

Coding Project 2

The purpose of this assignment was to understand the effects that sampling, bit- depth and filtering have on a continuous signal being digitized into discrete values.

Figure 1: Sampling at Different Frequencies

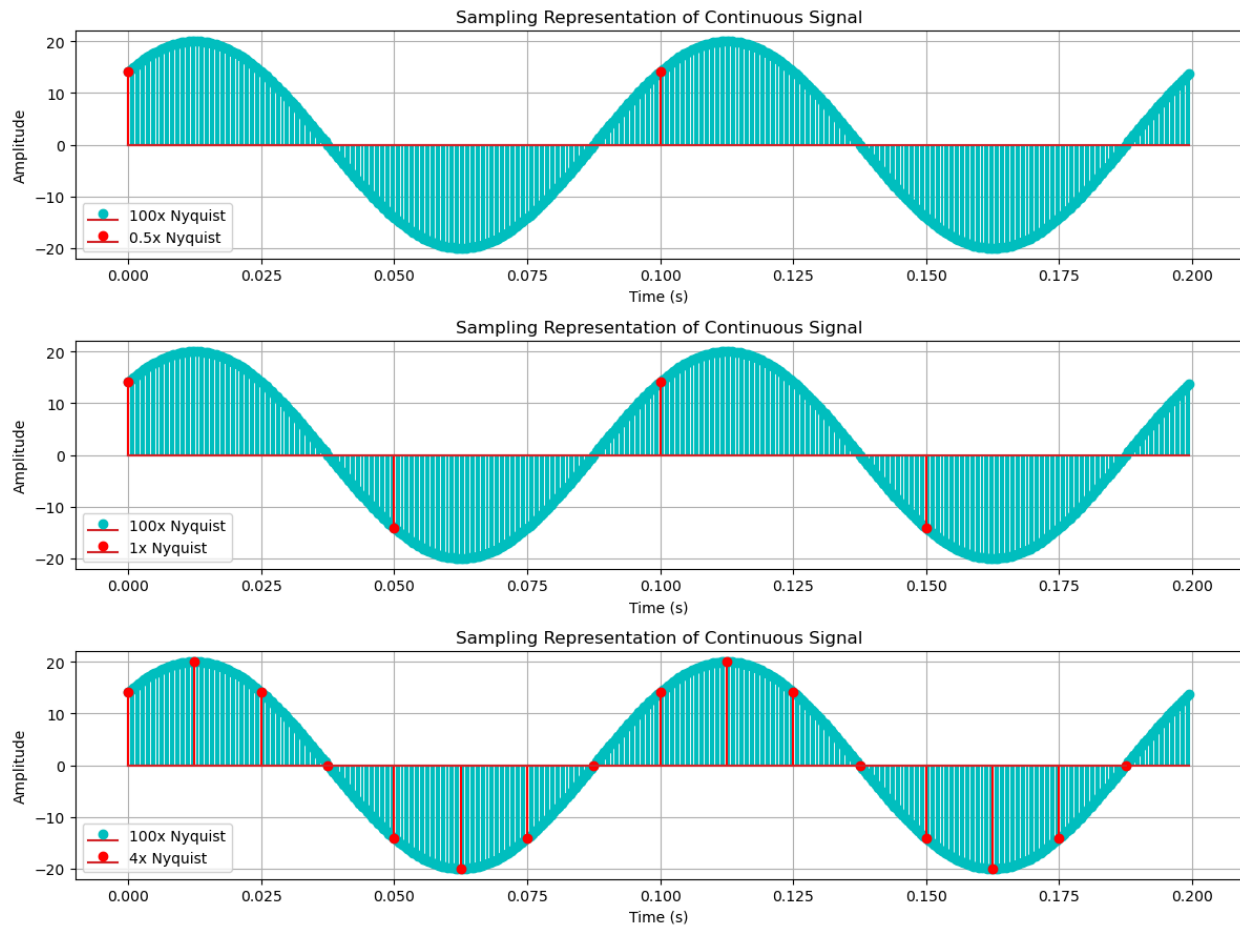


Figure 1 shows a continuous signal being sampled at different frequencies relative to the Nyquist frequency, plotted on top of our over-sampled signal that is sampled at 100x Nyquist. As shown, a signal that is sampled at 0.5x Nyquist is basically incapable of properly representing the original signal. At Nyquist ($f_s = 2 * f$), we can see that the signal is at least capable of being represented and understood, although not very accurately. Once we increase to 4x Nyquist, we see the most accurate representation of our signal of those we have observed.

