Problem Set 3

Applied Stats/Quant Methods 1

Due: November 19, 2022

Instructions

- Please show your work! You may lose points by simply writing in the answer. If the problem requires you to execute commands in `R`, please include the code you used to get your answers. Please also include the `.R` file that contains your code. If you are not sure if work needs to be shown for a particular problem, please ask.
- Your homework should be submitted electronically on GitHub.
- This problem set is due before 23:59 on Sunday November 19, 2023. No late assignments will be accepted.

In this problem set, you will run several regressions and create an add variable plot (see the lecture slides) in `R` using the `incumbents_subset.csv` dataset. Include all of your code.

Question 1

We are interested in knowing how the difference in campaign spending between incumbent and challenger affects the incumbent's vote share.

1. Run a regression where the outcome variable is `voteshare` and the explanatory variable is `difflog`.

   Answer

   After we load our dataset into our working environment, we execute our regression model in which the incumbent's vote share is explained by the difference in campaign spending between incumbent and challenger.

   Step 1: We begin by outlining our hypotheses.

Null hypothesis: A difference in campaign spending between incumbents and challengers has no impact on incumbent vote share

Alternative hypothesis: A difference in incumbent campaign spending either increases (or decreases) their vote share

$$H_0 : \beta = 0$$
$$H_A : \beta \neq 0$$

Step 2: We run our regression in R

```
model <- lm(inc.sub$voteshare ~ inc.sub$difflog, data=inc.sub)
```

We then run a summary to check the coefficients

```
summary(model)

Call:
lm(formula = inc.sub$voteshare ~ inc.sub$difflog, data = inc.sub)

Residuals:
Min       1Q   Median       3Q      Max
-0.26832 -0.05345 -0.00377  0.04780  0.32749

Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept)     0.579031   0.002251  257.19   <2e-16 ***
inc.sub$difflog 0.041666   0.000968   43.04   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.07867 on 3191 degrees of freedom
Multiple R-squared:  0.3673,Adjusted R-squared:  0.3671
F-statistic:  1853 on 1 and 3191 DF,  p-value: < 2.2e-16
```

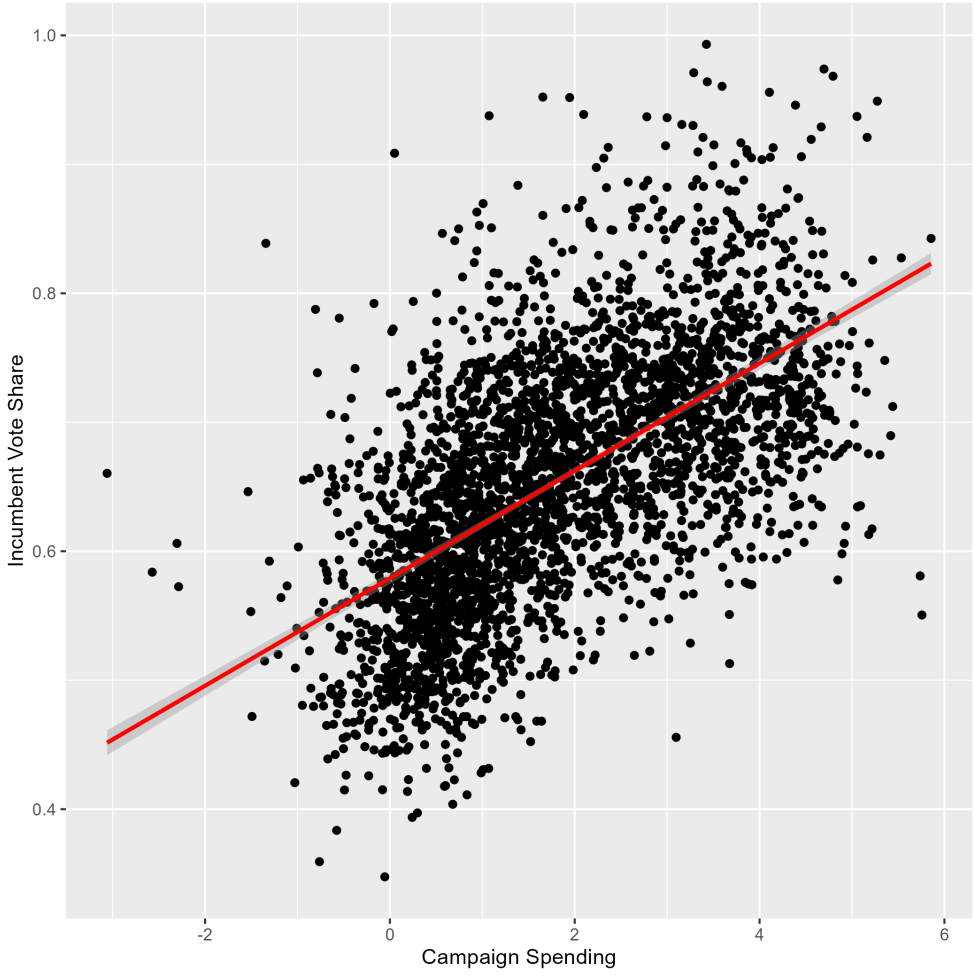
Step 3: Conclusions:

