Ball on Plate

Group 4

MECA 482-01-7834 - Control System Design

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I. Introduction

The ball and plate is a system with two degrees of freedom that must recognize the motion of the ball and adjust its orientation to ensure that the ball stays in the middle of the plate. This is done by using two servo motors attached to the bottom of the plate to control the x-y-z position of the ball. The position of the ball is measured by an overhead vision sensor that tracks the position of the ball in real time and provides live video feedback. A mathematical model of the ball and plate system and a model of the PD system was integrated into Matlab/Simulink to run Coppelia Sim and demonstrate a simulation of the entire system. The PD controller measures the error in the system and considers the position and velocity of the ball. Link to all documentation and MATLAB files related to the project can be found on the github page.

II. Modeling

The system shown in Figure 1 has two servo motors that control the x-y-z motion of the plate. The angle of the plate affects the movement of the ball. The mathematical model was derived based on this system. For the ball to stay on the plate, the force of gravity must be equal to the force of the ball's momentum.

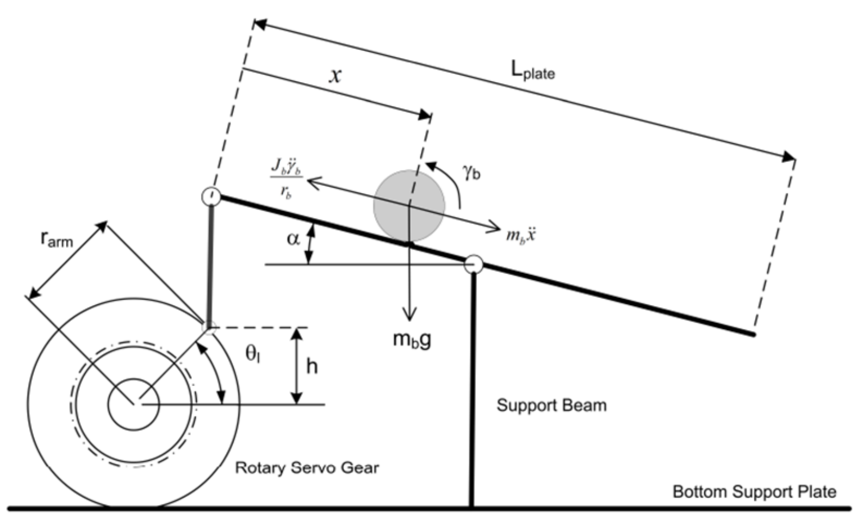

Figure 1. Free Body Diagram of the Ball and Plate System.

Deriving the mathematical model starts with the free body diagram and the first law of motion in Eq.(1). Using the motion equations, it is possible to then calculate the torque with Eq.(2) and linear acceleration with Eq.(3). The next step is to find the transfer function. Eq.(4) represents the relationship of the angle of the plate to the ball's position and Eq.(5) is the laplace transform of the linear equation of motion. With this we can derive the closed-loop transfer function in Eq.(6) where kp is the proportional gain and kd is the derivative gain. Computing the values for these will bring the settling time and overshoot specifications. Eq.(7) represents the calculation for percent overshoot.

	$m_b\ddot{x}(t) = \sum F = F_{x,t} - F_{x,r}$	Eq.(1)
	$\tau_b = J_b\ddot{\gamma}_b(t)$	Eq.(2)
	$\ddot{x}(t) = \frac{m_b\, g\, \sin\alpha(t)\, r_b^2}{m_b\, r_b^2 + J_b}.$	Eq.(3)
	$P_{bb}(s) = \frac{X(s)}{\Theta_l(s)}$	Eq.(4)
	$P_{bb}(s) = \frac{X(s)}{\Theta_l(s)} = \frac{K_{bb}}{s^2}$	Eq.(5)
	$\frac{X(s)}{X_d(s)} = \frac{K_{bb}\, k_p}{s^2 + K_{bb}\, k_d\, s + K_{bb}\, k_p}$	Eq.(6)
	$PO = 100e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$	Eq.(7)

From the transfer function, using Matlab, the state-space representation of the system can be found and is shown in Figure 2.

$$\begin{bmatrix} -k_{bb}k_p & -k_{bb}k_p \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ k_{bb}k_p \end{bmatrix} [0]$$

A B C D

Figure 2. State-Space Transfer Function.

