DIPARTIMENTO INTERATENEO DI FISICA "M. MERLIN"

Tesi di laurea Magistrale in
"Nuclear, Subnuclear and Astroparticle Physics"

Search for $\tau \to 3\mu$ decays using τ leptons produced in D and B mesons decays in CMS experiment at LHC

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Chapter 1

Standard Model and new physics searches

The *Standard Model* (SM) of particle physics is the theory that we use to describe the fundamental blocks of the matter and the forces that govern their interaction. This theory includes three of the four known fundamental forces (the electromagnetic, weak and strong interactions), leaving aside the gravitational force, which is also the weakest one. This is the reasonn why this model provides extremely precise predictions even if it is not complete. The Standard Model was finalized only in mid-1970s and it has predicted a variety of physics phenomena and successfully explained almost all the experimental results obtained up to now.
However, it is evident that it is an "effective theory", and therefore it needs to be extended in order to include also the gravitation and to explain some experimental observations which are still unexplained.
For these reasons, in the last years some extensions of the SM have been developed and their predictions are continously under test in order to reject or to confirm these theories.

1.1 The Standard Model

1.1.1 Fundamental particles and gauge symmetries

The Standard Model classifies the particles according to their intrinsic angular momentum, the spin, dividing them in two main categories, as shown in Fig. 1.1.

- **Bosons**: particles with integer values of the spin. They follow Bose-Einstein statistics and are defined as the carriers of the fundamental interactions. The photon γ is the carrier of the electromagnetic force, the weak force is mediated by $W^{\pm}$ and Z bosons and the strong force is mediated by gluons. In the SM there is also another boson, called *Higgs* boson, which is not a force mediator but it is a scalar boson arising from a spontaneous symmetry breaking phenomenon, the mechanism responsible for particles masses.

- **Fermions**: particles with semi-integer values of the spin. They obey Fermi-Dirac statistics and some of them are the elementary matter blocks that constitute the observable matter. Fermions are divided in 2 groups: *leptons* (which have an integer electric charge) and *quarks* (characterized by a fractional electric charge).
  Each of these group is made up of 6 particles, paired in 3 generations (or families), as shown in the Fig. 1.1. Each of the three charged leptons: electron, muon and tau, has a corresponding neutrino, which has no electric charge. Each quark pair, whereas, is made up of a quark with electric charge equal to 2/3 and the other equal to -1/2.
  For each of this 12 different fermions there is a corresponding antiparticle, characterized by the same mass and spin values but opposite value for the other quantum numbers. Therefore, in total the model distinguish 24 fermions.

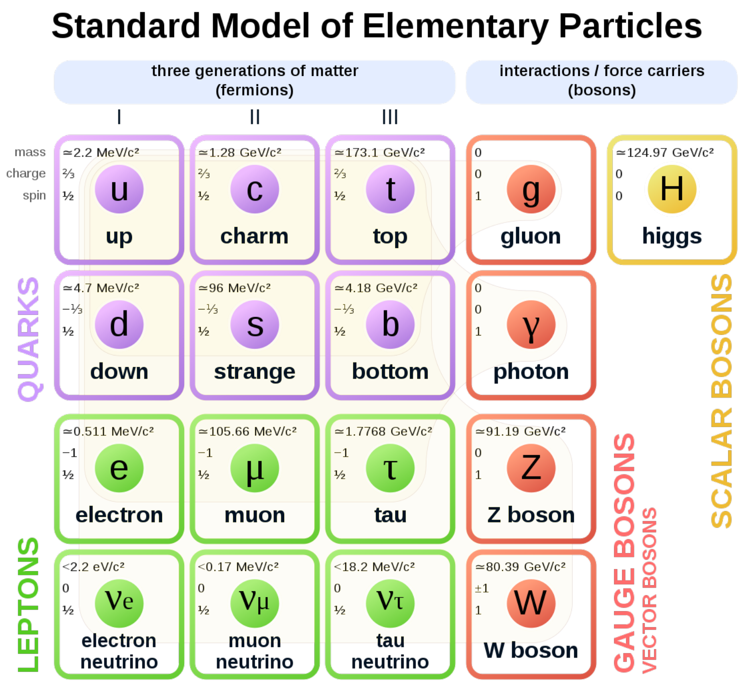

Figure 1.1: A summary table with all the SM particles: the fermions (on the left), which are the matter particles, divide into quarks (in violet) and leptons (in green), while the bosons (on the right), divide into the gauge bosons, i.e. the carriers of the interactions (in red) and the scalar Higgs boson (in yellow). For each particle, the mass, electric charge and spin values are reported.

The fundamental symmetry group that is gauged in the Standard Model is $\mathrm{SU(3)_C} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)_Y}$, where:

- $SU(3)_C$ is the *color group*, the symmetry group at the basis of the Quantum Cromo Dynamics (QCD), the theory that describes the interaction between quarks and gluons,

- $SU(2)_L \otimes$ $U(1)_Y$, where $SU(2)_L$ is the *weak isospin group* and $U(1)_Y$ is the *weak hypercharge group*. They describe the electroweak interactions. $U(1)_Y$ is different from $U(1)_{em}$, the symmetry group at the basis of the Quantum Electro Dynamics (QED), that describes the electromagnetic interactions. In fact the generator of $U(1)_{em}$ is the electromagnetic charge Q, while the hypercharge Y is the generator of $U(1)_Y$.

In the following, the theories from which the SM is born, i.e. the Quantum Chromodynamics and the Electro-weak theory, along with the Higgs mechanism that explains the masses of particles, will be explained in more detail. At the end, an overview of the whole SM will be given. (ma forse questo periodo va bene solo se lo metto al di fuori di questa sezione, nel "livello superiore".

Quantum Chromodynamics (QCD)

The Quantum Chromodynamics [1] describes the interaction between quarks and gluons. In this theory, a quantum number, called *color*, is used and each quark can have three different colors. However, in nature only colorless states can be observed, and this is explained by the fact that these observable states are singlets under rotations in color space $SU(3)_C$. A consequence of this so-called *confinement hypothesis*, is that quarks can never be observed as free (because they carry color) and they are always confined within color-singlet bound states.

The symmetry group $SU(3)_C$ is non-Abelian, and this is the reason why the QCD phenomenology is completely different from that of Quantum Electrodynamics (QED), which is describes the electromagnetic interactions and is based on the group $U(1)_{em}$, that is Abelian. One of the most relevant consequences of this, is the fact that gluons are self-interactive.
The generatos of $SU(3)_C$ are the matrices: $T^a = \frac{\lambda^a}{3}$, with $a = 1, ..., 8$ (the dimension of the group) and the λ^a are the *Gell-Mann matrices*. The group algebra is defined by: $[T^a,\ T^b] = if^{abc}T^c$.

Since quarks are Dirac particles, the Lagrangian density of free quarks can be written as in 1.1.

$$\mathcal{L}_q(x) = \sum_{j=1}^{f} \bar{q}^j(x)\ (i\gamma^\mu \partial_\mu - m_j)q^j(x) \tag{1.1}$$

where, for each possible flavour labelled by j, each matter field q^j is given by the three possible color states: $q^j = \begin{pmatrix} q_1^j \\ q_2^j \\ q_3^j \end{pmatrix}$

The Lagrangian in 1.1 is invariant under global transformations belonging to $SU(3)_C$, i.e. the transformations not depending on x, but not under local ones, which is a very important requirements in physics (in order to make our description not dependent on the point of the space where we are describing it!) A gauge transformation in $SU(3)_C$ can be written as: $q(x) \to q'(x) = U(x)q(x)$.

The gauge invariance can be restored by introducing 8 vector fields, the gluons, defined as: $G_\mu(x) = G^a_\mu(x)\frac{\lambda_a}{2}$, which are hermitian and traceless.
Moreover, the minimal coupling between the quarks and the gluons can be introduced by replacing the derivative with a covariant derivative D_μ, defined as: $\partial_\mu + ig_s G^a_\mu(x)\frac{\lambda_a}{2}$, being g_S the dimensionless coupling constant.
The Lagrangian density in 1.1 is invariant under gauge transformations iff the gluon fields transform as: $G_\mu(x) \to G'_\mu(x) = U(x)G_\mu(x)U^\dagger(x) - \frac{i}{g_s}U(x)\partial_\mu U^\dagger(x)$.
In addition, a term describing the gluon dynamics is still missing. In order to construct it, a *gluon field strength tensor* $G_{\mu\nu}$ must be introduced. It is defined as: $G_{\mu\nu}(x) = \partial_\mu G_\nu(x) - \partial_\nu G_\mu(x) + ig_s[G_\mu(x), G_\nu(x)]$ and its components are given by: $G_{\mu\nu}(x) = G^a_{\mu\nu}(x)\frac{\lambda_a}{2}$.

The QCD Lagrangian density, which is gauge invariant, is given by the equation 1.2.

$$\mathcal{L}_{QCD} = -\frac{1}{4}\, G^a_{\mu\nu}(x)G^{\mu\nu a}(x) + \sum_{j=1}^{f}\, \bar{q}^j(x)\, \left(i\gamma^\mu D_\mu - m_j\right)\, q_j(x) \tag{1.2}$$

SU(3)$_C$ is an exact symmetry: in 1.2 there is not a term $\propto G^\mu G_\mu$ because it would be $\propto m^2$ and it would break the invariance. Therefore the gluons, like it happens for the photons in QED (where also $U(1)_{em}$ is an exact symmetry), are massless, and they have a spin equal to 1.
Moreover, expanding the term $G_{\mu\nu}G^{\mu\nu}$ present in the Lagrangian, it is possible to find explicitly the contributions which describe the self-interaction of three or four gluons.

