Modeling Charged and Minty Particles

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Group 1 Lab 2B

Introduction

In this Experiment we will investigate charged and minty particles and test hypothesized models for their behavior. Before we proposed our model we observed the charges and the minty particles to gain a background to our investigation. We learned that Charged particles come in two forms which we will call positive and negative. Positive and negative charges attracted one another whereas similar charges repelled. It also seemed as if the force between charged particles increased with proximity and decreased with distance. Separately, Minty particles only came in one variety and had a more complex force-distance relationship. We noticed that the minty particle would oscillate between attraction and repulsion around a common 'natural length' where forces would diminish to zero. We also noticed that when minty particles came close to each other they had a far stronger force than when they were farther away.

Based on these background observations we hypothesize two models of force due to charged and minty particles. For charged particles our model predicts an inverse square relation with a constant multiple:

$$F = \frac{kQ_1Q_2}{r^2}$$

We proposed this inverse square relation because we felt that the attraction between charged particles was similar in behavior to that of gravity. For Minty particles we suspected a natural logarithm relation with a natural length and 'b' as a constant multiple:

$$F = b(ln(x) - ln(a))$$

Our natural logarithm model gives the desired zero force at the natural length, the directional change of force at the natural length, and the decreasing magnitude of force due to distance.

Methods

In our experiment, the independent variable is the distance between the two particles being tested. The dependent variable is the force being felt on one of the particles due to the other. The controlled variable is the pair of particles being tested--whether each one was positive, negative, or minty.

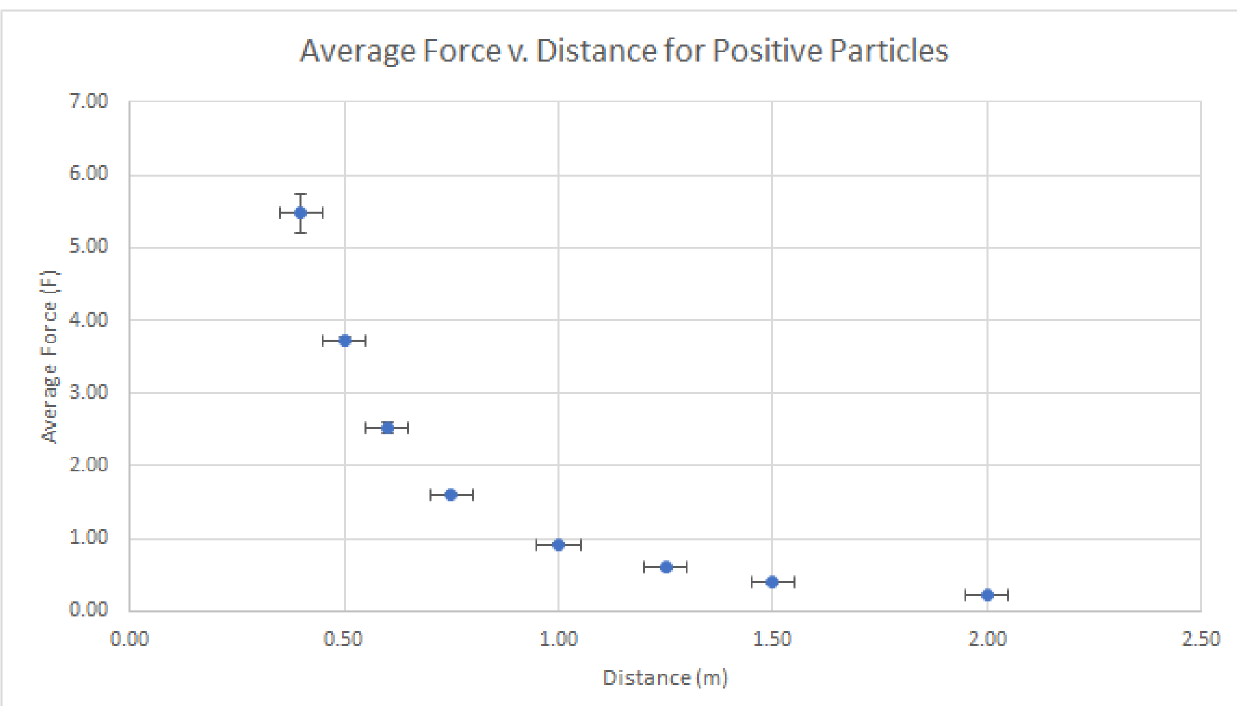
To determine the force between a pair of particles as a function of distance, we set up the following experiment in the VR space:

