(Q6)

Proof. We prove this by induction on the dimensions of B. Let $B \in \mathcal{M}_{1\times1}$. Then we calculate $\det M$ by cofactor expansion on row n, the bottommost row of M.

$$\det M = \sum_{j=1}^{n} M_{nj} \det(\tilde{M}_{nj})$$

Since every element on row n is zero except for the last element of row n which is B_{11}, $\det M$ simplifies to

$$M_{nn} \det(\tilde{M}_{nn}) = B_{11} \det(\tilde{M}_{nn})$$

We observe that $\tilde{M}_{nn} = A$, so the theorem holds for the case $B \in \mathcal{M}_{1\times1}$.
We now assume the theorem holds for $B \in \mathcal{M}_{1\times1}$, and prove it holds for $n+1$.
We calculate $\det M$ as:

$$\det M = \sum_{j=1}^{n} M_{nj} \det(\tilde{M}_{nj})$$

■