Electroweak theory

The electroweak theory is the product of the unification of the QED and the weak theory, carried out by Glashow in 1961 [2].
It is described by a symmetry group: SU(2)$_L\otimes$ U(1)$_Y$ where L and Y are the left-handed weak isospin and the hypercharge, related to the electromagnetic charge through the equation $Q = Y/2 + T3$. SU(2)$_L\otimes$ U(1)$_Y$ is not a "simple" group, therefore it is characterized by two coupling constants: g for SU(2)$_L$ and g' for U(1)$_Y$.

Hp: - neutrinos, leptons, quarks are massless (otherwise the Lagrangian later won't result gauge invariant!) A family of leptons (l) can be described with:

$\psi_L = \begin{pmatrix} \nu_l \\ l_L \end{pmatrix}_L$, $\psi_R = l_R$, where l_L is the left-handed part of a lepton while l_R is the right-handed part, respectively equal to: $l_L = \frac{1}{2}(1-\gamma_5)\ l$ $l_R = \frac{1}{2}(1+\gamma_5)\ l$ It is written in this way because χ_L: the left-handed lepton and the neutrino behave as a doublet under SU(2)$_L$, while the right-handed lepton behaves as a singlet. A right-handed neutrino is absent because in nature it has never been observed.

The same can be done with a quark family, but in this case all the quarks, unlike neutrinos, have a right-handed component. $\psi_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$, $\psi_R = u_R$, or d_r.

If I call: $\psi = \begin{pmatrix} \begin{pmatrix} \nu_l \\ l \end{pmatrix}_L \\ l_R \end{pmatrix}$

They transform under U(1)$_Y$ local as: $\psi \ \rightarrow \begin{pmatrix} e^{iy_L\alpha(x)} & 0 & 0 \\ 0 & e^{iy_L\alpha(x)} & 0 \\ 0 & 0 & e^{iy_R\alpha(x)} \end{pmatrix} \psi$

where, from experimental evidences, $y_L = -\ \frac{1}{2}$ and $y_R = -1$.

The Lagrangian of the electroweak theory is:

$$\mathcal{L} = \bar{\psi}_L \ i \ \gamma^\mu D_\mu \psi_L + \bar{\psi}_R \ i \ \gamma^\mu D_\mu \psi_R - \ \frac{1}{4} \ W_{\mu\nu} W^{\mu\nu} - \ \frac{1}{4} \ B_{\mu\nu} B^{\mu\nu} \tag{1.3}$$

where: D_μ is the covariant derivative, necessary for the Lagrangian in order to be invariant under local (gauge) transformations. It is given by:

$$D_\mu = \partial_\mu + igW^a_\mu T^a + ig' B_\mu Y \tag{1.4}$$

Under a local U(1)$_Y$ trasformation:

$$B_\mu(x) \ \rightarrow \ B'_\mu = B_\mu - \frac{i}{g'} \partial\alpha \tag{1.5}$$

with:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{1.6}$$

Additional definitions:

$$Z_\mu = \frac{gW^3_\mu - g' B_\mu}{\sqrt{g^2 + (g')^2}} = cos\theta_W W^3_\mu - sin\theta B_\mu \tag{1.7}$$

$$A_\mu = \frac{g' W^3_\mu + gB_\mu}{\sqrt{g^2 + (g')^2}} = sin\theta_W W^3_\mu + cos\theta B_\mu \tag{1.8}$$

where θ_W is called the *Weinberg angle* and: $cos\theta_W = \frac{g}{\sqrt{g^2+(g')^2}}$ and $sin\theta_W = \frac{g'}{\sqrt{g^2+(g')^2}}$

1.1.2 Spontaneous Symmetry Breaking

After the unification of the electromagnetic and weak interaction described in the previous section, there was still a not negligible problem in the theory. The matter particles involved, as well as the carriers of the weak interactions, were massless. Even if these latter gauge bosons, at the beginning of 1960's were not yet been discovered, there were expected to be quite massive because of the characteristics of the weak interaction (that, for its phenomenology, in the first description Fermi assumed to be point-wise). However, it was not

possible to add mass terms explicitly in the Lagrangian density because that would violate the gauge invariance, leading to a non-renormalizable theory!
In order to overcome this problem, Weinberg and Salam (in realtà: Only after the ground-breaking work of Sheldon Glashow, Abdus Salam, Steven Weinberg, and of Gerard 't Hooft with Martinus Veltmann, symmetry breaking in a unified electro-weak theory could finally be settled.) integrated in the theory a *Spontaneous Symmetry Breaking* (SSB) mechanism, hypothesized in 1964 independently by Higgs [3, 5] and by Englert and Brout [4], which was able to explain the mass of the particles without violating the gauge invariance.

A SSB occurs in physics when the vacuum state of a system (which is the lowest energy state) is not invariant under a symmetry transformation of the Lagrangian which describes that system. // there is a symmetry of the system that is not respected by the ground state. // Therefore this symmetry is said to be *spontaneously broken* and a degeneracy is always implied!

In order to introduce a SSB in the model described so far, we can consider two complex scalar fields ϕ_1 and ϕ_2, which form a doublet in $\mathrm{SU}(2)_L$: $\phi(x) = \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix} \rightarrow \phi'(x) = U(x)\phi(x)$ under a local transformation of $\mathrm{SU}(2)_L$.
Moreover, this doublet transforms under a local transformation of $\mathrm{U}(1)_Y$ as: $\phi(x) \rightarrow e^{iy_H\alpha(x)}\phi(x)$, where y_H is the so-called *Higgs hypercharge* and it has to be equal to 1/2.

The Lagrangian density which describes the dynamics of the scalar field ϕ is the one in 1.9.

$$\mathcal{L}_H = (\partial_\mu\phi^\dagger)(\partial^\mu\phi) \; - \; V(\phi^\dagger\phi) \tag{1.9}$$

A potential suitable for a spontaneous symmetry breaking is the one in 1.10.

$$V(\phi^\dagger\phi) = k\,(\phi^\dagger\phi) + \lambda\,(\phi^\dagger\phi)^2 = \frac{k}{2}\rho^2 + \frac{\lambda}{4}\rho^4 \tag{1.10}$$

with $k < 0$ and $\lambda > 0$. (having posed: $\frac{\rho(x)}{\sqrt{2}} = \sqrt{\phi^\dagger\phi}$)

Considering the potential as a function of ρ, if ρ is not dependent on x, it has 2 minima at: $\pm\rho_0 = \pm\sqrt{-\frac{k}{\lambda}}$, as shown in Fig. 1.2.

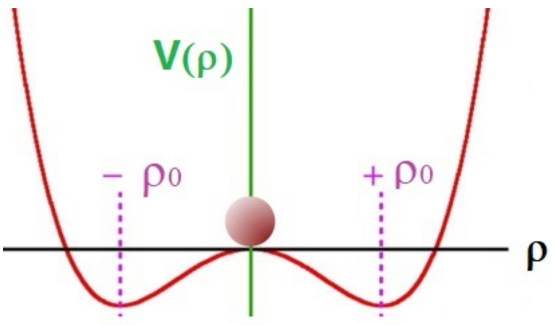

Figure 1.2: Bla

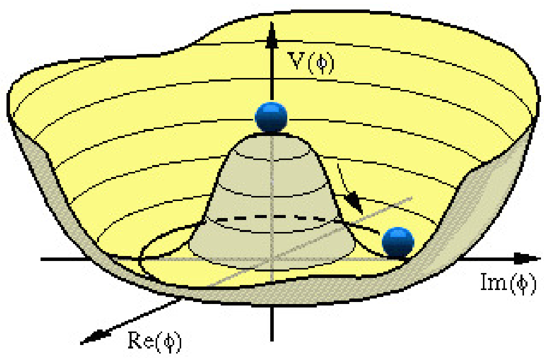

Figure 1.3: Bla

The Higgs field ϕ has 4 free parameters(the real and imaginary parts of the 2 scalar complex fields), therefore it adds 4 degrees of freedom to the theory: 3 are used to give mass to the $W^{\pm}$ and Z bosons, while the fourth is the one that gives mass to the Higgs boson. (When the field ϕ finds its ground state, three of these four free degrees freeze out and become the transverse polarization modes of the two oppositely charged W-bosons and the Z-boson. The mere existence of transverse polarization of a spin-1 vector particle is equivalent for it having gained mass, as Lorentz invariance for mass-less particles prohibits otherwise. One degree of freedom stays, which gives rise to an excitation mode of the field ϕ above its ground state; this excitation is exactly what yields the Higgs boson.) More effort by people like Gerald Guralnik, Carl Hagen and Tom Kibble [6] was needed to understand the properties of this mass generating mechanism

- PDG

From the four generators of the SU(2)L $\times$ U(1)Y gauge group, three are spontaneously broken, implying that they lead to non-trivial transformations of the ground state and indicate the existence of three massless Goldstone bosons identified with three of the four Higgs field degrees of freedom. The Higgs field couples to the Wμ and Bμ gauge fields associated with the SU(2)L $\times$ U(1)Y local symmetry through the covariant derivative appearing in the kinetic term of the Higgs Lagrangian, where The fourth generator remains unbroken since it is the one associated to the conserved U(1)em gauge symmetry, and its corresponding gauge field, the photon, remains massless.

