Impact Crater Saturation Simulation

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Contents

1 Part 1

1.1 Initial Assumptions

1.1.1 Constant Crater Size

Initially, we assume all craters to have the same radius. The circle formula (see equation 1) in determining which coordinates are effected by the impact. For those coordinates, the surface height value is decreased by a constant "crater depth".

$$(x-h)^2+(y-k)^2=r^2 \tag{1}$$

A visual representation of this formula can be seen in figure 1. The coordinates (h,k) determine where the center of the circle will be from a given origin determined by the coordinates (x,y). The variable r represents the circle radius.

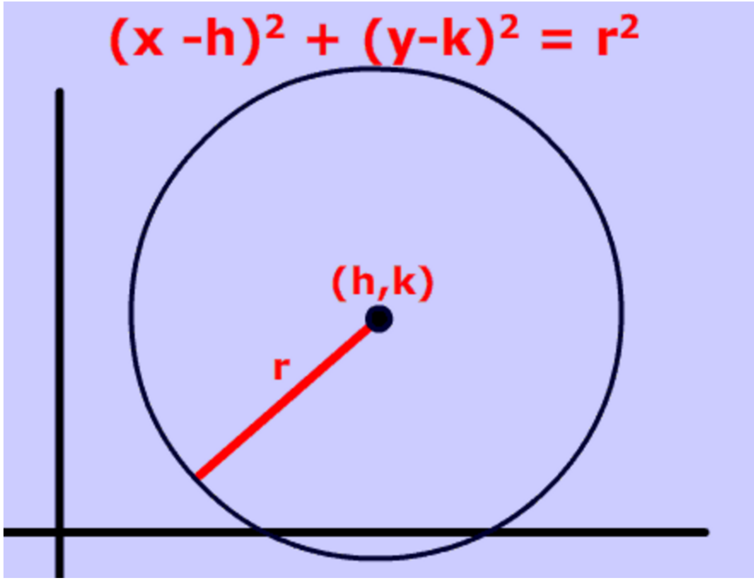


Figure 1: General Form of Circle Formula

A random number generator (randu) is used to get impact coordinates on the surface, where the surface boundaries are 0 to 500 for x and y dimensions. These coordinates are used in the circle formula (see equation 2) to make the crater and then cataloged and used again in the detection algorithm. The radius of all impacters are initially set to be 5 km, making 50 km craters.

$$(x-impact_x)^2+(y-impact_y)^2=r_{crater}^2 \tag{2}$$

1.1.2 All Impacters Are Spherical

Again, since we are using the circle formula, we were able to make the assumption that each impacter and crater were to be spherical. In real craters, this is not always the case, however it is a good approximation based on the way shocks propagate from a point source.

1.1.3 All Impacters Have the Same Velocity

This assumption is needed to ensure that the depth of each impact would remain constant. This is talked about further in the next assumption.

1.1.4 Depth & Shape Of Craters

Since all impacters have the same velocity, and the same size, this meant that each impact would have a constant depth. The depth was used to change the values in the impact surface (500 x 500 array) which allowed us to determine where a crater was when searching for them. The impact area is initially set to have a height of 1 km. When an impact occurs, the depth of the crater is then changed to a value of 0.5 km to indicate an impact, and crater rim is set to a depth below the surface of 0.25 km which will be discussed further later.

1.1.5 Average Impact Rate

It is assumed that the rate of impact is once per 1000 years. This assumption is a requirement from the assignment document. The time scale in our code goes from 0 to 10,000, where each integer step represents 1,000 years.

1.1.6 Crater Detection Criteria

The craters on the surface are detected when at least 30% of the rim is at the "rim depth". The number of detectable craters was plotted against time at 100,000 year intervals. A crater rim was created for the last 10% of the outer shell of the crater. The depth of the rim is 0.25 km, which is how we distinguish a rim from a crater which is 0.5 km deep respectively. The search algorithm searches for all craters on the surface where the detection criteria is met using a catalog of events, and those that do not meet this criteria are removed from the list of detectable craters. This prevents old "erased" craters from being evaluated after they are erased by later impacts. Cataloging current detectable craters is programmatically achieved by appending a small dictionary-type variable of each impact's parameters, which include all coordinates within crater, impact center, and radius, to an array of all detectable craters (the catalog). When a crater is physically erased by a newer, larger crater then it is removed from this catalog. Computing time is then significantly reduced.

1.1.7 Random Impact Locations on the Surface

A random number generator function was used to obtain coordinates between 0 and 500. Since the function in IDL only provides random numbers between 0 and 0.9999 so we then had to multiply these coordinates by 500 to get a range of random numbers between 0 and 499.

1.1.8 All Craters Are Cylindrical Impressions, Not Bowl Shaped

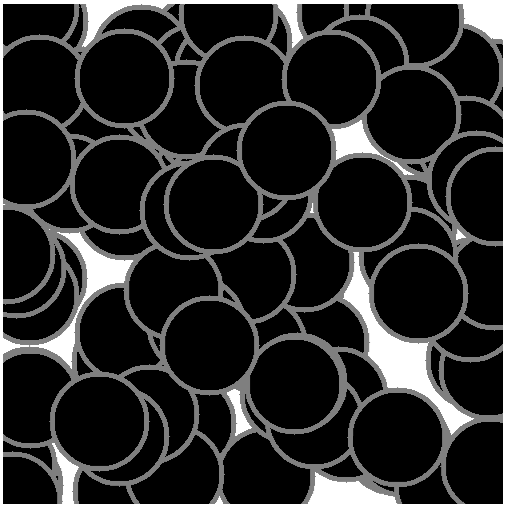
Since we are using a circle formula and depth to symbolize craters, the slope from the area down to the rim, and from the rim to the middle of the crater, is an instantaneous one. Craters normally have a smooth bowl shape to them, however our craters are very discrete in nature. This is fine because it is only an approximation, and the number of craters is what is really important, not how they actually appear in the z-direction where the z-direction represents height.

