

**(Q12)**

(a) True.

*Proof.* By definition of vector space sums,

$$W_1 + W_2 = \{w = w_1 + w_2 : w_1 \in W_1, w_2 \in W_2\}$$

Since  $W_1 + W_2 = W_1$ , for all  $w \in W_1 + W_2, w \in W_1$ , then  $w_1, w_2 \in W_1$  and thus  $W_2 \subseteq W_1$ . ■

(b) True.

*Proof.* Rearrange the set into a system of homogeneous equations. If there is one solution, then it has to be unique and thus trivial.

If there is more than one (nontrivial) solution, it can be expressed in the form

$$\sum_{i=1}^k t_i \mathbf{x}_i$$

Where  $\mathbf{x}$  is a basic solution and  $t_i$  is an arbitrary field element.

Since  $\mathbb{F}$  is infinite, there are infinite choices for  $t_i$ , and thus there are infinite solutions. ■

(c) False.

Let  $V = P_3(\mathbb{F})$  and  $S_1 = \{cx\}$  where  $c \in \mathbb{F}$ ,  $S_2 = \{x, x^2, x^3\}$ .

$\text{span } S_1$  is clearly a subset of  $\text{span } S_2$ , but  $S_1$  is clearly not a subset of  $S_2$ .