Hence, from the initial four degrees of freedom of the Higgs field, two are absorbed by the W$\pm$ gauge bosons, one by the Z gauge boson, and there is one remaining degree of freedom, H, that is the physical Higgs boson — a new scalar particle. The Higgs boson is neutral under the electromagnetic interactions and transforms as a singlet under SU(3)C and hence does not couple at tree level to the massless photons and gluons.

The fermions of the SM acquire mass through renormalizable interactions between the Higgs field and the fermions: the Yukawa interactions, (vedi PDG per la lagrangiana)

The SM Higgs boson is a CP-even scalar of spin 0. Its mass is given by mH = 2λ v, where λ is the Higgs self-coupling parameter in V (ϕ). The expectation value of the Higgs field,

The Higgs boson couplings to the fundamental particles are set by their masses; the SM Higgs couplings to fundamental fermions are linearly proportional to the fermion masses, whereas the couplings to bosons are proportional to the square of the boson masses.

etc. pag 183

Altro...
(the interaction strengths of fermions with the Higgs boson H turns out to be proportional to their measured mass. Mass is thus a dynamic property of elementary particles due to their interaction with the Higgs field ϕ. Mass is truly distinct from other intrinsic properties of particles like charge or spin. This is also true for the Higgs boson itself, which owes its

mass to its own interaction with the Higgs field.)

- From PDG

The CP-even neutral component of the Higgs field acquires a vacuum expectation value $\rho_0 \approx 246$ GeV, which sets the scale of electroweak symmetry breaking. Three massless Goldstone bosons are generated and are absorbed to give masses to the W and Z gauge bosons. The remaining component of the complex doublet becomes the Higgs boson – a new fundamental scalar particle.

as the SM Higgs boson is a scalar particle, at the quantum level it has sensitivity to high energy thresholds. Quite generally, the Higgs mass is affected by the presence of heavy particles and receives quantum corrections proportional to highest energy scale which destabilize the weak scale barring a large fine tuning of unrelated parameters. This is known as the hierarchy or naturalness problem. There are two broad classes of models addressing the naturalness problem1: one is based on a new fermion-boson symmetry in nature called supersymmetry (SUSY). in this case, the Higgs boson remains elementary and the corrections to its mass are screened at the scale at which SUSY [REF PDG] is broken and remain insensitive to the details of the physics at higher scales. These theories predict at least three neutral Higgs particles and a pair of charged Higgs particles. One of the neutral Higgs bosons, most often the lightest has properties that resemble those of the SM Higgs boson.

The other approach invokes the existence of strong interactions at a scale of the order of a TeV or above and induces strong breaking of the electroweak symmetry [REF PDG] In the original incarnation of this second approach, dubbed technicolor, the strong interactions themselves trigger EWSB without the need of a Higgs boson. In the original incarnation of this second approach, dubbed technicolor, the strong interactions themselves trigger EWSB without the need of a Higgs boson.

– The naturalness problem has been the prime argument for new physics at the TeV scale, and sizable effects on the Higgs boson properties were expected.

1.1.3 Finally, the SM

The Standard Model Lagrangian is given in 1.11. It contains, in sequence, the gauge Lagrangian (describing the dynamics of the free fields), the electroweak interaction-not sure-, the Higgs terms and the Yukawa Lagrangian.

$$\mathcal{L}_{SM} = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{1}{2}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{\psi}i\gamma^{\lambda}D_{\lambda}\psi + (D_{\lambda}\phi)^{\dagger}(D^{\lambda}\phi) - V(\phi) + \mathcal{L}_{Yukawa} + hermitianconju \tag{1.11}$$

where:

- $\mathcal{L}_{Yukawa}$ is the Yukawa Lagrangian, which describes the coupling between the Higgs field and massless quark and lepton fields. It is given by: $-g\bar{\psi}\phi\psi$
- the covariant derivative D_μ is defined as: $D_\mu = \partial_\mu + ig_s G^a_\mu F^a + igW^b_\mu T^b + ig' B_\mu Y$

- the matter fields are described by the multiplet: $\psi = \begin{pmatrix} \begin{pmatrix} \nu_l \\ l \end{pmatrix}_L & . \\ b'_R & \end{pmatrix}$

- the Higgs field ϕ is defined as: $\phi = \frac{\rho_0}{2} \begin{pmatrix} 0 \\ 1 + \frac{\rho'}{rho_0} \end{pmatrix}$

1.1.4 The discovery of the Higgs boson

The ATLAS and CMS experiments at the Large Hadron Collider, on the 4th of July 2012 announced that they had observed a new particle, (which seemed) consistent with the Higgs boson predicted by the SM, in the mass region around 125 GeV [7, 8].
In the successive years the properties of this particle have been studied i

(In particular, the SM Higgs boson couplings to fundamental fermions are linearly proportional to the fermion masses mf (yf = mf $/\nu$ where yf is the Yukawa coupling) whereas the couplings to bosons are proportional to the squares of the boson masses. As consequence, the dominant mechanism for Higgs boson production and decay involve the coupling of H to W, Z and/or the third generation quarks and leptons. The Higgs boson coupling to gluons is induced at leading order by a one-loop process in which H couples to a virtual t$\bar{t}$ pair. Likewise, the Higgs boson couplings to photon is also generated via loops, although in this case the one-loop graph with a virtual W+W- pair provides the dominant contribution and the one involving a virtual t$\bar{t}$ pair is subdominant.)

Beautiful plots collection

The distribution of the four-lepton reconstructed invariant mass with full Run 2 data collected at 13 TeV by CMS and ATLAS experiments is shown in Fig. 1.4.

The distribution of the diphoton reconstructed invariant mass with data collected by CMS and ATLAS experiments at 13 TeV is shown in Fig. 1.5.

1.1.5 Status of the Higgs boson physics - from PDG19

"Conclusion summary from PDG"
Milestone measurements have been performed: (i) a clear observation of the Higgs boson decay to taus has been made by the CMS experiment; (ii) unambiguous evidence for the Higgs boson decay to a pair of b quarks was provided by both the ATLAS and CMS experiments; (iii) the ATLAS and CMS experiments have also provided evidence for the production of the Higgs boson through the t$\bar{t}$ H mechanism, thus yielding direct evidence for the Yukawa coupling of the Higgs boson to top quarks, with a strength compatible with that of the Standard Model. These and all other experimental measurements are consistent with the EWSB mechanism of the Standard Model.

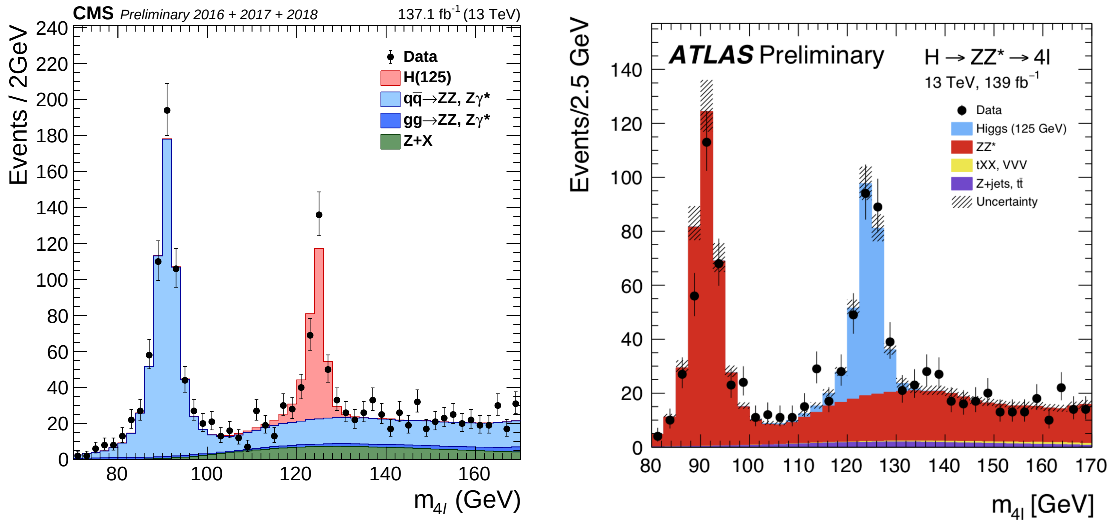

Figure 1.4: Distribution of the reconstructed four-lepton invariant mass m_{4l} in the low-mass range, with full Run 2 data, in CMS [9] (on the left) and ATLAS [10] (on the right) experiments. Points with error bars represent the data and stacked histograms represent expected distributions of the signal and background processes.

Many extensions of the SM at higher energies call for an enlargement of the EWSB sector. Hence, direct searches for additional scalar states can provide valuable insights on the dynamics of the EWSB mechanism. The ATLAS and CMS experiments have searched for additional Higgs bosons in the Run 2 data, and imposed constraints in broad ranges of mass and couplings for various extended Higgs scenarios.

