(Q1)

Theorem. If a matrix $A \in \mathcal{M}_{n\times n}$ has a zero row, then $\det A = 0$.

Proof. We prove this by induction on n.
First we consider $n = 1$. We then have a matrix $A = (0)$, whose determinant is clearly 0.
We then consider $n = 2$. We construct a matrix

$$A = \begin{pmatrix} 0 & 0 \\ a & b \end{pmatrix}$$

$\det A$ is then $0b - 0a = 0$.
We then assume $\det A$ for $A \in \mathcal{M}_{n\times n}$ is 0, and prove this also holds for the $n+1$ case.
The determinant of $A \in \mathcal{M}_{n+1\times n+1}$ is given by:

$$\sum_{j=1}^{n+1} (-1)^{1+j} A_{1j} \det(\tilde{A}_{1j})$$

We observe that $\det(\tilde{A}_{1j})$ is the determinant of an $n \times n$ matrix, so it will be 0 if it has a zero row.
We now consider two possibilities:
Case 1: The zero row of the matrix is the top row.
In this case, the term A_{1j} will always be 0. The determinant will then evaluate to:

$$\begin{aligned} &\sum_{j=1}^{n+1} (-1)^{1+j} A_{1j} \det(\tilde{A}_{1j}) \\ &= \sum_{j=1}^{n+1} (-1)^{1+j} 0 \det(\tilde{A}_{1j}) \\ &= 0 \end{aligned}$$

Case 2: The zero row of the matrix is from row 2 onwards.
In this case, the matrix $\tilde{A}_{1j} \in \mathcal{M}_{n\times n}$ will have a zero row, so $\det \tilde{A}_{1j} = 0$.
Then $\det A$ evaluates to:

$$\begin{aligned} &\sum_{j=1}^{n+1} (-1)^{1+j} A_{1j} \det(\tilde{A}_{1j}) \\ &= \sum_{j=1}^{n+1} (-1)^{1+j} A_{1j} 0 \\ &= 0 \end{aligned}$$

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