

# Sensors

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AE 483

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## GYROSCOPES

They measure the angular velocity:

$$\omega'_{0,1} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \begin{array}{l} \leftarrow ae^{483 \log. \omega-x} \\ \leftarrow ae^{483 \log. \omega-y} \\ \leftarrow ae^{483 \log. \omega-z} \end{array}$$

As you will see in lab, these measurements are so good (they have so little noise) that we might as well just use them as state estimates.

So, we ignore these EOM:

$$\dot{\omega}'_{0,1} = (J')^{-1} (\tau - \widehat{\omega}_{0,1} J' \omega'_{0,1}) \quad \leftarrow \text{allows us to ignore } \tau$$

And we treat  $\omega'_{0,1}$  as an input instead of a state.

$$\begin{bmatrix} \dot{\psi} \\ \dot{\theta} \\ \dot{\phi} \end{bmatrix} = N \omega'_{0,1}$$

# ACCELEROMETERS

They measure **specific force**:

EOM of drone is:

$$m\ddot{z} = f_z - mg$$

- ① Accelerometer measures  $1/m$  times all forces except gravity

$$\underbrace{\ddot{z} + g}_{\text{measurement}} = (1/m) f_z$$

- ② Accelerometer measures "acceleration without the gravity term"

$$\ddot{z} = \underbrace{(1/m) f_z}_{\text{measurement}} - g$$

EOM of proof mass is:

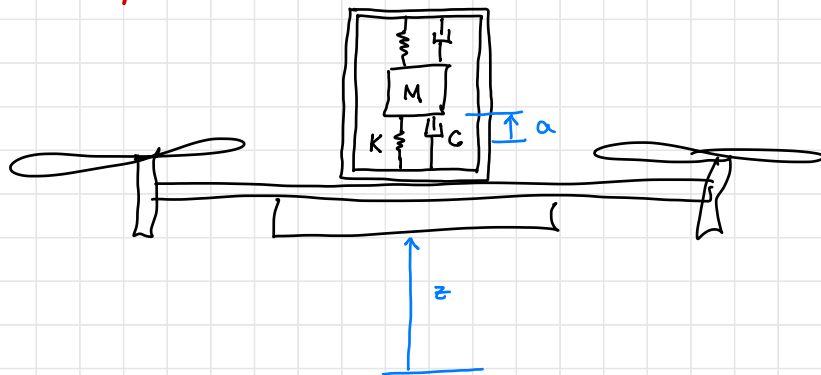
$$M(\ddot{z} + \ddot{a}) = -ka - c\dot{a} - Mg$$

If  $\ddot{z}$  is constant, then steady-state is:

$$M\ddot{z} = -ka - Mg$$

The measurement is:

$$a = \underbrace{\left(-\frac{M}{k}\right)}_{\text{this is a constant that is calibrated away}} \underbrace{(\ddot{z} + g)}_{\text{this is specific force}}$$



## ACCELEROMETERS

They measure **specific force**:

$$(R_1^0)^T \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ f_z \end{bmatrix} = m \dot{v}_{0,1}^1 + \hat{\omega}_{0,1}^1 m v_{0,1}^1$$

$$\underbrace{\dot{v}_{0,1}^1}_{\text{acceleration}} = (R_1^0)^T \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} + \boxed{(1/m) \begin{bmatrix} 0 \\ 0 \\ f_z \end{bmatrix} - \hat{\omega}_{0,1}^1 v_{0,1}^1}$$

This allows us to ignore  $f_z$  and replace it with  $\underline{a^{SF}}$ , which we treat as an input.