1.2 Physics Beyond the Standard Model

1.2.1 Shortcomings of the SM

Shortcomings of the SM [The Standard Model is a very successful theory and its predictions have been verified over a wide range of energies. Among them, a particular importance has the discovery of the Higgs boson by the ATLAS and CMS collaborations in 2012 [**??**arie]. However, the SM has several shortcomings, as described below. (and this is the reason why a lot of different theories BSM have been created in the last years to extend it.)

- First of all, the SM is not a complete theory: it unifies only 3 of the 4 fundamental interactions, leaving aside the gravitation (described successfully by the theory of General Relativity).
  (da modificare : To combine the quantum theory of the SM with general relativity, a quantum theory of gravity is necessary, which could be obtained by adding a particle carrying the gravitational force, a graviton. This proves to be difficult because of the way the gravity interacts with the geometry of spacetime. The strength of the gravitational force is much lower than the ones from the other three fundamental

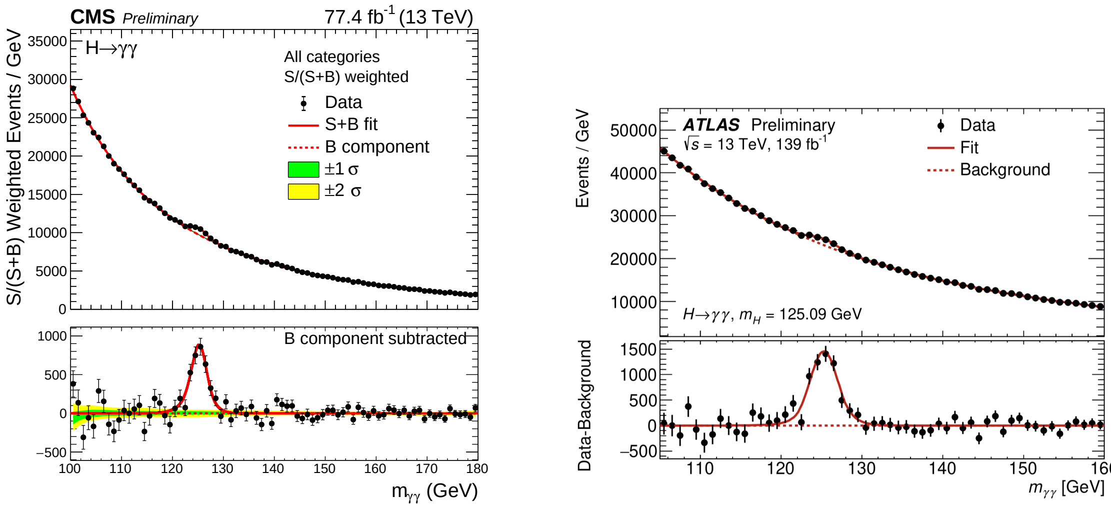

Figure 1.5: Distribution of the reconstructed four-lepton invariant mass $m_{2\gamma}$ in the low-mass range, with 2016 and 2017 data collected by CMS [11] (on the left) and the full Run 2 data collected by ATLAS [12] (on the right). The black dots represent the data.

forces. While those have a similar strength that shows an effect at the electroweak scale of O(100 GeV), the energy at which gravitational interactions become relevant is at the order of the Planck scale $E_P = 10^{19}$ GeV, which is defined by the Planck mass, $M_P = \sqrt{\frac{\hbar c}{G_N}}$, with G_N being the gravitational constant. The huge difference between the electroweak scale and the Planck scale is also known as a hierarchy problem.)

- "Naturalness" of the theory : (The hierarchy problem implies that the mass of the Higgs boson should be of the order of the Planck scale. In order for the observed mass value to be 125 GeV, a fine-tuned cancellation of the bare mass and the contri- bution from Feynman diagrams loop corrections, both of O(10^{19} GeV), is necessary.)

- In the original formulation of the SM, neutrinos were strictly massless, but experimental evidences have established the existence of neutrino oscillations, i.e. transitions between the different neutrino flavors. This is an evidence for neutrino mixing and nonzero neutrino masses [**??** Vanf 1].
(modifica: A mass term for the neutrinos can be added to the SM, but it is not clear if the small masses that the neutrinos must have can arise from the same electroweak symmetry breaking mechanisms than the masses for the other particles of the SM.)

- From astrophysical observations is known that the visible content of matter can only be approximately 5% of the total matter and energy content of our universe. Therefore, it has been hypothesized the existence of an unknown type of matter and energy, called *dark matter* and *dark energy*, which cannot be explained within the SM. The dark matter (25%) is assumed not to interact electromagnetically or by strong interactions, while the dark energy (70%) is thought to be responsible for the observed accelerated expansion of the universe.

- the CP-violating effects of the SM cannot explain the large difference between the

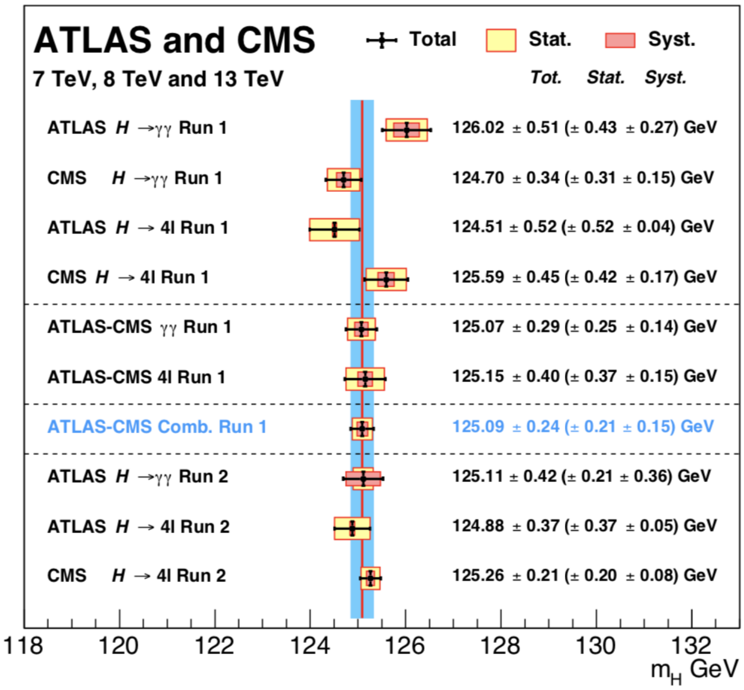

Figure 1.6: Summary of the CMS and ATLAS mass measurements in the $\gamma\gamma$ and ZZ channels in pp collisions during Run 1 and Run 2. (REF PDG19).

amount of matter and anti-matter in the universe [1].

- the SM has a large number of free parameters:
  - 17 (EW theory) = 2 coupling constants (g, g'), 2 provided by Higgs mechanism ($m_{\rho'}$, m_W (or m_Z) oppure mass and vacuum expectation value of the Higgs boson), 4 given by the Cabibbo-Kobayashi-Maskawa V_{CKM} matrix (3 angles + 1 phase) and 9 coupling constants of the different fermions with the Higgs field, in order to obtain the fermion masses (3 charged leptons + 6 quarks).
  - 1 (QCD) = 1 coupling constant (g_S)
  Including also the neutrino oscillations, there are other 7 parameters to take into account: 3 neutrino masses and 4 parameters for the PMNS matrix.
  In total 25 parameters: all of them have been measured!

- (da modificare: The SM coupling constants of the electromagnetic interaction, the weak interaction and the strong interaction have a similar value at an energy scale of O(1016 GeV). However, they do not converge to a single value as shown in Figure 1.1. In order to unify the coupling constants, an extension of the SM is necessary that changes the evolution above the electroweak scale.) Aggiungi immagine

(from all these considerations it is evident that there must be new physics at a scale beyond the electroweak scale. What is unknown, however, is the energy scale at which this new physics will manifest itself. It could be as high as the Planck scale, but this would mean that the hierarchy problem remains unsolved. Therefore, it is rather believed that there should be new physics at the TeV scale, at which a discovery with direct searches at the LHC could be possible.)

1.2.2 SM extensions

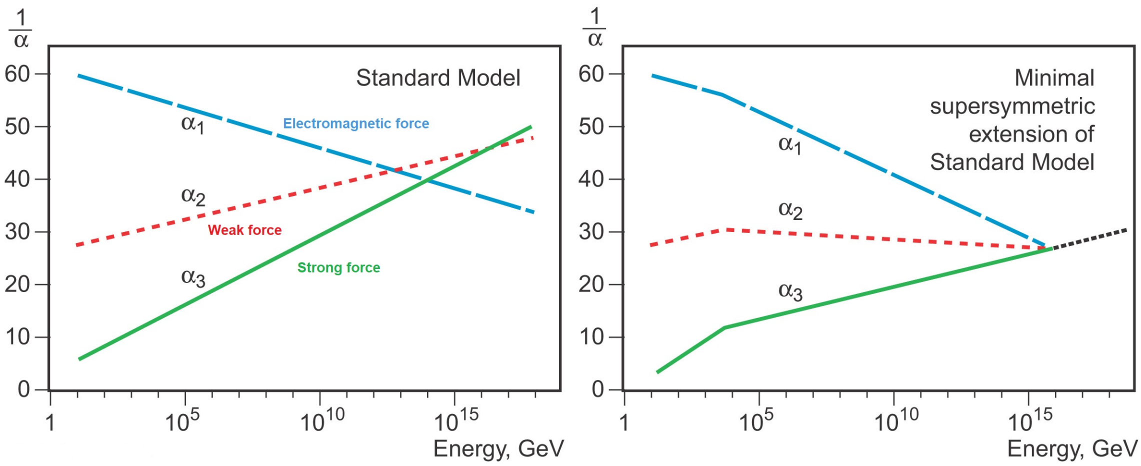

Figure 2.7: Evolution of the couplings $\alpha_i = \frac{g_i^2}{4\pi}$ as a function of the energy as predicted by the SM (on the left) and by the MSSM (on the right). The coupling constants of the strong weak and electromagnetic force are shown respectively in green, red and blue.

