Control Design

T. Bretl

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Write equations of motion in standard form:

$$\dot{s} = f(\underbrace{s}_{\text{state}},\ \underbrace{i}_{\text{input}},\ \overbrace{p}^{\text{parameter}})$$

← ordinary differential equations

Choose an equilibrium point:

$$0 = f(s_{eq},\ i_{eq},\ \overbrace{p_{eq}})$$

this is unconventional — parameter values are usually given and not chosen — but it makes the notation easier when we implement with code

Linearize equations of motion about equilibrium point:

$$\dot{x} = Ax + Bu$$

$u = i - i_{eq}$ ← input

$x = s - s_{eq}$ ← state

Constant:
A, B, s_{eq}, i_{eq}

Functions of time:
x, u, s, i

$$A = \left.\frac{\partial f}{\partial s}\right|_{(s_{eq},\, i_{eq},\, p_{eq})} \qquad B = \left.\frac{\partial f}{\partial i}\right|_{(s_{eq},\, i_{eq},\, p_{eq})}$$

$\dot{x} = Ax + Bu \qquad x = s - s_{eq} \qquad u = i - i_{eq}$

Design linear state feedback that makes the state go to zero

$u = -Kx \longleftarrow \underbrace{\dot{x} = Ax + B(-Kx) = (A - BK)x}$

if all eigenvalues of $A-BK$ have negative real part, then $\underbrace{x(t) \to 0}$ as $t \to \infty$

THIS MEANS: $s(t) \to s_{eq}$

LQR is a standard way to choose K

$$\boxed{\begin{aligned} &\underset{u}{\text{minimize}} \quad \int_0^\infty \left(x(t)^T Q\, x(t) + u(t)^T R\, u(t) \right) dt \\ &\text{subject to} \quad \dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0 \end{aligned}}$$

$K = lqr(A, B, Q, R)$

Q big $\Rightarrow$ x small

R big $\Rightarrow$ u small

SYSTEM $\dot{x} = Ax + Bu$ $x = s - s_{eq}$ $u = i - i_{eq}$

CONTROLLER $u = -Kx$

Add reference tracking

$$u = -K(x - x_{des})$$

$x(t) \to x_{des}$ as $t \to \infty$

desired value of x (what you want x to become, if not zero)

If you could have chosen any value for the jth element of s_{eq} and

1. still have an equilibrium point
2. still have produced the same A and B

then the jth element of x_{des} can be any non-zero number you want — all other elements of x_{des} must be zero.

SYSTEM $\dot{x} = Ax + Bu$ $\quad x = s - s_{eq}$ $\quad u = i - i_{eq}$

CONTROLLER $u = -K(x - x_{des})$

IMPLEMENTATION

$$u = -K(x - x_{des})$$

$$\underbrace{\begin{aligned} i - i_{eq} &= -K\left((s - s_{eq}) - (s_{des} - s_{eq})\right) \\ &= -K(s - s_{des}) \end{aligned}}$$

SOLVE FOR i

$$\boxed{i = i_{eq} - K(s - s_{des})}$$

a 4x12 matrix

$$i = i_{eq} - K(s - s_{des})$$

$$\begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \\ f_z \end{bmatrix} \qquad \begin{bmatrix} 0 \\ 0 \\ 0 \\ mg \end{bmatrix} \qquad \begin{bmatrix} o_x \\ o_y \\ o_z \\ \psi \\ \theta \\ \phi \\ v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \qquad \begin{bmatrix} o_{x,des} \\ o_{y,des} \\ o_{z,des} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Control Loop

get $\overbrace{o_x, o_y, o_z, \psi, \theta, \phi, v_x, v_y, v_z, \omega_x, \omega_y, \omega_z}^{\text{state}}$ and $\overbrace{o_{x_{des}}, o_{y_{des}}, o_{z_{des}}}^{\text{desired position}}$

choose $\underbrace{\tau_x, \tau_y, \tau_z, f_z}_{\text{input}}$ $\leftarrow$ $i = i_{eq} - K(s - s_{eq})$

choose $\underbrace{m_1, m_2, m_3, m_4}_{\text{motor power commands}}$ $\leftarrow$ $\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = P^{-1} \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \\ f_z \end{bmatrix}$