Figure 2: Magnitude of FFT for 100x, 0.5x, 1x, and 4x Nyquist

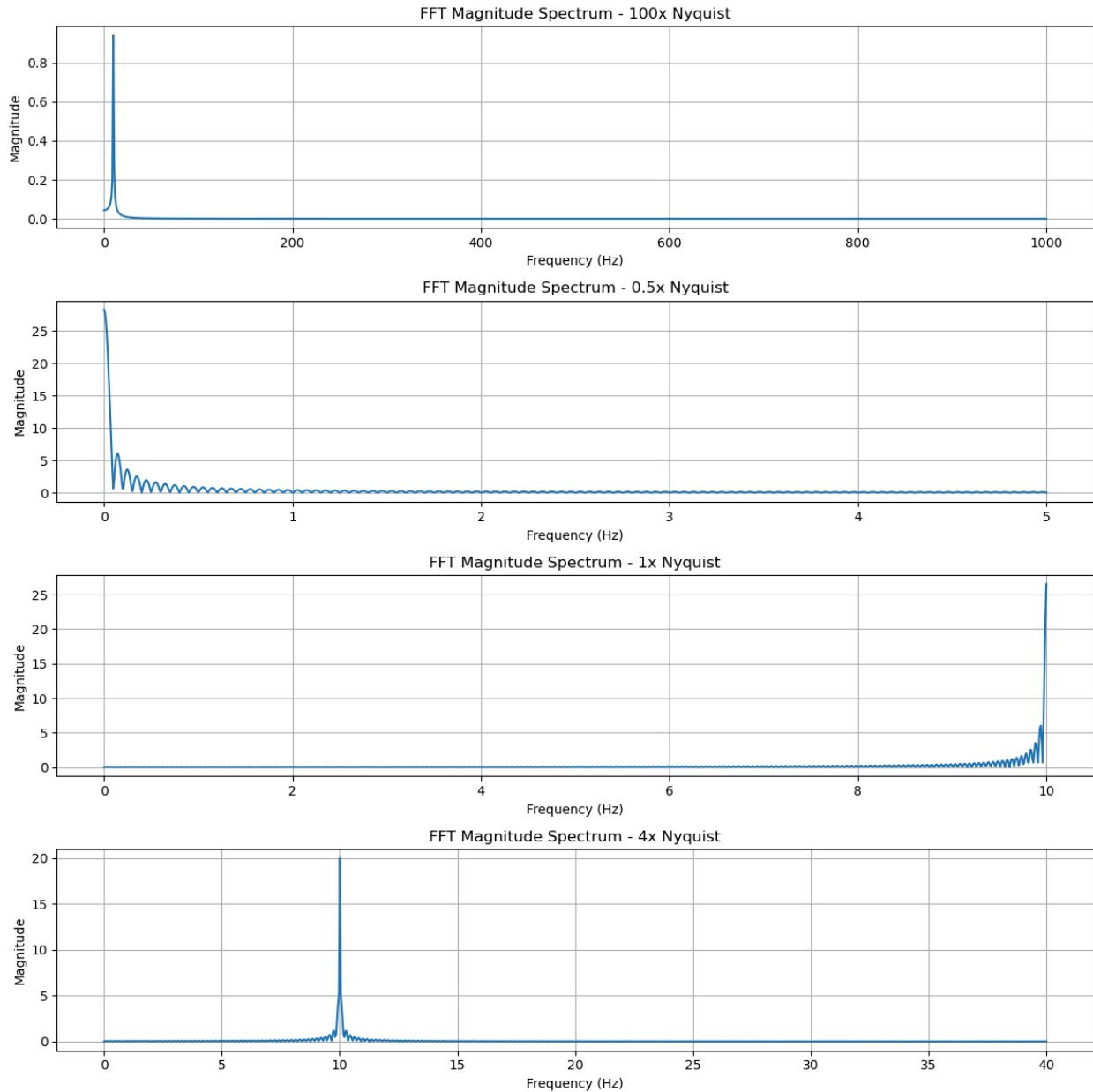
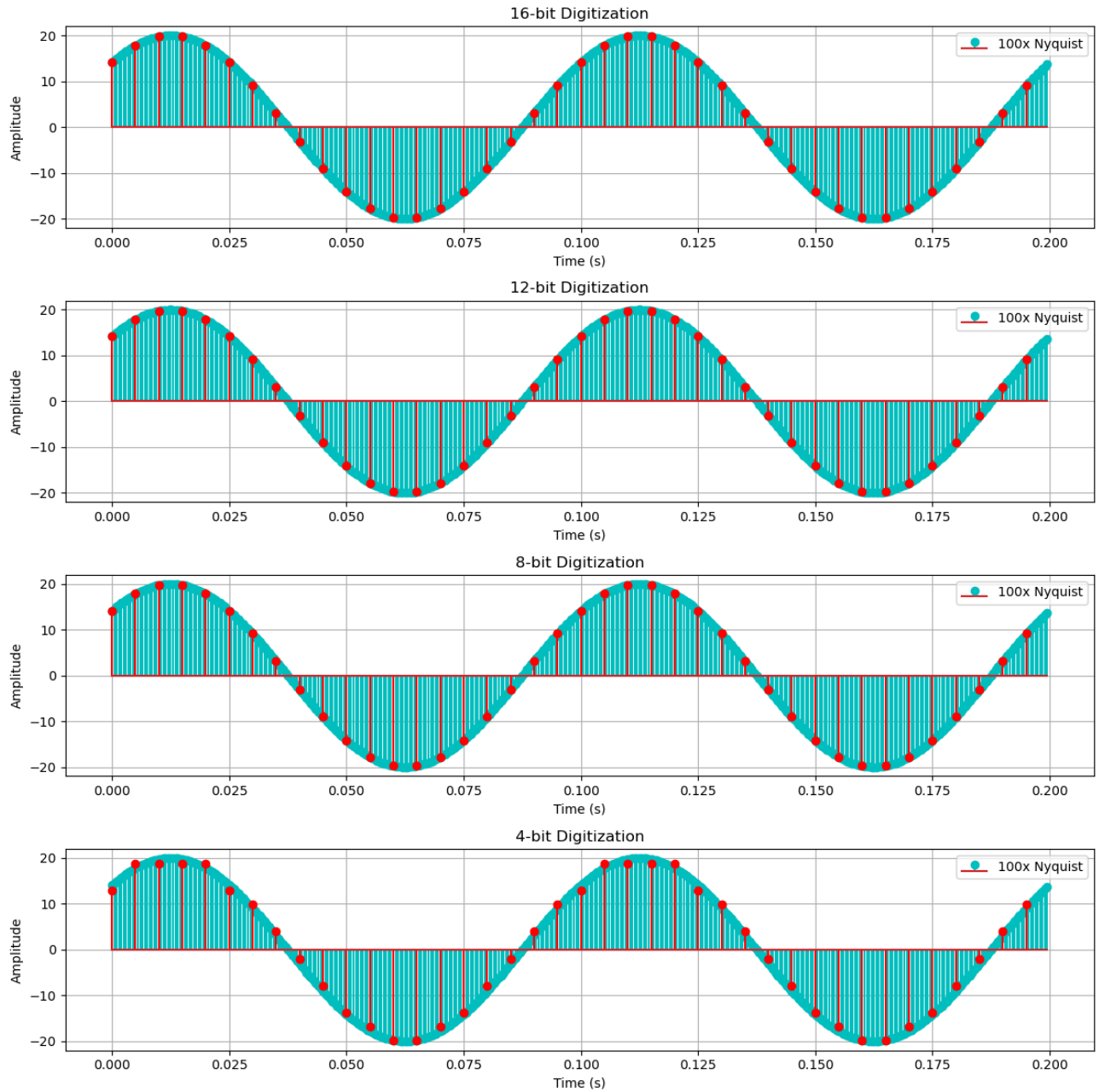