1.1.9 Saturation Criteria

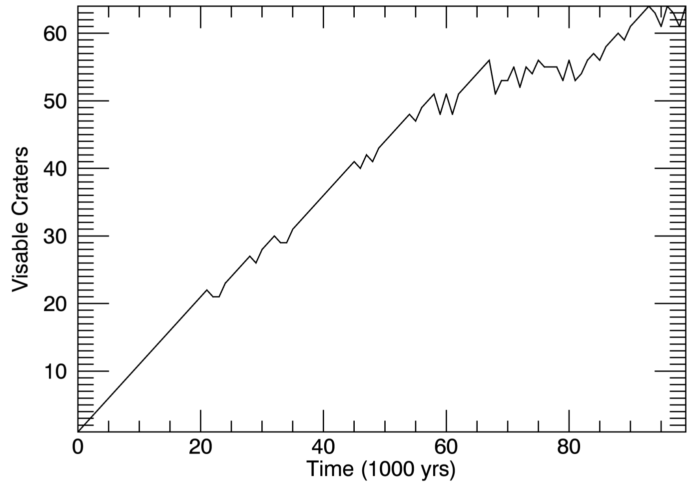
Craters will be simulated and accounted for throughout the entire simulation. A plot of visible craters as a function of time is plotted at every 100,000 year interval where the x-axis is time and the y-axis is the number of visible craters. The plot should increase linearly at the beginning and then after a certain amount of time, the slope of the line will level out as older craters are "erased" by newer, larger ones. When the saturation curve generally levels out (there is some variance in surface crater density across time), then we know how long it takes to achieve saturation for the impact area.

1.2 Plots

1.2.1 First Time Interval: 100,000 Years



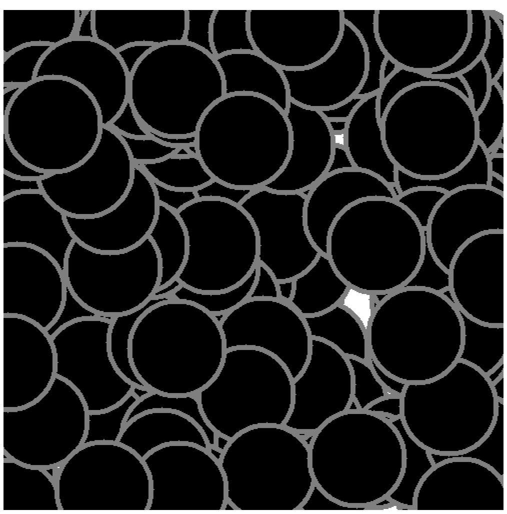
(a) Crater Area For 100,000 Years



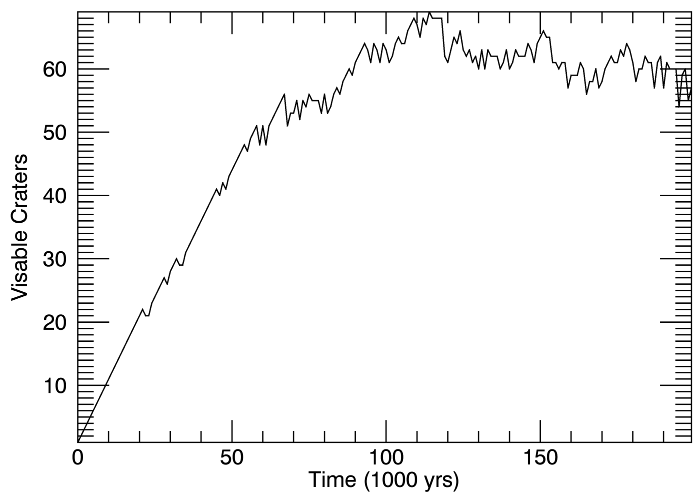
(b) Crater Saturation Curve for 100,000 Years

Figure 2: Crater Plots For Interval Of 100,000 Years

1.2.2 Second Time Interval: 200,000 Years



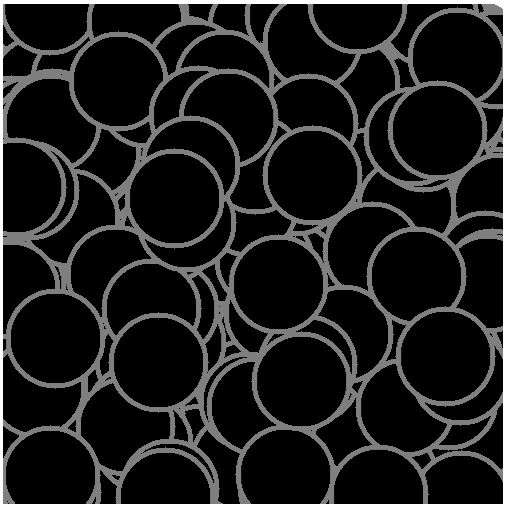
(a) Crater Area For 200,000 Years



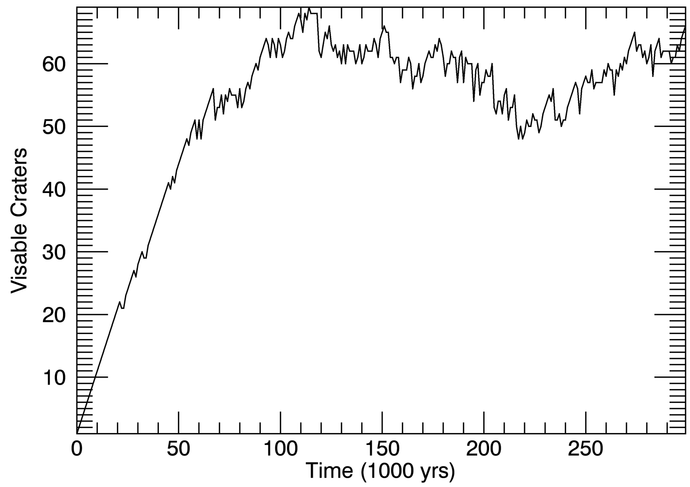
(b) Crater Saturation Curve for 200,000 Years