Neutrino oscillations - by Wiki

The so-called *neutrino oscillations* is the phenomenon whereby a neutrino created with a specific lepton family number ("lepton flavor": electron, muon, or tau) can later be measured to have a different lepton family number. This phenomenon was predicted by Bruno Pontecorvo in 1957 and it has been observed by a several experiments in several different contexts. it implies that the neutrino has a non-zero mass, which requires a modification to the Standard Model of particle physics.

The experimental discovery of neutrino oscillation, and thus neutrino mass, by the Super-Kamiokande Observatory and the Sudbury Neutrino Observatories was recognized with the 2015 Nobel Prize for Physics.

Neutrino oscillation arises from mixing between the flavor and mass eigenstates of neutrinos. That is, the three neutrino states that interact with the charged leptons in weak interactions are each a different superposition of the three (propagating) neutrino states of definite mass. Neutrinos are emitted and absorbed in weak processes in their flavor eigenstates[a] but travel as mass eigenstates. As a neutrino superposition propagates through space, the quantum mechanical phases of the three mass states advance at slightly different rates, due to the slight differences in their respective neutrino masses. This results in a changing superposition mixture of mass eigenstates as the neutrino travels. The electron flavor content of the neutrino will then continue to oscillate – as long as the quantum mechanical state maintains coherence. Since mass differences between neutrino flavors are

small in comparison with long coherence lengths for neutrino oscillations, this microscopic quantum effect becomes observable over macroscopic distances.
In contrast, due to their larger masses, the charged leptons (electrons, muons, and tau leptons) have never been observed to oscillate.

The original Pontecorvo's idea was included in a quantitative theory developed by Maki, Nakagawa, and Sakata in 1962 and further elaborated by Pontecorvo in 1967 [13].

This theory successfully explained the deficit observed in solar neutrinos the year later.

In its simplest form the neutrino oscillation is expressed as a unitary transformation relating the flavor and mass eigenbasis and can be written as:

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle$$

$$|\nu_i\rangle = \sum_\alpha U_{\alpha i} |\nu_\alpha\rangle$$

where:

- $|\nu_\alpha\rangle$ is a neutrino with definite flavor $\alpha =$ e (electron), μ (muon) or τ (tauon),
- $|\nu_i\rangle$ is a neutrino with definite mass $m_i, i = 1, 2, 3$.

$U_{\alpha i}$ is the Pontecorvo–Maki–Nakagawa–Sakata matrix (PMNS matrix), which is the analogue of the CKM matrix describing the mixing of quarks. It is a 3×3 matrix given by:
$\begin{pmatrix} U_{e_1} & U_{e_2} & U_{e_3} \\ U_{\mu_1} & U_{\mu_2} & U_{\mu_3} \\ U_{\tau_1} & U_{\tau_2} & U_{\tau_3} \end{pmatrix}$

Bla bla bla, se vuoi approfondisci ... (parametrizzazione matrice e risultati sperimentali up to now)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e_1} & U_{e_2} & U_{e_3} \\ U_{\mu_1} & U_{\mu_2} & U_{\mu_3} \\ U_{\tau_1} & U_{\tau_2} & U_{\tau_3} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \tag{2.12}$$

1.3 Lepton Flavor Violation (LFV)

1.3.1 LFV in SM

1.3.2 LFV in BSM

Supersymmetric Model with R-Parity Violation

The discrete R-parity has been introduced into Supersymmetry [16] because ...

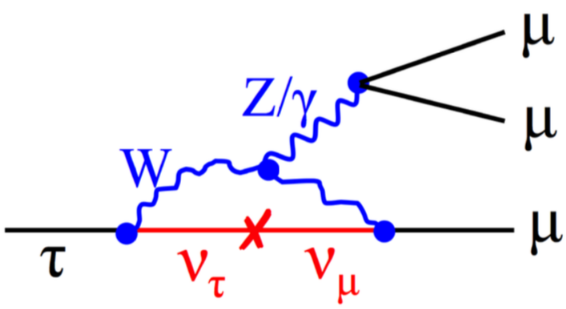

Figure 3.8: FD/SM.

It is defined as:

$$R_p = (-1)^R = \begin{cases} +1 & \text{for SM particles } (R-\mathit{even}) \\ -1 & \text{for SUSY particles } (R-\mathit{odd}) \end{cases} \tag{3.13}$$

where R is defined as $R = 3B + L + 2S$ where B and L are respectively the baryon and the lepton numbers, while S is the spin of the particle.

This parity guarantees that the lightest supersymmetric particle is stable and therefore it is a good candidate for dark matter.

In supersymmetric models with R-parity violation which are still possible with respect to the present experimental bounds, an additional non-diagonal Yukawa coupling between the charged leptons and sneutrinos allows lepton flavour violation in the charged lepton sector [17].
The contributing Feynman diagram to the decay $\tau \to \mu\mu$ is depicted in Fig. 3.9.

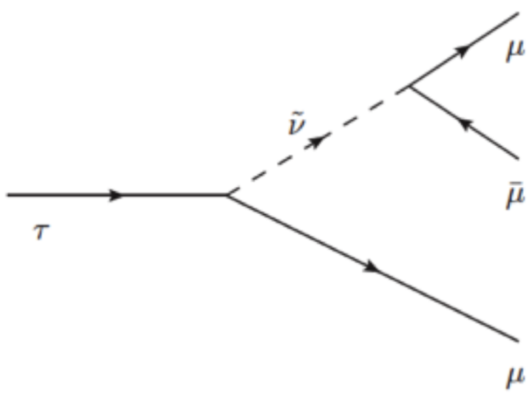

Figure 3.9: FD/BSM.

1.3.3 Analysis theorical intro

In nature there are no known symmetries that would forbid leptons decays in which the flavor is not conserved. These lepton-flavor violation (LFV) decays can be of the kind: $l \to l'\gamma$, $l \to 3l'$ and $l \to 2\mu e/2e\mu$ (where l and l' are leptons (e, μ, τ) with different flavor).

Among these possible decays, the second one was chosen to be searched for in the work of this thesis. In particular, the analysis I have performed had the purpose to search for $\tau \to 3\mu$ decays. Actually, in the SM with the neutrino oscillations, such kind of decays are possible and they are described in terms of Feynman diagrams as shown in Fig. 3.8.

However, this kind of decay is predicted with a small branching fraction: $\mathcal{B}(\tau \to 3\mu \sim \mathcal{O}(10^{-14})$. [ref. AN] In present day experiment a so low BR is not accessible! On the other hand, several theories that aim to extend the SM, called "Beyond Standard Model" (BSM) theories, allow these LFV decays with a BR which is enhanced of several orders of magnitude, resulting in cross section values which can be tested with current experiments[ref. AN]. An example of the decay mechanisms predicted for this channel by one BSM is shown in Fig. 3.9.

The chosen of this particular decay channel to investigate this kind of violation is motivated by several reasons:

1. Other kind of LFV decays are expected to be extremely suppressed, with a BR many orders of magnitude smaller! For example: $\mathcal{B}(\tau \to \mu\gamma \sim \mathcal{O}(10^{-40})$. [ref. AN]

2. LHC is a τ factory! the expected inclusive production cross section for pp collisions is about 2×10^{11} fb . [ref. AN]

3. CMS is brilliant in detecting muons

4. The enhancements predicted by BSMs may be larger for heavier-flavor lepton [ref. AN]

The search for this kind of decay has been already performed in the last years by different experiments, but no signal has never been observed, leading to the determination of an upper limit value for the BR of the process. The best experimental value set for this upper limit, up to now, is the one obtained by Belle experiment [ref. AN] : $\mathcal{B}(\tau \to 3\mu \leq 2.1 \cdot 10^{-8}$ at 90% confidence level [18]. A similar result was obtained by BaBar, another experiment operating, as Belle, at an e^+e^- collider and now decommissioned, that set the limit at $3.3 \cdot 10^{-8}$ [19].

Also LHC experiments have looked for this decay, with the following results:

- LHCb: 4.6 $\times 10^{-8}$ [20],
- ATLAS: 3.8 $\times 10^{-7}$ [21]
- CMS (only 2016 data - 33 fb^{-1}) : 8.8 $\times 10^{-8}$ - [ref. AN]

Chapter 2

The CMS experiment at LHC

Chapter 3

Events reconstruction in CMS

Chapter 4

Analysis

(This analysis was performed in CMS on the 2016 data: int. lumi of 33 fb^{-1} of pp collision at center of mass energy $\sqrt{s} = 13$ TeV. No signal has been observed, putting an upper limit on the branching fraction of the tau lepton decaying in 3 muons $\mathcal{B}(\tau \to 3\mu)$ of 8.8 $\times 10^{-8}$ at 90% confidence level. -¿ Si può citare il PAS ?)

