OBSERVER IMPLEMENTATION

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AE 483

Fall 2021

How *exactly* should we choose Q and R?

Define:

$$e = Cx + Du - y$$
$$Q = \mathrm{diag}(q_1, q_2, \ldots)$$

Suppose σ_i is the standard deviation of e_i (as you found in experiment). Then a good choice is:

$$q_i = (1/\sigma_i)^2$$

← inverse of error variance

Why? If σ_i is small, then q_i will be big, and so you are saying you trust the ith sensor a lot — which is exactly what you *should* do if your model of this sensor has low noise.

← this is just intuition — it is possible to prove that this choice of Q is, in fact, optimal

LQR is a standard way to choose L

$$\underset{d,n}{\text{minimize}} \quad \int_{-\infty}^{0} \left(n(t)^T Q\, n(t) + d(t)^T R\, d(t) \right) dt$$

$$\text{subject to} \quad \begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + d(t) \\ y(t) &= Cx(t) + Du(t) + n(t) \end{aligned}$$

The solution is:

$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} + Du - y)$$

where

$$L = \mathrm{lqr}(A^T, C^T, R^{-1}, Q^{-1})^T$$

$$K = \mathrm{lqr}(A, B, Q, R)$$

Q big $\Rightarrow$ n small $\Rightarrow$ TRUST SENSORS

R big $\Rightarrow$ d small $\Rightarrow$ TRUST DYNAMIC MODEL

Choose R in exactly the same way, by looking at the error

$$A x + Bu - \dot{x}$$

in the linearized EOMs.

Is it possible to avoid converting between s, i, o and x, u, y when implementing an observer?

YES

This is an observer: $$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} - y)$$

$$\frac{d}{dt}(\hat{s} - s_{eq}) = \dot{\hat{s}} - 0 = \dot{\hat{s}}$$

$\hat{s} - s_{eq}$ $\qquad$ $i - i_{eq}$ $\qquad$ $o - o_{eq}$

This is an equivalent way to write the same observer:

$$\dot{\hat{s}} = A(\hat{s} - s_{eq}) + B(i - i_{eq}) - L\Big(C(\hat{s} - s_{eq}) + D(i - i_{eq}) - (o - o_{eq})\Big)$$

this is often easier to implement in practice (it avoids the need to define x, u, y in code)

How do I update the estimate of a state that is <u>not</u> observable?

The best you can do is integrate:

$$\dot{\hat{x}} = A\hat{x} + Bu$$

For example, since o_x is not observable, then we must integrate the linearized EOM for o_x:

$$\dot{\hat{o}}_x = v_x$$

We approximate the integral as:

$$\hat{o}_x(t+\Delta t) = \hat{o}_x(t) + \Delta t \, v_x(t)$$

How do I update the estimate of a state that <u>is</u> observable?

We use our observer:

$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} + Du - y)$$

For example, consider o_z (an observable state). You should find the row of L corresponding to o_z is:

$$\begin{bmatrix} 0 & 0 & \ell \end{bmatrix}$$

for some number ℓ. So, from the linearized EOM for o_z and the linearized measurement equation for r, we find:

$$\dot{\hat{o}}_z = v_z - \ell(o_z - r)$$

We approximate the integral as:

$$\hat{o}_z(t + \Delta t) = \hat{o}_z(t) + \Delta t \left(v_z(t) - \ell \left(\underbrace{o_z(t) - r(t)} \right) \right)$$

these error terms are often used more than once, so it is best to precompute them - we might call this one r_{err}