(Q7)
(a)

Proof. First assume P is invertible, so T_P is an isomorphism. Let β be a basis of $\mathbb{F}^n$, and $\beta' = T(\beta)$. Since T is an isomorphism, β' is linearly independent and a spanning set for $\mathbb{F}^n$. So $P = [I_{\mathbb{F}^n}]_\beta^{\beta'}$.
Now let $P = [I_{\mathbb{F}^n}]_\beta^{\beta'}$ for two bases β, β' of $\mathbb{F}^n$. Then T_P is a linear map mapping $\mathbb{F}^n \to \mathbb{F}^n$. ■

(b)

Proof. First assume $A \sim B$. Then $\exists P^{-1} \in \mathcal{M}_{n\times n}$ such that $B = P^{-1}AP$. Since P is invertible, $P = [I_{\mathbb{F}^n}]_\beta^{\beta'}$ for some bases β, β'. Then

$$B = [I_n]_\beta^{\beta'}[T]_\beta[I_n]_\beta^{\beta'} = [I_n]_\beta^{\beta'}[T]_\beta = [I_n]_{\beta'}$$

Now suppose $\exists \beta' : B = [I_n]_\beta^{\beta'}$. Then

$$B = [I_n]_\beta^{\beta'}[T]_\beta = [I_n]_\beta^{\beta'}[T]_\beta[I_n]_\beta^{\beta'}$$

where $[T]_\beta = A$, which fulfills the definition of similarity. ■