(Q10)
(a)

Proof. We intend to show that for any two vectors $w_1 \in W_1$ and $w_2 \in W_2$, $\exists W_2 \subseteq V : w_1 + w_2 \in V$ and $W_1 \cap W_2 = \{0\}$.
Let B be the basis of V and B_1 the basis of W_1, and $n = \dim V$.
Then $B = \{v_1, v_2, \dots v_n\}$ and $B_1 = \{u_1, u_2, \dots u_k\}$. Then by the Replacement Theorem, we have

$$B = \{u_1, u_2, \dots u_k, v_{k+1}, v_{k+2}, \dots v_n\}$$

Let $B_2 = B \setminus B_1 = \{v_{k+1}, \dots v_n\}$. Since B is a basis, all its constituent vectors are linearly independent and thus $B_2 \subseteq B \implies B_2$ is linearly independent.
Let $W_2 = \operatorname{span} B_2$. By Theorem 20, W_2 is a subspace of V.
Since $B_1 \cap B_2 = \phi$, no vector from W_1 can be expressed in terms of vectors from B_2, thus the only common element W_1 and W_2 have is $\mathbf{0}$. ■

(b) Since $B_2 = B \setminus B_1$, $|B_2| = |B| - |B_1|$ and thus $\dim W_2 = \dim V - \dim W_1$.
(c) Yes.

Proof. Assume $\exists W_3 : V = W_1 \oplus W_2 = W_1 \oplus W_3$. We aim to show that $W_2 = W_3$.
We then have

$$V = \{v \in V = w_1 + w_2 : w_1 \in W_1 \text{ and } w_2 \in W_2\}$$
$$V = \{v \in V = w_1 + w_3 : w_1 \in W_1 \text{ and } w_3 \in W_3\}$$

Then $\forall v \in V, v - w_1 = w_2$ and $v - w_2 = w_3$.
Since every vector in W_2 has a corresponding vector in W_3 and vice-versa, by mutual subset inclusion, $W_2 = W_3$. ■