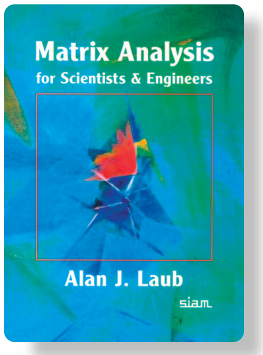
Mathematical Systems Theory I: Modelling, State Space Analysis, Stability and Robustness [Book Review]

Article *in* IEEE Control Systems Magazine · May 2006

DOI: 10.1109/MCS.2006.1615277 · Source: IEEE Xplore



SIAM Publications,
Philadelphia, 2005,
US$25.20
ISBN 0-89871-576-8

who are not mathematics majors cannot relate this approach to their engineering or physics courses, which may involve linear systems and eigenvalue and singular-value problems.

2) The abstract-plus-coordinate-based approach, which includes vector spaces, homomorphisms, and eigenvalues of endomorphisms, but with a parallel treatment of coordinate vectors and matrices as representations of linear mappings. Combined with an introduction to software such as MATLAB, such a course (which consists of approximately 64 lectures of 45 minutes each in my institution) presents the field in depth while relating the contents to real-world problems. I teach the course in this way for mathematics students and I believe in this approach.

3) The reduced abstract-plus-coordinate-based approach. Forced by the unfortunate development that many engineering schools reduce the mathematics contents of their curricula to the absolute minimum, the number of hours available for a basic linear algebra course has become limited. Currently, in my institution, this course occupies approximately 32 lectures of 45 minutes each. This allotment is not enough, and I strongly feel that this situation is a disaster for the development of science and technology. In teaching such a course, the instructor is forced to strip away the abstract theory as much as possible and teach the basics that are most needed by future engineers and scientists.

WHERE DOES THIS BOOK FIT IN?

The textbook *Matrix Analysis for Scientists and Engineers* provides an exact fit for the third category. The book can be used for such a reduced course while providing the students with the content that they need.

The 13 chapters of the book cover vector spaces, linear transformations, pseudoinverses, singular-value decomposition, linear systems, inner products and norms, least-squares problems, eigenvalue and eigenvectors, canonical forms, linear differential and difference equations, generalized eigenvalue problems, and Kronecker products. For engineers and scientists, these topics are the most important topics from linear algebra.

The author does a great job in concisely presenting the results. The explanations are accompanied by examples that are easy to understand as well as many exercises that are good for reinforcing understanding. With these features, the book is ideal for a reduced course.

A single point of criticism that I have concerning this book relates to my own experience in teaching this course. I would have slightly reordered the presentation and moved the singular-value decomposition and the pseudoinverse to after the chapter on canonical forms.

CONCLUSIONS

For engineering and science students, the modern curriculum limits the amount of time devoted to a basic course in linear algebra or matrix theory. For such a course, *Matrix Analysis for Scientists and Engineers* is an ideal textbook due to its topical coverage, clarity, examples, and exercises.

—Volker Mehrmann

REVIEWER INFORMATION

Volker Mehrmann is a professor at TU Berlin specializing in numerical mathematics. His research interests are in the areas of applied and numerical linear algebra, control theory, and the theory and numerical solution of differential-algebraic equations. He is editor-in-chief of *Linear Algebra and Its Applications* and author of several books, including a basic introductory textbook on numerical analysis for scientists and engineers (in German).

Mathematical Systems Theory I: Modelling, State Space Analysis, Stability and Robustness

By DIEDERICH HINRICHSEN and ANTHONY J. PRITCHARD

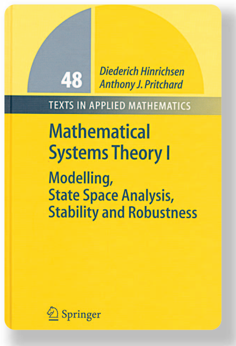
SYSTEMS THEORY

The basic input-state-output concept of a system introduced by Kalman [1] has become a cornerstone of control theory. In the first chapter of [1], we are told that

> The terms "systems," "system concepts," "systems approach," and "systems science" are used so widely and so broadly today that they tend to connote fuzzy thinking. For us, however, a *system*, or more correctly a *dynamical system*, is a precise mathematical object; the study of systems theory is then largely, although not entirely, a branch of mathematics.

