Pixel geolocation algorithm for satellite scanner data

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PIXEL GEOLOCATION ALGORITHM FOR SATELLITE SCANNER DATA

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Abstract

This work describes the geolocation determination of remote sensing data, utilizing a satellite configuration that supports a sensor designed to scan the surface of the Earth and it presents mathematical algorithm for determining the position of field scanning satellite records, especially those of imaging. In this paper, it is presented a relationship that give us the coordinates of the point of intersection of the line of sight scanning system and the Earth's surface as a function of: terrestrial ellipsoid surface, satellite position, satellite velocity, satellite attitude (spatial situation) and orientation of the scanner. The input parameters include the orbital state and attitude information of the satellite and the look vector of the remote sensing sensor. The process for calculating the pixel geolocation (geodetic latitude and longitude) starts with the calculation of the Instantaneous Field-Of-View (IFOV) matrix in sensor coordinates. Then, several rotations are required to obtain the IFOV in the Earth Centered Inertial (ECI) coordinate system. First there is the sensor-to-satellite rotation that obtains the IFOV relative to the satellite. Next there is the satellite-to-orbital (geodetic nadir pointing) rotation that obtains the IFOV relative to the path of the satellite. The transformation between the scan pixel and the ECEF pixel is expressed in terms of a series of consecutive matrix transformations applied to the line of sight vector. Finally, for any scan pixel, we obtain ECEF coordinates (by intersection of the IFOV with the ellipsoid used to model Earth) and then geodetic coordinates (geodetic longitude and latitude).

Key words: *Earth Observation, Geodesy, Mathematics, Remote Sensing*

INTRODUCTION

A very important issue, but, that can be solved quite easily, is to locate on the Earth's surface of image-points obtained with a satellite scanning system. This problem, has been first studied by Edward F. Puccinelli in 1976 and it presents mathematical formulation for determining the position of the scanning field of satellite records (especially those from remote sensing) (Puccinelli, 1976). In other words, we must find a relationship which gives us the coordinates of the point of intersection between the line of sight scanning system and the Earth's surface, as a function of: terrestrial ellipsoid surface, satellite position, satellite speed, spatial satellite situation ("attitude") and scanner orientation.

In general, a satellite scanner data system consists of three components:

* the sensor data,
* the geolocation information,
* the relationships between data and geolocation.

In this paper we do not proposed to analyze the pixel geolocation accuracy for different types of remote sensing sensors. As we know, there are many factors which influence geolocation, among them the most important are:

* range sampling frequency accuracy,
* stability of signal propagation in ionosphere and troposphere,
* Earth tides (solid earth tides caused by Earth deformations due to gravitational forces of Sun and Moon, pole tides caused by changes in Earth‘s rotational axis due to polar motion, etc).

The satellite is moving, by Kepler’s laws, in an elliptical orbit with the center of the Earth at one of the foci of the ellipse. The velocity is not constant and varies according to the position of the satellite in its orbit. The Earth is rotating with respect to the orbital plane of the satellite, so the motion of the satellite with respect to the Earth’s surface is quite complex.

The basic logic of a scanning sensor is the use of a mechanism to sweep a small field of view (known as an instantaneous field of view - IFOV) in a west to east direction at the same time the satellite is moving in a north to south direction. Together this movement provides the means of composing a complete raster image of the environment. A simple scanning technique is to use a rotating mirror that can sweep the field of view in a consistent west to east fashion. There are several satellite systems in operation today that collect imagery that is subsequently distributed to users. Several of the most common systems are described in (Dou et al., 2013). Each type of satellite data offers specific characteristics that make it more or less appropriate for a particular application.

The sensor data, which are the direct instrument measurements from which information about the Earth can be derived, are the major component of the swath data. The nature of the measurement may vary from instrument to instrument. Normally, sensor data will be processed by scientific algorithms for retrieving useful information. The sensor data type, as stored in digital format, can have the following standard formats (FGDC, 1999):

* ASCII representation of numerical values,
* 8, 16, 32 and n-bit (n>0) binary integers,
* 32 and 64 bits binary floats.

The elementary data structures for storing both the sensor data and the geolocation information are tables, arrays, or combinations of tables and arrays. A single swath structure can contain any number of tables and multidimensional arrays.

The geolocation information has a special role. It allows identification of the geographical location on the Earth surface corresponding to the data measurements for an individual pixel. Every swath is required to contain some geolocation component and geolocation information can be stored as a table, as a series of arrays, or as a combination of a table and arrays.

As the FGDC standard mentions (FGDC, 1999) "*A swath is produced when an instrument scans perpendicular to a moving point. Perpendicular, in this context, means close to, but not necessarily precisely at, a 90° angle. The path of this point, along which time or a time-like variable increases or decrease monotonically, is defined as the 'Track' dimension (sometimes referred to as 'along track'). The direction of the scan, which is perpendicular to the 'Track' dimension, is called the 'Cross-Track' dimension. Determining geolocation depends on knowing which array dimensions correspond to the 'Track' and 'Cross-Track' conceptual dimensions".*

The swath concept is shown in Figure 1, and it can be applied to measurements from a variety of platforms, including satellite, aircraft, and surface.

