Chapter 10 Answers

Highlights

- Review these functions
  - complex
  - Re
  - Im
  - Conj
  - Mod
  - Arg
- Don't forget these functions
  - matrix
  - rbind
  - seq
  - ifelse
  - %%
  - %/%

10.1 Write a single R command that calculates the product of the complex numbers $2-3i$ and $3+4i$. Check th correctness of the result by hand.

```
(2-3i)*(3+4i)
```

```
## [1] 18-1i
```

```
# or, lets get crazy

my.complex.multiplier = function (c1,c2) {
    # Manual FOIL method
    # Re(c1)*Re(c2) + Re(c1)*Im(c2) + Im(c1)*Re(c2) + Im(c1)*Im(c2)
    # But, Re and Im only return non-imaginary components, so
    # REAL = Re(c1)*Re(c2) * (-1)*(Im(c1)*Im(c2))
    # IMAGINARY = Re(c1)*Im(c2) + Im(c1)*Re(c2)
    return(complex(real = Re(c1)*Re(c2) - Im(c1)*Im(c2),
                   imaginary = Re(c1)*Im(c2) + Im(c1)*Re(c2)))

}

xc1 = complex(real = 2, imaginary = -3)
xc2 = complex(real = 3, imaginary = 4)
my.complex.multiplier(xc1,xc2)
```

```
## [1] 18-1i
```

```
# and confirm:
xc1*xc2
```

```
## [1] 18-1i
```

10.2 What is returned by the following R command? Check the correctness of the result by hand.

```
(2 - 3i) ^ 2
```

```
(2-3i)^2
```

```
## [1] -5-12i
```

```r
(2-3i)*(2-3i)
```

```
## [1] -5-12i
```

```r
my.complex.multiplier(2-3i,complex(real=2,imaginary=-3))
```

```
## [1] -5-12i
```

10.3 Write R commands that calculate $(1-i)^n$, where $i = \sqrt{-1}$, for $n = 4, 8, 16, 20, 24$. Use the results to write a general mathematical expression for $(1-i)^n$, where n is a multiple of 4.

```r
(1-1i)^(seq(4,24,by=4))
```

```
## [1]    -4+0i    16+0i   -64+0i   256+0i -1024+0i  4096+0i
```

```r
n<-seq(4,24,by=4)
ifelse((n%/%4)%%2,-1,1)*4^(n%/%4) + 0i
```

```
## [1]    -4+0i    16+0i   -64+0i   256+0i -1024+0i  4096+0i
```

or,

Let $m = n \text{ div } 4, then$
$$\begin{cases} -4^m + 0i & \text{where } m \text{ is odd} \\ 4^m + 0i & \text{where } m \text{ is even} \end{cases}$$

10.4 Write R commands to create a 5 x 5 matrix named w whose elements are complex numbers. The real part of each element is the row number; the imaginary part of each element is the column number. Compute the matrix that results from multiplying each element of w by its conjugate.

```r
seed <- complex(real=1,imaginary=1:5)
w=matrix(rbind(seed,seed+1,seed+2,seed+3,seed+4),5,5)
w
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,] 1+1i 1+2i 1+3i 1+4i 1+5i
## [2,] 2+1i 2+2i 2+3i 2+4i 2+5i
## [3,] 3+1i 3+2i 3+3i 3+4i 3+5i
## [4,] 4+1i 4+2i 4+3i 4+4i 4+5i
## [5,] 5+1i 5+2i 5+3i 5+4i 5+5i
```

```r
Conj(w)
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,] 1-1i 1-2i 1-3i 1-4i 1-5i
## [2,] 2-1i 2-2i 2-3i 2-4i 2-5i
## [3,] 3-1i 3-2i 3-3i 3-4i 3-5i
## [4,] 4-1i 4-2i 4-3i 4-4i 4-5i
## [5,] 5-1i 5-2i 5-3i 5-4i 5-5i
```

```r
w*Conj(w)
```

```
##       [,1]  [,2]  [,3]  [,4]  [,5]
## [1,]  2+0i  5+0i 10+0i 17+0i 26+0i
## [2,]  5+0i  8+0i 13+0i 20+0i 29+0i
## [3,] 10+0i 13+0i 18+0i 25+0i 34+0i
## [4,] 17+0i 20+0i 25+0i 32+0i 41+0i
## [5,] 26+0i 29+0i 34+0i 41+0i 50+0i
```

```r
w[3,3]*Conj(w[3,3])
```

```
## [1] 18+0i
```

```r
w[1,2]*Conj(w[1,2])
```

```
## [1] 5+0i
```