We have evidence to support the view that a one unit increase in spending leads to a 0.04 unit increase in vote share for the incumbent party. The estimated coefficient is statistically differentiable from zero at the $\alpha = 0.05$ level because the p-value < 0.05 ($\approx$2e-16).

2. Make a scatterplot of the two variables and add the regression line.

```
1
2 # Scatter plot
3 scatter <-
4   ggplot(data = inc.sub,
5          mapping = aes(x = difflog,
6                        y = voteshare)) +
7   geom_point() +
8   labs(x = "Campaign Spending",
9        y = "Incumbent Vote Share") +
10
11  geom_smooth(method='lm',col="red")
12
13 ggsave(scatter, file = "vote_share_incumbent_scatter.png")
14
15 # Print plot object
```

Figure 1: Incumbent Vote Share as compared to Campaign Spending. Using ggplot.



3. Save the residuals of the model in a separate object.

```
vote_residuals <- model$residuals
vote_residuals
```

4. Write the prediction equation.

$$\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i$$

$$\hat{Y}_i = 0.579031 + 0.041666(Campaign\,Spending_i)$$

On average, with every additional dollar spent on the campaign, we can expect the vote share for the incumbent Party to increase by 0.041666 scale points.

Question 2

We are interested in knowing how the difference between incumbent and challenger's spending and the vote share of the presidential candidate of the incumbent's party are related.

1. Run a regression where the outcome variable is `presvote` and the explanatory variable is `difflog`.

   Answer

   After we load our dataset into our working environment, we execute our regression model in which the presidential candidate of the incumbent's party vote share is explained by the difference in campaign spending between incumbent and challenger.

   Step 1: We begin by outlining our hypotheses.

   Null hypothesis: A difference in campaign spending between incumbents and challengers has no impact on the presidential candidate of the incumbent's party vote share

   Alternative hypothesis: A difference in campaign spending between incumbents and challengers either increases (or decreases) the vote share for the presidential candidate of the incumbent's party.

   $$H_0 : \beta = 0$$
   $$H_A : \beta \neq 0$$

   Step 2: We run our regression in R

```
model_pres <- lm(inc.sub$presvote ~ inc.sub$difflog, data=inc.sub)
```

   We then run a summary to check the coefficients

```
summary(model_pres)
```

```
Call:
lm(formula = inc.sub$presvote ~ inc.sub$difflog, data = inc.sub)

Residuals:
```

```
Min       1Q   Median       3Q      Max
-0.32196 -0.07407 -0.00102  0.07151  0.42743

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)     0.507583   0.003161  160.60   <2e-16 ***
inc.sub$difflog 0.023837   0.001359   17.54   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.1104 on 3191 degrees of freedom
Multiple R-squared:  0.08795,Adjusted R-squared:  0.08767
F-statistic: 307.7 on 1 and 3191 DF,  p-value: < 2.2e-16
```

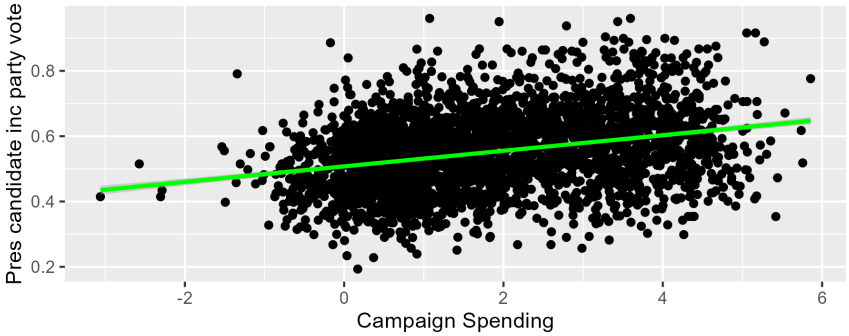
Step 3: Conclusions:

