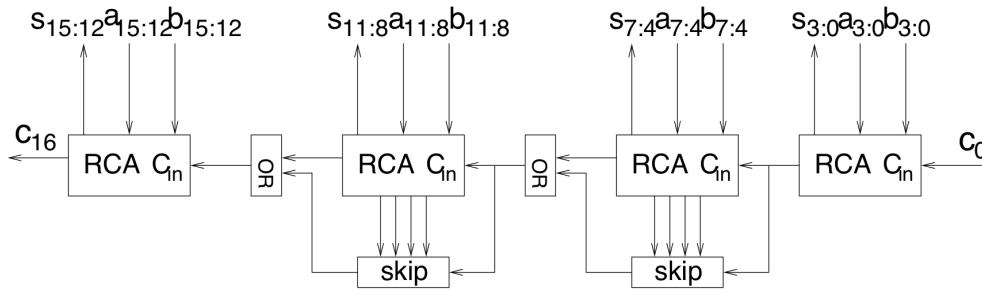


HW 4

①



* only use skip if p value is all 1's in block

$$X = 0-1111-0001-0101-010, Y = 1-0101-0111-0110-010, C_0 = 0$$

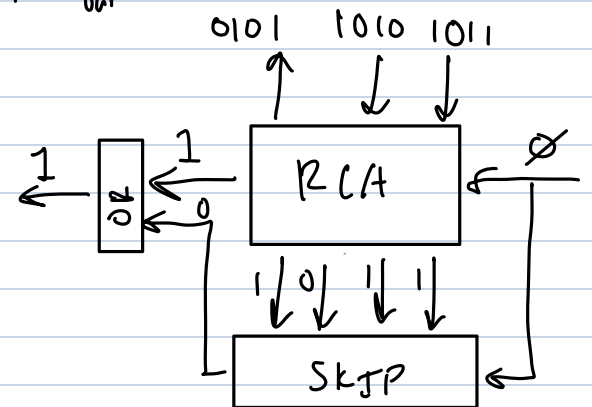
$$X_{3:0} + Y_{3:0} = \begin{array}{r} 1010 \\ + 0010 \\ \hline 1100 \end{array} \quad S_{3:0} = 1100, C_{out} = C_4 = 0$$

$$(Use RCA) X_{7:4} + Y_{7:4} = \begin{array}{r} 1010 \\ + 1011 \\ \hline 0101 \end{array} \quad S_{7:4} = 0101, C_{out} = 1$$

$$p_{7:4} = X_{7:4} + Y_{7:4} = 1011$$

$$SKIP = p_{7:4} \cdot C_4 = 1 \cdot 0 \cdot 1 \cdot 1 \cdot 0 = 0$$

$$OR = C_8 = 2 + 0 = 1$$

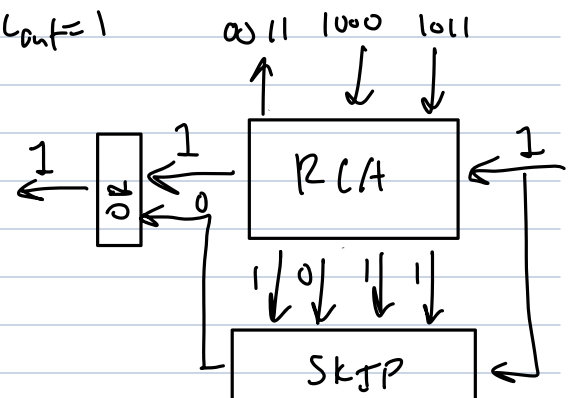


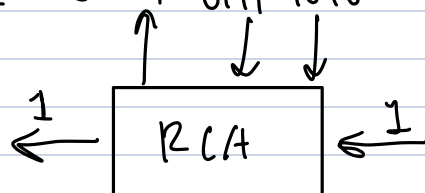
$$(use skip) X_{11:8} + Y_{11:8} = \begin{array}{r} 1000 \\ + 1011 \\ \hline 0011 \\ + 1 (C_8) \\ \hline 0100 \end{array} \quad S_{11:8} = 0011, C_{out} = 1$$

$$p_{11:8} = X_{11:8} + Y_{11:8} = 1011$$

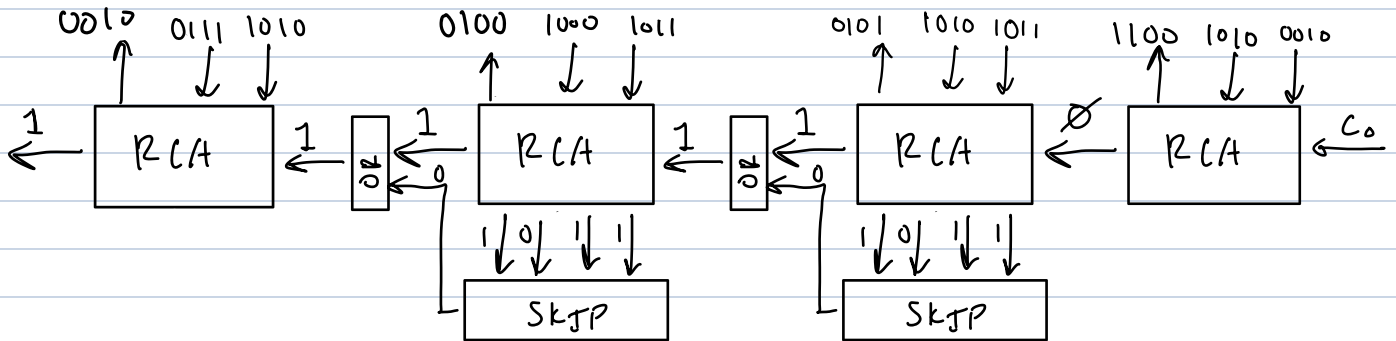
$$SKIP = p_{11:8} \cdot C_8 = 1 \cdot 0 \cdot 1 \cdot 1 \cdot 1 = 0$$

