The goal of this discussion section if to get familiar to definitions and tools for optimization. Participation in discussion section counts as 5% of the grade. Completion of the worksheets counts as 20% of the grade. **Submit your worksheet work by March 31st at 2:59pm.**

Consider the functions $J_1 : K_1 \to \mathbb{R}$, $J_2 : K_2 \to \mathbb{R}$ such that for $v \in \mathbb{R}^3$,

$$J_1(v) = b^\intercal v, \qquad J_2(v) = \frac{1}{2} v^\intercal A v - b^\intercal v,$$

where the matrix $A \in \mathbb{R}^{3\times3}$, and the vector $b \in \mathbb{R}^3$ are defined by

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}, \quad b = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}.$$

We consider the sets $K_1 = \{v = \langle v_1, v_2, v_3 \rangle \in \mathbb{R}^3 | v_2 + v_3 \leq 4,\ v_1 \leq 4,\ v_1, v_2, v_3 \geq 0\}$, and $K_2 = \{v = \langle v_1, v_2, v_3 \rangle \in \mathbb{R}^3 | v_2 + v_3 = 0,\ v_1 \geq 0\}$. The goal is to study the problems

$$\max_{v \in K_1} J_1(v), \qquad \min_{v \in K_2} J_2(v)$$

1. Sketch the sets K_1, K_2. You may use your computer to graph (any tool you'd like).
2. Show that K_1 and K_2 are convex sets.
3. Is K_1 bounded ?
4. Show that J_1 is a concave function and that J_2 is a strictly convex function.
5. Conclude if we can find a unique solution for each problem or not.
6. Submit your work on Catcourses under the assignment `Worksheet 9` **as a .pdf**.