

(Q8)

Theorem. Let $p \in P(\mathbb{C})$ be a polynomial with real coefficients. If a is a root of p , then \bar{a} is a root of p .

Proof. The standard form of a polynomial $p(x)$ is:

$$p(x) = c_0x^0 + c_1x^1 + c_2x^2 + \dots + c_nx^n$$

Then by the definition of root, if a is a root, then

$$\begin{aligned} p(a) &= c_0a^0 + c_1a^1 + c_2a^2 + \dots + c_na^n \\ &= \sum_{j=0}^n c_j(a)^j = 0 \end{aligned}$$

Then we have

$$\begin{aligned} \overline{\sum_{j=0}^n c_j(a)^j} &= \sum_{j=0}^n \overline{c_j(a)^j} = \bar{0} \\ &= \sum_{j=0}^n c_j(\bar{a})^j = 0 \end{aligned}$$

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