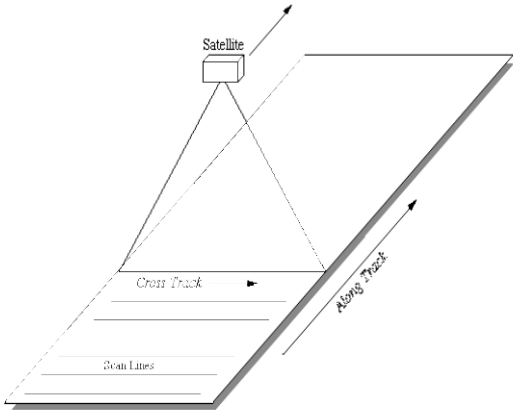


Figure 1. View of a simple swath (FGDC, 1999)

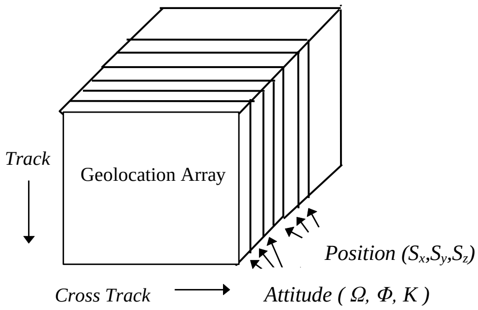


Figure 2. Geolocation Array containing Attitude and Position planes (FGDC, 1999)

When attitude and ephemeris data are provided for a swath structure describing measurements from satellite or airborne platforms, the following information is required to enable the user to calculate both geolocation and viewing geometry for the measurements:

* Date and time;
* Satellite attitude roll, pitch, and yaw angles (Ω, Φ, K);
* Satellite position vector $\vec{S}$ (S_x,S_y,S_z);
* Satellite velocity vector $\vec{v}$ (V_x,V_y,V_z), which is an optional information because it can be calculated.

Each set of date/time and geolocation are attached to an individual measurement (pixel) in the scan. The data set may provide the satellite attitude and position information directly or it may contain other information from which attitude and position can be calculated. The velocity vector may be calculated from successive values of position vector and time and thus need not be provided explicitly. Other information used in the process of converting from focal plane coordinates to latitude and longitude, such as transformations from individual component coordinate systems to instrument (along-track, along-scan, and nadir) coordinates or instrument to satellite coordinates is normally provided separately. The relationship between the satellite and orbital coordinate systems is defined by the satellite attitude.

The **orbital coordinate system** (Figure 3, extracted from *NASA, Goddard Space Flight Center 2011 - Joint Polar Satellite System (JPSS) Ground Project*) is centered on the satellite and its orientation is based on the platform position in inertial space. The origin is the satellite's center of mass with the Z_{orb} axis pointing from the satellite center of mass to the direction perpendicular to the reference ellipsoid. The Y_{orb} axis is the normalized cross product of the Z_{orb} axis and the instantaneous (inertial) velocity vector. It corresponds to the direction of the negative of the instantaneous angular momentum vector direction. The X_{orb} axis is the cross product of the Y_{orb} and Z_{orb} axes.

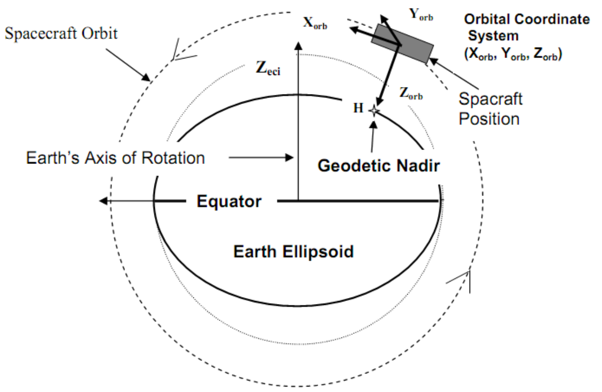


Figure 3. Orbital coordinate system (NASA, 2011)

The transformation from satellite to orbital coordinates is a three-dimensional rotation matrix with the components of the rotation matrix being functions of the satellite roll, pitch, and yaw attitude angles. The nature of the functions of roll ω, pitch φ, and yaw κ depends on how they are generated by the attitude control system.

The **Earth Centered Inertial (ECI) coordinate system** (described in detail by NIMA in 1997) has its origin at the Earth's center of mass. The Z axis corresponds to the mean north celestial pole of epoch J2000.0. The X axis is based on the mean vernal equinox of epoch J2000.0. The Y axis is the vector product of the Z and X axes. This coordinate system is shown in Figure 4.

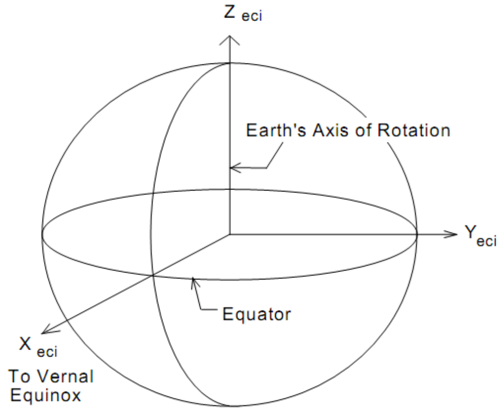


Figure 4. Earth Centered Inertial (ECI) Coordinate System

To determine the coordinates of the scan pixel, we work in ECEF coordinate system. This is a right-handed Cartesian frame of reference having its origin at the centre of the Earth. The Z axis is directed along the rotation axis towards the North pole, and the X and Y axes lie in the plane of the equator; the X axis lies in the plane of the Greenwich meridian, and the Y axis completes the right-handed set (Figure 5).

