h)(s) &= g(s) + h(s)\\ f(s) + g(s) + h(s) &= f(s) + (g+h)(s)\end{aligned}$$

■

(c)

Proof. Set $s \in S$ as arbitrary, and $f, g \in \mathcal{F}$. Then

$$\begin{aligned}c(f(s)) = (cf)(s)&, c(g(s)) = (cg)(s)\\ (cf)(s) + (cg)(s) &= c(f(s)) + c(g(s))\\ &= c(f(s) + g(s))\\ &= c((f+g)(s))\end{aligned}$$

■

(d)

Proof. Let $s \in S$ be arbitrary, and $f \in \mathcal{F}$. Then

$$\begin{aligned}(f+\mathbf{0})(s) &= f(s) + \mathbf{0}(s)\\ &= f(s) + \mathbf{0}_V\\ &= f(s)\end{aligned}$$

■

(e)

Proof. Let $s \in S$, $g = -f$. Then

$$\begin{aligned}(f+g)(s) &= (f-f)(s)\\ &= f(s) - f(s)\\ &= \mathbf{0}_V\end{aligned}$$

■