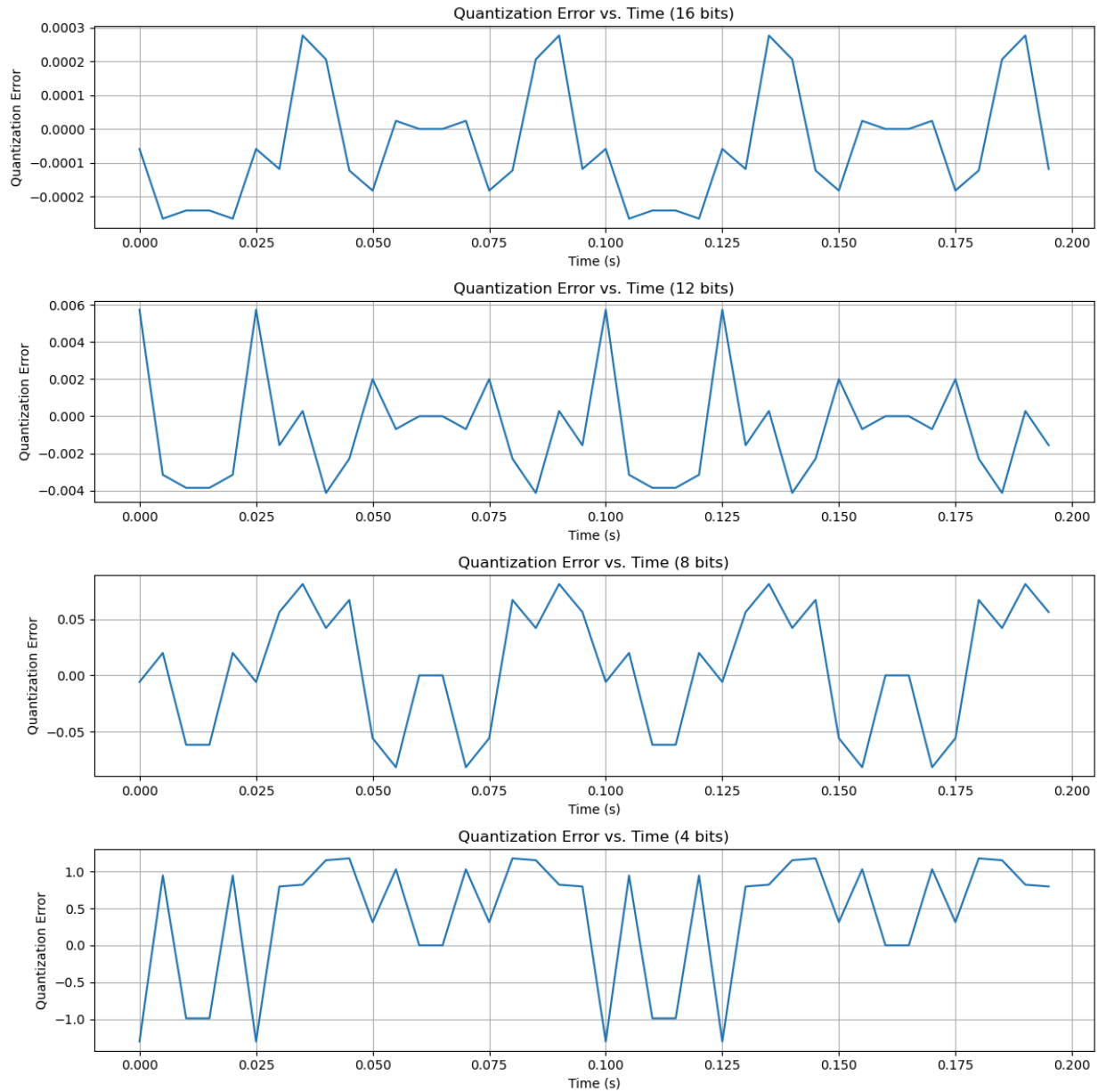
Figure 2 depicts the results of taking the fast fourier transform (FFT) of each of our signals (including 100x Nyquist for visual). Below Nyquist (0.5x Nyquist), there is no frequency being recognized by the FFT since there is not an apparent signal being translated at this sampling rate. Once we reach Nyquist and beyond, the FFT correctly displays 10 Hz as the frequency of our signal. As the sampling rate increases, as does the precision of measurement of this frequency value. 1x and 4x Nyquist have some noise present near the spike (sinc function), while the 100x appears as an almost immediate spike.