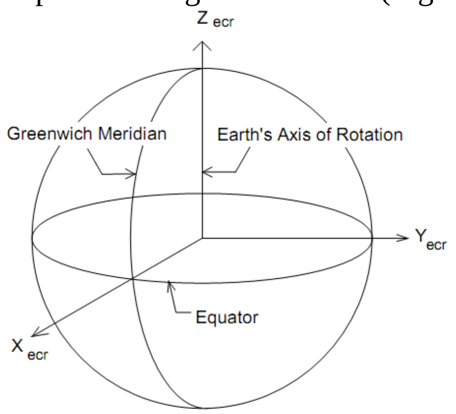


Figure 5. Earth-Centered-Earth-Fixed (ECEF) Coordinate System

The transformation between the scan frame and the ECEF frame can be expressed in terms of a series of consecutive matrix transformations applied to the line of sight vector. In principle this would be done by applying the transformations to each pixel. In practice, to reduce the processing overhead, they are carried out for a subset of tie point pixels, and the coordinates of intermediate pixels are

determined by linear interpolation in scan number and scan angle. That is, the pixel latitude and longitude are regarded as functions of scan number and pixel number, and are interpolated accordingly.
The transformation from ECI to ECEF coordinates is a time varying rotation due primarily to Earth rotation but also containing more slowly varying terms for precession, astronomic nutation, and polar wander. The ECI to ECEF rotation matrix can be expressed as a composite of four transformations (all these transformation terms were described in detail by NIMA in 1997):

$$T_{ECI \rightarrow ECEF} = A \cdot B \cdot C \cdot D$$

where:

A - Polar Motion
B - Sidereal Time
C - Astronomical Nutation
D - Precession

For many purposes it is necessary to control the satellite with respect to translation and rotation. This is realized through an ***attitude control system***. The attitude of a satellite may be defined as its rotational orientation in space with respect to an inertial frame (ECI), like in Figure 6.

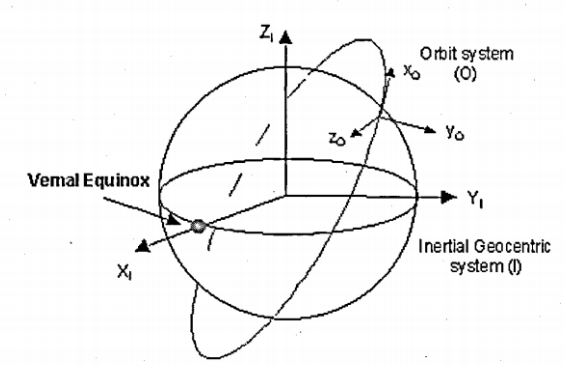


Figure 6. Orbit system and ECI Coordinate System

Components of an active attitude control system may be: accelerometers, gyroscopes, star sensors and GPS arrays.

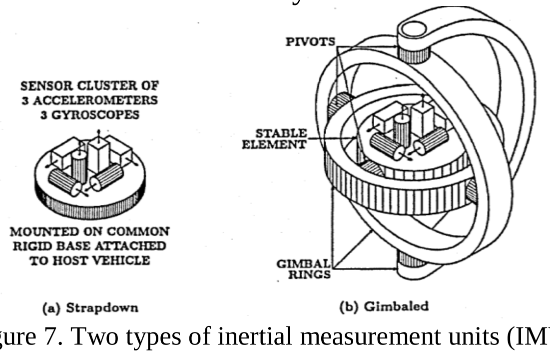


Figure 7. Two types of inertial measurement units (IMU)

Accelerometers are most suitable for sensing translations. Satellites transmit the elements of the orbit, as part of their data message. These are Keplerian elements with periodic terms added to account for solar radiation and gravity perturbations. Periodic terms are added for argument of perigee, geocentric distance and orbit inclination.

MATERIALS AND METHODS

The present paper uses the theoretical and practical experience of public use by NASA and ESA in the last years of international researches in geolocation of remote sensing data. I have used also my previous expertise and applications in some digital photogram-metric projects in Romania.
The method of pixel geolocation is based on the general shape of the Earth ellipsoid of rotation

$$\frac{X^2+Y^2}{a^2}+\frac{Z^2}{b^2}-1=0$$

and the general form of the parametric equations

$$X = X(B,L)$$
$$Y = Y(B,L)$$
$$Z = Z(B)$$

and are obtained parametric equations of the ellipsoid of rotation:

$$\begin{cases} X = \dfrac{a\cos B\cos L}{W} = \dfrac{c\cos B\cos L}{V}; \\ Y = \dfrac{a\cos B\sin L}{W} = \dfrac{c\cos B\sin L}{V}; \\ Z = \dfrac{a(1-e^2)\sin B}{W} = \dfrac{c(1-e^2)\sin B}{V}. \end{cases}$$

Where :

$$c = \frac{a^2}{b} \qquad \frac{a}{W} = \frac{c}{V}$$

$$W = (1-e^2\sin^2 B)^{1/2}$$

$$V = (1+e'^2\sin^2 B)^{1/2}$$

the first eccentricity is:

$$e^2 = \frac{a^2-b^2}{a^2} = 1-\frac{b^2}{a^2}$$

and the second eccentricity is:

$$e'^2 = \frac{a^2-b^2}{b^2} = \frac{a^2}{b^2}-1$$

To solve the proposed problem we choose the coordinates system OXYZ based on a global equatorial orthogonal Cartesian system ("global geocentric system," as E. Grafarend called). The origin O of the system is considered in the vicinity of the center of mass of the Earth and its axis: 0X axis is parallel to the local meridian of Greenwich, 0Z axis is parallel to the axis of the world and the axis 0Y is perpendicular to X0Z. This global geodetic system (for example WGS84) is shown below in the Figure 8.