III. Sensor Calibration

This system relies on an overhead camera to convert the physical location of the ball into a set of coordinates. Starting with the dimensions of the plate, there is a coordinate system to build a boundary for the ball and a coordinate system to locate the exact point of the ball on the plate. Those coordinates are sent to Simulink where servo adjustments are made to center the ball. The calibration system establishes the plate as a plane with a coordinate system, allowing for data to be extracted. Figure 3 shows the coordinate system of the plate viewed by the camera. Adjustments were made to the calibration code provided to link Matlab and Coppelia.

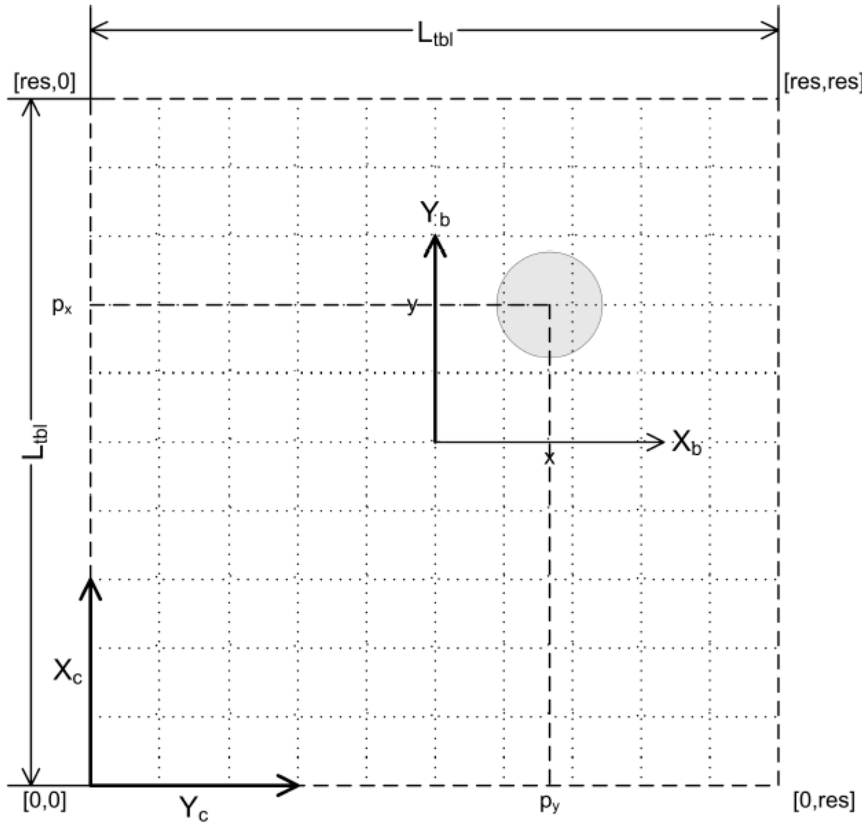

Figure 3. Coordinate system of the plate.

The simulink provided for the camera calibration model is in the following figure. The camera calibration block should be modified with the correct size of the camera image.