4.1 Preliminary considerations

4.1.1 Production of τ leptons at LHC

As said previously, the LHC collider is a τ factory! These leptons can be produced in different processes, leading to an estimation of the total INCLUSIVE number of them of sim 2×10^{11} fb.

(qui ci sarebbe la tabella con i vari processi e il relativo numero di leptoni ... quella della Florida è riferita alla luminosità del 2016)!

In pp collisions, τ leptons are produced mainly from hadrons containing c or b quarks (in particular from D and B mesons and only rarely from c/b-baryons) and from the decays of W and Z bosons: $W \to \tau\nu_\tau$ and $Z \to \tau^+\tau^-$.

Tau leptons can be originated also by B meson decaying in D mesons that in turn decays in tau lepton + neutrino. These “indirect” decays can be distinguished from the “direct/prompt” ones (from directly produced D mesons) because they are characterized by larger transverse momenta and displaced vertices.

The expected inclusive number of τ coming from different processes are reported in Tab. 4.1.

The relative τ yields are listed in Tab. 4.2.

Process1	Process2	N. of τ for L =?
$pp \to c\bar{c} + ...$	$D \to \tau\nu_\tau$ (95% D_s, 5% $D^\pm$)	000
$pp \to b\bar{b} + ...$	$B \to \tau\nu_\tau + ...$ (44% $B^\pm$, 45% B^0, 11% B_s^0)	000
	$B \to D(\tau\nu_\tau) + ...$ (98% D_s, 2% $D^\pm$)	000
$pp \to W + ...$	$W \to \tau\nu_\tau + ...$	000
$pp \to Z + ...$	$Z + ... \to \tau\tau + ...$	000

Table 4.1: List of processes from which τ leptons are produced, with their corresponding expected inclusive number for L = ? // The charge-conjugated states are included.

Mother meson	Quark composition	Meson mass (GeV)	Relative τ yield
D_s	$c\bar{s}$	1.97	72%
D^+	$c\bar{d}$	1.87	3%
B^+	$\bar{b}u$	5.28	11%
B^0	$\bar{b}d$	5.28	11%
B_s	$\bar{b}s$	5.37	3%

Table 4.2: List of mesons from which τ are produced with the relative tau yields. The charge-conjugated states are included.

4.1.2 Signal acceptance

4.2 Outline: search strategy

4.3 Data and MonteCarlo samples

4.3.1 Datasets

For the analysis described in this thesis, I have used data collected by CMS in pp collisions at a center of mass energy $\sqrt{s} = 13$ TeV during Run II in the whole 2017.
These datasets are listed in Tab. 4.3.
They correspond to a total integrated luminosity: $\mathcal{L} = 41.49\ fb^{-1}$.

2017 Datasets	Run range	Integr. Luminosity (fb^{-1})
/DoubleMuonLowMass/Run2017B-17Nov2017-v1/AOD	000	4.79
/DoubleMuonLowMass/Run2017C-17Nov2017-v1/AOD	000	9.63
/DoubleMuonLowMass/Run2017D-17Nov2017-v1/AOD	000	4.25
/DoubleMuonLowMass/Run2017E-17Nov2017-v1/AOD	000	9.30
/DoubleMuonLowMass/Run2017F-17Nov2017-v1/AOD	000	13.52
Whole 2017 data	000	41.49

Table 4.3: Datasets of the whole 2017 data-taking that have been used for the analysis described in this thesis.

The json file used for data processing is:
Collisions17/13TeV/ReReco/Cert$_2$94927 − 306462$_1$3TeV$_\mathrm{E}$OY2017ReReco$_\mathrm{C}$ollisions17$_\mathrm{J}$SON$_\mathrm{v}$1.txt.

4.3.2 MonteCarlo samples

The MC samples used in this analysis are listed in Tab. 4.4.

Simulated process	MC Dataset name
$\mathrm{D_s} \to \tau\nu_\tau \to 3\mu\ \nu_\tau$	/DsToTau$_\mathrm{T}$o3Mu$_\mathrm{M}$uFilter$_\mathrm{T}$uneCUEP8M1$_1$3TeV − pythia8/RunIIFall17DRPremix −
$\mathrm{B^0} \to \tau\nu_\tau \to 3\mu\ \nu_\tau$	/BuToTau$_\mathrm{T}$o3Mu$_\mathrm{M}$uFilter$_\mathrm{T}$uneCUEP8M1$_1$3TeV − pythia8/RunIISummer16DR80Premix − PUMori
$\mathrm{B^\pm} \to \tau\nu_\tau \to 3\mu\ \nu_\tau$	/BdToTau$_\mathrm{T}$o3Mu$_\mathrm{M}$uFilter$_\mathrm{T}$uneCUEP8M1$_1$3TeV − pythia8/RunIISummer16DR80PremixPUMorio
$\mathrm{D_s} \to \phi\pi \to 2\mu\ \pi$	/DsPhiPi/fsimone − crab$_\mathrm{c}$rab$_\mathrm{D}$sPhiPi$_1$3TeV$_\mathrm{R}$ECO − c3763c515b5d7d94a8

Table 4.4: MC samples used for the analysis described in this thesis.

The branching fractions of the B and D mesons decays are listed in Tab. 4.5.

Process	Branching ratio(BR)	Reference
$\mathrm{D_s} \to \tau\nu_\tau$	5.48 ± 0.23%	PDG [71]
$\mathrm{B^+} \to \tau\nu_\tau \mathrm{D}_0^*$	2.7 ± 0.3%	PDG [71]
other$\mathrm{B^+} \to \tau\nu_\tau \mathrm{X}$	0.7%	PYTHIA [68]
$\mathrm{B^0} \to \tau\nu_\tau \mathrm{D}^{+*}$	2.7 ± 0.3%	PDG [71]
other$\mathrm{B^0} \to \tau\nu_\tau \mathrm{X}$	0.7%	PYTHIA [68]
$\mathrm{B^+} \to \mathrm{D_s X}$	9.0 ± 1.5%	PDG [71]
$\mathrm{B^0} \to \mathrm{D_s X}$	10.3 ± 2.1%	PDG [71]
$\mathrm{D_s} \to \phi(\mu\mu)\pi$	$1.3(\pm\ 0.1 \cdot 10^{-5})$	PDG [71]

Table 4.5: MC samples used for the analysis described in this thesis.

4.4 Event selection

	Signal		Data
	$D_s \to \tau\nu_\tau$	$B^\pm/B^0 \to \tau...$	
Produced in pp collisions	000	000	
(with three muons in fiducial volume)	000	000	
L1/HLT trigger	000	000	
At least 3 GlobalMuon ($\mathrm{p_T} > 2$ GeV)	000	000	000
Trimuon candidate selection	000	000	000

Table 4.6: Bla Bla

The PRE-selections applied to choose the interesting (DsTau3Mu) events are the following:

1. Fired HLT : at least one HLT path among
    - HLT_DoubleMu3_Trk_Tau3mu_v∗

- HLT_DoubleMu3_TkMu_DsTau3Mu_v*
- HLT_DoubleMu3_Trk_Tau3mu_NoL1Mass_v*

2. At least 3 muons with :

   - $p_T > 0.5$
   - $|\eta| < 2.4$
   - innerTrack().isNonnull
   - charge $\neq 0$
   - innerTrack().hitPattern().numberOfValidPixelHits()> 0

3. At least 1 triplet with $|\text{charge}| = 1$

The selections applied on the triplets to choose the interesting (DsTau3Mu) ones are the following:

1. The χ^2 of the triplet vertex is in (0-15)
2. The 3 possible pairs of mu of the triplet have $\Delta R < 0.8$
3. The 3 possible pairs of mu of the triplet have $|\Delta Z| < 0.5$
4. VETO on mass of o.s. muons of the triplet compatible with ϕ meson
5. VETO on mass of o.s. muons of the triplet compatible with ω meson
6. Mass window ??
7. L1 ???
8. trigger matching requirements

If more than one triplet per event survives, it's chosen the one with the best (min) χ^2 value.

