Chapter 8 Questions

8.1 Write a one-line function named `reverse` with a single arument `x` that reverses the order of the elements in the vector `x`.

8.2 Write a functions namd `my.cos` (that calculates the cosine of the supplied angle) with two arguments. The first argument is named `angle` and the second argument is named `degrees`, which has a default value of `FALSE`. Test your function with the R commands

```
> my.cos(pi/2)
> my.cos(90, degrees = TRUE)
```

8.3 Write a function named `cube.root` that calculates the cube root(s) of its argument. Test your function with the R command

```
> cube.root(c(-8,8,729,1000000))
```

8.4 Consider the sample data values $x_1, x_2, \cdots, x_n$ and the associated sample order statistics, $x_{(1)}, x_{(2)}, \cdots, x_{(n)}$. The *sample truncated mean* (also known as the *sample trimmed mean*) is a measure of central tendency defined as

$$\bar{x} = \frac{x_{(k+1)} + x_{(k+2)} + \cdots + x_{(n-k)}}{n - 2k}$$

This is the arithmetic average of the data values with the k lowest and k highest observations removed. The truncated mean is less sensitive to outliers than the arithmetic mean and is hence known as a *robust estimator*. This estimator is used in sports that are evaluated by a panel of n judges in which the lowest and highest scores ($k = 1$) are discarded. Likewise, the truncated mean is used in calculating the London Interbank Offer Rate (LIBOR) when $n = 18$ interest rates are collected, and the lowest four and highest four interest rates ($k = 4$) are discarded. Assuming that $k < n/2$, write an R function named `tmean` with two arguments `x` and `k` that calculates the sample truncated mean of the elements in the vector `x` discarding the `k` lowest and `k` highest observations. Test your code with the R commands

```
> tmean(c(9.4, 9.6, 9.1, 9.5, 9.3), 1)
> tmean(1:18, 4)
```

8.5 Let $x_1, x_2, \cdots, x_n$ be the $n > 2$ elements in a vector `x`. Write an R function named `moveave` with a single argument `x` that returns a vector of length $n - 1$ whose elements are the moving averages

$$\frac{x_1 + x_2}{2}, \frac{x_2 + x_3}{2}, \frac{x_3 + x_4}{2}, \cdots, \frac{x_{n-1} + x_n}{2}$$

8.6 Let $x_1, x_2, \cdots, x_n$ denote the elements of the vector `x`. Write an R function named `L2` with a single vector argument `x` that calculates the L_2 norm

$$\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

Test your function with the R commands

```
> L2(c(3,4))
> L2(c(1,1,1))
```

8.7 Let $x_1, x_2, \cdots, x_n$ denote the elements of the vector `x`. Write an R function named `Lp` with a vector argument `x` and an integer argument `p` that calculates the $p-$norm

$$\left(\sum_{i=1}^{n}|x_i|^p\right)^{(1/p)}$$

Test your R function with the R commands

```
Lp(c(3,4),2)
Lp(c(1,1,1),3)
```

8.8 The built-in R function `mean` calculates the sample mean $\bar{x}$. Write R functions named `hmean, gmean,` and `qmean` that calculate the sample harmonic mean, geometric mean, and quadratic mean defined by

$$h = \left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}\right)^{-1}, \qquad g = \left(\prod_{i=1}^{n}x_i\right)^{1/n}, \qquad q = \sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}$$

where $x_1, x_2, \cdots, x_n$ are the data values. Test your functions with a vector of data values of your choice and verify the inequality

$$min\{x_1, x_2, \cdots, x_n\} \le h \le g \le \bar{x} \le q \le max\{x_1, x_2, \cdots, x_n\}$$

8.9 Let $x_1, x_2, \cdots, x_n$ be the n elements in the R vector `x`. Write an R function named `mad` (for *mean absolute deviation*) with a single argument `x` that calculates

$$\frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$$

where $\bar{x}$ is the sample mean.

note: this problem includes the $\frac{1}{n}$ correction from author's errata

8.10 Write an R function named `fifth` with a single vector argument `x` that calculates and returns the sample mean of the 5th, 10th, 15th, ... elements of `x`. You may assume that `x` has at least five elements.

8.11 The rectangular coordinate system uses x, the signed horizontal distance from the origin, and y, the signed vertical distance from the origin, to describe the point (x, y). The polar coordinate system, on the other hand, uses r, the signed distance from the origin, and θ, the signed angle measured counterclockwise from the polar axis, to describe the point (r, θ). Assume that θ is measured in radians.

- Write an R function named `polar2rect` with arguments `r` and `theta` that returns a two-element vector that contains the rectangular coordinates associated with the point (r, θ) in the polar coordinate system.
- Write an R function named `rect2polar` with arguments `x` and `y` that returns a two-element vector that contains the polar coordinates associated with the point (x, y) in the rectangular coordinate system.

8.12 Write a one-line function named `fourth` that raises its argument to the fourth power. Then apply the `formals, body,` and `environment` functions to `fourth` in order to isolate the arguments to the function, the R code that comprises the function, and the location of the objects used in the function.

Exercises taken from Chapter 8, "Learning Base R", by Lawrence M Leemis, ISBN 978-0-9829174-8-0