$$OR = C_{12} = 1 + 0 = 1$$



$$\begin{array}{r}
 X_{15:12} + Y_{15:12} = \begin{array}{r} 111 \\ 0111 \\ 1010 \\ \hline 0001 \\ + 1 \text{ (c12)} \\ \hline 0010 \end{array} \quad S_{15:12} = 0001, C_{out} = C_{16} = 1
 \end{array}$$


Final Result:



$$Sum = 1 - 0010 - 0100 - 0101 - 1100 = X + Y \text{ (sum matches)}$$

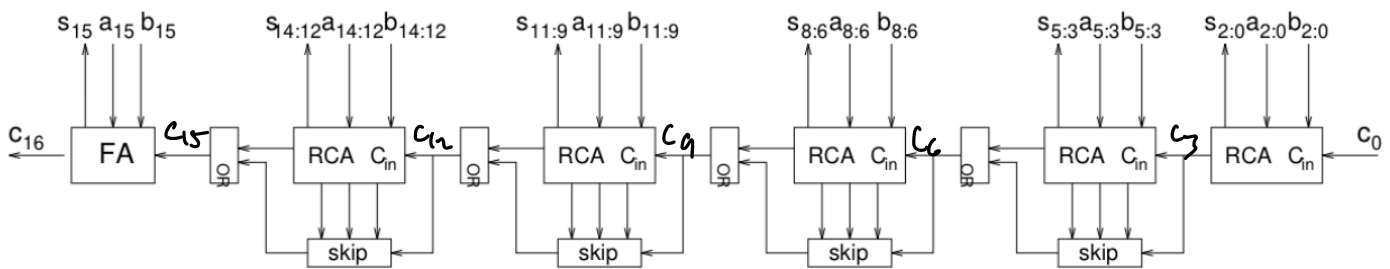
Gate Count: $9 \cdot n + 2 \cdot \left(\left\lceil \frac{n}{r} \right\rceil - 2 \right) = 9 \cdot 16 + 2 \left(\left\lceil \frac{16}{4} \right\rceil - 2 \right)$

$$= 148 \text{ gates}$$

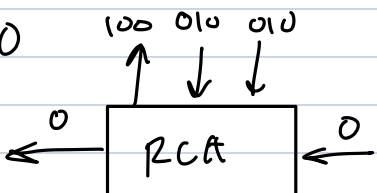
Delay Count: $4 \cdot r + 2 \left\lceil \frac{n}{r} \right\rceil = 4 \cdot 4 + 2 \left\lceil \frac{16}{4} \right\rceil$

$$= 24 \Delta$$

② C SLA optimized: $r = \sqrt{n/2} = \sqrt{16/2} = \sqrt{8} = 2.8 \approx 3$



$$X = 0 - 1111 - 0001 - 0101 - 010, Y = 1 - 0101 - 0111 - 0110 - 010, C_0 = 0$$

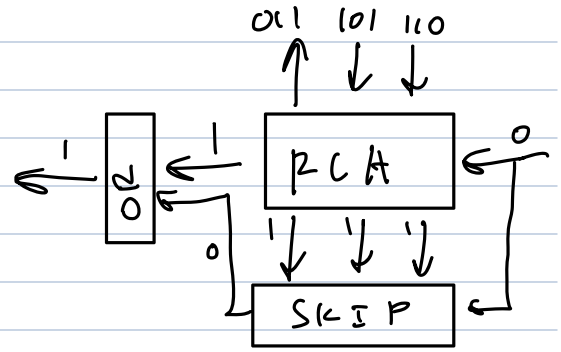
$$\begin{array}{r}
 X_{2:0} + Y_{2:0} = \begin{array}{r} 010 \\ + 010 \\ \hline 100 \end{array} \quad S_{2:0} = 100 \quad C_{out} = C_3 = 0
 \end{array}$$


$$-x_{5:3} + y_{5:3} = \begin{array}{r} 101 \\ + 110 \\ \hline 011 \end{array} \quad s_{5:3} = 011 \quad \text{Carry} = 1$$

$$\phi_{5:3} = x_{5:3} + y_{5:3} = 111$$

$$SKIP = p_{5:3} \cdot c_3 = 1 \cdot 1 \cdot 1 \cdot 0 = 0$$

$$OR = C_6 = 1 + 0 = 1$$

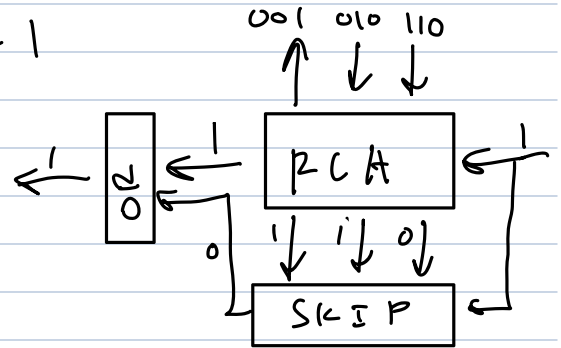


$$\begin{array}{r} \gamma \quad X_{8:6} + Y_{8:6} = \begin{array}{r} \begin{array}{r} 11 \\ 010 \\ + 110 \\ \hline 000 \\ + 000 \\ \hline 001 \end{array} \end{array} \quad S_{8:6} = 001 \quad \text{Cont} = 1 \end{array}$$

