Real Langevin

Complex Langevin

Fields

$$\phi$$

 $\phi_R + i\phi_I$

Action

$$S[\phi]$$

 $S[\phi_R + i\phi_I]$

Langevin step

$$\Delta \phi = K \Delta t + \eta(t)$$

$$\Delta \phi_R = K_R \Delta t + \eta_R(t)$$

$$\Delta \phi_I = K_I \Delta t + \eta_I(t)$$

Drift

$$K = -\frac{\delta S}{\delta \phi}$$

$$K_R = -\text{Re}\left[\frac{\delta S[\phi]}{\delta \phi}\right]$$

$$K_I = -\mathrm{Im} \left[\frac{\delta S[\phi]}{\delta \phi} \right]$$

Noise

$$\langle \eta^2 \rangle = 2\Delta t$$

$$\langle \eta_R^2 \rangle = 2N_R \Delta t$$

$$\langle \eta_I^2 \rangle = 2N_I \Delta t$$

$$N_R - N_I = 1$$

$$\langle \eta_R \rangle = \langle \eta_I \rangle = 0$$

 $\langle \eta \rangle = 0$