The goals of this discussion section are:

1. Be able to study equilibrium of some dynamical systems
2. Identify a model of population dynamics

Participation in discussion section counts as 5% of the grade. Completion of the worksheets counts as 20% of the grade. **Submit your worksheet work by February 17th at 2:59pm.**

The Lorenz equations form a simplified mathematical model for atmospheric convection. It is given by

$$\frac{dX}{dt} = \sigma(Y - X)$$
$$\frac{dY}{dt} = -XZ + \rho X - Y$$
$$\frac{dZ}{dt} = XY - \beta Z$$

with σ, ρ, β some constants. Those equations describe the rate of change of three quantities with respect to time: X is proportional to the rate of convection, Y to the horizontal temperature variation, and Z to the vertical temperature variation. The constants σ, ρ, β are system parameters proportional to the Prandtl number, Rayleigh number, and certain physical dimensions of the atmospheric layer.

1. Make some observations about the system using this code. In particular try the values given in Exercise 2.12 from `Math150_Chapter2.pdf`
2. Discretize the Lorenz equations using Runge-Kutta methods (of order 2 or order 4). You may take inspiration from this notebook and adjust, or use techniques discussed in Remark 2.4 from `Math150_Chapter2.pdf`. Explain your choice.
3. Fixing $\sigma = 10$, $\beta = \frac{8}{3}$, make observations for the following cases:
   - test for $0 < \rho < 1$
   - test for $0 < \rho < 20$, $\rho = 1$
   - test for $\rho = 28$

   We suggest to create different code cells in your .ipynb for each test, and write comments below each simulation.
4. Submit your work on Catcourses under the assignment `Worksheet 4` **as a .ipynb**. Do not forget to submit scans of the handwritten answers as well if they are not typed in the .ipynb.