We have evidence to support the view that a one unit difference in campaign spending leads to a 0.023837 unit change in vote share for the presidential candidate of the incumbent's party. The estimated coefficient is statistically differentiable from zero at the $\alpha = 0.05$ level because the p-value < 0.05 ($\approx$2e-16).

2. Make a scatterplot of the two variables and add the regression line.

```r
# Scatter plot
scatter_pres <-
  ggplot(data = inc.sub,
         mapping = aes(x = difflog,
                       y = presvote)) +
  geom_point() +
  labs(x = "Campaign Spending",
       y = "Pres candidate inc party vote") +

  geom_smooth(method='lm',col="green")

ggsave(scatter_pres, file = "vote_share_pres_scatter.png")
```

Figure 2: Presidential candidate of the incumbent's party vote share as compared to Campaign Spending. Using ggplot.



3. Save the residuals of the model in a separate object.

```
pres_residuals <- model_pres$residuals
pres_residuals
```

4. Write the prediction equation.

$$\hat{Y}_i = \hat{\alpha} + \hat{\beta}(X_i)$$

$$\hat{Y}_i = 0.507583 + 0.02383(Campaign\ Spending_i)$$

On average, with every additional dollar spent on the campaign, we can expect the vote share for the presidential candidate of the incumbent's party vote share to increase by 0.02383 scale points.

Question 3

We are interested in knowing how the vote share of the presidential candidate of the incumbent's party is associated with the incumbent's electoral success.

1. Run a regression where the outcome variable is `voteshare` and the explanatory variable is `presvote`.

   Answer

   After we load our dataset into our working environment, we execute our regression model in which we check for association between the vote share of the presidential candidate of the incumbent's party and the incumbent's electoral success.

   Step 1: We begin by outlining our hypotheses.

   Null hypothesis: There is no association between the vote share of the presidential candidate of the incumbent's party and the incumbent's electoral success

   Alternative hypothesis: There is an association between the vote share of the presidential candidate of the incumbent's party and the incumbent's electoral success

   $$H_0 : \beta = 0$$
   $$H_A : \beta \neq 0$$

   Step 2: We run our regression in R

   ```
   model <- lm(inc.sub$voteshare ~ inc.sub$presvote, data=inc.sub)
   ```

   We then run a summary to check the coefficients

   ```
   summary(model_pres)

   Call:
   lm(formula = inc.sub$voteshare ~ inc.sub$presvote, data = inc.sub)

   Residuals:
   ```

```
Min       1Q   Median       3Q      Max
-0.27330 -0.05888  0.00394  0.06148  0.41365

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)      0.441330   0.007599   58.08   <2e-16 ***
inc.sub$presvote 0.388018   0.013493   28.76   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08815 on 3191 degrees of freedom
Multiple R-squared:  0.2058,Adjusted R-squared:  0.2056
F-statistic:   827 on 1 and 3191 DF,  p-value: < 2.2e-16
```

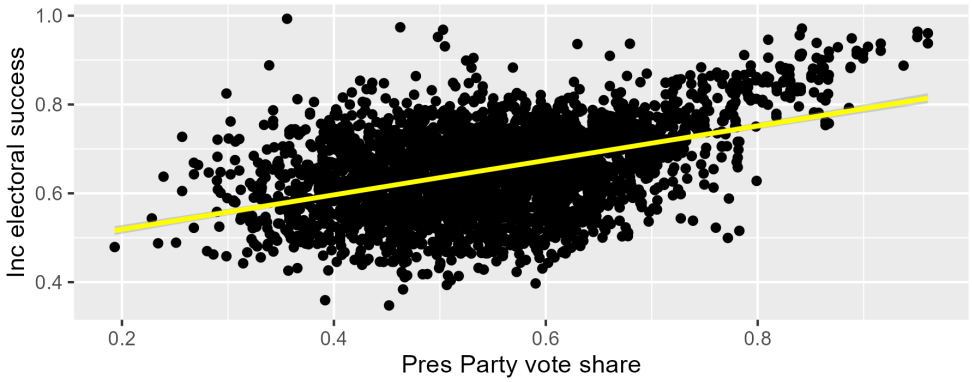
Step 3: Conclusions:

We have evidence to support the view that a one unit increase in the incumbent party's electoral success corresponds to a 0.388 unit scale increase in vote share for the President's Party. The estimated coefficient is statistically differentiable from zero at the $\alpha = 0.05$ level because the p-value < 0.05 ($\approx$2e-16).

