exacthessian

April 24, 2023

1 Exact Hessian

[1]:
```
from casadi import *
from numpy import *
import casadi as c
```

We will investigate the use of an exact Hessian with the help of the Rosenbrock function

[2]:
```
x=SX.sym('x')
y=SX.sym('y')
obj = (1-x)**2+100*(y-x**2)**2
constr = x**2+y**2
nlp={'x':vertcat(x,y), 'f':obj, 'g':constr}
```

We solve the problem with an exact Hessian (default)

[3]:

```python
solver = nlpsol('solver', 'ipopt', nlp)
sol = solver(lbx=-10, ubx=10, lbg=0, ubg=1)
print('Optimal solution (exact Hessian): %s' % sol['x'])
```

```
******************************************************************************
This program contains Ipopt, a library for large-scale nonlinear optimization.
 Ipopt is released as open source code under the Eclipse Public License (EPL).
         For more information visit https://github.com/coin-or/Ipopt
******************************************************************************

This is Ipopt version 3.14.11, running with linear solver MUMPS 5.4.1.

Number of nonzeros in equality constraint Jacobian...:        0
Number of nonzeros in inequality constraint Jacobian.:        2
Number of nonzeros in Lagrangian Hessian...:        3

Total number of variables...:        2
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        2
                     variables with only upper bounds:        0
Total number of equality constraints...:        0
Total number of inequality constraints...:        1
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        1
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
   0 1.0000000e+00 0.00e+00 2.00e+00  -1.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1 6.8309610e+01 0.00e+00 4.32e+02  -1.0 9.09e-01    -  1.36e-02 1.00e+00H  1
   2 6.2418830e+00 0.00e+00 7.25e+01  -1.0 3.06e-01    -  6.76e-01 1.00e+00F  1
   3 6.3023184e-02 0.00e+00 5.46e-01  -1.0 2.58e-01    -  9.97e-01 1.00e+00f  1
   4 6.5589711e-02 0.00e+00 5.27e-03  -1.7 4.84e-02    -  1.00e+00 1.00e+00h  1
   5 5.0619847e-02 0.00e+00 3.07e-01  -3.8 9.95e-02    -  9.05e-01 1.00e+00f  1
   6 4.6180852e-02 0.00e+00 2.45e-02  -3.8 3.71e-02    -  1.00e+00 1.00e+00h  1
   7 4.5822797e-02 0.00e+00 1.76e-04  -3.8 3.26e-03    -  1.00e+00 1.00e+00h  1
   8 4.5677137e-02 0.00e+00 3.59e-05  -5.7 1.20e-03    -  1.00e+00 1.00e+00h  1
   9 4.5676652e-02 0.00e+00 3.22e-10  -5.7 4.39e-06    -  1.00e+00 1.00e+00h  1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
  10 4.5674810e-02 0.00e+00 5.78e-09  -8.6 1.52e-05    -  1.00e+00 1.00e+00h  1

Number of Iterations...: 10

                                   (scaled)                 (unscaled)
Objective...:   4.5674810088672947e-02    4.5674810088672947e-02
Dual infeasibility...:   5.7761012971635439e-09    5.7761012971635439e-09
Constraint violation...:   0.0000000000000000e+00    0.0000000000000000e+00
```

```
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity…:   2.5919940506206774e-09    2.5919940506206774e-09
Overall NLP error…:   5.7761012971635439e-09    5.7761012971635439e-09


Number of objective function evaluations             = 14
Number of objective gradient evaluations             = 11
Number of equality constraint evaluations            = 0
Number of inequality constraint evaluations          = 14
Number of equality constraint Jacobian evaluations   = 0
Number of inequality constraint Jacobian evaluations = 11
Number of Lagrangian Hessian evaluations             = 10
Total seconds in IPOPT                               = 0.005

EXIT: Optimal Solution Found.
      solver  :   t_proc      (avg)   t_wall      (avg)    n_eval
       nlp_f  |  43.00us (  3.07us)  21.90us (  1.56us)        14
       nlp_g  |  82.00us (  5.86us)  34.20us (  2.44us)        14
  nlp_grad_f  |  49.00us (  4.08us)  23.40us (  1.95us)        12
  nlp_hess_l  |  40.00us (  4.00us)  19.80us (  1.98us)        10
   nlp_jac_g  |  36.00us (  3.00us)  18.60us (  1.55us)        12
       total  |  11.88ms ( 11.88ms)   5.94ms (  5.94ms)         1
Optimal solution (exact Hessian): [0.786415, 0.617698]
```