$$p_{8:6} = X_{8:6} + Y_{8:6} = 110$$

$$SKIP = p_{y:6} \cdot C_b = 1 \cdot 1 \cdot 0 \cdot 1 = 0$$

$$OR = C_q = 1 + 0 = 1$$

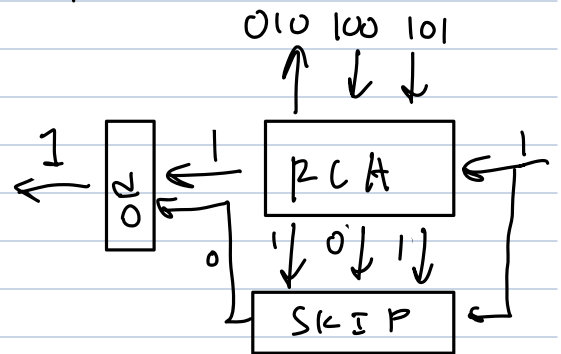


$$- X_{11:q} + Y_{11:q} = \begin{array}{r} 1 \ 0 \ 0 \\ + 1 \ 0 \ 1 \\ \hline 0 \ 0 \ 1 \\ + \quad \quad 1 \ (C_1) \\ \hline 0 \ 1 \ 0 \end{array} \quad S_{11:q} = 010, \text{ cont} = 1$$

$$\rho_{11:q} = x_{11:q} + y_{11:q} = 101$$

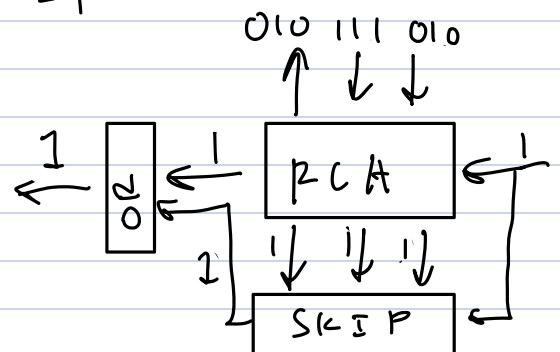
$$\text{SKIP} = P_{11;9} \cdot C_9 = 1 \cdot 0 \cdot 1 \cdot 1 = 0$$

$$OR = C_{12} = 1 + 0 = 1$$



$$-X_{14:12} + Y_{14:12} = \begin{array}{r} 111 \\ + 010 \\ \hline 001 \\ + 1 \\ \hline 010 \end{array} \quad S_{14:12} = 010, \text{Cont} = 1$$

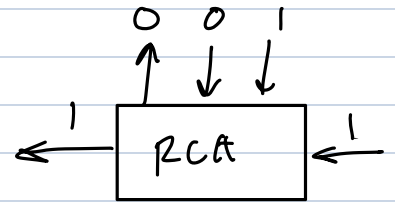
$$p_{14:12} = x_{14:12} + y_{14:12} = 111$$



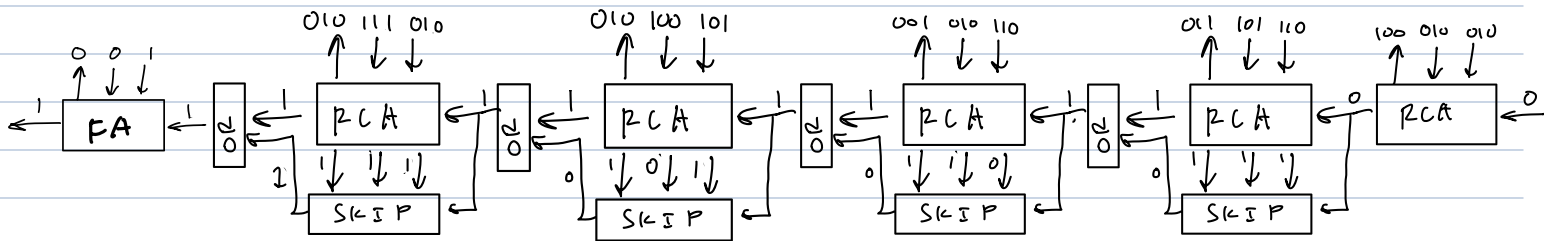
$$\text{SKIP} = p_{14:12} \cdot c_{12} = 1 \cdot 1 \cdot 1 \cdot 1 = 1$$

$$\text{OR} = c_{15} = 1 + 1 = 1$$

$$- X_{15} + Y_{15} = \begin{array}{r} 0 \\ + 1 \\ \hline 1 \\ + 1 \text{ (} c_{15} \text{)} \\ \hline 0 \end{array} \quad S_{15} = 0, \text{Carry} = c_{16} = 1$$