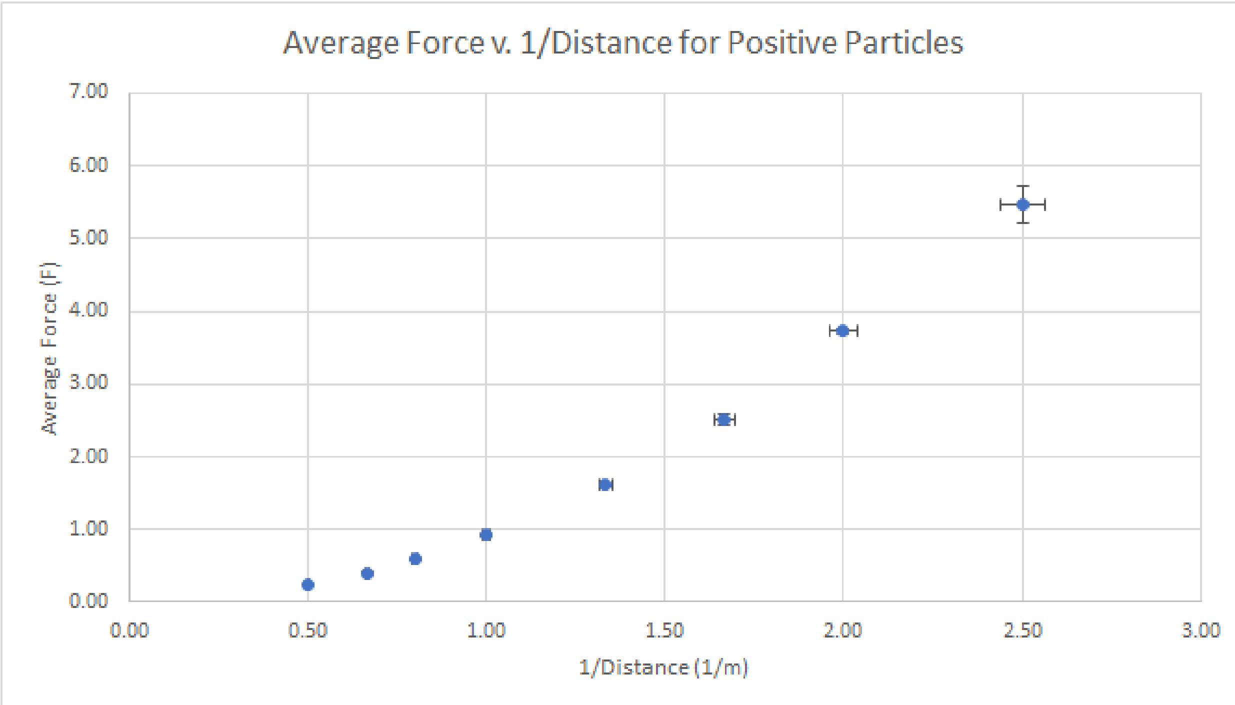
1. Pull out a virtual force sensor and attach one of the particles to it. Fix the pivot of the force sensor in place but let the particle on the end swing freely.
2. Place one end of the virtual measuring stick on the pivot of the force sensor and extend the other end to a certain distance. (The fact that the measuring stick is placed on the pivot instead of the particle will be accounted for later.)

