ipopt

April 18, 2023

1 nlpsol

[1]:
```
from casadi import *
from numpy import *
```

In this example, we will solve a few optimization problems with increasing complexity

2 Scalar unconstrained problem

$\min_x \quad (x-1)^2 \ \backslash$ subject to $-10 \leq x \leq 10 \ \backslash$

with x scalar

[2]:
```
x=SX.sym('x')
nlp = {'x':x, 'f':(x-1)**2}
```

[3]:
```
solver = nlpsol('solver', 'ipopt', nlp)
sol = solver(lbx=-10, ubx=10)
```

```
******************************************************************************
This program contains Ipopt, a library for large-scale nonlinear optimization.
 Ipopt is released as open source code under the Eclipse Public License (EPL).
         For more information visit https://github.com/coin-or/Ipopt
******************************************************************************

This is Ipopt version 3.14.11, running with linear solver MUMPS 5.4.1.

Number of nonzeros in equality constraint Jacobian…:        0
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian…:        1

Total number of variables…:        1
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        1
                     variables with only upper bounds:        0
Total number of equality constraints…:        0
Total number of inequality constraints…:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  1.0000000e+00 0.00e+00 2.00e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  8.2644627e-03 0.00e+00 1.53e-02  -1.0 9.09e-01    -  9.16e-01 1.00e+00f  1
   2  3.3212748e-06 0.00e+00 2.08e-17  -1.0 8.91e-02    -  1.00e+00 1.00e+00f  1
   3  9.3171258e-10 0.00e+00 7.11e-17  -2.5 1.79e-03    -  1.00e+00 1.00e+00f  1
   4  2.3111166e-12 0.00e+00 2.40e-17  -3.8 2.90e-05    -  1.00e+00 1.00e+00f  1
   5  3.4736752e-16 0.00e+00 5.92e-17  -5.7 1.50e-06    -  1.00e+00 1.00e+00f  1
   6  6.4072417e-22 0.00e+00 3.96e-17  -8.6 1.86e-08    -  1.00e+00 1.00e+00f  1

Number of Iterations…: 6

                                   (scaled)                 (unscaled)
Objective…:   6.4072416740416713e-22    6.4072416740416713e-22
Dual infeasibility…:   3.9625445352197450e-17    3.9625445352197450e-17
Constraint violation…:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity…:   2.5059073699316697e-09    2.5059073699316697e-09
Overall NLP error…:   2.5059073699316697e-09    2.5059073699316697e-09


Number of objective function evaluations             = 7
```

```
Number of objective gradient evaluations             = 7
Number of equality constraint evaluations            = 0
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 6
Total seconds in IPOPT                               = 0.003

EXIT: Optimal Solution Found.
      solver  :   t_proc      (avg)   t_wall      (avg)    n_eval
       nlp_f  |  46.00us (  6.57us)  17.60us (  2.51us)         7
  nlp_grad_f  |  33.00us (  4.12us)  16.00us (  2.00us)         8
  nlp_hess_l  |  20.00us (  3.33us)   9.10us (  1.52us)         6
       total  |   7.70ms (  7.70ms)   3.85ms (  3.85ms)         1
```

The solution is obviously 1:

[4]:
```
print(sol['x'])
assert(abs(sol['x']-1)<1e-9)
```

```
1
```

3 Constrained problem

$\min_x \quad (x-1)^T.(x-1)$ \ subject to $-10 \leq x \leq 10$ \ subject to $0 \leq x_1 + x_2 \leq 1$ \ subject to $ x_0 = 2$ \