FINAL RESULT:



$$\text{Sum} = 1_0010_0100_0101_1100 = X + Y \text{ (sum matches)}$$

Gate Count:

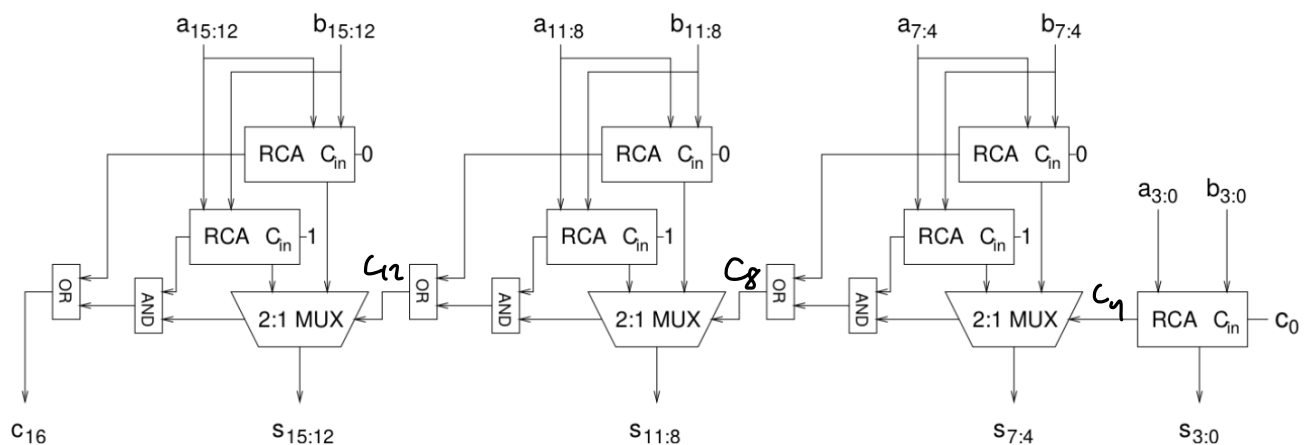
$$9 \cdot n + 2 \cdot \left(\left\lceil \frac{n}{r} \right\rceil - 2 \right) = 9 \cdot 16 + 2 \left(\left\lceil \frac{16}{3} \right\rceil - 2 \right)$$

$$= \boxed{152 \text{ gates}} \quad (2 \text{ gate for skip})$$

Delay Count: $r=3$ for first 5 blocks, then FA for remaining bit

$$\therefore \boxed{22\Delta}$$

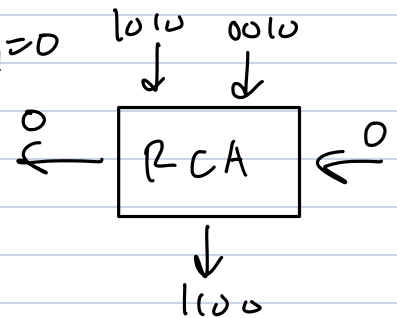
CSEA optimized: $r = \sqrt{n} = \sqrt{16} = 4$



$$X = 0-1111-0001-0101-010, Y = 1-0101-0111-0110-010, C_0 = 0$$

$$- X_{3:0} + Y_{3:0} = \begin{array}{r} 1010 \\ + 0010 \\ \hline 1100 \end{array}$$

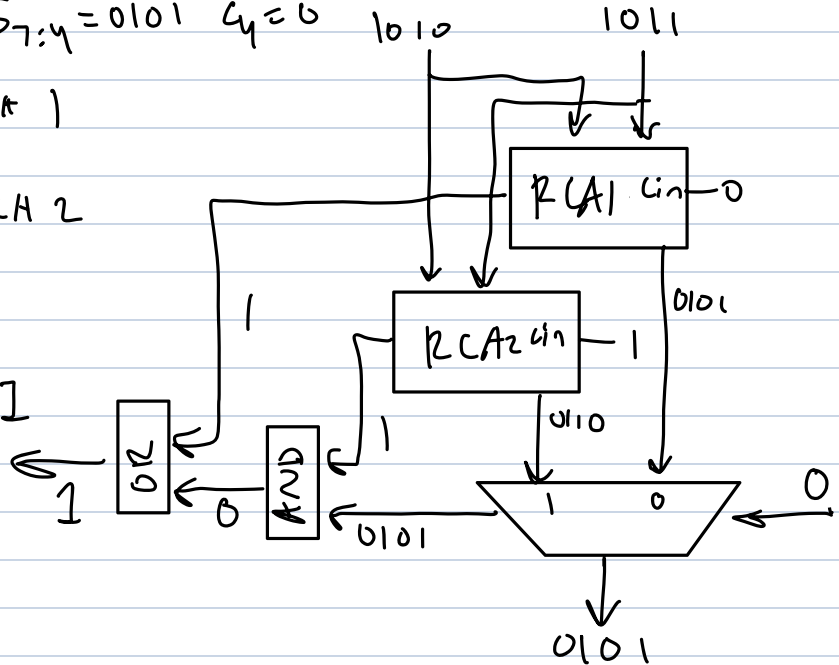
$$S_{3:0} = 1100 \quad C_{out} = C_4 = 0$$