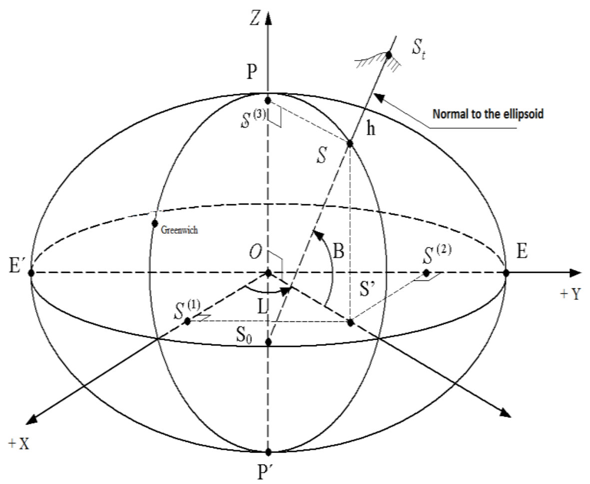


Figure 8. The World Geodetic System 1984 (WGS84)

The World Geodetic System 1984 (WGS84) models the Earth’s surface as an oblate spheroid (ellipsoid), which allows Cartesian Earth-Centered-Earth-Fixed (ECEF) positions on Earth’s surface to be represented using the angles geodetic longitude and geodetic latitude. The WGS84 was developed by the National Imagery and Mapping Agency (NIMA), now the National Geospatial-Intelligence Agency (NGA), and has been accepted as a standard for use in geodesy and navigation. NIMA expressed simply in 1997 the relationship between ECEF and geodetic coordinates WGS84 in its direct form.

The method, presented in this paper, can be applied to many types of sensors, shown in Figure 9, with some particularities, but I present only the case of linear array or scanners array passive imaging sensors. All sensors data must be geolocated based on the instrument, instrument mode of operation, and instrument coverage swath. The scan pixel center is defined as the geolocation point at the intersection of the look vector and the terrestrial ellipsoid surface.

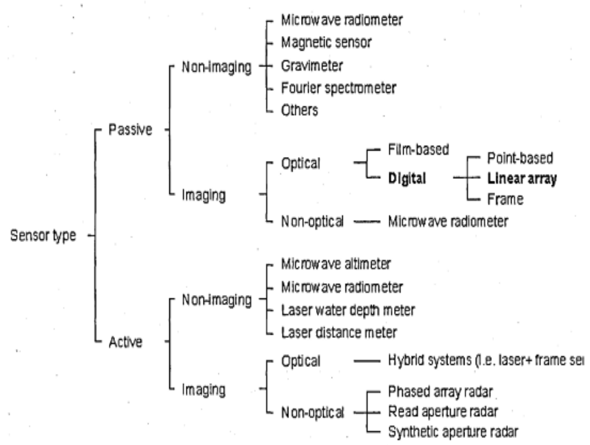


Figure 9. Classification of sensors for data acquisition

RESULTS AND DISCUSSIONS

Considering known the following elements:

- satellite position expressed by the vector $\vec{S}$ with components (S_x, S_y, S_z);
- satellite velocity on the orbit, characterized by the vector $\vec{v}$ with components (V_x , V_y , V_z);
- angles of roll (Ω), pitch (Φ) and yaw (K) measured around the three axes of the same name of satellite;
- Orientation of the pointing direction of the scanning system given by the vector $\vec{w}$ (w_1,w_2,w_3), where w_1,w_2,w_3 are angles measured between the axis scanning direction of the scanner and the satellite axis.

In Figure 10 are shown the two coordinate systems: global geocentric system **0XYZ** (considered basic coordinate system) and satellite coordinate system **o'xyz**.

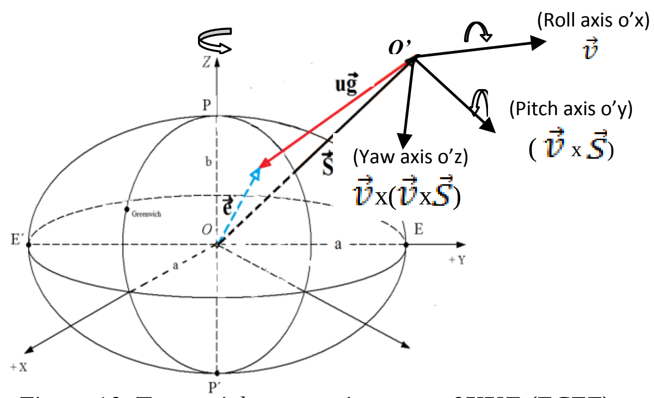


Figure 10. Terrestrial geocentric system **0XYZ** (ECEF) and satellite coordinate system **o'xyz**

The coordinates system of the satellite is as defined to the global coordinate system geocentric of the position vector. Roll axis (o'x) is chosen so as to coincide with the velocity vector axis $\vec{v}$, pitch axis (o'y) is taken so that the vector ($\vec{v}$x$\vec{S}$) results to be perpendicular to the orbit (and hence on the roll axis) and the yaw axis (o'z) is selected so as to coincide with the vector $\vec{v}$x($\vec{v}$x$\vec{S}$), so being perpendicular to the plane formed by the other two axes.

Position vector of the satellite $\vec{S}$ (S_x, S_y, S_z) and velocity vector $\vec{v}$ (v_x, v_y, v_z) as well as angles of yaw (k), pitch (Φ), roll (Ω) we use to orient the satellite axes to global geocentric system. First we determine the orientation of the axes of the satellite to the base coordinate system by calculating a matrix **D** whose columns represent the unit vectors (versors) of axes roll, pitch and yaw of the satellite, then we compute a matrix **M** with which we can apply corrections "attitude" (the three rotation axes introduced to satellite Ω, Φ, k,). Then we compute the orthogonal matrix **F = D · M** whose column vectors are the axes of roll (the first column of F) of the satellite in the basic coordinate system (after they were introduced rotations Ω, Φ and K).

