Eigenvalue-Based Time Delay Estimation of Repetitive Biomedical Signals

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Abstract

The time delay estimation problem associated with an ensemble of M misaligned, repetitive signals is revisited. Each observed signal is assumed to be composed of an unknown, delayed, deterministic signal corrupted by Gaussian, white noise at a SNR level. This work shows that maximum likelihood (ML) time delay estimation (TDE), $\hat{\boldsymbol{\theta}}_{\mathrm{ML}}$, can be viewed as the maximization of an eigenvalue ratio, $\hat{\boldsymbol{\theta}}_{\mathrm{ER}}$, where the eigenvalues are obtained from the ensemble correlation matrix. A suboptimal, one-step TDE, $\hat{\boldsymbol{\theta}}_{\mathrm{OS}}$, is proposed for initiating the ML estimator, based on the eigenvector of the inter-signal correlation matrix. The ML time delay estimator can be determined without the need for an intermediate estimate of the underlying, unknown signal as required in the Woody method, $\hat{\boldsymbol{\theta}}_{\mathrm{W}}$. Based on respiratory flow signals, simulations show that the variance of the TDE error, σ_e, for $\hat{\boldsymbol{\theta}}_{\mathrm{ER}}$ is almost the same as that for $\hat{\boldsymbol{\theta}}_{\mathrm{ML}}$, see Fig. 1 where σ_e dependency with SNR for three different ensemble size M are plotted evidencing equivalent results for both estimators. The results of a study to evaluate the computation time as a function of ensemble size M for three different sizes of the grid delay search δ are shown in Fig. 2. As expected, the computation time increases with M and with δ, justifying initialization strategies to reduce the search. The error σ_e as a function of SNR for $\hat{\boldsymbol{\theta}}_{\mathrm{OS}}$ and $\hat{\boldsymbol{\theta}}_{\mathrm{W}}$ is explored in Fig. 3, from where it is clearly visible these estimators are outperformed by the optimum $\hat{\boldsymbol{\theta}}_{\mathrm{ER}}$ and $\hat{\boldsymbol{\theta}}_{\mathrm{ML}}$ in Fig. 1. However, given its one step nature, those estimators are well-suited for the initialization of the maximum search in $\hat{\boldsymbol{\theta}}_{\mathrm{ER}}$, leading to substantial computation time savings as reported in Fig. 2. It is concluded that eigenanalysis of the ensemble correlation matrix not only provides valuable insight on how signal energy, jitter, and noise influence the TDE process, but it also leads to a one-step TDE which provides a substantial reduction in computation time.

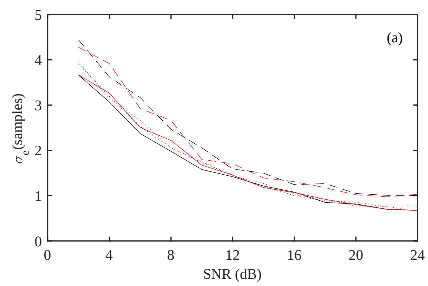

Figure 1. The σ_e as a function of SNR for $M = 10$ (solid line), $M = 20$ (dotted line), and $M = 50$ (dashed), using $\hat{\boldsymbol{\theta}}_{\mathrm{ER}}$ (black line) and $\hat{\boldsymbol{\theta}}_{\mathrm{ML}}$ (red line).

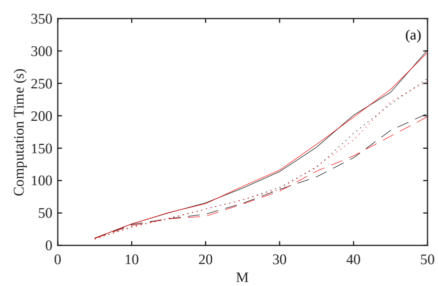

Figure 2. The computation time as a function of M, with δ set to 10 (dashed line), 20 (dotted line), and 100 (solid line), using $\hat{\boldsymbol{\theta}}_{\mathrm{ER}}$ (black line) and $\hat{\boldsymbol{\theta}}_{\mathrm{ML}}$ (red line).

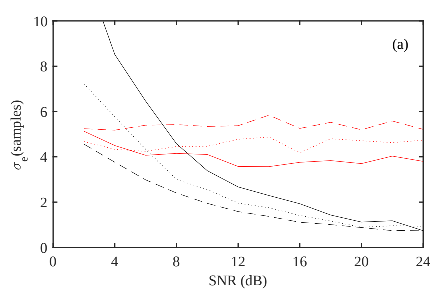

Figure 3. The time delay estimation error σ_e as a function of SNR in dB for $M = 10$ (solid line), $M = 20$, (dotted line), and $M = 50$ (dashed line), using $\hat{\boldsymbol{\theta}}_{\mathrm{W}}$ (black line) and $\hat{\boldsymbol{\theta}}_{\mathrm{OS}}$ (red line).