

exacthessian

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This file is part of CasADi.

CasADi -- A symbolic framework for dynamic optimization.
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1 Exact Hessian

```
[1]: from casadi import *  
     from numpy import *  
     import casadi as c
```

We will investigate the use of an exact Hessian with the help of the Rosenbrock function

```
[2]: x= SX.sym('x')  
     y= SX.sym('y')  
     obj = (1-x)**2+100*(y-x**2)**2  
     constr = x**2+y**2  
     nlp={ 'x':vertcat(x,y), 'f':obj, 'g':constr}
```

We solve the problem with an exact Hessian (default)

```
[3]: solver = nlpsol('solver', 'ipopt', nlp)
sol = solver(lbx=-10, ubx=10, lb=0, ub=1)
print('Optimal solution (exact Hessian): %s' % sol['x'])
```

```
*****
This program contains Ipopt, a library for large-scale nonlinear optimization.
Ipopt is released as open source code under the Eclipse Public License (EPL).
For more information visit https://github.com/coin-or/Ipopt
*****
```

This is Ipopt version 3.14.11, running with linear solver MUMPS 5.4.1.

```
Number of nonzeros in equality constraint Jacobian...:      0
Number of nonzeros in inequality constraint Jacobian.:      2
Number of nonzeros in Lagrangian Hessian...:           3
```

```
Total number of variables...:      2
      variables with only lower bounds:      0
      variables with lower and upper bounds:    2
      variables with only upper bounds:      0
```

```
Total number of equality constraints...:      0
Total number of inequality constraints...:      1
      inequality constraints with only lower bounds:      0
      inequality constraints with lower and upper bounds:    1
      inequality constraints with only upper bounds:      0
```

iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
0	1.0000000e+00	0.00e+00	2.00e+00	-1.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	6.8309610e+01	0.00e+00	4.32e+02	-1.0	9.09e-01	-	1.36e-02	1.00e+00H	1
2	6.2418830e+00	0.00e+00	7.25e+01	-1.0	3.06e-01	-	6.76e-01	1.00e+00F	1
3	6.3023184e-02	0.00e+00	5.46e-01	-1.0	2.58e-01	-	9.97e-01	1.00e+00f	1
4	6.5589711e-02	0.00e+00	5.27e-03	-1.7	4.84e-02	-	1.00e+00	1.00e+00h	1
5	5.0619847e-02	0.00e+00	3.07e-01	-3.8	9.95e-02	-	9.05e-01	1.00e+00f	1
6	4.6180852e-02	0.00e+00	2.45e-02	-3.8	3.71e-02	-	1.00e+00	1.00e+00h	1
7	4.5822797e-02	0.00e+00	1.76e-04	-3.8	3.26e-03	-	1.00e+00	1.00e+00h	1
8	4.5677137e-02	0.00e+00	3.59e-05	-5.7	1.20e-03	-	1.00e+00	1.00e+00h	1
9	4.5676652e-02	0.00e+00	3.22e-10	-5.7	4.39e-06	-	1.00e+00	1.00e+00h	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
10	4.5674810e-02	0.00e+00	5.78e-09	-8.6	1.52e-05	-	1.00e+00	1.00e+00h	1

Number of Iterations...: 10

	(scaled)	(unscaled)
Objective...:	4.5674810088672947e-02	4.5674810088672947e-02
Dual infeasibility...:	5.7761012971635439e-09	5.7761012971635439e-09
Constraint violation...:	0.0000000000000000e+00	0.0000000000000000e+00

```
Variable bound violation: 0.0000000000000000e+00 0.0000000000000000e+00
Complementarity...: 2.5919940506206774e-09 2.5919940506206774e-09
Overall NLP error...: 5.7761012971635439e-09 5.7761012971635439e-09
```

```
Number of objective function evaluations = 14
Number of objective gradient evaluations = 11
Number of equality constraint evaluations = 0
Number of inequality constraint evaluations = 14
Number of equality constraint Jacobian evaluations = 0
Number of inequality constraint Jacobian evaluations = 11
Number of Lagrangian Hessian evaluations = 10
Total seconds in IPOPT = 0.007
```

EXIT: Optimal Solution Found.

	solver	:	t_proc	(avg)	t_wall	(avg)	n_eval
	nlp_f		99.00us	(7.07us)	24.20us	(1.73us)	14
	nlp_g		181.00us	(12.93us)	42.06us	(3.00us)	14
	nlp_grad_f		107.00us	(8.92us)	25.58us	(2.13us)	12
	nlp_hess_l		82.00us	(8.20us)	19.14us	(1.91us)	10
	nlp_jac_g		103.00us	(8.58us)	19.07us	(1.59us)	12
	total		30.16ms	(30.16ms)	7.56ms	(7.56ms)	1

Optimal solution (exact Hessian): [0.786415, 0.617698]

Same problem but with limited memory BFGS

```
[4]: solver = nlpsol('solver', 'ipopt', nlp, {'ipopt.hessian_approximation':
↳ 'limited-memory'})
sol = solver(lbx=-10, ubx=10, lb=0, ub=1)
print('Optimal solution (BFGS): %s' % sol['x'])
```