Figure 3: Crater Plots For Interval Of 200,000 Years

1.2.3 Third Time Interval: 300,000 Years



(a) Crater Area For 300,000 Years



(b) Crater Saturation Curve for 300,000 Years

Figure 4: Crater Plots For Interval Of 300,000 Years

1.3 Plot Descriptions

1.3.1 First Time Interval: 100,000 Years

The subplot of the crater area and the subplot of the saturation curve for this time interval can be seen in figure 2. The subplot on the left, figure 2a, shows the craters and their rims as they appear for the first 100,000 years. The subplot on the right, figure 2b, shows our saturation curve. In this image, the saturation curve is still increasing, thus indicating that the impact area is not yet saturated. This makes sense because there is still white space in the crater area image.

1.3.2 Second Time Interval: 200,000 Years

The subplot of the crater area and the subplot of the saturation curve for this time interval can be seen in figure 3. The image of the crater area, figure 3a, shows that more of the white space is covered up. The saturation curve in figure 3b appears to be leveling out, however the slight decrease in the curve indicated that we needed to run the simulation further to see if there were any significant changes.

1.3.3 Third Time Interval: 300,000 Years

The subplot of the crater area and the subplot of the saturation curve for this time interval can be seen in figure 4. The image of the crater area in figure 4a no longer has any visible white space. From a qualitative standpoint, the impact area is now saturated. The saturation curve in figure 4b has reached a steady state and is oscillating about the saturation value. The number of craters that will produce saturation is about 60 for the given parameters described in the initial assumptions.

1.3.4 Conclusion

Saturation was reached in about 120,000 years with about 60 detectable craters. All craters were the same size which is why saturation was reached relatively quickly compared to part 2, which will be discussed in detail later. The constant diameter of 50 km for the craters also contributed to the quick saturation time. The minimum crater diameter per the requirement from the assignment sheet is 10 km.

2 Part 2: Assumption Change

2.1 Crater Size Assumption Change

In part 1, the size of the impacters and craters were assumed to be constant. Now a power law distribution describes the crater size frequency. The power law is defined as:

$$N_{cum} = cD^{-b} \tag{3}$$

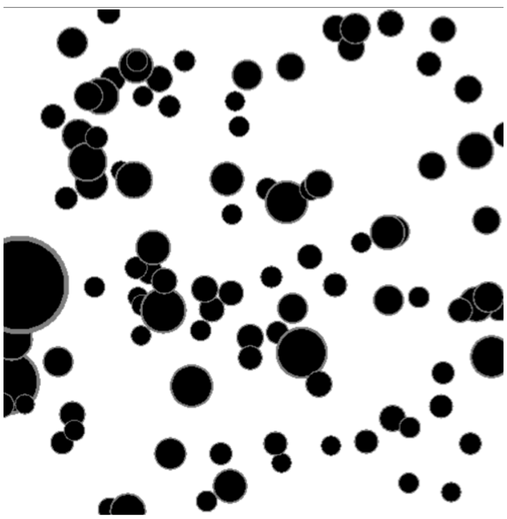
where N is the number of asteroids at a given size, c is a constant, D is the diameter of the asteroid, and b is the degree of the power polynomial.

We chose a value of 4 for b because that is commonly accepted measurement for the Kuiper Belt. The value for c is assumed to be 1, and the diameter D of the impacters was varied from 1 km to 30 km. This resulted in craters that were anywhere between 10 km and 300 km. We chose an upper limit of 300 km craters so as to not allow for complete erasure of all craters on the surface by one "Armageddon" sized impacter. This allows us to make a more simple comparison of saturation time between the two scenarios.

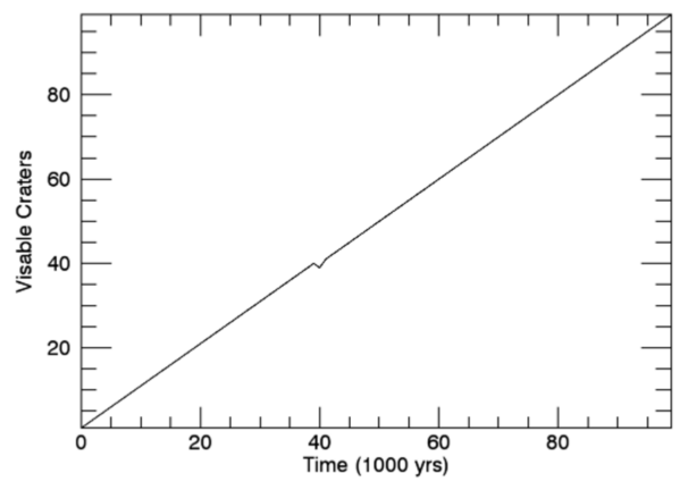
For the remaining assumptions, all are unchanged. The craters are still circular, velocity is the same for all impacters, and depth for all craters is constant, and saturation is still determined in the same manner described above in section. 1.1.9.