$$- X_{7:4} + Y_{7:4} = \begin{array}{r} 1010 \\ + 1011 \\ \hline 0101 - \text{RCA 1} \\ + 1 \\ \hline 0110 - \text{RCA 2} \end{array}$$

$$\text{AND} = 0101 \cdot 0110 = 0$$

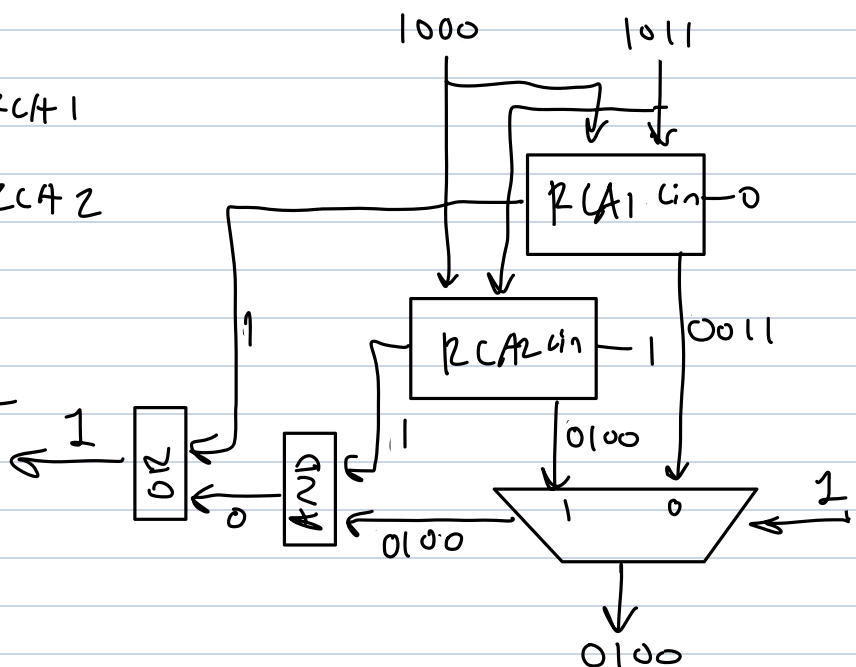
$$\text{OR} = C_8 = 1 + 0 = 1$$



$$- X_{11:8} + Y_{11:8} = \begin{array}{r} 1000 \\ + 1011 \\ \hline 0011 - \text{RCA 1} \\ + 1 \\ \hline 0100 - \text{RCA 2} \end{array}$$

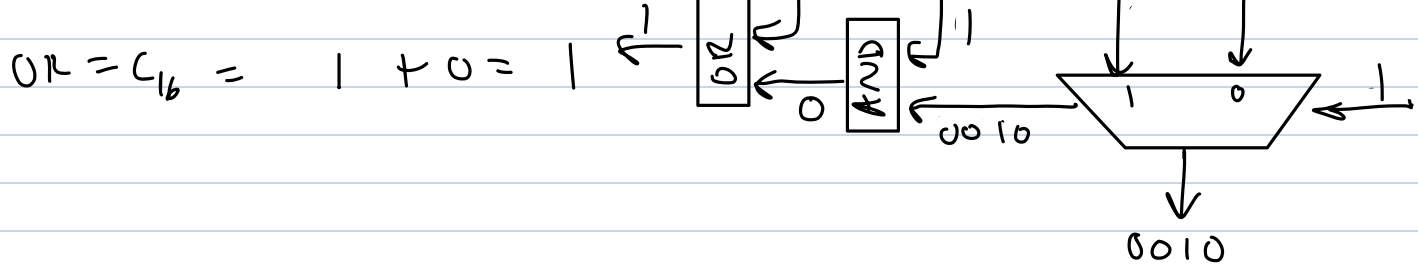
$$\text{AND} = 0 \cdot 100 \cdot 0100 = 0$$

