By pair or individually:

1. We want to model the growth of mosquito bites over the summer in Merced. Let's denote $y_k(t)$ the total number of bites after t days of the kth person. Let's consider the simplest model, the linear growth model:

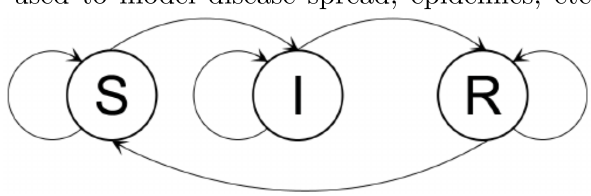
$$y_k'(t) = \lambda y_k(t), \quad y_k(0) = \alpha, \quad k = 1, \dots, N, 0 \leq t \leq 30.$$

   (a) What is the expression of the solution of this ODE ?

   (b) Data after $t = 30$ days has been collected and stored as (k, y_k), $k = 1, \dots, 100$, given in the `hwk5_data.csv` file. But there is some noise in the data. We assume that the growth rate λ follows a normal distribution $\mathcal{N}(\mu, \sigma)$. This means that the probability to observe $\lambda = \lambda_i$ is given by $p(\lambda_i, \mu, \sigma) = \frac{e^{-(\lambda_i - \mu)^2/2\sigma}}{\sqrt{2\pi\sigma}}$, and the maximum likelihood for μ, σ is given by (as done in class) is $\mathcal{F}(\text{data}) = \prod_{i=1}^{100} \frac{e^{-(\lambda_i - \mu)^2/2\sigma}}{\sqrt{2\pi\sigma}}$. If λ is a random variable following a normal distribution, using the expression of y, find the expression of the observations λ_i.

   (c) For $\alpha = 2$, using the data, and the lecture notes, find the MLE for μ and σ. How accurate are your estimates ?

2. The Susceptible-Infectious-Recovered (SIR) model is a well-known compartmental model that can be used to model disease spread, epidemics, etc. It can also be modeled as a Markov



chain:

   (a) Associate values (your choice BUT justify it) to each transition from the graph above and write the associate Transition matrix $\mathcal{P}$.

   (b) Suppose we use the SIR model to simulate the *Math 150 fever*: this infectious disease makes any infected person model everything all the time (write equations everywhere even on their body, simulate everything even on their smart phone, question every single aspect of their lives and discussing it with any encountered person). The above Markov chains the represents the transition to all state in a day time. Suppose that the current Math 150 class is distributed as follows: 60% susceptible, 10% infected, and 30% recovered. With your chosen values, what would happen after a semester (75 class days) ? You can change the values chosen in the previous question, as long as you explain your reasoning.

Submit on Catcourses (.pdf typed or a clean scan of your notes) by 5pm on 4/29. Don't forget to include your name(s) at the beginning.