

(Q4)

Proof. First suppose A is invertible. Then $\text{rank } A = \text{rank } T_A = n$. Then since $\text{rank } T_A = n$, T_A is an isomorphism and thus bijective.

Now assume T_A is invertible. Then $\text{rank } T_A = n = \text{rank } A$. Then since $\text{rank } A = n$, it is also invertible. ■