$$\text{OR} = C_2 = 1 + 0 = 1$$

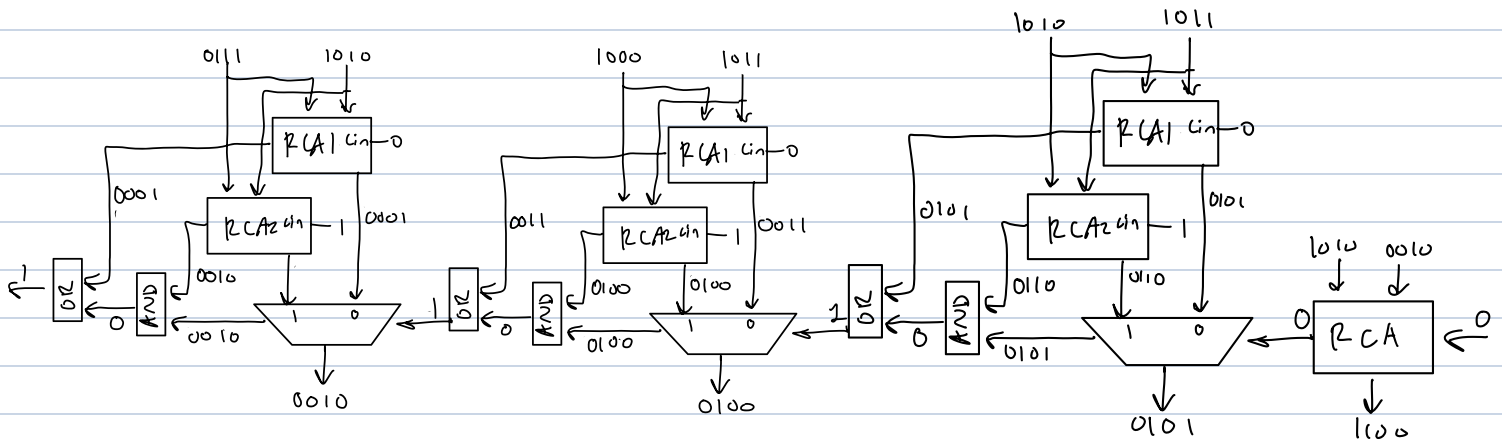


$$\begin{array}{r}
 -X_{15:12} + Y_{15:12} = \begin{array}{r} 111 \\ 0111 \\ 1010 \\ \hline 0001 - RCA1 \\ + 1 \\ \hline 0010 - RCA2 \end{array}
 \end{array}$$

$$AND = 0010 \cdot 0010 = 0$$



FINAL RESULT

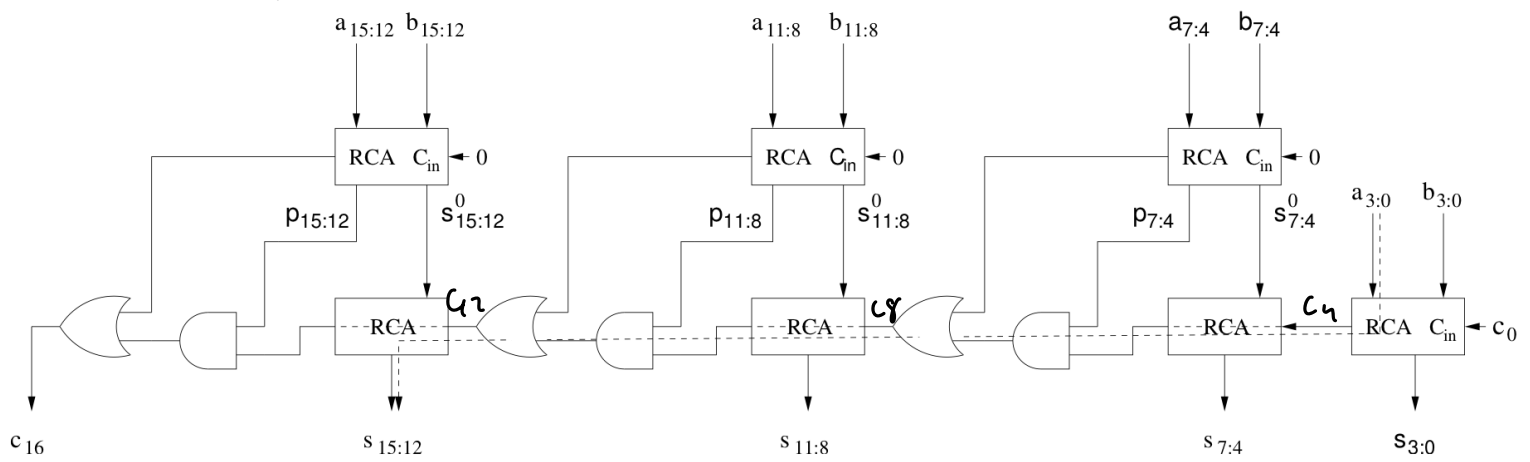


$$Sum = 1 - 0010 - 0100 - 0101 - 1100 = X + Y \text{ (sum matches)}$$

$$\begin{aligned}
 \text{Gate Count} &= 2^1 \cdot n - 12 \cdot r + 3 \cdot \left\lceil \frac{n}{r} \right\rceil - 3 \quad (n=16, r=4) \\
 &= 336 - 48 + 12 - 3 = \boxed{297 \text{ gates}}
 \end{aligned}$$

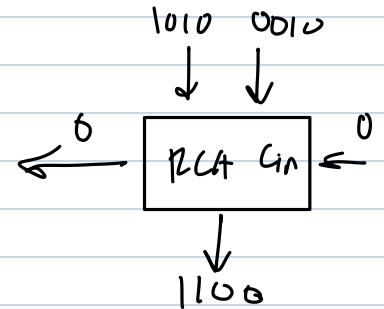
$$\begin{aligned}
 \text{Delay Count} &= 2r + 2 \cdot \frac{n}{r} + 2 \\
 &= 8 + 8 + 2 = \boxed{18 \Delta}
 \end{aligned}$$

CINA optimized = $r = \sqrt{n} = \sqrt{16} = 4$



$$X = 0-1111-0001-0101-010, Y = 1-0101-0111-0110-010, C_0 = 0$$

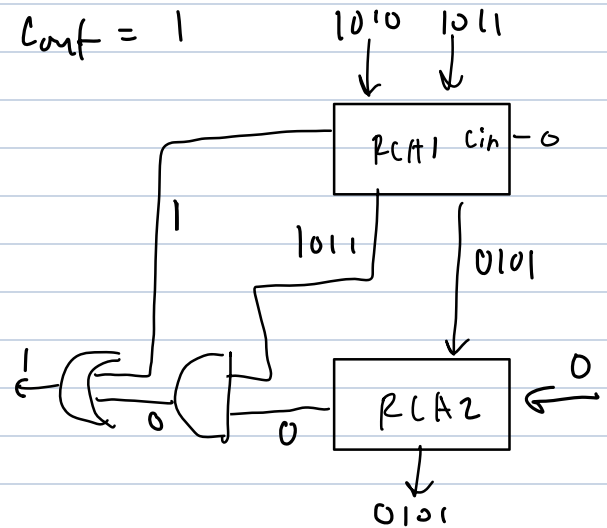
$$- X_{3:0} + Y_{3:0} = \begin{array}{r} 1010 \\ + 0010 \\ \hline 1100 \end{array} \quad S_{3:0} = 1100, C_4 = 0$$