This is Ipopt version 3.14.11, running with linear solver MUMPS 5.4.1.

```
Number of nonzeros in equality constraint Jacobian...: 0
Number of nonzeros in inequality constraint Jacobian.: 2
Number of nonzeros in Lagrangian Hessian...: 0
```

```
Total number of variables...: 2
      variables with only lower bounds: 0
      variables with lower and upper bounds: 2
      variables with only upper bounds: 0
Total number of equality constraints...: 0
Total number of inequality constraints...: 1
      inequality constraints with only lower bounds: 0
      inequality constraints with lower and upper bounds: 1
      inequality constraints with only upper bounds: 0
```

```
iter    objective    inf_pr    inf_du lg(mu)  ||d||  lg(rg) alpha_du alpha_pr  ls
```

0	1.0000000e+00	0.00e+00	2.00e+00	0.0	0.00e+00	-	0.00e+00	0.00e+00	0
1	8.1099664e-01	0.00e+00	8.51e+00	-5.2	1.67e+00	-	8.49e-01	1.24e-01f	4
2	7.8913241e-01	0.00e+00	7.56e+00	-0.7	3.79e-01	-	2.45e-01	1.00e+00F	1
3	5.1038399e-01	0.00e+00	2.00e+00	-1.6	1.47e-01	-	1.00e+00	1.00e+00h	1
4	9.4636921e-01	0.00e+00	1.66e+01	-1.7	2.10e-01	-	6.36e-01	1.00e+00H	1
5	4.3659026e-01	0.00e+00	8.99e-01	-2.6	1.70e-01	-	1.00e+00	1.00e+00f	1
6	3.9428196e-01	0.00e+00	8.08e-01	-3.2	5.30e-02	-	1.00e+00	1.00e+00h	1
7	1.5277780e+00	0.00e+00	3.32e+01	-4.2	3.31e-01	-	4.87e-01	1.00e+00H	1
8	3.5006028e-01	0.00e+00	1.56e+00	-3.3	5.71e-01	-	1.00e+00	1.00e+00f	1
9	3.1926755e-01	0.00e+00	1.63e+00	-3.9	1.60e-01	-	1.00e+00	1.00e+00h	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
10	2.7958742e-01	0.00e+00	4.47e+00	-4.1	8.31e+00	-	1.00e+00	1.15e-02f	4
11	4.1529404e-01	0.00e+00	1.74e+01	-4.6	3.79e-01	-	7.49e-01	1.00e+00H	1
12	1.9552435e-01	0.00e+00	7.94e+00	-3.4	1.93e-01	-	1.00e+00	1.00e+00f	1
13	1.0284176e-01	0.00e+00	1.20e+00	-4.6	5.10e-02	-	1.00e+00	1.00e+00f	1
14	7.9168874e-02	0.00e+00	1.74e+00	-6.0	1.24e-01	-	1.00e+00	1.00e+00f	1
15	6.4711554e-02	0.00e+00	3.50e+00	-7.3	1.52e-01	-	1.00e+00	1.00e+00h	1
16	5.5234143e-02	4.93e-03	3.01e+00	-4.6	1.34e-01	-	1.00e+00	3.44e-01h	1
17	4.7148548e-02	0.00e+00	9.32e-01	-4.3	7.45e-03	-	1.00e+00	1.00e+00h	1
18	4.5698648e-02	0.00e+00	1.46e-01	-6.0	6.27e-03	-	1.00e+00	8.55e-01h	1
19	4.5674859e-02	0.00e+00	1.07e-03	-6.7	1.78e-04	-	1.00e+00	9.95e-01h	1
iter	objective	inf_pr	inf_du	lg(mu)	d	lg(rg)	alpha_du	alpha_pr	ls
20	4.5674809e-02	0.00e+00	1.79e-07	-8.8	1.52e-06	-	1.00e+00	1.00e+00h	1
21	4.5674808e-02	0.00e+00	2.24e-09	-11.0	1.39e-08	-	1.00e+00	1.00e+00h	1

Number of Iterations...: 21

	(scaled)	(unscaled)
Objective...:	4.5674807514535586e-02	4.5674807514535586e-02
Dual infeasibility...:	2.2446746822391006e-09	2.2446746822391006e-09
Constraint violation...:	0.0000000000000000e+00	0.0000000000000000e+00
Variable bound violation:	0.0000000000000000e+00	0.0000000000000000e+00
Complementarity...:	1.0000913016783289e-11	1.0000913016783289e-11
Overall NLP error...:	2.2446746822391006e-09	2.2446746822391006e-09

Number of objective function evaluations	= 36
Number of objective gradient evaluations	= 22
Number of equality constraint evaluations	= 0
Number of inequality constraint evaluations	= 36
Number of equality constraint Jacobian evaluations	= 0
Number of inequality constraint Jacobian evaluations	= 22
Number of Lagrangian Hessian evaluations	= 0
Total seconds in IPOPT	= 0.038

EXIT: Optimal Solution Found.

solver	t_proc	(avg)	t_wall	(avg)	n_eval
nlp_f	228.00us	(6.33us)	55.08us	(1.53us)	36

nlp_g		465.00us	(12.92us)	104.70us	(2.91us)	36
nlp_grad_f		176.00us	(7.65us)	45.06us	(1.96us)	23
nlp_jac_g		151.00us	(6.57us)	36.28us	(1.58us)	23
total		151.75ms	(151.75ms)	38.12ms	(38.12ms)	1

Optimal solution (BFGS): [0.786415, 0.617698]