4.5 Signal Normalization

4.5.1 Events triggered by DoubleMu L1 seeds

$$\mathrm{N_{sig(D)}} = \mathcal{L}\ \sigma(pp \to D_s)\ \mathcal{B}(D_s \to \tau\nu_\tau)\ \mathcal{B}(\tau \to 3\mu)\ \mathcal{A}_{3\mu(D)}\ \epsilon_{reco}^{3\mu}\ \epsilon_{trig}^{2\mu} \tag{5.1}$$

$$\mathrm{N} = \mathcal{L}\ \sigma(pp \to D_s)\ \mathcal{B}(D_s \to \phi\pi \to \mu\mu\pi)\ \mathcal{A}_{2\mu\pi}\ \epsilon_{reco}^{2\mu\pi}\ \epsilon_{trig}^{2\mu} \tag{5.2}$$

$$\mathrm{N_{sig(D)}} = N\ \cdot \frac{\mathcal{B}(D_s \to \tau\nu_\tau)}{\mathcal{B}(D_s \to \phi\pi \to \mu\mu\pi)}\ \frac{\mathcal{A}_{3\mu(D)}}{\mathcal{A}_{2\mu\pi}}\ \frac{\epsilon_{reco}^{3\mu}}{\epsilon_{reco}^{2\mu\pi}}\ \mathcal{B}(\tau \to 3\mu) \tag{5.3}$$

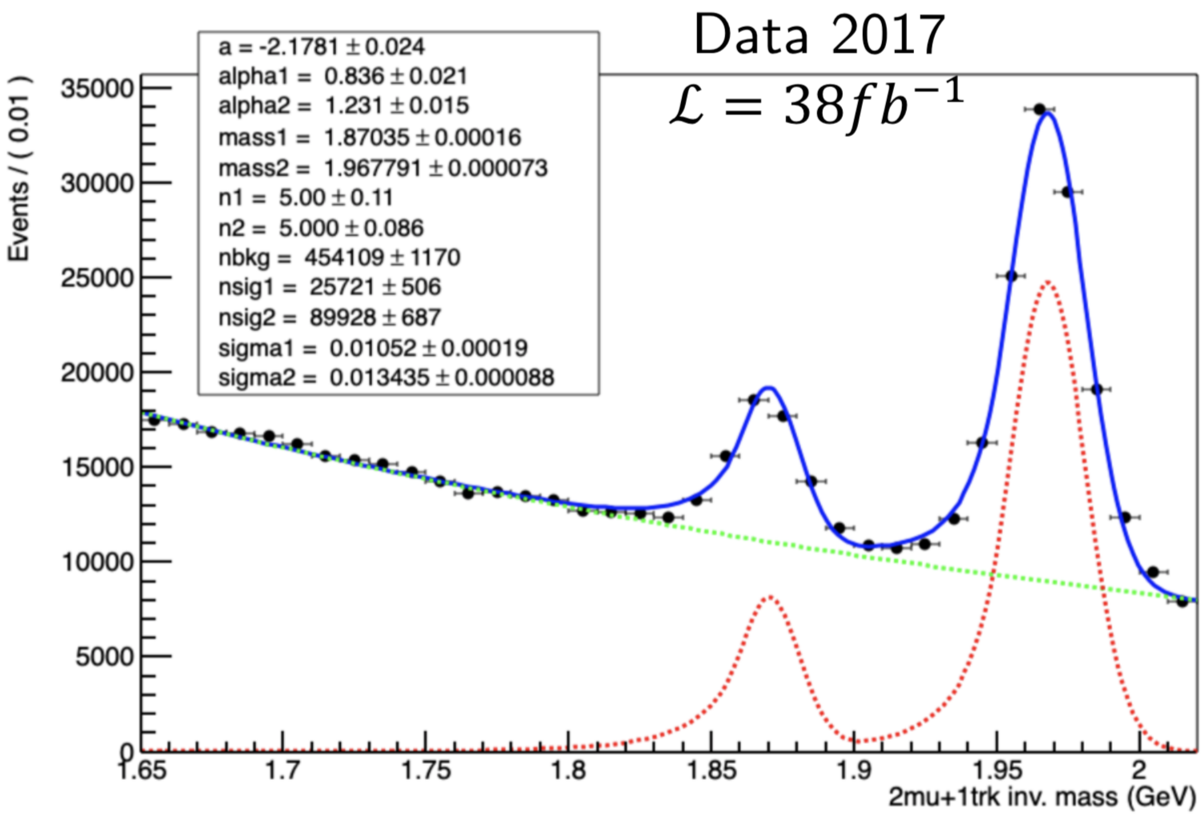

Figure 5.1: NDs-total.

Run	Fitted $D_s \to \phi(\mu\mu)\pi$ yields (fb^{-1})	Data/MC ratio
2017 B	000	000
2017 C	000	000
2017 D	000	000
2017 E	000	000
2017 F	000	000
Whole 2017	000	000

Table 4.7: Bla Bla

4.5.2 B/D ratio

$$\mathrm{N}_{\mathrm{sig(B)}} = \mathcal{L}\ \sigma(pp \to B)\ \mathcal{B}(B \to \tau + ...)\ \mathcal{B}(\tau \to 3\mu)\ \mathcal{A}_{3\mu(B)}\ \epsilon^{3\mu}_{reco}\ \epsilon^{2\mu}_{trig} \tag{5.4}$$

$$f \ = \ \frac{\sigma(pp \to B)\ \mathcal{B}(B \to D_s + ...)}{\sigma(pp \to D_s)} \tag{5.5}$$

$$\mathrm{N}_{\mathrm{sig(B)}} = N \cdot\ f \cdot \frac{\mathcal{B}(B \to \tau + ...)}{\mathcal{B}(D_s \to \phi\pi \to \mu\mu\pi)\ \mathcal{B}(B \to D_s + ...)}\ \frac{\mathcal{A}_{3\mu(B)}}{\mathcal{A}_{2\mu\pi}}\ \frac{\epsilon^{3\mu}_{reco}}{\epsilon^{2\mu\pi}_{reco}}\ \mathcal{B}(\tau \to 3\mu) \tag{5.6}$$

4.5.3 Control channel

MC validation with $D_s \to \phi(\mu\mu)\pi$.

PRE-selections applied on the events:

1. Fired HLT path: $HLT\_DoubleMu3\_Trk\_Tau3mu\_v*$
2. at least 1 triplet with:
   - 2 muons with:
     - $p_T > 0.5$
     - $|\eta| < 2.4$
     - innerTrack().isNonnull
     - charge $\neq 0$
     - innerTrack().hitPattern().numberOfValidPixelHits() > 0
   - 1 track with:
     - $p_T > 2$
     - $|\eta| < 2.4$
     - charge $\neq 0$
     - hitPattern().trackerLayersWithMeasurement() > 5
     - hitPattern().pixelLayersWithMeasurement() ≥ 1
3. the triplet must have:
   - mass in (0.5 - 10) GeV
   - $|\text{charge}| = 1$

The selections applied on the triplets are:

1. L1 trigger trigger fired: $L1\_DoubleMu0er1p5\_SQ\_OS\_dR\_Max1p4$
2. the 2 muons are global and different from the track
3. the χ^2 of the triplet vertex is in [0,15]
4. the 2 muons have opposite charge
5. the 2 muons invariant mass is in [1, 1.04] GeV
6. the longitudinal IP $<$ 20 cm and the transverse IP $<$ 0.3 cm
7. trigger matching requirements:
   - Mu01_dRtriggerMatch < 0.03
   - Mu02_dRtriggerMatch < 0.03
   - Tr_dRtriggerMatch < 0.03

If more than one triplet per event survives, it's chosen the one with the best (min) χ^2 value.

4.6 Multivariate Analysis (MVA)

4.6.1 Introduction to MVA: why to use it?

The Multivariate analysis has assumed progressively a central role in high energy physics searches [73]. In this field, this kind of analysis aims to exploit as much information as possible from the characteristics of each event in order to distinguish between event types (eg. signal and background).
In order to understand how this analysis works and which are the improvements in using it with respect to "traditional" analysis performed by successive cuts, let's look at the scatter plots in Fig. 6.2. They show the distribution of two variables x_1 and x_2 which represent two out of a potentially large number of quantities measured for each event, with different decision boundaries (cuts, lineary boundary, non linear boundary). The signal events are indicated with blue circles while the red triangles represent the background ones.

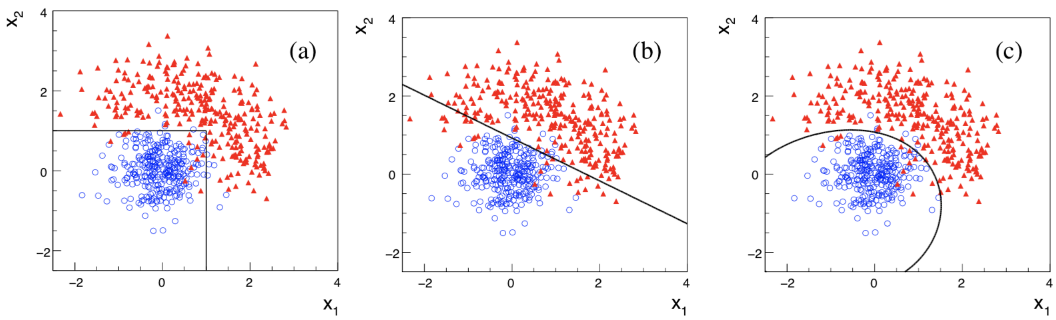

Figure 6.2: Scatter plots of two variables corresponding to two hypotheses: signal (blue) and background (red). Event selection could be based, e.g., on (a) cuts, (b) a linear boundary, (c) a nonlinear boundary. [73].

It is evident that rectangular and diagonal cuts are not as good as nonlinear boundaries in classifying the events. In general, the best decision boundary is a surface in the n-dimensional space of input variables, which can be represented by an equation of the form $y(\boldsymbol{x}) = y_{cut}$, where y_{cut} is some constant. Events are classified as signal if they are on one side of the boundary, e.g., $y(\boldsymbol{x}) \leq y_{cut}$, could represent the acceptance region and $y(\boldsymbol{x}) > y_{cut}$ could be the rejection region.
- oppure utilizzo come "scalar test" statistics (vedi 14-15 Cowan per more details)

4.6.2 Boosted Decision Tree (BDT)

Decision Tree

A decision tree is a binary tree structured classifier, defined by a collection of successive cuts on the set of input variables. It is schematically represented in Fig. 6.3.

