KinsolSolver

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1 KinsolSolver

[1]:
```
from casadi import *
from numpy import *
from pylab import *
```

We will investigate the working of rootfinder with the help of the parametrically exited Duffing equation.

$\ddot{u} + \dot{u} - \epsilon(2\mu\dot{u} + \alpha u^3 + 2ku\cos(\Omega t))$ with $\Omega = 2 + \epsilon\sigma$. \

The first order solution is $u(t) = a\cos(\frac{1}{2}\Omega t - \frac{1}{2}\gamma)$ with the modulation equations: \ $\frac{da}{d\epsilon t} = -\left[\mu a + \frac{1}{2}ka\sin\gamma\right]$ \ $a\frac{d\gamma}{d\epsilon t} = -\left[-\sigma a + \frac{3}{4}\alpha a^3 + ka\cos\gamma\right]$ \

We seek the stationair solution to these modulation equations.

Parameters

[2]:
```
eps   = SX.sym("eps")
mu    = SX.sym("mu")
alpha = SX.sym("alpha")
k     = SX.sym("k")
sigma = SX.sym("sigma")
params = [eps,mu,alpha,k,sigma]
```

Variables

[3]:
```
a     = SX.sym("a")
gamma = SX.sym("gamma")
```

Equations

[4]:
```
res0 = mu*a+1.0/2*k*a*sin(gamma)
res1 = -sigma * a + 3.0/4*alpha*a**3+k*a*cos(gamma)
```

Numerical values

[5]:
```
sigma_  = 0.1
alpha_  = 0.1
k_      = 0.2
params_ = [0.1,0.1,alpha_,k_,sigma_]
```

We create a Function instance

[6]:
```
f=Function("f", [vertcat(a,gamma),vertcat(*params)],[vertcat(res0,res1)])
opts = {}
opts["strategy"] = "linesearch"
opts["abstol"] = 1e-14
```

Require $a > 0$ and $\gamma < 0$

[7]:
```
opts["constraints"] = [2,-2]
s=rootfinder("s", "kinsol", f, opts)
```

Initialize $[a,\gamma]$ with a guess and solve

[8]:
```
x_ = s([1,-1], params_)
print("Solution = ", x_)
```

```
Solution =  [1.1547, -1.5708]
```

Compare with the analytic solution:

[9]:
```
x = [sqrt(4.0/3*sigma_/alpha_),-0.5*pi]
print("Reference solution = ", x)
```

```
Reference solution =  [1.1547005383792515, -1.5707963267948966]
```

We show that the residual is indeed (close to) zero

[10]:
```
residual = f(x_, params_)
print("residual = ", residual)
```

```
residual =  [4.16334e-15, 8.34363e-15]
```

[11]:
```
for i in range(1):
  assert(abs(x_[i]-x[i])<1e-6)
```

Solver statistics

[12]:
```
print(s.stats())
```

```
{'n_call_jac_f_z': 0, 'success': True, 't_proc_jac_f_z': 0.0, 't_wall_jac_f_z':
0.0, 'unified_return_status': 'SOLVER_RET_UNKNOWN'}
```