Calculate the matrix D of satellite axes orientation:

$$\mathbf{D} = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix} = (\underset{\text{versor}}{\overrightarrow{C_1}} \quad \underset{\text{versor}}{\overrightarrow{C_2}} \quad \underset{\text{versor}}{\overrightarrow{C_3}})$$

$$\overrightarrow{C_1} = \frac{\vec{v}}{|\vec{v}|} = \frac{v_x\vec{\imath} + v_y\vec{\jmath} + v_z\vec{k}}{\sqrt{v_x^2 + v_y^2 + v_z^2}} = \frac{v_x}{VLG}\vec{\imath} + \frac{v_y}{VLG}\vec{\jmath} + \frac{v_z}{VLG}\vec{k}$$

If we note: $\sqrt{v_x^2 + v_y^2 + v_z^2} = \boldsymbol{VLG}$

Then $\boldsymbol{D_{11}} = \frac{v_x}{VLG}$; $\boldsymbol{D_{21}} = \frac{v_y}{VLG}$; $\boldsymbol{D_{31}} = \frac{v_z}{VLG}$

$$\overrightarrow{C_2} = (\overrightarrow{C_1} \text{ x } \vec{S})/|\vec{S}| = \frac{\vec{v}}{|\vec{v}|} \text{ x } \frac{\vec{S}}{|\vec{S}|} = \frac{1}{|\vec{v}|\,|\vec{S}|} \cdot (\vec{v} \text{ x } \vec{S})$$

$$= \frac{1}{\sqrt{v_x^2 + v_y^2 + v_z^2}\ \sqrt{S_x^2 + S_y^2 + S_z^2}} \cdot \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ v_x & v_y & v_z \\ S_x & S_y & S_z \end{vmatrix}$$

$$= \frac{v_y S_z - S_y v_z}{VLG \cdot SLG}\vec{\imath} + \frac{S_x v_z - v_x S_z}{VLG \cdot SLG}\vec{\jmath} + \frac{v_x S_y - S_x v_y}{VLG \cdot SLG}\vec{k}$$

Where we denoted $\sqrt{S_x^2 + S_y^2 + S_z^2} = \boldsymbol{SLG}$

$$\boldsymbol{D_{12}} = \frac{v_y S_z - S_y v_z}{VLG \cdot SLG}; \quad \boldsymbol{D_{22}} = \frac{S_x v_z - v_x S_z}{VLG \cdot SLG};$$

$$\boldsymbol{D_{32}} = \frac{v_x S_y - S_x v_y}{VLG \cdot SLG}$$

$$\overrightarrow{C_3} = \overrightarrow{C_1} \text{ x } \overrightarrow{C_2} = \begin{vmatrix} i & j & k \\ D_{11} & D_{21} & D_{31} \\ D_{12} & D_{22} & D_{32} \end{vmatrix} =$$

$$= (\boldsymbol{D_{21}D_{32} - D_{22}D_{31}})\,\vec{\imath} + (\boldsymbol{D_{12}D_{31} - D_{11}D_{32}})\,\vec{\jmath} + (\boldsymbol{D_{11}D_{22} - D_{12}D_{21}})\,\vec{k}$$

$$\boldsymbol{D_{13} = D_{21}D_{32} - D_{22}D_{31}}$$
$$\boldsymbol{D_{23} = D_{12}D_{31} - D_{11}D_{32}}$$
$$\boldsymbol{D_{33} = D_{11}D_{22} - D_{12}D_{21}}$$

Thus, based on the above relations, we can calculate all nine elements of the matrix D. The next step is the determination of the elements of rotation matrix M. Matrix M has 9 elements, including 3 independent parameters namely Euler angles (ψ, φ, θ). As is well known, the relative position of the two reference systems consisting of mutually orthogonal axes, which have the same origin is perfectly defined by the Euler angles:

- ψ = angle of precession, defined by the line $0'x_1$ and the nodes line O’N (junction between plan $O'x_1y_1$ with O'xy plan);
- φ = angle of its own rotation, between O'N and O'x;
- θ = the rotation angle determined by the axes $O'z_1$ and O'z.

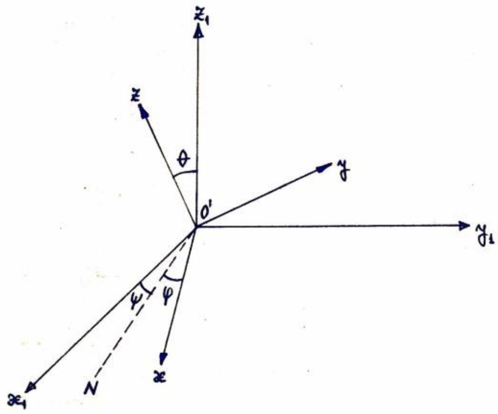


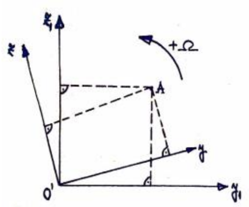
Figure 11. Euler’s angles (ψ, φ, θ)

In the problem studied we will denote the three independent parameters Ω, Φ, K, representing the angles of roll, pitch and respectively yaw of the satellite. O'xyz axis represents the initial position of the satellite at a given time ***t*** (position errors affected by the roll, pitch and yaw). $O'x_1y_1z_1$ axis represents the position of the satellite after they were applied to the three successive rotations Ω, Φ and K.

a) First apply rotation Ω:

Rotation Ω is done in the plane y0'z, positive sense being indicated by the arrow in the figure below:

$$\begin{cases} x_1 = x \\ y_1 = y\cos\Omega - z\sin\Omega \\ z_1 = y\sin\Omega + z\cos\Omega \end{cases}$$



