OBSERVER DESIGN

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AE 483

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Write equations of motion in standard form:

$$\dot{s} = f(s, i, p)$$

parameter

input

state

← ordinary differential equations

Choose an equilibrium point:

$$0 = f(s_{eq}, i_{eq}, p_{eq})$$

this is unconventional — parameter values are usually given and not chosen — but it makes the notation easier when we implement with code

Linearize equations of motion about equilibrium point:

$$\dot{x} = Ax + Bu$$

$u = i - i_{eq}$ ← input

$x = s - s_{eq}$ ← state

Constant:
A, B, s_{eq}, i_{eq}

Functions of time:
x, u, s, i

$$A = \left.\frac{\partial f}{\partial s}\right|_{(s_{eq}, i_{eq}, p_{eq})} \qquad B = \left.\frac{\partial f}{\partial i}\right|_{(s_{eq}, i_{eq}, p_{eq})}$$

Write measurement equations in standard form:

$$m = g(s, i, p)$$

← Different people use different variables to stand for different things. For example, I might use "o" (for "output") and "h" rather than "m" and "g" to avoid confusion with mass and gravity.

Linearize about equilibrium point:

$$y = Cx + Du$$

$u = i - i_{eq}$

$x = s - s_{eq}$

$y = m - g(s_{eq}, i_{eq}, p_{eq})$

$$C = \left.\frac{\partial g}{\partial s}\right|_{(s_{eq}, i_{eq}, p_{eq})} \qquad D = \left.\frac{\partial g}{\partial i}\right|_{(s_{eq}, i_{eq}, p_{eq})}$$

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}\right\}$$ SYSTEM

$$u = -K\hat{x}$$

$$\dot{\hat{x}} = A\underset{\text{state estimate}}{\hat{x}} + Bu - \overset{\text{gain matrix}}{L}(C\hat{x} + Du - y) \quad \}$$ OBSERVER

$$\left.\begin{aligned} \hat{x}(0) &= \text{some guess} \\ &\vdots \\ \hat{x}(t+\Delta t) &\approx \hat{x}(t) + \Delta t\Big(A\hat{x}(t) + Bu(t) - L\big(C\hat{x}(t) + Du(t) - y(t)\big)\Big) \end{aligned}\right\}$$ how to implement it

$$\left.\begin{aligned} \frac{d}{dt}(\underbrace{\hat{x}-x}_{\text{error in the state estimate}}) &= \dot{\hat{x}} - \dot{x} \\ &= \big(A\hat{x} - \cancel{Bu} - L(C\hat{x} + Du - y)\big) - (Ax + \cancel{Bu}) \\ &= A\hat{x} - L(C\hat{x} + \cancel{Du} - \overbrace{Cx - \cancel{Du}}) - Ax \\ &= \underbrace{(A - LC)}_{} (\hat{x} - x) \end{aligned}\right\}$$ why it works

if all eigenvalues of $A-LC$ have negative real part, then $\hat{x}(t) - x(t) \to 0$ as $t \to \infty$

LQR is a standard way to choose L

$$\underset{d,\,n}{\text{minimize}} \quad \int_{-\infty}^{0} \left(n(t)^T Q\, n(t) + d(t)^T R\, d(t) \right) dt$$

$$\text{subject to} \quad \begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + d(t) \\ y(t) &= Cx(t) + Du(t) + n(t) \end{aligned}$$

The solution is:

$$\dot{\hat{x}} = A\hat{x} + Bu - L\,(C\hat{x} + Du - y)$$

where

$$L = \text{lqr}(A^T, C^T, R^{-1}, Q^{-1})^T$$

$K = \text{lqr}(A, B, Q, R)$

Q big $\Rightarrow$ n small $\Rightarrow$ TRUST SENSORS

R big $\Rightarrow$ d small $\Rightarrow$ TRUST DYNAMIC MODEL