Stochastic Processes in Economics

In this homework I chose 2 transition matrices the first one is :

1 Absorbing Markov chain

Like in the course I decide to compute an absorbing matrices of the following form:

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1-x & 0 & x & 0 & 0 \\ 0 & 1-x & 0 & x & 0 \\ 0 & 0 & 1-x & 0 & x \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$\forall x \in [0, 1]$, I choose a value of $x = \frac{1}{7}$

The initial distribution of chain is the vector : $\nu = \begin{pmatrix} 1/10 & 2/10 & 4/10 & 2/10 & 1/10 \end{pmatrix}$

1.1 Theoretical values of an absorbing Markov chain

We know that we can rewrite the matrices as $P = \begin{pmatrix} Q & R \\ 0 & I_{N-r} \end{pmatrix}$

The fundamental matrices of the absorbing chain is :

$$F = \left(I_4 - \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1-1/7 & 0 & 1/7 & 0 \\ 0 & 1-1/7 & 0 & 1/7 \\ 0 & 0 & 1-1/7 & 0 \end{pmatrix} \right)^{-1}$$

the average time passed in an state i before absorption is given by : $\mathbb{E}_{\nu'(\tau)} = \nu' \sum_{j=1}^{q} f_{ij}$ with :$\nu'_i = \nu_i$ if $P_{ii} \neq 1$

theoretically we should expect an average of :

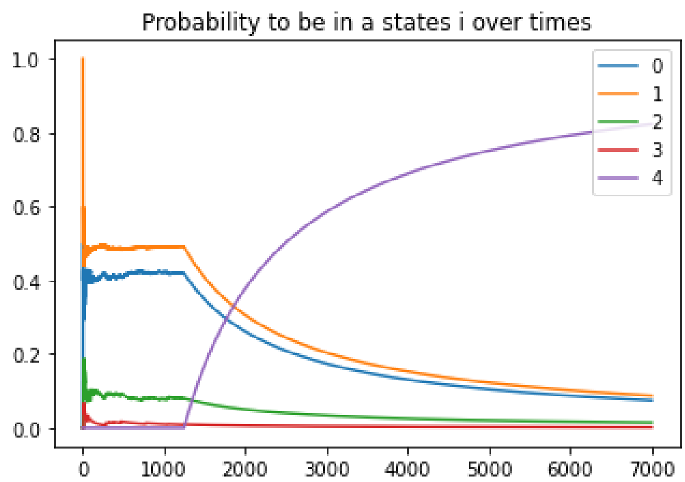
$$\mathbb{E}_{\nu(\tau)} = \sum_{i=1}^{q} \nu'_i f_{ij} = \begin{pmatrix} 221.5 & 258.3 & 42.7 & 6.3 \end{pmatrix}$$

We can deduce the average time before absorption is $\sum_{i=1} \mathbb{E}_{\nu(\tau)} = 528.8$

1.2 Empirical result absorbing matrices

First lets describe what I did for the absorbing matrices in the program, I decide to ran an Markov chain simulation of a **length: n = 7000**, and I repeat the **simulation 800 more times**.

I get the following result :



For the 1st simulation I have that the first stage who reached the absorbing state is the 1245^{th} step. With respect to the initial distribution

The Average times before absorption for all the simulation is 523 which is quite close from the theoretical expectation.

The average times in a states with the initial distribution probability is here : $\begin{pmatrix} 218.7 & 255.5 & 42.8 & 6.3 \end{pmatrix}$ for a length of the Markov chain: 7000 and : 800 simulation of random walk

2 Asymptotic Distribution of an regular matrices

the following matrices is an regular and irreducible matrices $P = \begin{pmatrix} 0.7 & 0.1 & 0.2 \\ 0.2 & 0.1 & 0.7 \\ 0.2 & 0.3 & 0.5 \end{pmatrix}$

with the initial distribution chain given by $\nu = \begin{pmatrix} 0.2 & 0.3 & 0.5 \end{pmatrix}$

2.1 THEORETICAL VALUES OF REGULAR MARKOV CHAIN

Theorem For any regular Markov chain and any initial distribution we have the following property :

$$\pi P = \pi$$

with $\sum_{i=1}^{n} \pi_i = 1$

it's tell us that whenever the initial is the distribution of this type of Markov chain is independent from their initial distribution ν. and it's converge to a stationary distribution when $n \to +\infty$

so we have : $lim_{n \to +\infty} \mathbb{P}_{\nu}(X_n = i) = \mathbb{P}_{\pi}(X_n = i) = \pi_i$

$$\pi_i = \begin{pmatrix} 0.63809524 & 0.12380952 & 0.23809524 \end{pmatrix}$$

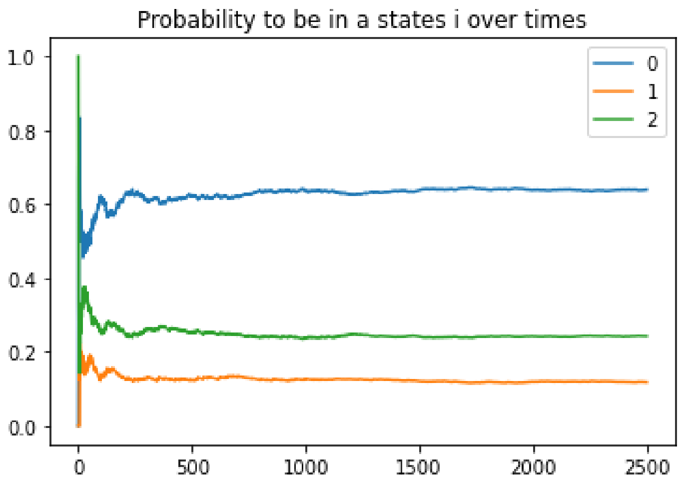
the average return times are given by : $\mathbb{E}_i(\tau_i) = \frac{1}{\pi_i}$

After computation we should have an average return times $\mathbb{E}_i(\tau_i) = \frac{1}{\pi_i} = \begin{pmatrix} 1.56716418 & 8.07692308 & 4.2 \end{pmatrix}$

2.2 Empirical result of the regular matrices

for this matrices same as before i ran an simulation of a **length: n = 2500**, and I repeat the simulation 500 more times.

I have from the simulation de following result :



Times spent in the matrices B for the 1st simulation in a state is : $\begin{pmatrix} 1623 & 293 & 584 \end{pmatrix}$.

Average times spent in the matrices for 2500 step is : $\begin{pmatrix} 1595.02 & 309.052 & 595.928 \end{pmatrix}$

Average probability times spent in the matrices for 500 and 2500 step in the simulation is : $\begin{pmatrix} 0.638008 & 0.1236208 & 0.2383712 \end{pmatrix}$

Average return times in the matrices B in state i is : $\begin{pmatrix} 1.56737847 & 8.08925359 & 4.19513767 \end{pmatrix}$