(Q2)
(a)

Proof. First assume $[A, B] = O_n$. Then $AB - BA = \mathbf{O}_n \implies AB = BA$.
Then assume $AB = BA$. Then by definition, $[A, B] = AB - BA = \mathbf{O}_n$. ∎

(b)

Proof. By definition, $[B, A] = BA - AB$. Then

$$-(BA - AB) = -[B, A] = (AB - BA) = [A, B]$$

∎

(c)

Proof. By definition of $[A, B]$, we expand the given expressions:

$$\begin{aligned}
[A, [B, C]] &= [A, (BC - CB)] = A(BC - CB) = (BC - CB)A \\
&= ABC - ACB - BCA + CBA \\
[C, [A, B]] &= [C, (AB - BA)] = C(AB - BA) - (AB - BA)C \\
&= CAB - CBA - ABC + BAC \\
[B, [C, A]] &= [B, (CA - AC)] = B(CA - AC) - (CA - AC)B \\
&= BCA - BAC - CAB + ACB
\end{aligned}$$

Adding all 3 expanded expressions together, we see that we can form pairs of matrix products and their negatives:

$$\begin{aligned}
&(ABC - ABC) + (ACB - ACB) \\
&+(BAC - BAC) + (BCA - BCA) \\
&+(CAB - CAB) + (CBA - CBA) \\
&= \mathrm{O}_{n \times n}
\end{aligned}$$

∎