↑  
"acceleration without the gravity term"

↑  
call this  $a^{SF}$

## ACCELEROMETERS

They measure **specific force**:

$$a_{sf} = (1/m) \begin{bmatrix} 0 \\ 0 \\ f_z \end{bmatrix} - \hat{\omega}_{0,1}^1 v_{0,1}^1 = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

if you want these, add code to your controller that logs sensors  $\rightarrow$  acc.x and sensors  $\rightarrow$  acc.y  
 $\leftarrow$  ac483log, a\_z

$$\approx \begin{bmatrix} 0 \\ 0 \\ f_z/m \end{bmatrix}$$

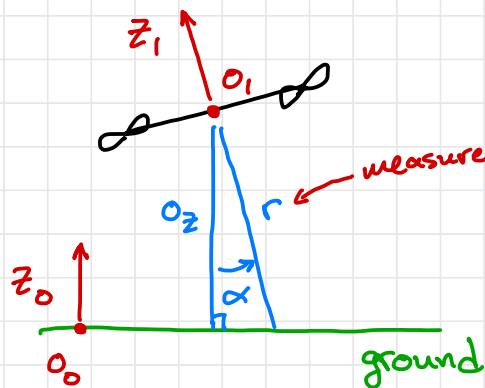
near hover (by linearization)  
so, near hover, the x and y components of  $a_{sf}$  tell us nothing!

# "Z RANGER"

It measures the distance along  $-z_1$  from  $o_1$  to the closest surface.

Assume that surface is "the ground" (flat, horizontal, zero height).

$$R_1^0 = \begin{bmatrix} x_1 \cdot x_0 & y_1 \cdot x_0 & z_1 \cdot x_0 \\ x_1 \cdot y_0 & y_1 \cdot y_0 & z_1 \cdot y_0 \\ x_1 \cdot z_0 & y_1 \cdot z_0 & \underbrace{z_1 \cdot z_0} \end{bmatrix}$$



$$\frac{o_z}{r} = \cos \alpha = \frac{z_1 \cdot z_0}{|z_1| |z_0|} = z_1 \cdot z_0 = \cos \phi \cos \theta$$

$$r = \frac{o_z}{\cos \phi \cos \theta}$$

$$\leftarrow ae^{483/\log \cdot r}$$

$$R_1^0 = \begin{bmatrix} \cos(\psi) \cos(\theta) & \sin(\phi) \sin(\theta) \cos(\psi) - \sin(\psi) \cos(\phi) & \sin(\phi) \sin(\psi) + \sin(\theta) \cos(\phi) \cos(\psi) \\ \sin(\psi) \cos(\theta) & \sin(\phi) \sin(\psi) \sin(\theta) + \cos(\phi) \cos(\psi) & -\sin(\phi) \cos(\psi) + \sin(\psi) \sin(\theta) \cos(\phi) \\ -\sin(\theta) & \sin(\phi) \cos(\theta) & \cos(\phi) \cos(\theta) \end{bmatrix}$$

# "OPTICAL FLOW"

velocity of point on ground

$$n_x \approx k_{\text{flow}} \left( \frac{v_x - o_z \omega_y}{o_z} \right)$$

$\approx 483 \log. n - x$

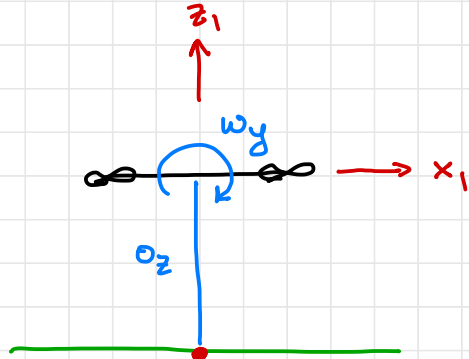
constant parameter that depends on shutter time, pixel size, etc.

perspective projection

it measures the distance that points move from one image to the next, in the "x" and "y" directions

$$n_y \approx k_{\text{flow}} \left( \frac{v_y + o_z \omega_x}{o_z} \right)$$

$\approx 483 \log. n - y$



## Flowdeck Measurement Model

This illustration explains how the height from the VL53L1x sensor and flow from the PMW3901 sensor are combined to calculate velocity. This has been implemented by the work of [3] and can be found in `kalman_core.c` in the function `kalman_coreUpdateWithFlow()`.