2.2 Plots

2.2.1 First Time Interval: 100,000 Years



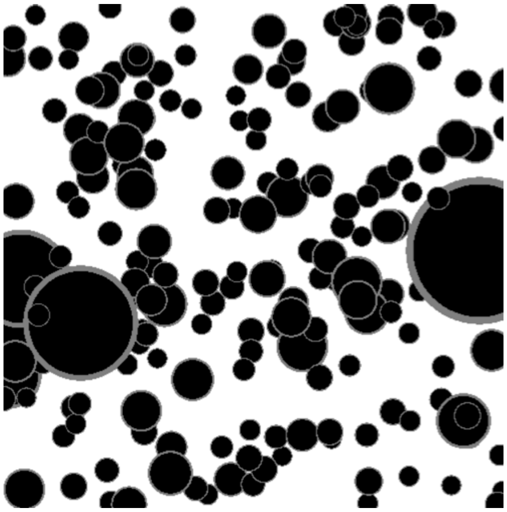
(a) Crater Area For 100,000 Years



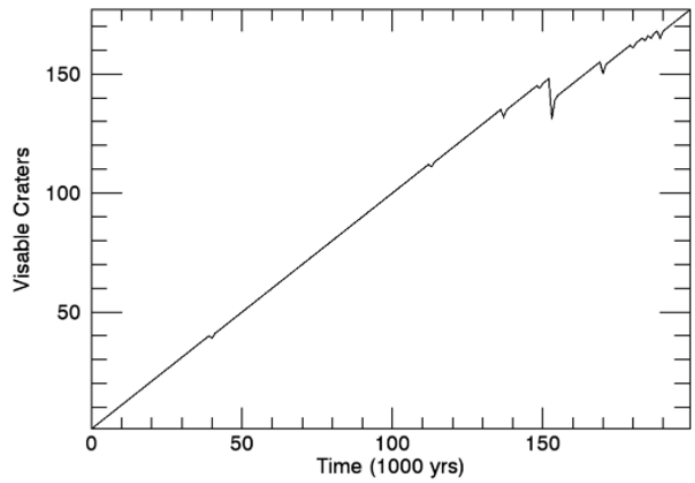
(b) Crater Saturation Curve for 100,000 Years

Figure 5: Crater Plots For Interval Of 100,000 Years

2.2.2 Second Time Interval: 200,000 Years



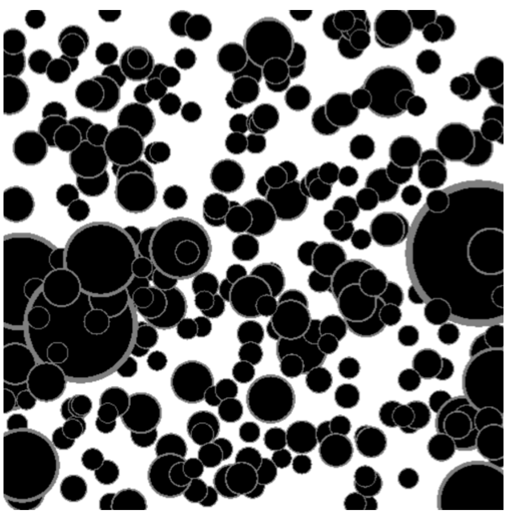
(a) Crater Area For 200,000 Years



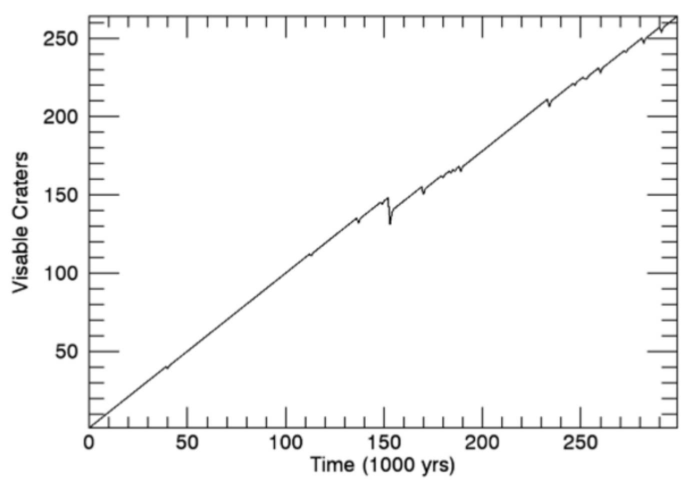
(b) Crater Saturation Curve for 200,000 Years

Figure 6: Crater Plots For Interval Of 200,000 Years

2.2.3 Third Time Interval: 300,000 Years



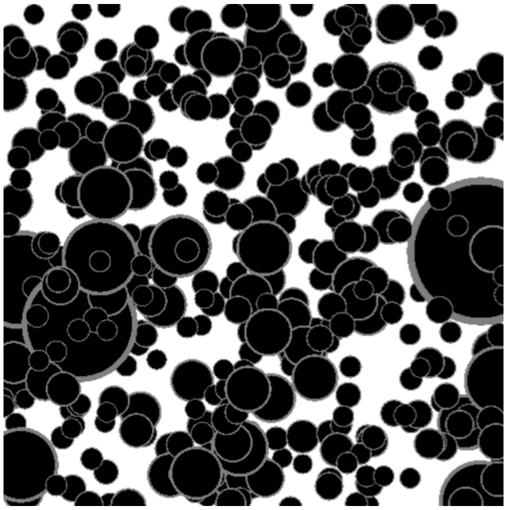
(a) Crater Area For 300,000 Years



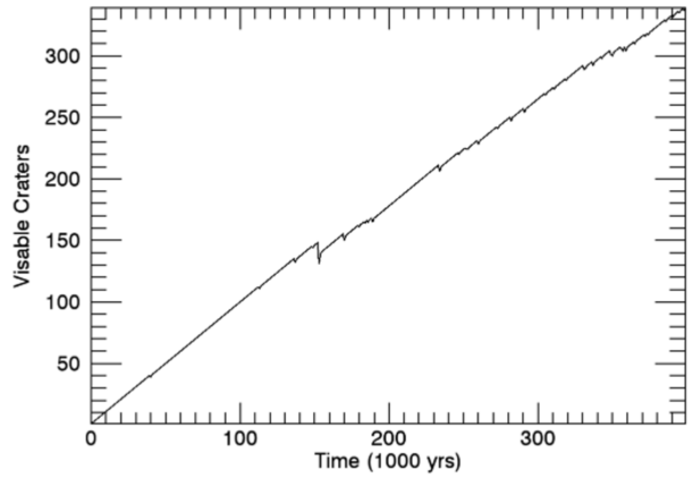
(b) Crater Saturation Curve for 300,000 Years