Starting from the entire sample of training events in the root node, out of all of the

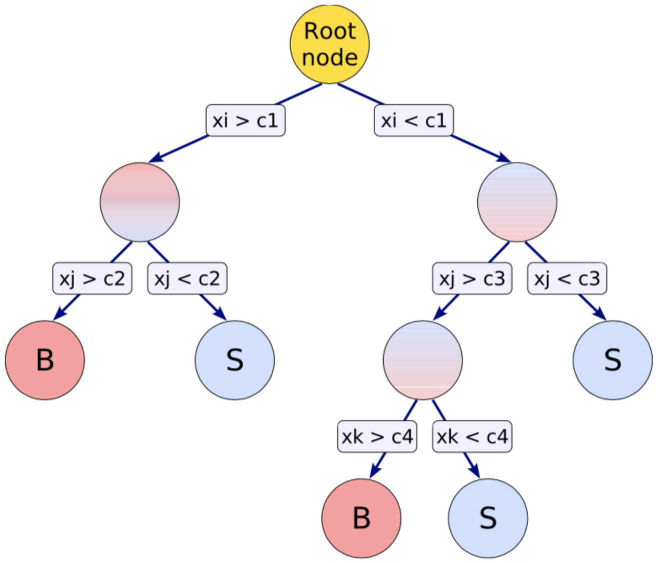

Figure 6.3: Scheme of a decision tree. A sequence of binary splits using the discriminating variables xi is applied to the data, starting from the root node. Each split uses the variable that, at that node, discriminates in the best way signal(S) and background(B). The leaf nodes at the bottom end of the tree are labeled (S or B) depending on the majority of events that end up in the respective nodes [74].

possible input variables, one finds the ones that provide the best separation between signal and background by use of a single cut. Then, repeated left/right (yes/no) decisions are taken on one single variable at a time until a stop criterion is fulfilled.
In this way, the phase space is split into many regions that are eventually classified as signal or background, depending on the majority of training events that end up in the final leaf node.
Therefore, the difference of this classifier, with respect to the case of rectangular cuts, is that whereas a cut-based analysis is able to select only one hypercube as region of phase space, the decision tree is able to split the phase space into a large number of hypercubes, each of which is identified as either "signal-like" or "background-like". For classification trees, the path down the tree to each leaf node represents an individual cut sequence that selects signal or background depending on the type of the leaf node.

A shortcoming of decision trees is their instability with respect to statistical fluctuations in the training sample from which the tree structure is derived.
This problem can be overcome by using a Boosted Decision Tree.

Boosting the tree...

The boosting of a decision tree extends this concept from one tree to several trees, which form a *forest*.
The trees are derived from the same training ensemble by reweighting events (procedure

called “boosting”), and are finally combined into a single classifier which is given by an average of the individual decision trees. This boosting increases the statistical stability of the classifier and is able to drastically improve the separation performance compared to a single decision tree.
(In many cases, the boosting performs best if applied to trees that, taken individually, have not much classification power. These so called “weak classifiers” are small trees, limited in growth to a typical tree depth of as small as two, depending on the how much interaction there is between the different input variables. By limiting the tree depth during the tree building process (training), the tendency of overtraining for simple decision trees which are typically grown to a large depth and then pruned, is almost completely eliminated.)
A number of boosting algorithms have been developed, and these differ primarily in the rule used to update the weights; the one employed in the BDT used in this analysis is the so-called *AdaBoost* (adaptive boost), which is explained in detail in [62].

Training a decision tree

(ridurre: in molte parti è una ripetizione di quanto già detto! Accorpare al precedente!)

The training of a decision tree is the process that defines the splitting criteria for each node. It starts with the root node, where an initial splitting criterion for the full training sample is determined. The split results in two subsets of training events that each go through the same algorithm of determining the next splitting iteration. This procedure is repeated until the whole tree is built. At each node, the split is determined by finding the variable and corresponding cut value that provides the best separation between signal and background. The node splitting stops once it has reached the minimum number of events which is specified in the BDT configuration (option nEventsMin). The leaf nodes are classified as signal or background according to the class the majority of events belongs to. A variety of separation criteria can be configured to assess the performance of a variable and a specific cut requirement. Because a cut that selects predominantly background is as valuable as one that selects signal, the criteria are symmetric with respect to the event classes. All separation criteria have a maximum where the samples are fully mixed, i.e., at purity p = 0.5, and fall off to zero when the sample consists of one event class only.

Since the splitting criterion is always a cut on a single variable, the training procedure selects the variable and cut value that optimises the increase in the separation index between the parent node and the sum of the indices of the two daughter nodes, weighted by their relative fraction of events. The cut values are optimised by scanning over the variable range with a granularity that is set via the option nCuts.

PROVARE!!! The default value of nCuts=20 proved to be a good compromise between computing time and step size. Finer stepping values did not increase noticeably the performance of the BDTs. However, a truly optimal cut, given the training sample, is determined by setting nCuts=-1. This invokes an algorithm that tests all possible cuts on the training sample and finds the best one.

Variable ranking

A ranking of the BDT input variables is derived by counting how often the variables are used to split decision tree nodes, and by weighting each split occurrence by the separation gain-squared it has achieved and by the number of events in the node.
This measure of the variable importance can be used for a single decision tree as well as for a forest.

Performance

PUNTO a FAVORE: Decision trees are also insensitive to the inclusion of poorly discriminating input variables (which is not the case of neural networks, for example).

Boosted decision trees have become increasingly popular in particle physics in recent years. One of their advantages is that they are relatively insensitive to the number of input variables used in the data vector x. Components that provide little or no separation between signal and background are rarely chosen as for the cut that provides separation, i.e., to split the tree, and thus they are effectively ignored. Decision trees have no difficulty in dealing with different types of data; these can be real, integer, or they can simply be labels for which there is no natural ordering (categorical data). Furthermore, boosted decision trees are surprisingly insensitive to overtraining. That is, although the error rate on the test sample will not decrease to zero as one increases the number of boosting iterations (as is the case for the training sample), it tends not to increase. Further discussion of this point can be found in [75]. – copia-e-incolla- END

- TMVA

The Toolkit for Multivariate Analysis (TMVA) provides a ROOT-integrated [74] environment for the application of multivariate classification techniques.
All TMVA techniques belong to the family of *supervised learning* algorithms. They make use of training events, for which the desired output is known, to determine the mapping function that describes a decision boundary (classification). This function can contain various degrees of approximations and may be a single global function, or a set of local models.
A typical TMVA classification consists of two independent phases: the *training phase*, where the multivariate methods are trained, tested and evaluated, and an *application phase*, where the chosen methods are applied to the concrete classification or regression problem they have been trained for.

Input variable	Importance
var	000
var	000
var	000
var	000
var	000
var	000
var	000
var	000
var	000
var	000

Table 4.8: Bla Bla

Parameter	Setting (default)
NTrees	800
nCuts	20
MaxDepth	3
BoostType	AdaBoost

Table 4.9: Bla Bla

4.6.3 BDT training

4.6.4 Event categorization

4.7 Evaluation of systematics

Bla Bla [76]

Source of uncertainty	Yield	Shape
Uncertainty on D_s normalization [3.5%]	000	
Uncertainty on measuring f (B/D ratio) [10%]	000	
Uncertainty on n. of events triggered by trimuon trigger [12%]	000	
Relative uncertainty in $\mathcal{B}(D_s \to \phi\pi \to \mu\mu\pi)$ [8%]	000	
Relative uncertainty in $\mathcal{B}(D_s \to \mu\nu)$ [4%]	000	
Relative uncertainty in $\mathcal{B}(B \to D_s + ...)$ [16%]	000	
Relative uncertainty in $\mathcal{B}(B \to \tau + ...)$ [11%]	000	
Uncertainty on the ratio of acceptances $\mathcal{A}_{sig}$ / $\mathcal{A}_{2\mu\pi}$ [1%]	000	
Muon reconstruction efficiency [1.5%]	000	
BDT cut efficiency [5%]	000	
Muon momentum scale uncertainty [0.2%]	-	yes
Muon momentum resolution uncertainty [10%]	-	yes

Table 4.10: Bla Bla

4.8 Results

$$q_\mu = -2\ \ln \frac{\mathcal{L}\ (\mathrm{obs}|\ \mu \cdot \mathrm{s} + \mathrm{b},\ \hat{\hat{\theta_\mu}})}{\mathcal{L}\ (\mathrm{obs}|\ \hat{\mu} \cdot \mathrm{s} + \mathrm{b},\ \hat{\theta})} \tag{8.7}$$

$$\mathcal{L}(\mathrm{data}|\ \mu\mathrm{s} + \mathrm{b})\ \sim\ e^{-(\mu S + B)}\ \prod_i \mathcal{P}(\mathrm{x_i}|\ \mu\mathrm{s} + \mathrm{b}) \tag{8.8}$$

$$\mathrm{CL_s}\ =\ \frac{\mathrm{P}\ (\mathrm{q}_\mu \geq \mathrm{q}_\mu^{\mathrm{obs}}\ |\mu \cdot\ \mathrm{s} + \mathrm{b})}{\mathrm{P}\ (\mathrm{q}_\mu \geq \mathrm{q}_\mu^{\mathrm{obs}}\ |\ \mathrm{b})} \leq 0.1 \tag{8.9}$$

	Signal		Background	
	sub-category 1	sub-category 2	sub-category 1	sub-category 2
Category A	000	000	000	000
Category B	000	000	000	000
Category C	000	000	000	000

Table 4.11: Bla Bla

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