Figure 3: N-Bit Digitization of Continuous Signal at Different Bit Levels



Although it is not extremely apparent in these graphs specifically, Figure 3 shows the representation of the $10 \times$ Nyquist-sampled signal being digitized at 16-, 12-, 8- and 4-bit levels. The 16-, 12-, and 8-bit plots do not show much deviation from one another, but we will see the differences in these in the following quantization error plots. The 4 bit graph, however, does show the effect that lower bit levels have on a signal as the points tend to deviate from the original points of the oversampled graph more so.

Figure 4: Quantization Error at Different Bit Levels



As previously mentioned, the idea of higher bit levels giving a more accurate representation of our signal can be shown by Figure 4. As the number of bits levels decreases by 4 at each plot, the quantization error increases by about a factor of 10. Thus, a 16-bit representation of a signal has error between -0.0002 and 0.0003 while a 4-bit representation of signal has error ranging from -1.0 to 1.0, a whole unit measurement of amplitude off from our oversampled data.

Figure 5: Quantization Error of 12-Bit Signal with Low Pass Filtering and Downsampling

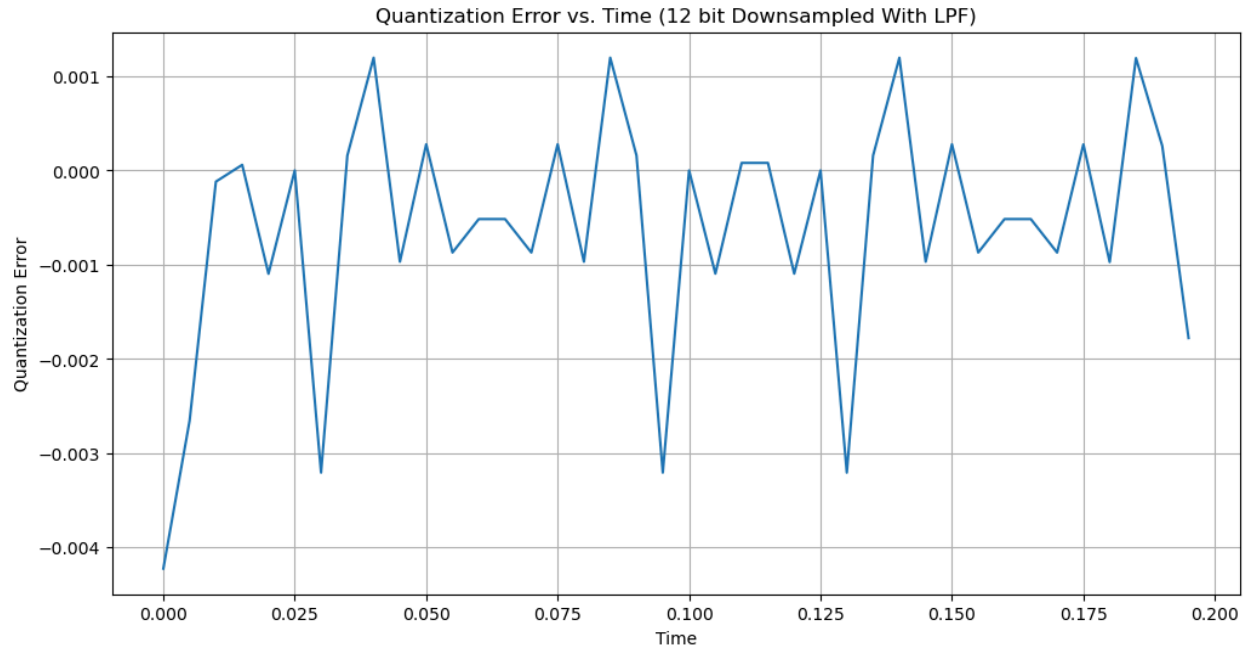


Figure 5 shows the quantization error of a 12-bit signal that has endured low pass filtering to $10 \times \text{Nyquist}$ of the sinusoid, and downsampling to $10 \times \text{Nyquist}$. Here, the error ranges from mostly -0.003 to 0.001 units of amplitude. In contrast, the error of a 12-bit signal in Figure 4 ranged from about -0.004 to 0.006. While not extreme, the difference of error in the two processes is significant enough to prefer the latter method of using a LPF and downsample approach.

Filtering reduces quantization noise by limiting high frequencies that contribute to the quantization error and produce a less-accurate signal. Additionally, oversampling allows for a larger record of measurements across a signal that produce less noise and a better representation of the original continuous signal. This, along with more bit values for smaller increments between levels, leads to less quantization noise as well.