The goals of this discussion section are:

1. Be able to study equilibrium of some dynamical systems
2. Identify a model of population dynamics

Participation in discussion section counts as 5% of the grade. Completion of the worksheets counts as 20% of the grade. **Submit your worksheet work by February 10th at 2:59pm.**

1. We consider an Initial Value Problem based on the Gompertz growth model, defined by

$$\frac{dy}{dt} = \alpha y \ln\left(\frac{M}{y}\right), \quad y(0) = 2.$$

   (a) The solution of the above IVP is of the form $y(t) = Ae^{\ln(B)e^{Ct}}$, with A, B, C some constants to find. Determine A, B, C.

   (b) Implement the analytic solution of the three growth models: exponential ($\frac{dy}{dt} = \alpha y$), logistic ($\frac{dy}{dt} = \alpha y(1 - y/M)$), and Gompertz ($\frac{dy}{dt} = \alpha y \ln\left(\frac{M}{y}\right)$) using Python. Load the file `data.cvs`. You may load this data, and plot it, using the following commands:

```
import pandas as pd
from matplotlib.pyplot import plot as plt
file = 'data.csv'
df = pd.read_csv(file)
t = df.iloc[:,0]
x = df.iloc[:,1]
plt(t,x,'x')
```

   Which model characterizes better the data ? Explain your reasoning, and all attempts you have considered.

2. Work on the exercise 2.3 from the typed notes `Math150_Chapter2.pdf`
3. Submit your work on Catcourses under the assignment `Worksheet 3` **as a .ipynb**. Do not forget to submit scans of the handwritten answers as well if they are not typed in the .ipynb.