with $x \in \mathbf{R}^n$

[5]:
```
n = 5
```

[6]:
```
x=SX.sym('x',n)
```

Note how we do not distinguish between equalities and inequalities here

[7]:
```
nlp = {'x':x, 'f':mtimes((x-1).T,x-1), 'g':vertcat(x[1]+x[2],x[0])}
```

[8]:
```
solver = nlpsol('solver', 'ipopt', nlp)
sol = solver(lbx=-10, ubx=10, lbg=[0,2], ubg=[1,2])
```

```
This is Ipopt version 3.14.11, running with linear solver MUMPS 5.4.1.

Number of nonzeros in equality constraint Jacobian…:        1
Number of nonzeros in inequality constraint Jacobian.:        2
Number of nonzeros in Lagrangian Hessian…:        5

Total number of variables…:        5
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        5
```

```
                     variables with only upper bounds:        0
Total number of equality constraints…:        1
Total number of inequality constraints…:        1
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        1
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  5.0000000e+00 2.00e+00 2.00e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  2.7716739e+00 0.00e+00 1.16e+00  -1.0 2.00e+00    -  3.75e-01 1.00e+00f  1
   2  1.5112117e+00 0.00e+00 4.53e-01  -1.0 1.31e+00    -  9.44e-01 6.62e-01f  1
   3  1.5183915e+00 0.00e+00 1.11e-16  -1.7 3.46e-02    -  1.00e+00 1.00e+00f  1
   4  1.5009100e+00 0.00e+00 2.03e-04  -3.8 1.73e-02    -  9.88e-01 1.00e+00f  1
   5  1.5000028e+00 0.00e+00 2.22e-16  -5.7 9.07e-04    -  1.00e+00 1.00e+00f  1
   6  1.5000000e+00 0.00e+00 2.04e-16  -8.6 2.85e-06    -  1.00e+00 1.00e+00f  1

Number of Iterations…: 6

                                   (scaled)                 (unscaled)
Objective…:   1.4999999925191814e+00    1.4999999925191814e+00
Dual infeasibility…:   2.0424918385704731e-16    2.0424918385704731e-16
Constraint violation…:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity…:   2.5191812953934411e-09    2.5191812953934411e-09
Overall NLP error…:   2.5191812953934411e-09    2.5191812953934411e-09


Number of objective function evaluations             = 7
Number of objective gradient evaluations             = 7
Number of equality constraint evaluations            = 7
Number of inequality constraint evaluations          = 7
Number of equality constraint Jacobian evaluations   = 7
Number of inequality constraint Jacobian evaluations = 7
Number of Lagrangian Hessian evaluations             = 6
Total seconds in IPOPT                               = 0.003

EXIT: Optimal Solution Found.
      solver  :   t_proc      (avg)   t_wall      (avg)    n_eval
       nlp_f  |  28.00us (  4.00us)  13.20us (  1.89us)         7
       nlp_g  |  42.00us (  6.00us)  16.90us (  2.41us)         7
  nlp_grad_f  |  35.00us (  4.38us)  17.20us (  2.15us)         8
  nlp_hess_l  |  19.00us (  3.17us)   9.60us (  1.60us)         6
   nlp_jac_g  |  24.00us (  3.00us)  11.70us (  1.46us)         8
       total  |   7.04ms (  7.04ms)   3.52ms (  3.52ms)         1
```

$ 2 x_0 2$ is not really as bad it looks. Ipopt will recognise this situation as an equality constraint.

The solution is obviously [2,0.5,0.5,1,1]:

[9]:
```
print(sol['x'])
for (i,e) in zip(list(range(n)),[2,0.5,0.5,1,1]):
  assert(abs(sol['x'][i]-e)<1e-7)
```

```
[2, 0.5, 0.5, 1, 1]
```

4 Problem with parameters

$\min_x \quad (x-a)^2 \setminus$ subject to $-10 \leq x \leq 10 \setminus$

with x scalar

[10]:
```
x=SX.sym('x')
a=SX.sym('a')
a_ = 2
nlp={'x':x, 'p':a, 'f':(x-a)**2}
```

