CW1 FOR INF2B YICHAO LIANG S1735938

EXERCISE 1.

1. Line 1, 2, 3 — $O(1)$ time each, executed once.

Line 4, — $O(1)$ time each, executed T.length − P.length = $n-m$ times.

Line 5, 10 — $O(1)$ time each, executed $n-m$ times.

Line 6, 7, 8, 9 — $O(1)$ time. No. of executions depends on for-test, m. For fixed m, for-loop at 6 takes at most m interations.

Thus, for fixed m. line 6-9 take at most $O(m)+O(m)+O(1)+O(1) = O(m)$.

Line 5a takes, therefore, $O(m)+O(1)+O(1) = O(m)$

There are $n-m$ different s values. Hence

line 4-5a takes $O(n-m)\,O(m) = O((n-m)m)$

We write it as $O((n-m+1)m)$ to take care of the case when $m=n$.

Then together Naive-Matcher (T, P) takes

$O((n-m+1)m) + O(1) + O(1) = O((n-m+1)m)$.

Hence proves $T_{NM}(n,m) = O((n-m+1)m)$

2. On a Bad instance

For $s = 0 \cdots n-m$, Naive-Matcher uses m executions to check if the pattern exist

Then $T_{Naive\text{-}Matcher}(n) \geq \sum_{s=0}^{n-m} cm = c\sum_{s=0}^{n-m} m = c(n-m)m$

Thus $T_{Naive\text{-}Matcher}(n) = \Omega(n-m+1)m$, with $+1$ to ensure it would be zero when $n=m$.

3. $T_{NM}(n,m) = \Theta((n-m+1)m)$ S1735938

Since $T_{NM}(n,m) = \Omega((n-m+1)m) = O((n-m+1)m)$

EXERCISE 2

1. since $c = c_1 + \cdots + c_n$, $\hat{c} = \hat{c}_1 + \cdots + \hat{c}_n$,

then $\hat{c} = c_1 + \Phi(1) - \Phi(1-1) + \cdots + c_n + \Phi(n) - \Phi(n-1)$

$= c + \Phi(n) - \Phi(0)$

Because $\Phi(0)=0$, and $\Phi(n)$ is the value of q just after the body of the for loop of line 6 in KMP-Matcher has executed for $i=n$, which is non-negative

Thus $c \le \hat{c}$

2.(a) Every time the while loop body is executed, q is assigned value $\pi(q)$ which is less than q. Then $q_1 + w_s \le \Phi(s-1)$ means the q value after the while loop terminates plus the number of times executed is at most the value of q before the while loop execution, because each time the loop executes $(w_s \leftarrow w_s+1)$, q value losses at least one (the π function)

Since $q_0 = \Phi(s-1)$,

Thus $q_1 + w_s \le q_0$, $q_1 + w_s - q_0 \le 0$

(b) After line 8 is executed $q_2 - q_1 \le 1$, since either $q_2 = q_1$ or $q_2 = q_1 + 1$.

Thus $1 + q_2 - q_1 \le 1 + 1 = 2$. $1 + q_2 - q_1 \le 2$

(c) After the if statement on line 9 is executed, q can be assigned value $\pi(q)$ which is less than q. Therefore q_3 is at most q_2 because when if evalutes true, $q_3 < q_2$, when false, $q_3 = q_2$. Thus $q_3 - q_2 \le 0$, $2 + q_3 - q_2 \le 2$

(d) We know $w_s + q_1 - \Phi(s-1) \le 0$ so $w_s + q_1 \le \Phi(s-1)$,

$1 + q_2 - q_1 \le 2$ so $q_2 \le q_1 + 1$,

$2 + \Phi(s) - q_2 \le 2$ so $\Phi(s) \le q_2$,

Since $\hat{c}_s = c_s + \Phi(s) - \Phi(s-1)$, $c_s \le w_s + 1 + 2$

$$\hat{c}_s \le w_s + 1 + 2 + \Phi(s) - \Phi(s-1) \le \Phi(s-1) - q_1 + 1 + 2 + \Phi(s) - \Phi(s-1) \le$$

$$-q_1 + 1 + 2 + \Phi(s) \le -q_2 + 4 + \Phi(s) \le -\Phi(s) + 4 + \Phi(s) \le 4$$

Thus $\hat{c}_s = O(1)$ for all s (indeed $\hat{c}_s \le 4$)