2. Make a scatterplot of the two variables and add the regression line.

```r
scatter_vote <-
  ggplot(data = inc.sub,
         mapping = aes(x = inc.sub$presvote,
                       y = inc.sub$voteshare)) +
  geom_point() +
  labs(x = "Pres Party vote share",
       y = "Inc electoral success") +

  geom_smooth(method='lm',col="yellow")

ggsave(scatter_vote, file = "vote_share2_scatter.png")
```

Figure 3: President Party's Vote Share as compared to Incumbent's electoral success. Using ggplot.



3. Save the residuals of the model in a separate object.

```
new_vote_residuals <- model_vote$residuals
```

4. Write the prediction equation.

$$\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i$$

$$\hat{Y}_i = 0.441330 + 0.388(Incumbent\,Vote\,Share_i)$$

On average, with every one unit of electoral success for incumbent party, we can expect the vote share for the President's Party to increase by 0.388 scale points.

With regard to association, the slope is positive, indicating a positive association, and we have evidence to accept the alternative hypothesis. The estimated coefficient is statistically differentiable from zero at the $\alpha = 0.05$ level because the p-value < 0.05 ($\approx$2e-16). The slope does not however tell us the strength of the association.

To do so we can look at the correlation estimate, which comes to 0.4536672, indicating a moderate strength linear association between the two variables .

```
correlation <- cor.test(inc.sub$presvote, inc.sub$voteshare)
```

```
Pearson's product-moment correlation
```

```
data:  inc.sub$presvote and inc.sub$voteshare
t = 28.757, df = 3191, p-value < 2.2e-16
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
0.4256781 0.4807891
sample estimates:
cor
0.4536672
```

If we look at the R^2 value, which is a related measure of association, it is 0.2058, indicating that X can explain 20 per cent of the variability of Y.

Question 4

The residuals from part (a) tell us how much of the variation in `voteshare` is *not* explained by the difference in spending between incumbent and challenger. The residuals in part (b) tell us how much of the variation in `presvote` is *not* explained by the difference in spending between incumbent and challenger in the district.

1. Run a regression where the outcome variable is the residuals from Question 1 and the explanatory variable is the residuals from Question 2.

   Null hypothesis: There is no association between the residuals

   Alternative hypothesis: There is an association between the residuals

   $$H_0 : \beta = 0$$
   $$H_A : \beta \neq 0$$

   Step 1: Run the regression ...

```
model_residuals <- lm(vote_residuals ~ pres_residuals, data=inc.sub)

summary(model_residuals)
```

```
Call:
lm(formula = vote_residuals ~ pres_residuals, data = inc.sub)

Residuals:
Min       1Q   Median       3Q      Max
-0.25928 -0.04737 -0.00121  0.04618  0.33126

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)    -5.934e-18  1.299e-03    0.00        1
pres_residuals  2.569e-01  1.176e-02   21.84   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.07338 on 3191 degrees of freedom
Multiple R-squared:   0.13,Adjusted R-squared:  0.1298
F-statistic:   477 on 1 and 3191 DF,  p-value: < 2.2e-16
```

Check smaller values :

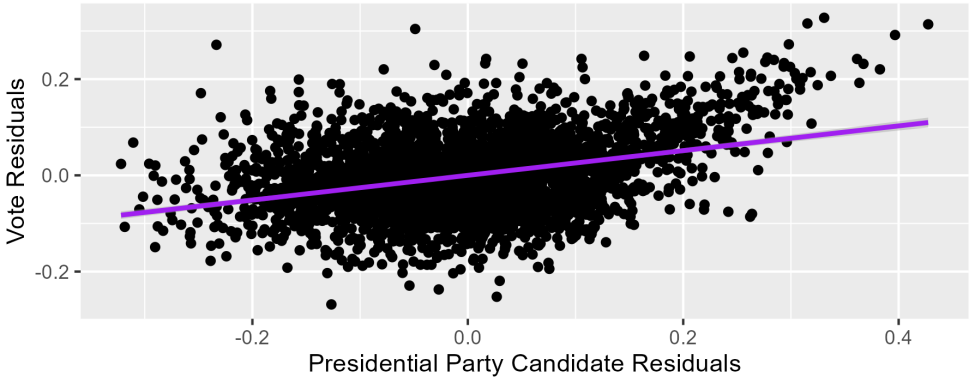
```
> sprintf("%.20f",2.569e-01)
[1] "0.25690000000000001723"

> sprintf("%.20f",-5.934e-18)
[1] "-0.00000000000000000593"
```

