The SYN Projection

AIPS++ note 175

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1 Introduction

The SYN (synthesis) projection provides an exact coordinate description for any co-planar synthesis array. This includes east-west arrays and VLA snapshots. It includes SIN and NCP as special cases.

SYN has been included in the FITS WCS proposal as a generalization of the SIN projection.

2 Derivation of the SYN projection

From the basic synthesis equation, the phase term in the Fourier exponent is

$$\text{phase} = (\mathbf{e} - \mathbf{e_0}) \cdot \mathbf{B} \tag{1}$$

where $\mathbf{e}$ and $\mathbf{e_0}$ are the unit vectors pointing towards a point in the field and the field centre, $\mathbf{B}$ is a baseline vector, and we measure phase in rotations so that we don't need to carry factors of 2π. We can write

$$\text{phase} = p_u u + p_v v + p_w w \tag{2}$$

where (u, v, w) are components of the baseline vector in a coordinate system with the w-axis pointing from the geocentre towards the source and the u-axis lying in the J2000.0 equatorial plane, and

$$\begin{aligned} p_u &= -\cos\theta \sin\phi \\ p_v &= -\cos\theta \cos\phi \\ p_w &= \sin\theta - 1 \end{aligned} \tag{3}$$

are the coordinates of $(\mathbf{e} - \mathbf{e_0})$, where (ϕ, θ) are the longitude and latitude of $\mathbf{e}$ in the (left-handed) native coordinate system of the projection with the pole towards $\mathbf{e_0}$. Now, for a planar array we may write

$$n_u u + n_v v + n_w w = 0 \tag{4}$$

where (n_u, n_v, n_w) are the direction cosines of the normal to the plane. Then

$$w = -\frac{n_u u + n_v v}{n_w} \tag{5}$$

Combining (2) and (5) we have

$$\text{phase} = [p_u - \frac{n_u}{n_w} p_w]u + [p_v - \frac{n_v}{n_w} p_w]v \tag{6}$$

From equations (3) and (6) the equations for the "SYN" projection for a planar synthesis array are thus

$$\begin{aligned} x &= -[\cos\theta\sin\phi + p_1(\sin\theta - 1)] \\ y &= -[\cos\theta\cos\phi + p_2(\sin\theta - 1)] \end{aligned} \tag{7}$$

where

$$\begin{aligned} p_1 &= n_u/n_w \\ p_2 &= n_v/n_w \end{aligned} \tag{8}$$

3 SYN projection equations in equatorial coordinates

If (α, δ) and (α_0, δ_0) are the J2000.0 right ascension and declination of $\mathbf{e}$ and $\mathbf{e_0}$ then

$$\begin{aligned} \cos\theta\sin\phi &= \cos\delta\sin(\alpha - \alpha_0) \\ \cos\theta\cos\phi &= - \sin\delta\cos\delta_0 + \cos\delta\sin\delta_0\cos(\alpha - \alpha_0) \\ \sin\theta &= \sin\delta\sin\delta_0 + \cos\delta\cos\delta_0\cos(\alpha - \alpha_0) \end{aligned} \tag{9}$$

These may be substituted into equations (7) to obtain the SYN projection equations in J2000.0 equatorial coordinates.

4 Special cases of the SYN projection: SIN and NCP

Note in equations (7) that since theta is approximately 90° the terms involving p_1 and p_2 are small; neglecting them, as is usually done, gives us the equations for the "SIN" projection:

$$\begin{aligned} x &= -\cos\theta\sin\phi \\ y &= -\cos\theta\cos\phi \end{aligned} \tag{10}$$

From equations (7), for an array which lies in the J2000.0 equatorial plane, we have

$$\begin{aligned} n_u &= 0 \\ n_v &= \cos\delta_0 \\ n_w &= \sin\delta_0 \end{aligned} \tag{11}$$

where δ_0 is the declination of the field centre, whence

$$\begin{aligned} p_1 &= 0 \\ p_2 &= \cot\delta_0 \end{aligned} \tag{12}$$

and

$$\begin{aligned} x &= -[\cos\theta\sin\phi] \\ y &= -[\cos\theta\cos\phi + \cot\delta_0(\sin\theta - 1)] \end{aligned} \tag{13}$$

These are the equations for the "NCP" projection. To first order the difference between equations (10b) and (13b) is

$$\frac{r^2}{2}\cot\delta_0 \tag{14}$$

where r is the distance from the field centre in radians. This amounts to nearly $1'$ for a position 1° from the field centre at $\delta_0 = 30^\circ$.

5 Correction for precession

The plane of an east-west array coincides with the apparent equatorial plane at the date of the observation and this is tilted slightly with respect to the J2000.0 equatorial plane. If (α_p, δ_p) are the J2000.0 right ascension and declination of the apparent pole then

$$\begin{aligned} n_u &= -\cos\delta_p\sin(\alpha_p - \alpha_0) \\ n_v &= \sin\delta_p\cos\delta_0 - \cos\delta_p\sin\delta_0\cos(\alpha_p - \alpha_0) \\ n_w &= \sin\delta_p\sin\delta_0 + \cos\delta_p\cos\delta_0\cos(\alpha_p - \alpha_0) \end{aligned} \tag{15}$$

These may be substituted directly into equations (7) and (8). Precession from 1990 to 2000 amounts to about $3'$. For $\alpha_p = \alpha_0$ and $\delta_0 = 30°$ we get

$$\begin{aligned} p_1 &= 0 \\ p_2 &= 1.72857 \end{aligned}$$

Equations 12 with no precession correction give

$$\begin{aligned} p_1 &= 0 \\ p_2 &= 1.73205 \end{aligned}$$

For a position $1°$ from the field centre at this declination the difference between these amounts to about 0".1.

6 Field shifts

A phase shift may be applied to the visibility data at the time a map is synthesized in order to translate the field centre. If the phase shift applied to the visibilities is

$$\text{phase shift} = q_u u + q_v v + q_w w \tag{16}$$

where (q_u, q_v, q_w) is constant then equation (2) becomes

$$\text{phase} = (p_u - q_u)u + (p_v - q_v)v + (p_w - q_w)w \tag{17}$$

whence equation (6) becomes

$$\text{phase} = [(p_u - q_u) - p_1(p_w - q_w)]u + [(p_v - q_v) - p_2(p_w - q_w)]v \tag{18}$$

Equations (7) become

$$\begin{aligned} x &= -[\cos\theta \sin\phi + p_1(\sin\theta - 1)] - [q_u - p_1 q_w] \\ y &= -[\cos\theta \cos\phi + p_2(\sin\theta - 1)] - [q_v - p_2 q_w] \end{aligned} \tag{19}$$

From which we see that the field centre is shifted by

$$\begin{aligned}\Delta x &= q_u - p_1 q_w \\ \Delta y &= q_v - p_2 q_w\end{aligned} \tag{20}$$

The shift is applied to the coordinate reference pixel. For the SIN projection $(p_1, p_2) = (0, 0)$ and the shift is just

$$\begin{aligned}\Delta x &= q_u \\ \Delta y &= q_v\end{aligned} \tag{21}$$

For the NCP projection the shift is

$$\begin{aligned}\Delta x &= q_u \\ \Delta y &= q_v - q_w \cot \delta_0\end{aligned} \tag{22}$$

In the general case the correction for precession, although small, applies systematically to the whole field. For a shift of 1° at $\alpha_0 = \alpha_p$, and $\delta_0 = 30^\circ$ the whole map is shifted by about 0".1 in declination.