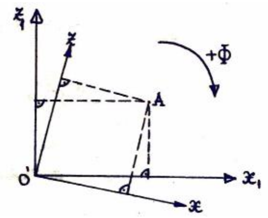
Or in matrix form:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = R_\Omega \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow R_\Omega = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\Omega & -\sin\Omega \\ 0 & \sin\Omega & \cos\Omega \end{pmatrix}$$

b) Apply the rotation Φ:

Rotation Φ shall be made in the plan xo'z (already rotated by angle Ω) with the positive sense showed the figure:

$$\begin{cases} x_1 = x\cos\Phi + z\sin\Phi \\ y_1 = y \\ z_1 = z\cos\Phi - x\sin\Phi \end{cases}$$



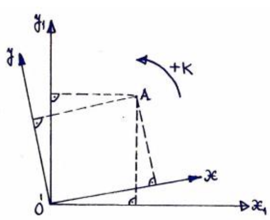
Or in matrix form:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = R_\Phi \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow R_\Phi = \begin{pmatrix} \cos\Phi & 0 & \sin\Phi \\ 0 & 1 & 0 \\ -\sin\Phi & 1 & \cos\Phi \end{pmatrix}$$

c) Apply the rotation K:

The rotation K shall be made in the plan x0'y (around the yaw axis of the satellite).

$$\begin{cases} x_1 = x\cos k - y\sin k \\ y_1 = y\cos k + x\sin k \\ z_1 = z \end{cases}$$



Or in matrix form:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = R_k \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow R_K = \begin{pmatrix} \cos K & -\sin K & 0 \\ \sin K & \cos K & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

As is known, the matrices of rotation are multiplied in reverse order to carry out rotations, as follows:

$$x_1^{(1)} = R_\Omega \cdot x$$

$$x_1^{(2)} = R_\Phi \cdot x_1^{(1)}$$

$$x_1^{(3)} = R_k \cdot x_1^{(2)}$$

$$x_1^{(3)} = x_{1\,final} = R_k \cdot x_1^{(2)} = R_k\, R_\Phi\, x_1^{(1)} = R_K \cdot R_\Phi \cdot R_\Omega \cdot x$$

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = R_K \cdot R_\Phi \cdot R_\Omega \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

As noted above relation $R_K \cdot R_\Phi \cdot R_\Omega = M$, where M represents rotation matrix result (including all three single rotations: Ω, Φ and K).

$$\mathbf{M} = R_k \cdot R_\Phi \cdot R_\Omega = \begin{pmatrix} \cos K & -\sin K & 0 \\ \sin K & \cos K & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot$$

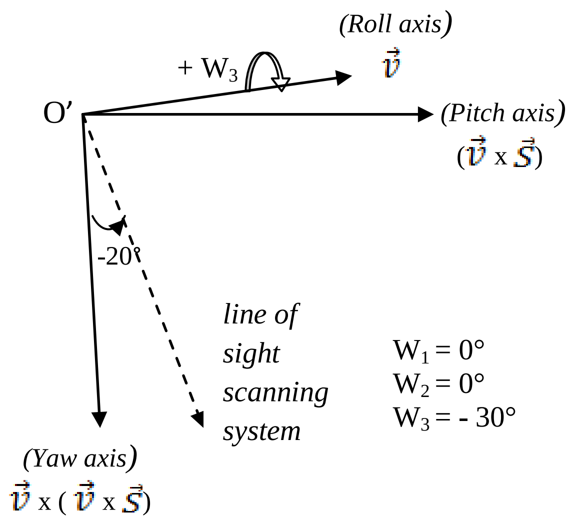
$$\cdot \begin{pmatrix} \cos\Phi & 0 & \sin\Phi \\ 0 & 1 & 0 \\ -\sin\Phi & 1 & \cos\Phi \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\Omega & -\sin\Omega \\ 0 & \sin\Omega & \cos\Omega \end{pmatrix} =$$

$$= \begin{pmatrix} \cos k\cos\Phi & -\sin k & \cos k\sin\Phi \\ \sin k\cos\Phi & \cos k & \sin k\sin\Phi \\ -\sin\Phi & 0 & \cos\Phi \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\Omega & -\sin\Omega \\ 0 & \sin\Omega & \cos\Omega \end{pmatrix}$$

$$= \begin{pmatrix} \cos k\cos\Phi & \cos k\sin\Phi - \sin k\cos\Omega & \cos k\sin\Phi\cos\Omega + \sin k\sin\Omega \\ \sin k\cos\Phi & \sin k\sin\Phi\sin\Omega + \cos k\cos\Omega & \sin k\sin\Phi\cos\Omega - \cos k\sin\Omega \\ -\sin\Phi & \cos\Phi\sin\Omega & \cos\Phi\cos\Omega \end{pmatrix}$$

Then, calculating the matrix **F = D · M** we only orient the satellite axes to global geocentric coordinate system 0XYZ (WGS84). Continue order to work in the same coordinates system will have to orient and order the line of scanning system towards the global geocentric coordinates system. So, we need to specify rotations of yaw, pitch and roll to obtain a coincidence between the yaw axis ($\vec{v}$x($\vec{v}$x$\vec{S}$)) and the line of sight scanning system. All rotations are usually screw (right hand rule) and they will be used to form the vector $\vec{W}$ ($w_1w_2w_3$). Since these rotations are made in the same manner as corrections "attitude" (K, Φ, Ω), the matrix M’ can be formed, because is the same as M except to that it has the angle W_1 rather than the angle K, of the angle W_2 instead of angle Φ and angle W_3 instead of the angle Ω. In most cases the line of sight of the scanning system is located in the plan defined by the pitch and yaw axes of the satellite. For example, the figure below shows the turning axis to align the line of sight vector is only necessary to -20° rotation around the axis of the roll.