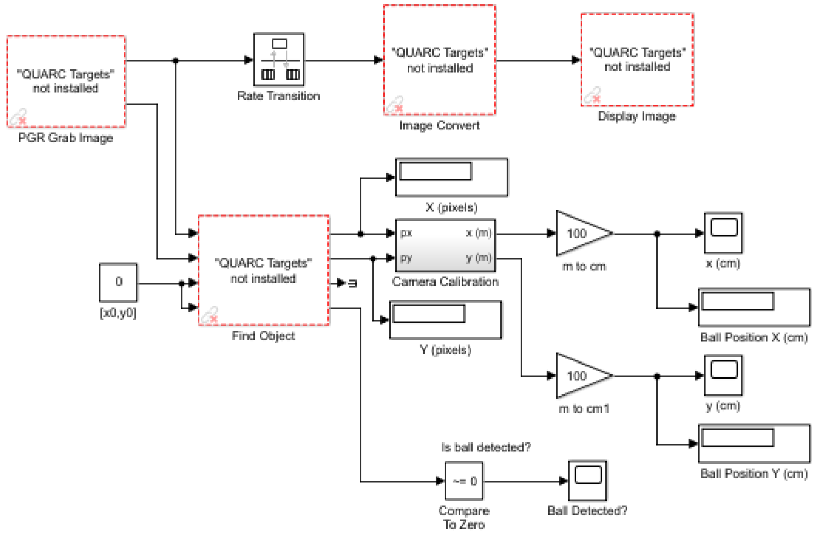

Figure 4. Camera Calibration.

The simulink model provided in the courseware resources is used to configure the PGR Grab Image QUARC block as well as the Camera Calibration subsystem. These subsystems ensure that the system will have the correct x and y ball position measurements.

IV. Controller Design and Simulations

Proportional Derivative (PD) Controller

To improve the stability of the ball on plate control system a proportional derivative controller was applied to control the damping within the system's response as well as to help predict the ball's position and velocity. The controller accomplishes this by correlating the derivative to the response error and the system's response to the error. The function of the PD controller is where K_P is the proportional factor and K_D is the derivative factor. In order to model in a block the PD controller the function would need to be converted into s-domain.

V. Checklist

Mathematical model of the system via Matlab / Simulink:

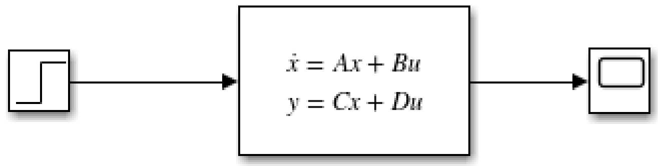

Figure 5. Model of the State-Space System in Simulink.

Running this model provides the step response shown in Figure 6.

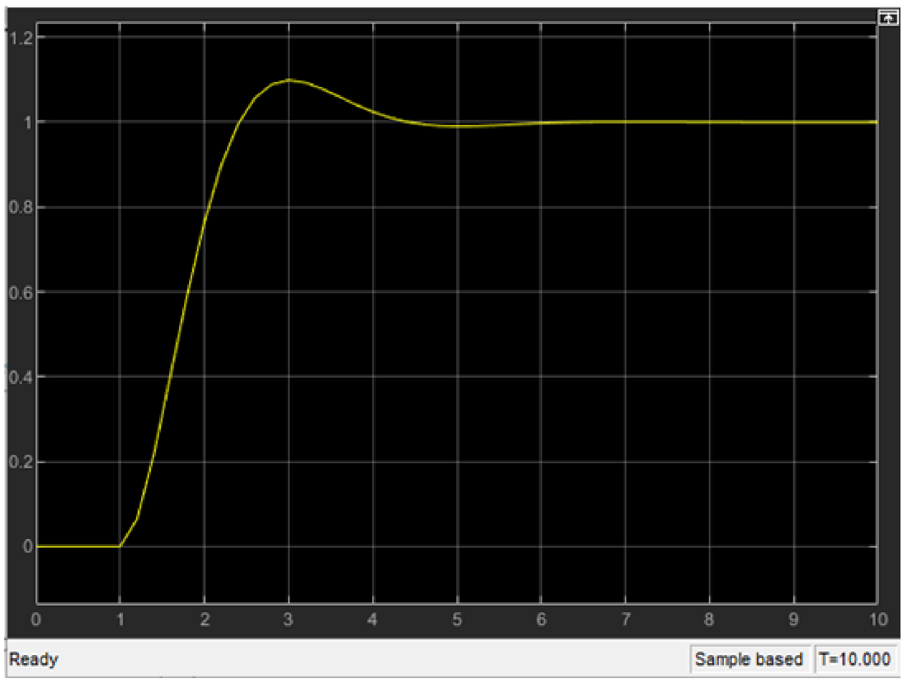

Figure 6. Continuous-Time State-Space Model.

