**(Q5)**

*Proof.* Since this system has a unique solution, it has to be trivial, (i.e. $x, y = 0$)
We first assume $ad - bc \neq 0$. We then have a matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

By row-reduction:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \to \begin{pmatrix} ad & bd \\ bc & bd \end{pmatrix} \to \begin{pmatrix} ad - bc & 0 \\ bc & bd \end{pmatrix} \to \begin{pmatrix} 1 & 0 \\ bc & bd \end{pmatrix}$$

Then $x = 0$, and by substitution into the original system $y = 0$.
We then assume the converse, that there is a trivial solution $x, y = 0$.
Then $\{(a, c), (b, d)\}$ is linearly independent.
Then $d(a, c) \neq c(b, d) \implies (ad, ac) \neq (bc, bd)$. ∎