Figure 7: Crater Plots For Interval Of 300,000 Years

2.2.4 Fourth Time Interval: 400,000 Years



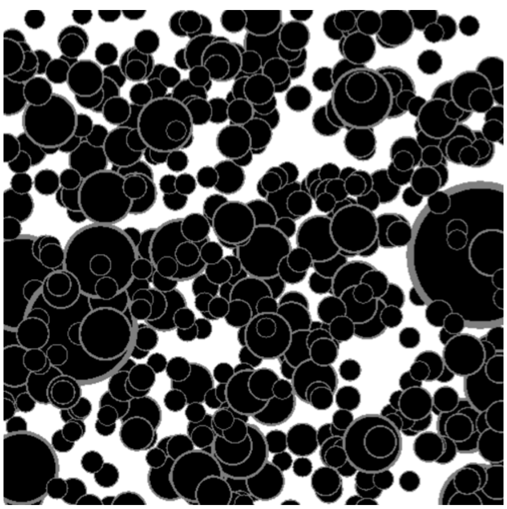
(a) Crater Area For 400,000 Years



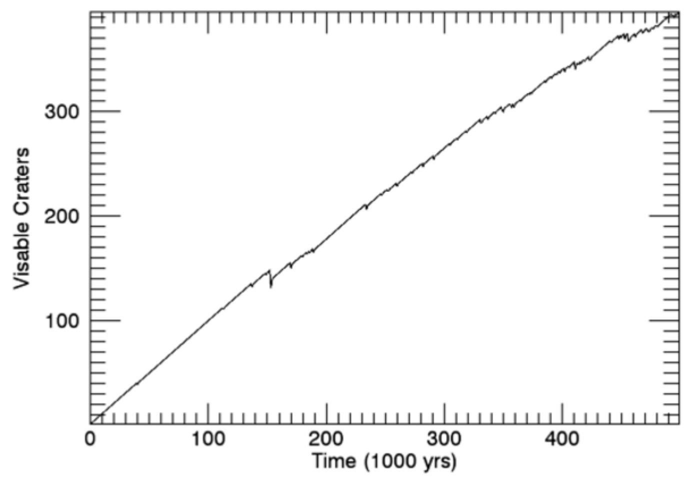
(b) Crater Saturation Curve for 400,000 Years

Figure 8: Crater Plots For Interval Of 400,000 Years

2.2.5 Fifth Time Interval: 500,000 Years



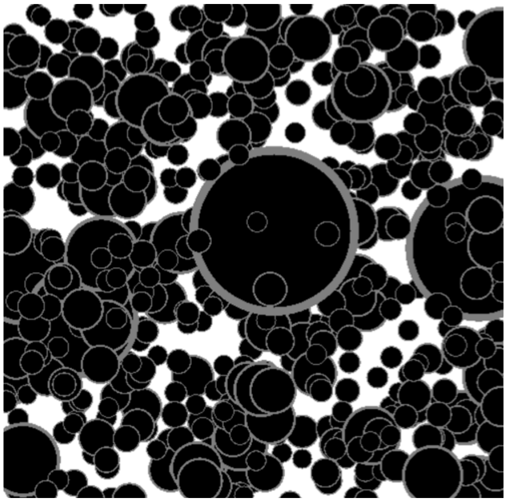
(a) Crater Area For 500,000 Years



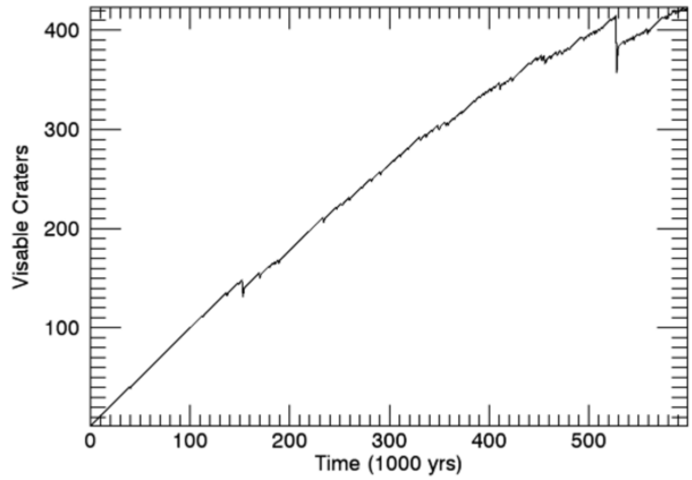
(b) Crater Saturation Curve for 500,000 Years