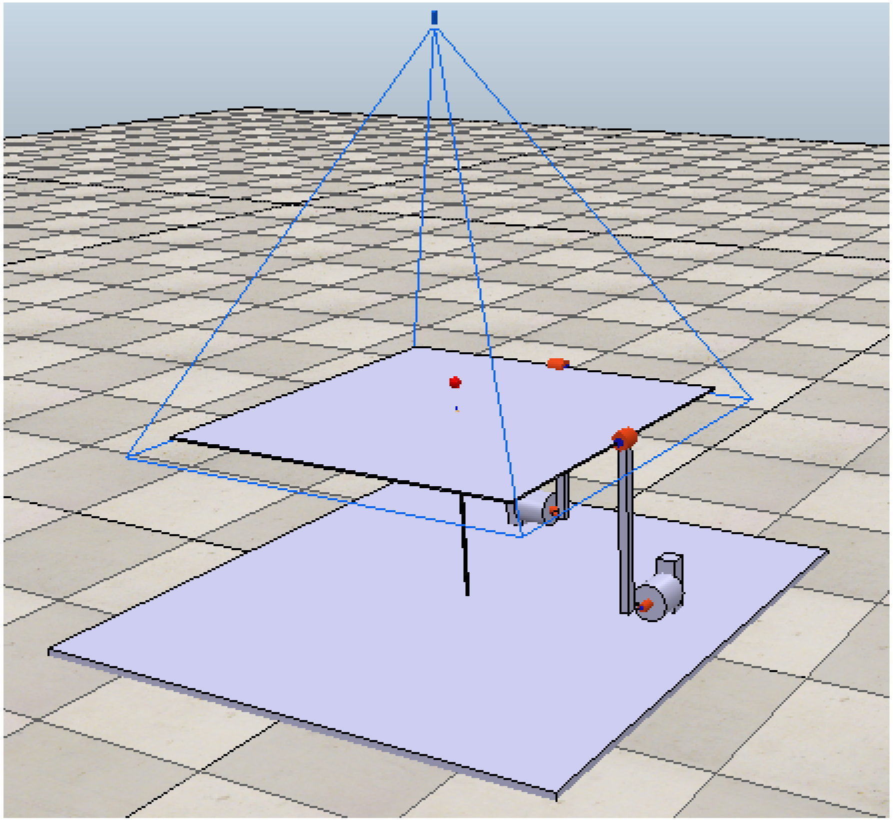

Figure 7. Model of the System in CoppeliaSim.

VI. Conclusion

The goal of this project was to complete a simulation of a working ball and plate system. Beginning with a free body diagram and deriving the mathematical model by hand, that model was then converted into Matlab code, then into a simulink model. Matlab and Simulink were connected to Coppelia to then run the simulation of the ball and plate. Our model did not work perfectly in the simulation. Since we derived our mathematical model in state space, it was difficult to get the code to run well with Coppelia.

VII. Appendix

Appendix A: MATLAB Code

```
syms s K1 K2 d2 d3 T

A = [K1+K2 -K2 0; -K2 ((d2*s)+K2) -d2*s; 0 -d2*s d3*s];

B = [T; 0; 0];

X = inv(A)*B;

X2 = X(2);

pretty(simplify(X2))

TF = X2 / T;

b = [0 K2*d3];

a = [K1*K2*d3 (-K1*d2^2)-(K2*d2^2)+(K1*d2*d3)+(K2*d2*d3)];

[A,B,C,D] = tf2ss(b,a)
```

VIII. References

[1] Hernández-Guzmán, Victor Manuel, and Ramón Silva-Ortigoza. *Automatic control with experiments*. Cham, Switzerland: Springer, 2019.

[2] Nise, Norman S. *Control Systems Engineering*. Hoboken, NJ: Wiley. 2011.

Quanser. *Two DOF Ball Balancer*. Quanser Incorporated. 2013