Within the control community, systems theory is typically thought of as the foundation for control analysis. Indeed, it is common practice to intertwine systems and control in a work on systems theory [2], [3]. An alternative approach is embodied in the general systems theory of von Bertalanffy and others, who define the field in much more general terms [4].

The approach taken by Hinrichsen and Pritchard in *Mathematical Systems Theory I* is, as decreed above, that systems theory is a branch of mathematics. Consequently, their text is a mathematics book and a fairly heady one at that. This statement



Springer, 2005, US$79.95
ISBN 3-540-44125-5

should not be taken as suggesting that the authors disregard the importance of the interconnections between systems and control theory. On the contrary, the second volume in this set will be devoted to issues of control. The presentation in the current volume, while presumably laying the groundwork for the next volume, can be taken on its own as a treatment of systems theory. In this effort the authors succeed admirably, although discussions of control topics are occasionally held at bay somewhat artificially, as when observability is introduced as an unmotivated algebraic condition.

This book provides a thorough description of the theory of linear systems. It may come as a surprise that a text on the graduate level would focus almost exclusively on linear systems. The common perception is that students would already have covered the subject as presented in undergraduate texts such as [5]. The treatment in *Mathematical Systems Theory I* goes far deeper than the typical understanding.

As a consequence, the required background for the text is not a mastery of linear systems theory but rather a grounding in the mathematical tools required to digest mathematically sophisticated results, such as a solid foundation in linear algebra, analysis, and differential equations as well as some knowledge of functional analysis. The reader should expect to see the use of tools such as the Arzela-Ascoli theorem and isometric embeddings of Banach spaces. The approach is unabashedly mathematical, with most discussions proceeding from the general treatment to specific cases. Although examples are presented throughout, it is clear that the goal of the text is to address the mathematical results, not to develop their engineering applications.

CONTENTS

The first chapter introduces various systems and models. Population dynamics and economics are included alongside standard examples from mechanics, electronics, and digital systems. The discussion of digital switching networks is a welcome inclusion that provides a solid context for the discrete-time results that follow. The examples from mechanics and circuit theory are accompanied by a general discussion of Lagrangian and Hamiltonian approaches as well as a derivation of Maxwell's equations. The presentation of the material in this introductory chapter indicates the level of sophistication expected from the reader. These topics are followed by an application that is rarely treated in a general text, namely, heat transfer. This material helps motivate the subsequent discussions on infinite-dimensional systems.

Chapter 2, on state-space theory, begins with a well-crafted intuitive introduction to dynamical systems. This discussion is followed by a careful definition of a system in the spirit of [1]. Several examples are chosen to illustrate the concept, with automata and delay systems included along with the standard pendulum and satellite. The discussion of linear systems is far deeper than the usual treatment. For example, the careful discussion of eigenmotions provides a rigorous definition of modes, a term that is sometimes used in a vague sense. Linear infinite-dimensional systems are introduced as well, and the authors are careful to point out the differences from the finite-dimensional case, including the resulting complications. Transfer matrices are discussed and the irrational transfer functions derived from infinite-dimensional systems are represented by an example of a system with time delay. Although the frequency response is defined, the classical single-input, single-output tools for their study are mentioned only in passing. A section is devoted to sampling and approximation, leading to a proof of the sampling theorem. This section is followed by a discussion of approximation of continuous systems by discrete systems and methods for numerical simulation, including an interesting treatment of the potential pitfalls of applying multistep methods to controlled systems.

Chapter 3, which covers stability theory, begins with an historical overview of the development of the mathematical notion of stability. Stability is first defined abstractly, specifically in terms of nonlinear flows on metric spaces. Lyapunov's direct method, which is developed in the same general setting (again with a welcome historical introduction), is illustrated by application to a delay equation. The results are specialized to time-varying, finite-dimensional systems and then to time-invariant systems. This coverage is quite complete, including a discussion of instability results as well as computation of regions of attraction. The presentation then turns to linear systems with a discussion of Lyapunov and Bohl exponents, the latter being the analog of Lyapunov exponents for addressing uniform behavior of time-varying systems. Finally, stability criteria for polynomials are presented in a compendium of algebraic and analytical results that are rarely covered in a course in control theory. The discussions provide deep insight into stability algorithms that are commonly used. The two applications worked in this section illustrate the most frequently used tools in this area, namely, the Routh criterion and the Jury test. Overall, this chapter provides an elegant display of mathematics.