The slope is equal to 0.2569. So we do not have enough evidence to reject the null hypothesis in this instance. Note that the estimated coefficient is statistically differentiable from zero at the $\alpha = 0.05$ level because the p-value < 0.05 ($\approx$2e-16).

2. Make a scatterplot of the two residuals and add the regression line.

```
1
2 # Scatter plot
3 scatter_residuals <-
4   ggplot(data = inc.sub,
5          mapping = aes(x = pres_residuals,
6                        y = vote_residuals)) +
7   geom_point() +
8   labs(x = "Presidential Party Candidate Residuals",
9        y = "Vote Residuals") +
10
11  geom_smooth(method='lm',col="purple")
12
13 ggsave(scatter_residuals, file = "residuals_scatter.png")
```

Figure 4: Comparison of Residuals Using ggplot.



3. Write the prediction equation.

$$\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i$$

$$(Inc\,Vote\,Share\,Residuals)_i = 0 + 0.2569(Pres\,Party\,Vote\,Share\,Residuals_i)$$

Question 5

What if the incumbent's vote share is affected by both the president's popularity and the difference in spending between incumbent and challenger?

1. Run a regression where the outcome variable is the incumbent's `voteshare` and the explanatory variables are `difflog` and `presvote`.

   Run the regression in R ....

```
multi_model <- lm(inc.sub$voteshare ~ inc.sub$difflog + inc.sub$presvote, data=inc.
summary(multi_model)

Residuals:
Min       1Q   Median       3Q      Max
-0.25928 -0.04737 -0.00121  0.04618  0.33126

Coefficients:
Estimate    Std. Error t value Pr(>|t|)
(Intercept)      0.4486442  0.0063297   70.88   <2e-16 ***
inc.sub$difflog  0.0355431  0.0009455   37.59   <2e-16 ***
inc.sub$presvote 0.2568770  0.0117637   21.84   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.07339 on 3190 degrees of freedom
Multiple R-squared:  0.4496,Adjusted R-squared:  0.4493
F-statistic:  1303 on 2 and 3190 DF,  p-value: < 2.2e-16
```

2. Write the prediction equation.

$$Y_i = \hat{\alpha} + \beta_1 X_i + \beta_2 X_{2_i}$$

$$(Inc\,Vote_{\,i}) = 0.4486442 + 0.0355431(Campaign\,Spending_i) + 0.2568770(Pres\,Vote\,Share_{2_i})$$

3. What is it in this output that is identical to the output in Question 4? Why do you think this is the case?

   The value of **Residual Standard Error** is 0.073 in both Q4 and Q5. The Residual Standard Error is an estimate of the standard deviation of ε. It is roughly the average amount that the response will deviate from the true regression line.

   There is also a **coefficient of 0.2569** in both cases.

   Why? Recall that:

   In Q1, we found the linear relationship between voteshare (Y) and difflog (X1). The residual from this regression is voteshare, after taking out the linear effects of difflog.

   In the second step, we replaced voteshare by presvote (X2), so the residual is the part of presvote, that is not linearly related to difflog (X1).

   In Q4 we looked for the linear relationship between the Y residual and the X2 residual, and found a coefficient for the slope equal to 0.2569. This represents the effect of X2 on Y after taking out the effects of X1 from both Y and X2.

   In Q5, the regression coefficient of 0.2569 represents the effect of presvote (X2) on voteshare (Y) after also taking out the effects of difflog (X1).

   This can help explain why the coefficient of 0.2569 and residual standard errors are the same as in Q4.

   Chatterjee and Hadi (2012) note that the simple and multiple regression coefficients are not the same unless the predictor values are uncorrelated.