Figure 9: Crater Plots For Interval Of 500,000 Years

2.2.6 Sixth Time Interval: 600,000 Years



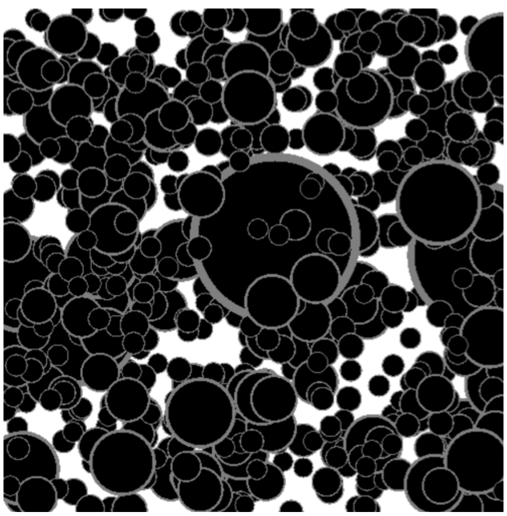
(**a**) Crater Area For 600,000 Years



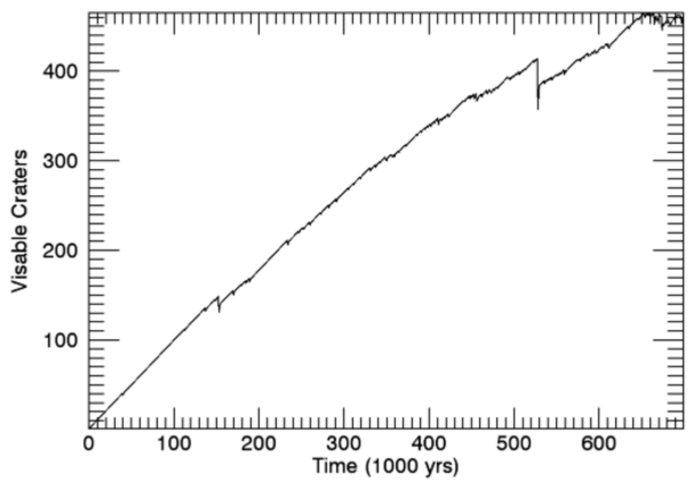
(**b**) Crater Saturation Curve for 600,000 Years

Figure 10: Crater Plots For Interval Of 600,000 Years

2.2.7 Seventh Time Interval: 700,000 Years



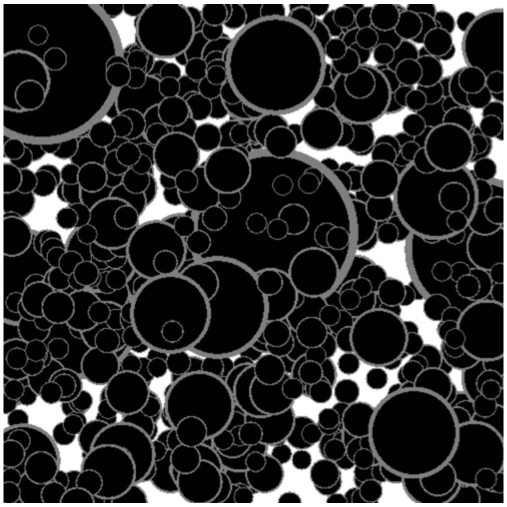
(**a**) Crater Area For 700,000 Years



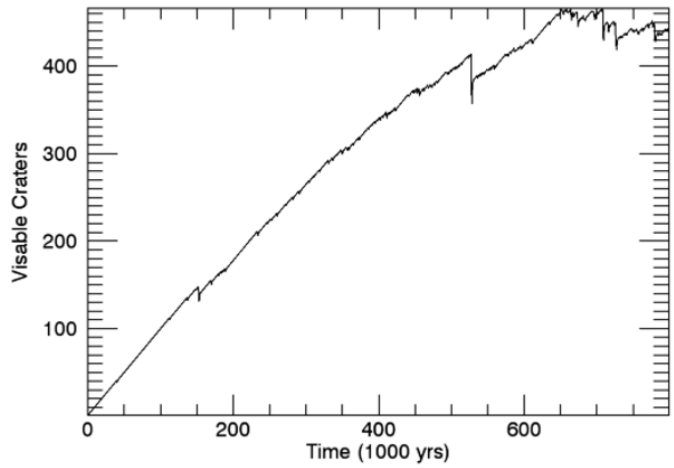
(**b**) Crater Saturation Curve for 700,000 Years

Figure 11: Crater Plots For Interval Of 700,000 Years

2.2.8 Eighth Time Interval: 800,000 Years



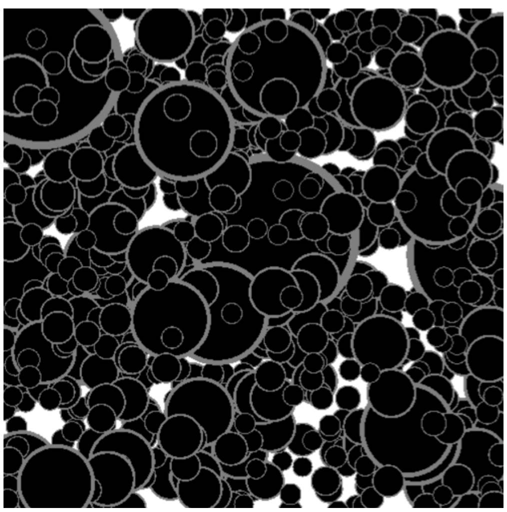
(a) Crater Area For 800,000 Years



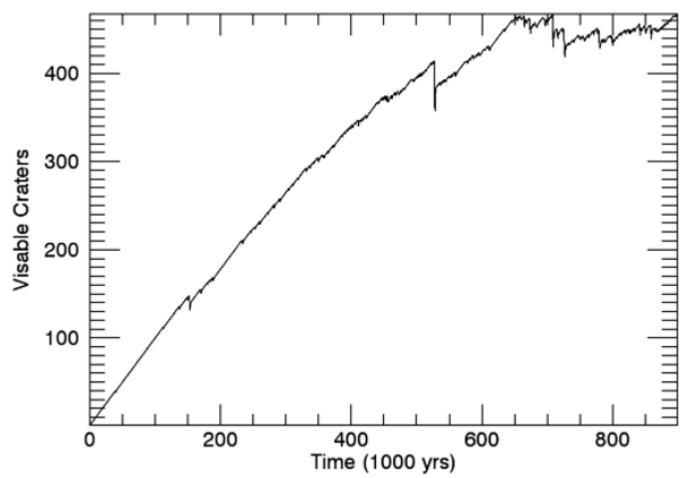
(b) Crater Saturation Curve for 800,000 Years

Figure 12: Crater Plots For Interval Of 800,000 Years

2.2.9 Ninth Time Interval: 900,000 Years



(a) Crater Area For 900,000 Years



(b) Crater Saturation Curve for 900,000 Years

Figure 13: Crater Plots For Interval Of 900,000 Years

2.3 Plot Descriptions

2.3.1 First Time Interval: 100,000 Years

Figure 5 shows the crater plots for time interval of 100,000 years for our power law distribution simulation. The first subplot, figure 5a, shows the impact area during this time period. The area has a lot of white space still, so qualitatively we can see that it is not yet saturated. Figure 5b on the right shows the saturation plot. The slope of the plot is constantly increasing which quantitatively shows that the area is not yet saturated.