Chapter 4 addresses the behavior of indicators of stability under perturbations. Beginning with a discussion of the roots of polynomials, the authors then turn to eigenvalues of matrices. The singular value decomposition is introduced, and the behavior of singular values under perturbations is addressed. This discussion leads naturally to a corresponding treatment of structured singular values in the context of μ-analysis. The chapter ends with a detailed treatment of the computation of eigenvalues and singular values. While the discussion of condition number follows naturally from the rest of the material in the chapter, the lengthy treatment of computational algorithms (complete with pseudocode) seems a bit out of place.

The last chapter continues the discussion from Chapter 4 on the consequences of perturbations. Spectral value sets for families of matrices are introduced, and their computation is discussed. Stability radii (generalizations of gain margins) are then addressed in a treatment that relies heavily on the authors' own research. The discussion next turns to a subject often left untreated, namely, stability bounds on transient behavior and the corresponding robustness analysis.

CONCLUSIONS

In the preface to [1], the authors state that they have "tried to convey the fundamental notion that systems theory is not simply a branch of applied analysis, but provides a source of problems and intuition for a *rich interplay between algebra and analysis*." The book under review is a celebration of that interplay.

I heartily recommend this book as a remarkably complete reference for researchers doing theoretical work involving linear systems. The book is a unique compilation of the mathematics underlying the field, and I look forward to seeing the author's treatment of synthesis problems in the second volume.

More care must be taken in recommending the use of this text for students. There is no doubt that parts of the text could be successfully used for a graduate course in a mathematics program; in fact, exercises are provided throughout to facilitate that use. The authors indicate that they have also had success using the first few chapters in undergraduate classes. However, it is my experience that undergraduates typically balk at the level of rigor required by Sontag's text [6]. *Mathematical Systems Theory I*, which is written in a similar vein, is considerably more demanding and would undoubtedly invoke a similar reaction.

The book is a welcome addition to the library of work on systems theory and will no doubt serve as a valuable reference in many studies. Most readers will come to the book with control-theoretic applications in mind, and while the connections to those applications are often left implicit, the text skillfully highlights the rigorous basis on which control engineering rests. Even von Bertalanffy, a biologist who applied his general system theory to psychology and the social sciences, is quick to remind us that "systems theory . . . is preeminently a mathematical field" [4]. It is those mathematical foundations that have been carefully documented by the authors in *Mathematical Systems Theory I*.

—Brian Ingalls

REVIEWER INFORMATION

Brian Ingalls is an assistant professor in the Department of Applied Mathematics at the University of Waterloo. He received his Ph.D. in mathematics from Rutgers University in 2001 and held a postdoctoral fellowship at the California Institute of Technology in 2001–2002. His main research interests lie in the application of systems and control theory to biochemical regulation. He is an associate editor of *IEE Proceedings Systems Biology* and serves on the editorial board of the *International Journal of Applied Mathematical Analysis and Applications*.

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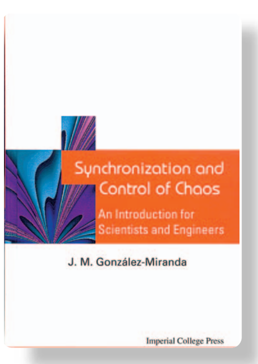

Imperial College Press, London, 2004, 212 pp + xi, US$52.00
ISBN 1-86094-488-4

Synchronization and Control of Chaos: An Introduction for Scientists and Engineers

By J.M. GONZÁLEZ-MIRANDA

CHAOS CONTROL

Chaos control refers to manipulating the dynamical behavior of a chaotic system, in which the goal is to suppress chaos when it is harmful or to enhance or create chaos when it is beneficial. Chaos synchronization, on the other hand, refers to the task of enabling or disabling the dynamical synchrony of several connected chaotic systems by means of control techniques or through specially designed coupling configurations. These topics are addressed in the book *Synchronization and Control of Chaos: An Introduction for Scientists and Engineers* by J.M. González-Miranda.

Numerous research monographs, textbooks, and edited volumes are devoted to the subjects of synchronization and control of chaos. Examples include [1]–[10], although none of these works are included in the 217 citations of the book. Since the preface states that "The purpose of this book is to provide a systematic and broad account of that research for a wide audience," I was interested in seeing how these topics are presented in a book intended as an introduction for scientists and engineers.

CONTENTS

The book has eight chapters, none of which include any exercises.

Chapter 1 begins with an introduction to standard examples of free and damped second-order linear mechanical harmonic oscillators, evolving to one external-force driven and