$$- X_{7:4} + Y_{7:4} = \begin{array}{r} 1010 \\ + 1011 \\ \hline 0101 \end{array} \quad S_{7:4} = 0101, C_{out} = 1$$

$$P_{7:4} = X_{7:4} + Y_{7:4} = 1011$$

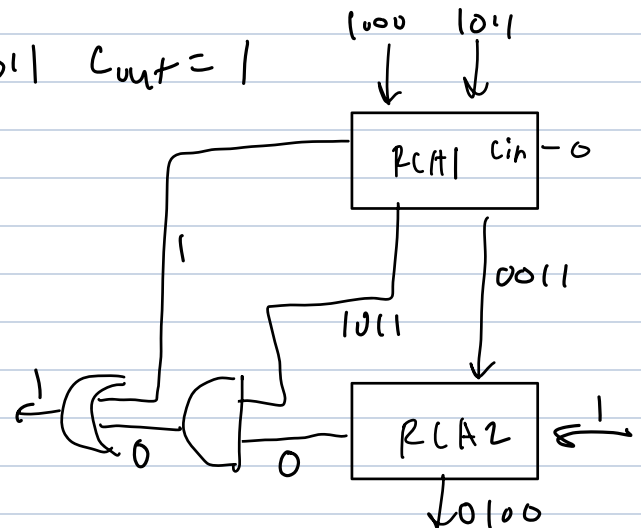
$$C_8 = 1$$



$$- X_{11:8} + Y_{11:8} = \begin{array}{r} 1000 \\ + 1011 \\ \hline 0011 \end{array} \quad S_{11:8} = 0011, C_{out} = 1$$

$$P_{11:8} = X_{11:8} + Y_{11:8} = 1011$$

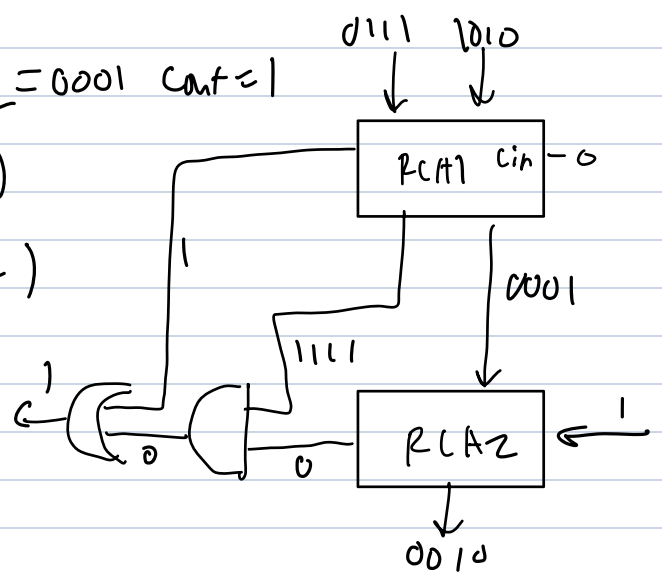
$$C_{12} = 1$$



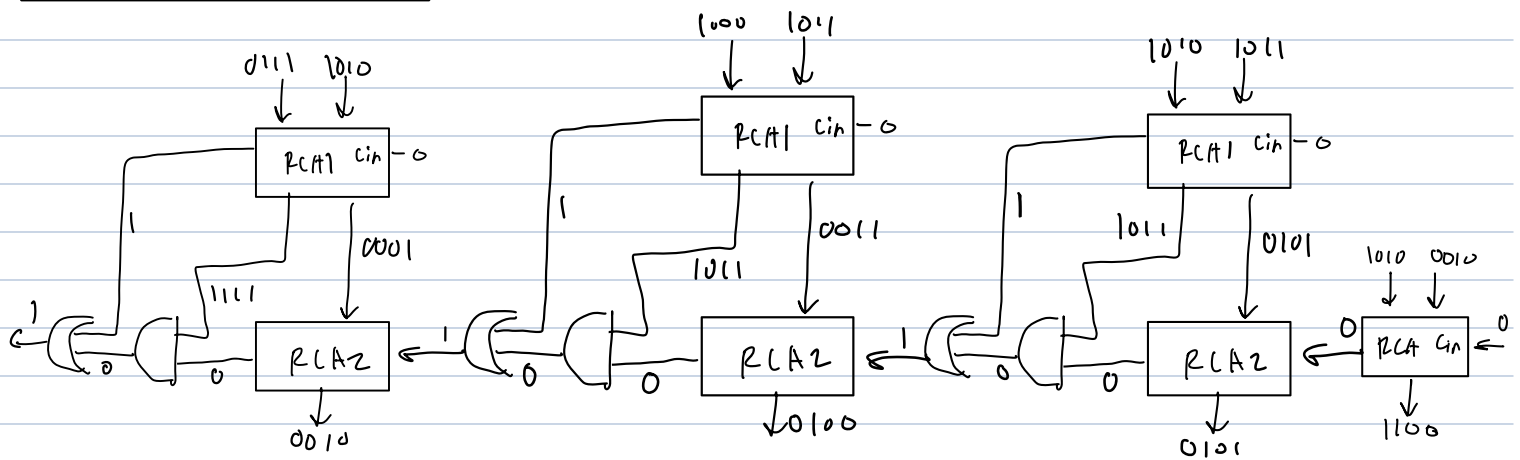
$$-X_{15:12} + Y_{15:12} = \begin{array}{r} 0111 \\ + 1010 \\ \hline 0001 \text{ (PLA1)} \\ + 0001 \\ \hline 0010 \text{ (PLA2)} \end{array} \quad S_{15:12} = 0001 \text{ Cont} = 1$$