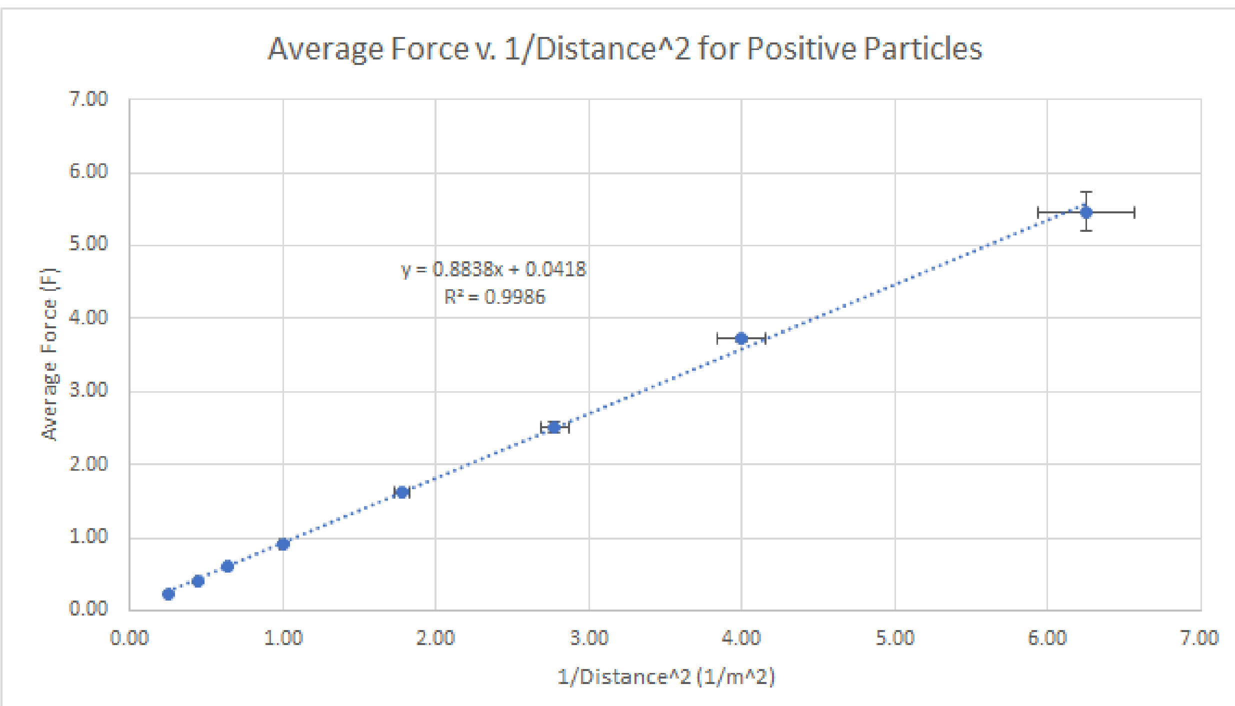
3. Move the free particle so that its center coincides with the other end of the meterstick, peering inside the free particle to ensure its internal crosshairs are centered on the meterstick's end.
4. Fix the free particle in place so the force exerted upon it by the particle on the sensor does not accelerate it.
5. After the particle on the sensor has achieved equilibrium, record the distance between the center of the particle in space and the pivot on the force sensor. Add 10 cm if the two particles are repelled, or subtract 10 cm if they attract, to account for the length of the arm of the force sensor and record this distance. This recorded value is the distance between the centers of the two particles.
6. Record the force displayed on the force sensor. If the particles are attracted instead of repelled, negate the force before recording it.