Introducing rotations W_1, W_2, W_3 through the rotation matrix **M'**, we obtain an orthogonal matrix **G = F • M'**, whose columns are the new versors of the axes of roll, pitch and yaw.



Because we are interested in only the versor $\vec{g}$ of the line of sight of scanner (which is the same as versor of the satellite yaw axis) we will work with the third column of the matrix M'.

By noting $\vec{m}$ the third column of M', it will have the following form:

$$\vec{m} = \begin{pmatrix} \cos w_1 \cdot \sin w_2 \cdot \cos w_3 + \sin w_1 \cdot \sin w_3 \\ \sin w_1 \cdot \sin w_2 \cdot \cos w_3 - \cos w_1 \cdot \sin w_3 \\ \cos w_2 \cdot \cos w_3 \end{pmatrix}$$

Result $\vec{g}$ = **F**$\cdot\vec{m}$ = **D · M ·** $\vec{m}$, which represents the versor (unit vector) of the scanner line of sight expressed in global geocentric coordinate system. Because the line of sight vector scanning system to be fully defined we must determine the size (module), in addition to direction and its meaning. For the size determination **u** of the line of sight vector of scanner we will work with parametric equations of the ellipsoid of rotation:

$$E(u_1, u_2) = \begin{pmatrix} a \cdot \cos u_1 \cdot \cos u_2 \\ a \cdot \sin u_1 \cdot \cos u_2 \\ b \cdot \sin u_2 \end{pmatrix}$$

and parametric equations of a straight line in space:

$$\begin{cases} \mathbf{x = S_x + u \cdot g_x} \\ \mathbf{y = S_y + u \cdot g_y} \\ \mathbf{z = S_z + u \cdot g_z} \end{cases}$$

Where: $\mathbf{g_x, g_y, g_z}$ are the cosines guiding of the line of sight regard to scanning system.

Equalizing the two parametric representations (intersecting the line of sight scanning system with terrestrial ellipsoid surface) we obtain:

$$\begin{cases} \mathbf{S_x + u \cdot g_x = a \cos u_1 \cos u_2} \\ \mathbf{S_y + u \cdot g_y = a \sin u_1 \cos u_2} \\ \mathbf{S_z + u \cdot g_z = b \sin u_2} \end{cases}$$

By multiplying the third equation with **(a/b)**, then rising to the square part of each equation, and by adding together all three equations, we obtain a quadratic equation in **u** (removing the parameters $\mathbf{u_1}$ and $\mathbf{u_2}$).

$$\begin{cases} (S_x + u \cdot g_x)^2 = (a \cos u_1 \cos u_2)^2 \\ (S_y + u \cdot g_y)^2 = (a \sin u_1 \cos u_2)^2 \\ (\frac{a}{b} S_z + u \cdot \frac{a}{b} g_z)^2 = (a \sin u_2)^2 \end{cases}$$

$$\begin{cases} S_x^2 + 2u\, S_x\, g_x + u^2\, g_x^2 = a^2 \cos^2 u_1 \cos^2 u_2 \\ S_y^2 + 2u\, S_y\, g_y + u^2\, g_y^2 = a^2 \sin^2 u_1 \cos^2 u_2 \\ \frac{a^2}{b^2} S_z^2 + 2u\, \frac{a^2}{b^2}\, S_z\, g_z + u^2 \cdot \frac{a^2}{b^2}\, g_z^2 = a^2 \sin^2 u_2 \end{cases}$$

$$(S_x^2 + S_y^2 + \frac{a^2}{b^2} S_z^2) + 2(S_x\, g_x + S_y\, g_y + \frac{a^2}{b^2} S_z\, g_z)\, u$$

$$+(g_x^2 + g_y^2 + \frac{a^2}{b^2} g_z^2)\, u^2 =$$

$$= a^2 (\cos^2 u_1 \cos^2 u_2 + \sin^2 u_1 \cos^2 u_2 + \sin^2 u_2)$$

The coefficient of a^2 in the right member is equal to 1, because:

$\cos^2 u_1 \cos^2 u_2 + \sin^2 u_1 \cos^2 u_2 + \sin^2 u_2 =$
$\cos^2 u_2 (\cos^2 u_1 + \sin^2 u_1) + \sin^2 u_2 = 1$

$$\frac{b^2 (S_x^2 + S_y^2) + a^2 S_z^2}{b^2} + \frac{b^2 (S_x g_x + S_y g_y) + a^2 S_z g_z}{b^2} \cdot 2u$$

$$+ \frac{b^2 (g_x^2 + g_y^2) + a^2 g_z^2}{b^2}\, u^2 = \frac{a^2 \cdot b^2}{b^2}$$

Since $b \neq 0$ results:

$$[b^2(g_x^2+g_y^2)+a^2g_z^2]\,\mathbf{u^2} + 2[b^2(S_xg_x+S_yg_y) + a^2S_zg_z]\,\mathbf{u} + [b^2(S_x^2+ S_y^2) + a^2(S_z^2-b^2)] = 0$$

Written in a shortened form, the above equation becomes a simple quadratic equation in the parameter **u**:

$$A\mathbf{u}^2 + 2B\mathbf{u} + C = 0$$

Provided the argument of the square root is positive, which will always be the case in practice, both solutions are real and positive, and the one that we require is the smaller of the two, which we denote by $\mathbf{u}_{min}$; this will be the one corresponding to the negative sign. The other solution then defines the point of emergence of the line of sight at the far side of the Earth. (If the quantity under the square root is negative, the solutions of the equation are complex. This case would arise if the line of sight did not intersect the terrestrial ellipsoid, and will never occur in the normal course of pixel geolocation with the satellite in yaw steering mode).