Same problem but with limited memory BFSG

[4]:
```python
solver = nlpsol('solver', 'ipopt', nlp, {'ipopt.hessian_approximation':
 ↪'limited-memory'})
sol = solver(lbx=-10, ubx=10, lbg=0, ubg=1)
print('Optimal solution (BFGS): %s' % sol['x'])
```

```
This is Ipopt version 3.14.11, running with linear solver MUMPS 5.4.1.

Number of nonzeros in equality constraint Jacobian…:        0
Number of nonzeros in inequality constraint Jacobian.:        2
Number of nonzeros in Lagrangian Hessian…:        0

Total number of variables…:        2
                     variables with only lower bounds:        0
                variables with lower and upper bounds:        2
                     variables with only upper bounds:        0
Total number of equality constraints…:        0
Total number of inequality constraints…:        1
        inequality constraints with only lower bounds:        0
   inequality constraints with lower and upper bounds:        1
        inequality constraints with only upper bounds:        0

iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
```

```
   0  1.0000000e+00 0.00e+00 2.00e+00   0.0 0.00e+00    -  0.00e+00 0.00e+00   0
   1  8.1099664e-01 0.00e+00 8.51e+00  -5.2 1.67e+00    -  8.49e-01 1.24e-01f  4
   2  7.8913241e-01 0.00e+00 7.56e+00  -0.7 3.79e-01    -  2.45e-01 1.00e+00F  1
   3  5.1038399e-01 0.00e+00 2.00e+00  -1.6 1.47e-01    -  1.00e+00 1.00e+00h  1
   4  9.4636921e-01 0.00e+00 1.66e+01  -1.7 2.10e-01    -  6.36e-01 1.00e+00H  1
   5  4.3659026e-01 0.00e+00 8.99e-01  -2.6 1.70e-01    -  1.00e+00 1.00e+00f  1
   6  3.9428196e-01 0.00e+00 8.08e-01  -3.2 5.30e-02    -  1.00e+00 1.00e+00h  1
   7  1.5277780e+00 0.00e+00 3.32e+01  -4.2 3.31e-01    -  4.87e-01 1.00e+00H  1
   8  3.5006028e-01 0.00e+00 1.56e+00  -3.3 5.71e-01    -  1.00e+00 1.00e+00f  1
   9  3.1926755e-01 0.00e+00 1.63e+00  -3.9 1.60e-01    -  1.00e+00 1.00e+00h  1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
  10  2.7958742e-01 0.00e+00 4.47e+00  -4.1 8.31e+00    -  1.00e+00 1.15e-02f  4
  11  4.1529404e-01 0.00e+00 1.74e+01  -4.6 3.79e-01    -  7.49e-01 1.00e+00H  1
  12  1.9552435e-01 0.00e+00 7.94e+00  -3.4 1.93e-01    -  1.00e+00 1.00e+00f  1
  13  1.0284176e-01 0.00e+00 1.20e+00  -4.6 5.10e-02    -  1.00e+00 1.00e+00f  1
  14  7.9168874e-02 0.00e+00 1.74e+00  -6.0 1.24e-01    -  1.00e+00 1.00e+00f  1
  15  6.4711554e-02 0.00e+00 3.50e+00  -7.3 1.52e-01    -  1.00e+00 1.00e+00h  1
  16  5.5234143e-02 4.93e-03 3.01e+00  -4.6 1.34e-01    -  1.00e+00 3.44e-01h  1
  17  4.7148548e-02 0.00e+00 9.32e-01  -4.3 7.45e-03    -  1.00e+00 1.00e+00h  1
  18  4.5698648e-02 0.00e+00 1.46e-01  -6.0 6.27e-03    -  1.00e+00 8.55e-01h  1
  19  4.5674859e-02 0.00e+00 1.07e-03  -6.7 1.78e-04    -  1.00e+00 9.95e-01h  1
iter    objective    inf_pr   inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
  20  4.5674809e-02 0.00e+00 1.79e-07  -8.8 1.52e-06    -  1.00e+00 1.00e+00h  1
  21  4.5674808e-02 0.00e+00 2.24e-09 -11.0 1.39e-08    -  1.00e+00 1.00e+00h  1

Number of Iterations…: 21

                                   (scaled)                 (unscaled)
Objective…:   4.5674807514535586e-02    4.5674807514535586e-02
Dual infeasibility…:   2.2446746822391006e-09    2.2446746822391006e-09
Constraint violation…:   0.0000000000000000e+00    0.0000000000000000e+00
Variable bound violation:   0.0000000000000000e+00    0.0000000000000000e+00
Complementarity…:   1.0000913016783289e-11    1.0000913016783289e-11
Overall NLP error…:   2.2446746822391006e-09    2.2446746822391006e-09


Number of objective function evaluations             = 36
Number of objective gradient evaluations             = 22
Number of equality constraint evaluations            = 0
Number of inequality constraint evaluations          = 36
Number of equality constraint Jacobian evaluations   = 0
Number of inequality constraint Jacobian evaluations = 22
Number of Lagrangian Hessian evaluations             = 0
Total seconds in IPOPT                               = 0.027

EXIT: Optimal Solution Found.
      solver  :   t_proc      (avg)   t_wall      (avg)    n_eval
       nlp_f  |  96.00us (  2.67us)  49.90us (  1.39us)        36
```

```
       nlp_g  | 251.00us (  6.97us)  89.50us (  2.49us)        36
  nlp_grad_f  |  82.00us (  3.57us)  41.40us (  1.80us)        23
   nlp_jac_g  |  65.00us (  2.83us)  30.30us (  1.32us)        23
       total  |  54.50ms ( 54.50ms)  27.26ms ( 27.26ms)         1
Optimal solution (BFGS): [0.786415, 0.617698]
```