7. Conduct 3 trials of steps 2-6 for each distance. Record the average of the measured forces across these trials.
8. Repeat steps 1-7 for various distances.

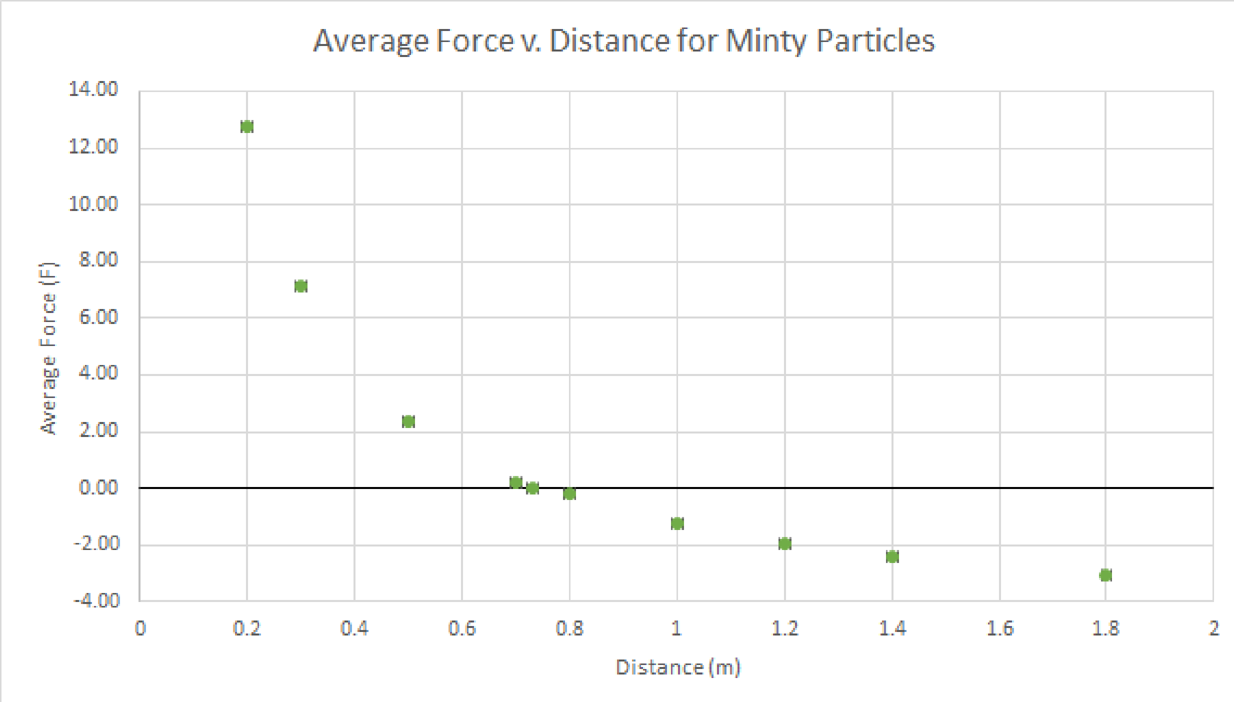
We conducted our initial experiment using a positive and another positive particle and our final experiment using a minty and another minty particle. Based on our hypothesis for our initial experiment, we expected the force between the positive particles to be positive always and to be proportional to the inverse square of the distance between them. Based on our hypothesis for our final experiment, we expected the force between the minty particles to be positive when less than some critical distance away and negative when greater than that distance away, with the force being proportional to the natural logarithm of the distance.