Ground coordinates of a scan pixel in global geocentric system are even components of the vector $\vec{e}$ (represented in Figure 7) obtained by the sum of other two vectors $\vec{S}$ and $u\vec{g}$:

$$\vec{e} = \vec{S} + \mathbf{u}\vec{g}$$

$$\begin{cases} \mathbf{X = e_x = S_x + ug_x} \\ \mathbf{Y = e_y = S_y + ug_y} \\ \mathbf{Z = e_z = S_z + ug_z} \end{cases}$$

So, the above relations express the ground coordinates of a scan pixel in ECEF coordinate system. Finally, from the ECEF cartesian coordinates of the pixel we can derive its geodetic longitude (L) and geodetic latitude (B) from Figure 8, using the formulas:

$$L = \arctan (Y/X)$$

$$B= (1-e^2)^{-1} \arctan (Z/U), \text{ where } \quad U = \sqrt{X^2 + Y^2}$$

CONCLUSIONS

Geolocation of satellite data is a standard part of the post-launch calibration process. For the data to be of value, it is critical that the measured parameters be correctly mapped to the surface of the Earth.

The input parameters include the orbital state and attitude information of the satellite and the look vector of the sensor. The proposed algorithm agrees with the navigation product Satellite Tool Kit (STK), within 0.5 m in ideal situations. STK is a software tool that allows engineers and scientists to design and develop complex dynamic simulations of real-world problems, especially those relating to satellites, spacecrafts, and aircrafts. STK is the same software that space companies use to determine where to place satellites on orbit and to find their satellites once launched.

Satellites and their missions play a critical part in our everyday lives. Everything we do somehow is now connected to satellites in space. We use satellites to communicate, conduct banking transactions, navigate our way around a city – or the country for that matter, forecast the weather, protect our national security, create precise maps, examine the oceans and seas, analyze the Sun activity, map the galaxy, the list is practically endless. The more we know about how satellites work and the environment they operate in, the better we will be in determining additional ways we can use these unique assets in the future. STK will excite users about space and space operations, and should motivate them want to learn more about this critical part of our infrastructure. *STK Aerospace Education Program was designed to educate users on the exciting aspects of satellites, satellite orbits, the types and locations of orbits, and satellite missions using Analytical Graphics Incorporated (AGI) state of the art computer application, Satellite ToolKit (STK)*(CAP-STK Aerospace Program).

Georeferencing an analog or digital photograph is dependent on the interior geometry of the sensor as well as the spatial relationship between the sensor platform and the ground. The single vertical aerial photograph is the simplest case; we can use the internal camera model and six parameters of exterior orientation (X, Y, Z, roll, pitch, and yaw) to extrapolate a ground coordinate for each identifiable point in the image. We can either compute the exterior orientation parameters from a minimum of 3 ground control points using space resection equations, or we can use direct measurements of the exterior orientation parameters obtained from GPS and IMU.

The internal geometry of design of a spaceborne multispectral sensor is quite different from an aerial camera. The Figure 12 shows below six types of remote sensing systems, comparing and contrasting those using scanning mirrors, linear pushbroom arrays, linear whiskbroom areas, and frame area arrays.

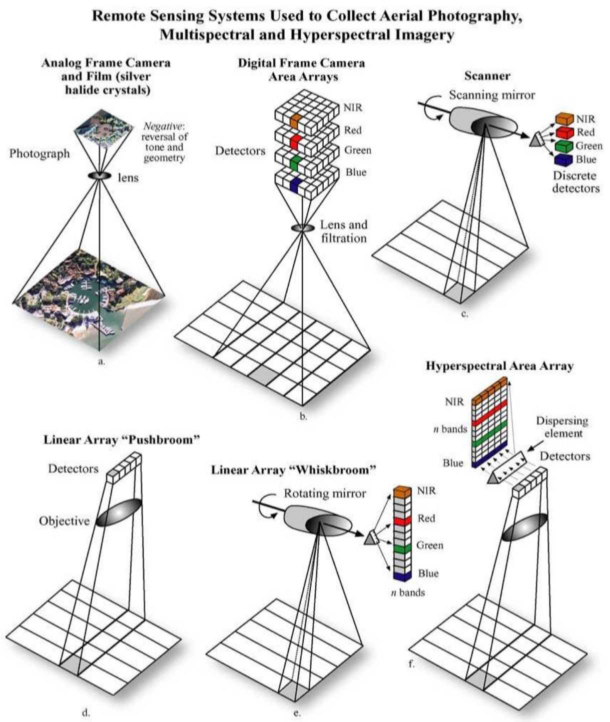


Figure 12. Sensor types of remote sensing systems (Jensen, 2007)

A linear array, or pushbroom scanner, is used in many spaceborne imaging systems, including SPOT, IRS, QuickBird, OrbView, IKONOS, etc. The position and orientation of the sensor are precisely tracked and recorded in the platform ephemeris. However, other geometric distortions, such as skew caused by the rotation of the Earth, must be corrected before the imagery can be referenced to a ground coordinate system.

Each pixel of imagery is captured at a unique moment in time, corresponding with an instantaneous position and attitude of the aircraft. When direct georeferencing is integrated with these sensors, each single line of imagery has the exterior orientation parameters needed for rectification. However, without direct georeferencing, it is impossible to reconstruct the image geometry; the principles of space resection only apply to a rigid two-dimensional image.

We can relate that the units of satellite position error (meters in space) and platform attitude or instrument pointing error (arc seconds) to absolute geolocation error (meters on the ground). Earth location errors are scan angle dependent, growing larger with increasing scan (or view) angle. NASA describes in (NASA, 2011) the corresponding growth of the viewed ground pixel, which also increases in size with increasing scan angle.

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