2.3.2 Second Time Interval: 200,000 Years

Figure 6 shows the crater plots for time interval of 200,000 years for our power law distribution simulation. The first subplot, figure 6a, shows the impact area during this time period. The area has a lot of white space still, so qualitatively we can see that it is not yet saturated. Figure 6b on the right shows the saturation plot. The slope of the plot is constantly increasing which quantitatively shows that the area is not yet saturated.

2.3.3 Third Time Interval: 300,000 Years

Figure 7 shows the crater plots for time interval of 300,000 years for our power law distribution simulation. The first subplot, figure 7a, shows the impact area during this time period. The area has a lot of white space still, so qualitatively we can see that it is not yet saturated. Figure 7b on the right shows the saturation plot. The slope of the plot is constantly increasing which quantitatively shows that the area is not yet saturated.

2.3.4 Fourth Time Interval: 400,000 Years

Figure 8 shows the crater plots for time interval of 400,000 years for our power law distribution simulation. The first subplot, figure 8a, shows the impact area during this time period. The area has a lot of white space still, so qualitatively we can see that it is not yet saturated. Figure 8b on the right shows the saturation plot. The slope of the plot is constantly increasing which quantitatively shows that the area is not yet saturated.

2.3.5 Fifth Time Interval: 500,000 Years

Figure 9 shows the crater plots for time interval of 500,000 years for our power law distribution simulation. The first subplot, figure 9a, shows the impact area during this time period. The area has a lot of white space still, so qualitatively we can see that it is not yet saturated. Figure 9b on the right shows the saturation plot. The slope of the plot is constantly increasing which quantitatively shows that the area is not yet saturated.

2.3.6 Sixth Time Interval: 600,000 Years

Figure 10 shows the crater plots for time interval of 600,000 years for our power law distribution simulation. The first subplot, figure 10a, shows the impact area during this time period. The area has a lot of white space still, so qualitatively we can see that it is not yet saturated. Figure 10b on the right shows the saturation plot. The slope of the plot is constantly increasing which quantitatively shows that the area is not yet saturated.

2.3.7 Seventh Time Interval: 700,000 Years

Figure 11a shows the crater plots for the time interval of 700,000 years. For approximately the first 600,000 years, the saturation curve in figure 11b increases linearly at a constant rate. However, from approximately 600,000 years to 700,000 years the curve begins to level out and imply that saturation has occurred. However, we decided to run the simulation a bit further to ensure saturation occurred. The plot of the impact area, figure 11a, on the left, shows the impact area. There is still white space on this plot unlike in part one where there was no white space in the impact area when saturation was reached.

2.3.8 Eighth Time Interval: 800,000 Years

Figure 11b shows the saturation curve for the eighth time interval of 800,000 years. The curve ceases to increase between 700,000 years to 800,000 years. Thus, we can quantitatively say that the impact area is saturated and has been since 625,000 years. Figure 11a shows the impact area for this time interval of 800,000 years.

2.3.9 Ninth Time Interval: 900,000 Years

Figure 12b shows the saturation curve for the ninth time interval of 900,000 years. Though in previous sections it was determined that saturation was reached at 625,000 years, we decided to run the simulation for one more iteration of 100,000 years just to see what would happen. Unsurprisingly, the saturation curve remained constant. The impact area shown in figure 12a has slightly less white space, however it remains similar to figure 11a and figure 10a.

2.3.10 Conclusion

In part one the impact area was saturated in only 120,000 years because the diameter of the crater was constant at 50 km. In part 2 the craters were allowed to be any size diameter as long as they were at least 10 km in diameter and smaller than 300 km. This is why the saturation time in part two was more than twice that of part one. When there are a larger number of smaller impacters, the same impact area will take longer to be saturated. This physically represents a planetary surface that is in a much older region of an impacter neighborhood. In these older impacter regions, there are many more smaller objects which can strike a surface. In younger regions, there are relatively larger impacters, which would result in a higher density of large craters. This is the primary difference between the two simulations. In the first simulation, the saturation time was quicker, appropriate for a higher density of large impacters. In the second simulation, the saturation time was much longer since the size frequency distribution allowed for smaller craters which have very little effect on the overall saturation.

From this simulation, we see evidence that the older a system is, the longer it would take for a planetary surface to become saturated with craters. This supports the idea that when we have images of a planetary surface with a power law distribution of crater sizes where the slope of the distribution in log-log space is shallow, we are seeing a relatively young surface. And for a deeper slope in this distribution, we are looking at a relatively older surface.