$$D_{15:12} = X_{15:12} + Y_{15:12} = 1111$$

$$C_{16} = 1$$



FINAL RESULT



$$\text{Sum} = 1 - 0010 - 0100 - 0101 - 1100 = X + Y \quad (\text{sum matches})$$

$$\text{Gate Count} = 13n + 2 \left\lceil \frac{n}{r} \right\rceil - 2 - 4r$$

$$= 208 + 8 - 2 - 16 = \boxed{198 \text{ gates}}$$

$$\text{Delay Count} = 3 \cdot r + 2 \left\lceil \frac{n}{r} \right\rceil + 1$$

$$= 12 + 8 + 1 = \boxed{21 \Delta}$$

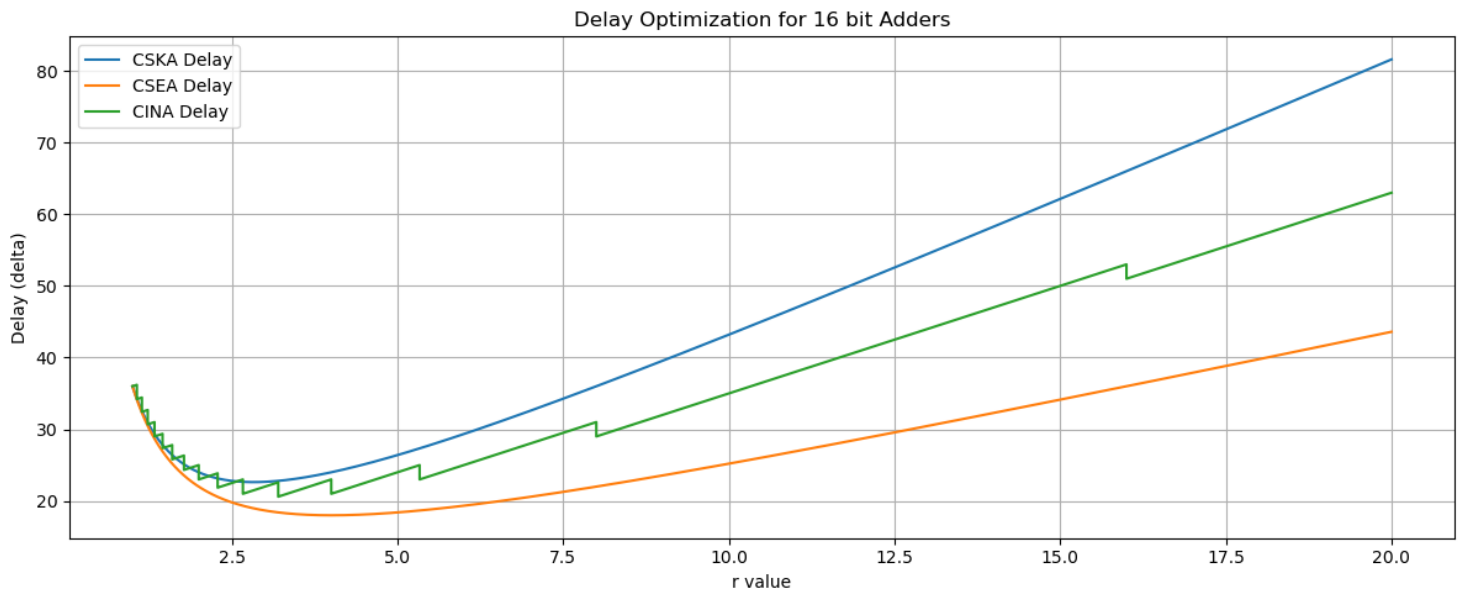
Comparison

	Gates	Delay
CSKA:	152	22 Δ
CSEA:	297	18 Δ
CINA:	198	21 Δ

- Fastest: CSEA, slowest: CSKA (not by much)
- Largest: CSEA (significantly), smallest: CSKA

- ③ * Included in the CSAM16.zip file are the following:
- CSAM.do
 - CSAM16.out
 - CSAM16.sv
 - t6-CSAM.sv

EXTRA CREDIT: Plotting analysis from Question 2 in python
- python file included in submission



X-value at the minimum y-value(CSKA): 2.83
X-value at the minimum y-value(CSEA): 4.0
X-value at the minimum y-value(CINA): 3.2

} r-values used for blocks in question ②