

# KinsolSolver

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This file is part of CasADi.

CasADi -- A symbolic framework for dynamic optimization.  
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## 1 KinsolSolver

```
[1]: from casadi import *  
from numpy import *  
from pylab import *
```

We will investigate the working of rootfinder with the help of the parametrically excited Duffing equation.

$\ddot{u} + \dot{u} - \epsilon(2\mu\dot{u} + \alpha u^3 + 2ku \cos(\Omega t))$  with  $\Omega = 2 + \epsilon\sigma$ . \

The first order solution is  $u(t) = a \cos(\frac{1}{2}\Omega t - \frac{1}{2}\gamma)$  with the modulation equations: \  $\frac{da}{d\epsilon t} = -[\mu a + \frac{1}{2}ka \sin \gamma]$  \  $a \frac{d\gamma}{d\epsilon t} = -[-\sigma a + \frac{3}{4}\alpha a^3 + ka \cos \gamma]$  \

We seek the stationary solution to these modulation equations.

Parameters

```
[2]: eps = SX.sym("eps")
      mu = SX.sym("mu")
      alpha = SX.sym("alpha")
      k = SX.sym("k")
      sigma = SX.sym("sigma")
      params = [eps,mu,alpha,k,sigma]
```

Variables

```
[3]: a = SX.sym("a")
      gamma = SX.sym("gamma")
```

Equations

```
[4]: res0 = mu*a+1.0/2*k*a*sin(gamma)
      res1 = -sigma * a + 3.0/4*alpha*a**3+k*a*cos(gamma)
```

Numerical values

```
[5]: sigma_ = 0.1
      alpha_ = 0.1
      k_ = 0.2
      params_ = [0.1,0.1,alpha_,k_,sigma_]
```

We create a Function instance

```
[6]: f=Function("f", [vertcat(a,gamma),vertcat(*params)],[vertcat(res0,res1)])
      opts = {}
      opts["strategy"] = "linesearch"
      opts["abstol"] = 1e-14
```

Require  $a > 0$  and  $\gamma < 0$

```
[7]: opts["constraints"] = [2,-2]
      s=rootfinder("s", "kinsol", f, opts)
```

Initialize  $[a,\gamma]$  with a guess and solve

```
[8]: x_ = s([1,-1], params_)
      print("Solution = ", x_)
```

Solution = [1.1547, -1.5708]

Compare with the analytic solution:

```
[9]: x = [sqrt(4.0/3*sigma_/alpha_),-0.5*pi]
      print("Reference solution = ", x)
```

Reference solution = [1.1547005383792515, -1.5707963267948966]

We show that the residual is indeed (close to) zero

```
[10]: residual = f(x_, params_)
      print("residual = ", residual)
```

```
residual = [4.16334e-15, 8.34363e-15]
```

```
[11]: for i in range(1):
      assert(abs(x_[i]-x[i])<1e-6)
```

Solver statistics

```
[12]: print(s.stats())
```

```
{'n_call_jac_g_x': 0, 'success': True, 't_proc_jac_g_x': 0.0, 't_wall_jac_g_x':
0.0, 'unified_return_status': 'SOLVER_RET_UNKNOWN'}
```