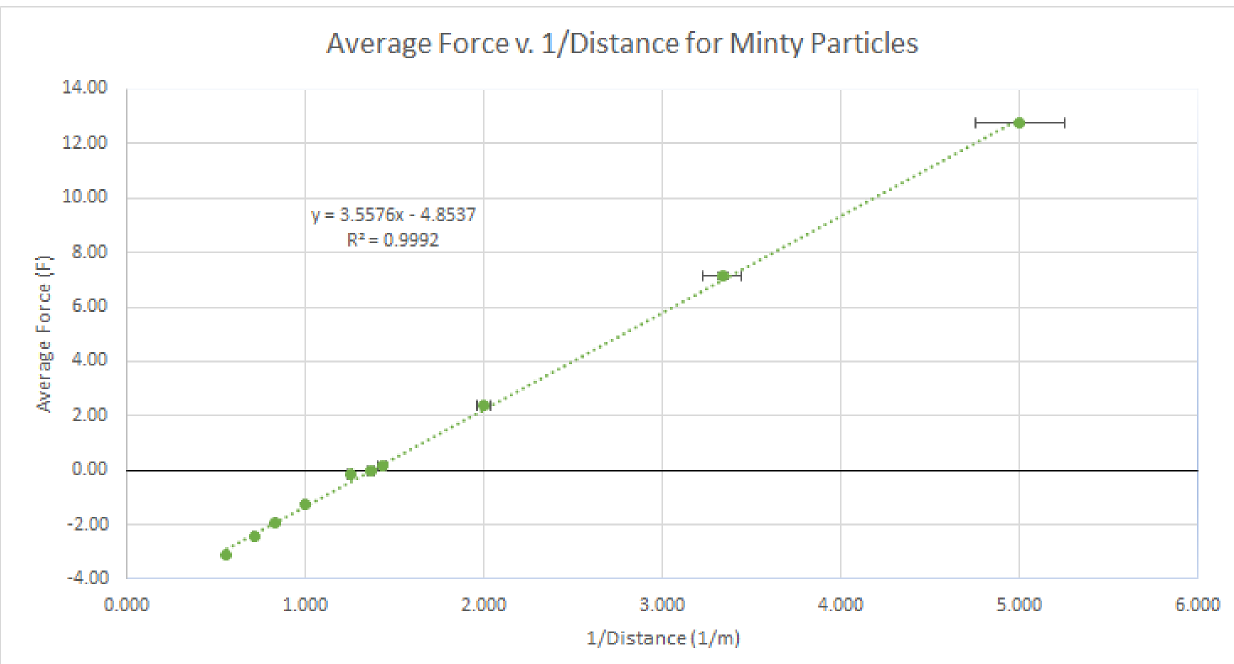
Graphs

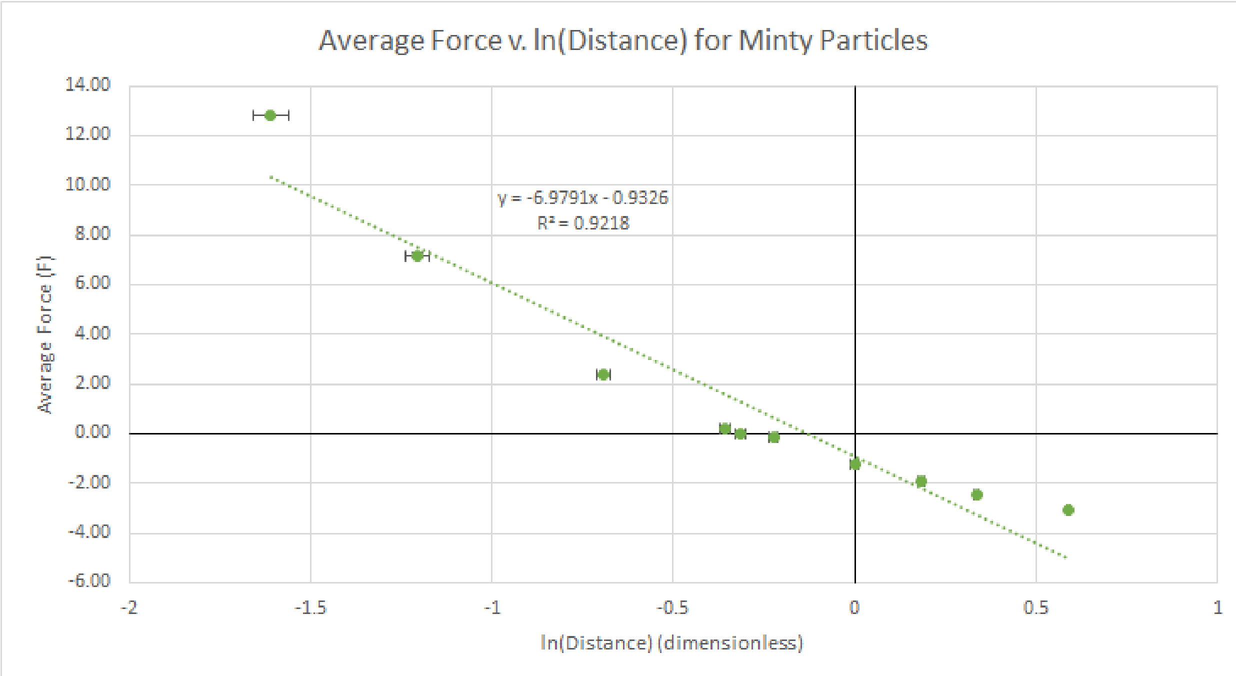



Average Force v. 1/Distance for Positive Particles
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
Average Force (F)
0.00
0.50
1.00
1.50
2.00
2.50
3.00
1/Distance (1/m)


Average Force v. 1/Distance^2 for Positive Particles
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
Average Force (F)
y = 0.8838x + 0.0418
R² = 0.9986
0.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
1/Distance^2 (1/m^2)


Average Force v. Distance for Minty Particles
Average Force (F)
Distance (m)
14.00
12.00
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Average Force v. 1/Distance for Minty Particles
y = 3.5576x - 4.8537
R² = 0.9992
Average Force (F)
1/Distance (1/m)
14.00
12.00
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
0.000
1.000
2.000
3.000
4.000
5.000
6.000


Average Force v. ln(Distance) for Minty Particles
y = -6.9791x - 0.9326
R² = 0.9218
Average Force (F)
ln(Distance) (dimensionless)
14.00
12.00
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
-2
-1.5
-1
-0.5
0
0.5
1

Data Analysis

For the first experiment, with positive particles, when average force was graphed against distance, the curve was hyperbolic. Our first linearization was thus to graph average force against 1/distance. This yielded a parabolic curve, so our second linearization was to graph average force against 1/distance^2. This yielded a straight line with slope 0.88 and very good precision (R^2 = 0.9986), indicating that the force F exerted by a positive particle on another positive particle some distance r away is approximately given by:

$$F = (0.88 \text{ F m}^2) / r^2$$

The y-intercept of the trendline was near-zero and so ignored.

