

(Q7)

(a)

$$(3 + 2i)z = (2 + i)(-2i)$$

$$(3 + 2i)z = 2 - 4i$$

$$z = (2 - 4i) \cdot \frac{3 - 2i}{3^2 + 2^2} = \frac{-(2 + 16i)}{13}$$

(b)

$$z^6 = -125$$

$$z^6 = 125e^{i(\pi+2k\pi)}$$

$$z = \sqrt[6]{125}e^{i\frac{\pi}{6} + \frac{2k\pi}{6}}$$

$$z = x + yi : x = \sqrt{5} \cos\left(\frac{\pi}{6} + \frac{2k\pi}{6}\right)$$

$$y = \sqrt{5} \sin\left(\frac{\pi}{6} + \frac{2k\pi}{6}\right)$$

$\Downarrow$

$$x = \sqrt{5} \cos\left(\frac{3\pi}{6}\right), \sqrt{5} \cos\left(\frac{5\pi}{6}\right), \dots, \sqrt{5} \cos\left(\frac{13\pi}{6}\right)$$

$$= 0, \frac{\sqrt{15}}{2}, \frac{\sqrt{15}}{2}, 0, \frac{\sqrt{15}}{2}, \frac{\sqrt{15}}{2}$$

$$y = \sqrt{5} \sin\left(\frac{3\pi}{6}\right), \sqrt{5} \sin\left(\frac{5\pi}{6}\right), \dots, \sqrt{5} \sin\left(\frac{13\pi}{6}\right)$$

$$y = \sqrt{5}, \pm \frac{\sqrt{5}}{2}, -\sqrt{5}, \mp \frac{\sqrt{5}}{2}$$

$\Downarrow$

$$z = \pm\sqrt{5}i, \frac{\sqrt{15}}{2} \pm \frac{\sqrt{5}}{2}$$

(c)

$$e^z = 1 - \sqrt{3}i$$

$$e^x e^{iy} = \sqrt{1+3}e^{i\frac{\pi}{3}} = 2e^{i\frac{\pi}{3}}$$

$$\therefore e^x = 2 \implies x = \ln 2, y = -\frac{\pi}{3}$$

$$\therefore z = \ln 2 - \frac{\pi}{3}i$$