[11]:
```
solver = nlpsol('solver', 'ipopt', nlp)
sol = solver(lbx=-10, ubx=10, p=a_)
```

```
This is Ipopt version 3.14.11, running with linear solver MUMPS 5.4.1.

Number of nonzeros in equality constraint Jacobian…:        0
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian…:        1

Total number of variables…:        1
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        1
                     variables with only upper bounds:        0
Total number of equality constraints…:        0
Total number of inequality constraints…:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  4.0000000e+00 0.00e+00 4.00e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  3.3057851e-02 0.00e+00 5.64e-02  -1.0 1.82e+00    -  8.45e-01 1.00e+00f  1
   2  2.8894889e-05 0.00e+00 1.53e-16  -1.0 1.76e-01    -  1.00e+00 1.00e+00f  1
   3  4.4774726e-09 0.00e+00 1.08e-16  -2.5 5.31e-03    -  1.00e+00 1.00e+00f  1
   4  9.8332283e-12 0.00e+00 2.02e-16  -3.8 6.38e-05    -  1.00e+00 1.00e+00f  1
   5  1.4777036e-15 0.00e+00 3.79e-18  -5.7 3.10e-06    -  1.00e+00 1.00e+00f  1
   6  2.7255842e-21 0.00e+00 1.26e-17  -8.6 3.84e-08    -  1.00e+00 1.00e+00f  1

Number of Iterations…: 6

                                   (scaled)                 (unscaled)
```

```
Objective…:   2.7255841628426173e-21    2.7255841628426173e-21
Dual infeasibility…:   1.2569571890559792e-17    1.2569571890559792e-17
Constraint violation…:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity…:   2.5059124009153784e-09    2.5059124009153784e-09
Overall NLP error…:   2.5059124009153784e-09    2.5059124009153784e-09


Number of objective function evaluations             = 7
Number of objective gradient evaluations             = 7
Number of equality constraint evaluations            = 0
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 6
Total seconds in IPOPT                               = 0.003

EXIT: Optimal Solution Found.
      solver  :   t_proc      (avg)   t_wall      (avg)    n_eval
       nlp_f  |  39.00us (  5.57us)  16.70us (  2.39us)         7
  nlp_grad_f  |  35.00us (  4.37us)  16.10us (  2.01us)         8
  nlp_hess_l  |  21.00us (  3.50us)   9.60us (  1.60us)         6
       total  |   6.11ms (  6.11ms)   3.06ms (  3.06ms)         1
```

The solution is obviously a:

[12]:
```
print(sol['x'])
assert(abs(sol['x']-a_)<1e-9)
```

```
2
```

The parameter can change inbetween two solve calls:

[13]:
```
sol = solver(lbx=-10, ubx=10, p=2*a_)
```

```
This is Ipopt version 3.14.11, running with linear solver MUMPS 5.4.1.

Number of nonzeros in equality constraint Jacobian…:        0
Number of nonzeros in inequality constraint Jacobian.:        0
Number of nonzeros in Lagrangian Hessian…:        1

Total number of variables…:        1
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        1
                     variables with only upper bounds:        0
Total number of equality constraints…:        0
Total number of inequality constraints…:        0
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        0
```

```
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0  1.6000000e+01 0.00e+00 8.00e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  1.3223140e-01 0.00e+00 1.95e-01  -1.0 3.64e+00    -  7.31e-01 1.00e+00f  1
   2  3.6066256e-04 0.00e+00 1.04e-16  -1.0 3.45e-01    -  1.00e+00 1.00e+00f  1
   3  1.0362906e-06 0.00e+00 3.71e-16  -1.7 1.80e-02    -  1.00e+00 1.00e+00f  1
   4  5.6293833e-11 0.00e+00 8.15e-17  -3.8 1.01e-03    -  1.00e+00 1.00e+00f  1
   5  7.7214018e-15 0.00e+00 3.56e-16  -5.7 7.42e-06    -  1.00e+00 1.00e+00f  1
   6  1.4239938e-20 0.00e+00 3.69e-16  -8.6 8.78e-08    -  1.00e+00 1.00e+00f  1

Number of Iterations…: 6

                                   (scaled)                 (unscaled)
Objective…:   1.4239938056866796e-20    1.4239938056866796e-20
Dual infeasibility…:   3.6930385037381279e-16    3.6930385037381279e-16
Constraint violation…:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity…:   2.5059305083202185e-09    2.5059305083202185e-09
Overall NLP error…:   2.5059305083202185e-09    2.5059305083202185e-09


Number of objective function evaluations             = 7
Number of objective gradient evaluations             = 7
Number of equality constraint evaluations            = 0
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 0
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 6
Total seconds in IPOPT                               = 0.003

EXIT: Optimal Solution Found.
      solver  :   t_proc      (avg)   t_wall      (avg)    n_eval
       nlp_f  |  77.00us (  5.50us)  32.30us (  2.31us)        14
    nlp_grad  |   6.00us (  6.00us)   3.00us (  3.00us)         1
  nlp_grad_f  |  69.00us (  4.31us)  31.80us (  1.99us)        16
  nlp_hess_l  |  39.00us (  3.25us)  18.30us (  1.53us)        12
       total  |   6.28ms (  6.28ms)   3.14ms (  3.14ms)         1
```

The solution is obviously 2*a:

[14]:
```
print(sol['x'])
assert(abs(sol['x']-2*a_)<1e-9)
```

```
4
```