For the second experiment, with minty particles, when average force was graphed against distance, the curve was again hyperbolic. Our linearization was thus to graph average force against 1/distance. This yielded a straight line. A trendline with slope 3.56 fit the data with extreme precision (R^2 = 0.9992). However, in this case, the y-intercept was significantly far from zero and could not be ignored. Thus the force F exerted by a minty particle on another minty particle some distance r away is approximately given by:

$$F = (3.56 \text{ F m}) / r - 4.85 \text{ F}$$

We recognized in the introduction that minty particles have some equilibrium distance a at which the force between them is zero, meaning:

$$0 = (3.56 \text{ F m}) / a - 4.85 \text{ F}$$

$$(3.56 \text{ F m}) / a = 4.85 \text{ F}$$

$$a = 0.73 \text{ m}$$

And so the force between two minty particles is given by

$$F = (3.56 \text{ F m}) * (1 / r - 1 / 0.73 \text{ m})$$

This 0.73 m value was also confirmed during experimentally--the force sensor read 0.00 F when the particles were 0.73 m apart.

For completeness, we also linearized the data for the second experiment according to the model given in our hypothesis, graphing average force against the natural logarithm of the distance. However, this curved was pronouncedly nonlinear. The trendline fit much less precisely (R^2 = 0.9218). Thus the natural-log-model we hypothesized is supported by the data much less than the 1/r-model.

Uncertainty

We quantified the uncertainty in the distance measurement by attempting to measure the distance between a force meter and a fixed particle multiple times. We found that we could reliably measure to ±0.01 m each time, and used this as our distance uncertainty. We quantified the uncertainty in the force measurement by taking the maximum of the difference between the forces recorded in each trial and the average force across all three trials.

Unfortunately, we needed to propagate the distance uncertainty through linearizations. To do this, we worked out the following trick: Assume the linearization function is f(x), with x being the original variable and f being the linearized variable. The uncertainty of the original at some value

x_0 is $\pm\delta x_0$, and we wish to know the linearized uncertainty $\pm\delta f_0$ at $f(x_0)$. If we assume these uncertainties are small, then we can treat them as differentials. Then we get that $\delta f_0 = f'(x_0)\, \delta x_0$, where $f'(x_0)$ is the derivative of $f(x)$ at $x = x_0$. Thus we can propagate uncertainty through a linearization by multiplying the original uncertainty by the derivative of the linearization function at the measured value. For example, the uncertainty in 1/r around the measurement r = 0.80 m ± 0.01 m is

$$\pm 0.01 \text{ m} * (-1 / (0.80 \text{ m})^2) = \pm 0.16 \text{ m}^{-1}$$

since the derivative of 1/r is $-1/r^2$. However, in many cases, the uncertainties (especially in average force) are too small to even see on the graph.

Conclusion

In this two-part lab, we explored the relationship between force and distance of separation for different types of particles. Our hypothesis was accurate for the first experiment, but our hypothesis for the second experiment was not as consistent with the data as the model we discussed later.

Our prediction on the first section, that two positive particles would repel according to Coulomb's law, was validated by the results of that experiment. Force varied directly with the inverse of the squared distance between the charges, and did so to a high degree of accuracy. Our results were, within the margin of error in our methods of measuring, consistent with Coulomb's law. When we found that force versus the inverse of distance squared produced a linear relationship, we found a slope that was in units of force times distance squared (as it encompassed Coulomb's constant times the product of the two unknown charges in the experiment). That constant slope was about .88.

In the second portion of the lab, when studying the minty particle interactions, our hypothesis using a natural logarithm did not fit the data as accurately as we had predicted. More accurately, the force varied according to a constant, 3.56 force units per meter, times the difference of the inverse of the radius and the inverse of a natural distance, .73 meters.

In mathematical terms, this is:

$$F = k\,(1/r - 1/a) = 3.56\,(1/r - 1/.73) = 3.56/r - 4.88$$

Thus, the graph was linear when force was plotted versus the inverse of r, with an r^2 of .999. This indicates a high accuracy and shows that our data is consistent with an inverse relationship between force and distance. The force was zero at a natural distance of .73. At lower distances, the force was repulsive, while it was attractive at higher distances. This explains the nature of the minty particle's motion when moving freely.

Some ways that error could be minimized and/or the experiment could be improved are:

1. to use an automatic measuring tool for the distances between particles, as we generally had a larger degree of human error while measuring distance than force;
2. or, short of that, to test a wider range distances, especially at the higher end (longer distances produce a smaller margin of error on the distance measurement);
3. and to vary the control variables in each experiment (ex.: to vary the type/charge of particle the test charge was influenced by).

However, overall our error was quite small on both of these experiments, and our data strongly indicate a fit for the respective models we found given our chosen controls.