3 Code

```
pro crater2, PLD = pld
  ;closewin ; this line just closes all open IDL graphics windows currently open.

  ; 1) We have a square portion of planetary surface 500km^2, subject to impacts at average rate of once per 1000yrs.
  impact_site = fltarr(500,500)+1 ;500x500km area of planetary surface with thickness of 1km.

  ; We will be counting which craters are detectable after each impact (basically a function of time) and at each 1000 year time step we will plot how many are visible at that time. The number of detected craters as a function of time will be stored as an array in the 'crater history' array.
  crater_history =[]

  ; A single saturation plot is generated, which is continually updated with each impact (expensive in computing time, but allows you to see the impact of each crater, which is useful when a power law distribution for asteroid sizes is used and an uncommonly large asteroid impacts the surface). We could very well reserve the plotting for once every 1 Myrs, but seeing it in "real time" is much more fun!
  plt = window(dimensions = [500,500])

  ; A single image window is also used to see when and where each impact occurs. This also costs computing time, but we already covered the motivation behind that.
  im1 = window(dimensions = [500,500])
  im1.window.setcurrent


  ; 2) Time is incremented in 1000 yr steps, with one impact per time step, starting at t equals zero.
  t = 0

  ; In order to track each impact location and test for its detectability, we will save some parameters for each impact to an array, where each array element is a dictionary of an impact's parameters (explained below).
  impact_catalog = []



  while t lt 1e4 do begin; hey, we gotta stop somewhere :/

    ; The 'xlist' and 'ylist' hold the coordinates of the impact site within the bounds of each crater.
    xlist = []
    ylist = []

    ; Here is where we can vary the sizes of the impactors. If the keyword 'pld' (power law distrobution) is set, then we have a powerlaw probability distrobution of impactor size, given by the value provided of this keyword.
    if not keyword_set(pld) then R_asteroid = 5 else randomp, R_asteroid , pld,1,range_x = [1,30]

    ; THE crater IS GENERATED  Oh shit, the sky is falling!
    ; As per class discussion, the crater radius generated by the impactor is 10 times the impactor radius.
    R_crater = 10*R_asteroid; in kilometers

    ; NOTE: At this point we reiterate that the crater depth is assumed to be a constant 0.5 km. This will be the same for all impacts.

```

```
; THE LOCATION OF THE IMPACT IS GENERATED   Oh no! That metor is headed straight for that truck! Dont worry, its a Toyota Tundra.  ]B)

; The impact locations are generated randomly for the x and y coordinates on the surface. The '500' ensures we are returned a number between 0 and 500 for the x and y coordinates, since the random function returns only floating values up to .9999
impact_locx = round(randomu(!null)*500) & impact_locy = round(randomu(!null)*500)


; THE DEPTH OF THE crater IS APPLIED TO THE SURFACE
; For every coordinate on the planetary impact surface, we test to see if the coordinates fall within the crater location.
for x =0,499 do begin; in km
  for y =0,499 do begin; km

    ; We can simply use the circle plotting formula from high school geometry class to determine which coordinates are effected by the impact. All locations within the radius of the crater are set to be crater depth of 0.5 km.
    if sqrt((x impact_locx)^2+(y impact_locy)^2) lt R_crater then begin
      ; reduce the surface by amount of crater within crater diameter (carve out huge mass)
      impact_site[x,y]= 0.5
    endif

    ; Critical to the detection effort, we create an inner rim of the crater that is .25 km below the planetary surface.

    ; In other words, the thickness of our planetary impact "slab" is reduced to .75 km.

    ; The extent of the crater is the last 90% of the crater radius.
    if sqrt((x impact_locx)^2+(y impact_locy)^2) ge R_crater*.9 and \$
      sqrt((x impact_locx)^2+(y impact_locy)^2) le R_crater then begin
      impact_site[x,y]= .75

      ; Now that we have coordinates for the impact, we save them to the 'xlist' and 'ylist'.
      xlist = [xlist, x]
      ylist = [ylist, y]
    endif
  endfor
endfor



; THE IMPACT CATALOG IS UPDATED WITH crater METADATA
; The impact will be cataloged! Each entry will be a dictionary variable with the x,y coords within the crater, impact location 'r', and
impact_catalog = [impact_catalog, dictionary('x',xlist, 'y',ylist, 'center',[impact_locx, impact_locy],'r',R_crater) ]

; THE crater IS IDENTIFIED BY THE RIM
; How many craters can we see now, after this latest impact?
observed_craters = 0
; Test the visibility for each crater still in the catalog of (visible)craters, with a counter...named 'counter'.
counter = 0

foreach crater_rim,impact_catalog do begin
  ; Measure the height where each crater's rim should be, according to the location of the rim.
  crater_rim_height = impact_site[crater_rim.x,crater_rim.y]
  ; To detect a crater, at least 30\% of the rim must be at a value of 0.75km (above the 'mantle'?)
  crater_rim_observability = n_elements(where(crater_rim_height eq 0.7500))*1.0/n_elements(crater_rim_height)*100

  ; if the crater rim is 30\% detectable, then we add one to the count of detected craters, else we remove it from the catalog of crators since we dont need to search for visibiliy of this crater anymore. This corresponds to a crater being "erased" by another impact.
  if crater_rim_observability gt 30 then observed_craters +=1
  if crater_rim_observability le 30 then begin
    print, "imapact # ", strcompress(counter), " has observability of ", strcompress(crater_rim_observability), " centered at ", strcompress(crater_rim.R)
    print, "Removed a crater from observed counts."
    ;print, "Stddev(crater_rim_height) = ", Stddev(crater_rim_height)
```

```
        ;stop
        remove,counter, impact_catalog
      endif
      counter+=1; move to the next crator in the catalog
    endforeach

    print, "# Observed craters:", strcompress(observed_craters)
    crater_history = [crater_history, observed_craters]
    print, "# Impacts to date:", strcompress(t+1)

    t+=1; move to the next 1000 yrs when another impact happens

    if t mod 100 eq 0 then begin
      im1.window.setcurrent
      img = image(impact_site, /current, margin = 0)
      plt.window.setcurrent
      pl = plot(indgen(t+1, start= 0),crater_history, /overplot, xtitle ='Time (1000 yrs)',
    ytitle = "Visable Craters")
      stop
    endif

  endwhile


end;
```