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Minkowski stochastic quantization

Hiromichi Nakazato and Yoshiya Yamanaka*
Department of Physics, Waseda University, Tokyo 160, Japan
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Stochastic quantization in Minkowski space is discussed in detail. The Fokker-Planck equation corresponding to the complex Langevin equation is derived and solved explicitly in the case of free scalar fields. It turns out that Minkowski stochastic quantization can be formulated in terms of a real positive probability.

In the path-integral quantization, expectation values of physical quantities are calculated by

$$\langle f[\phi]\rangle = \int \mathscr{D}\phi f[\phi]e^{iS[\phi]} \Big/ \int \mathscr{D}\phi e^{iS[\phi]} , \tag{1}$$

where $S[\phi]$ is an action in Minkowski space. It is, however, well known that the expression (1) is formal and ill defined. The usual remedy of this is to rotate the time axis to the imaginary axis (Wick rotation). Then instead of (1), we consider the formula

$$\langle f[\phi]\rangle = \int \mathscr{D}\phi f[\phi]e^{-\bar{S}[\phi]} \Big/ \int \mathscr{D}\phi e^{-\bar{S}[\phi]} , \tag{2}$$

using the real positive weight factor $\mathscr{D}\phi \exp(-\bar{S})$. Thus relativistic quantum field theory is reduced to the form of a partition function in statistical mechanics and its Monte Carlo simulations are extensively performed to give nonperturbative results. Here the Euclidean hypothesis is crucial for simulations because then we have a probability measure. It is impossible to apply the same procedure directly to the formula (1).

Contrary to the above situation, nothing seems to prohibit us from quantizing a system in Minkowski space directly by the stochastic quantization method, which was proposed by Parisi and Wu[1] and has subsequently been developed by many other people under the Euclidean hypothesis. The basic equation of the stochastic quantization, i.e., the Langevin equation, reads, in the original Euclidean form, as

$$\frac{\partial}{\partial t}\phi(x,t) = -\left.\frac{\delta \bar{S}[\phi]}{\delta\phi(x)}\right|_{\phi(x)=\phi(x,t)} + \eta(x,t) , \tag{3}$$

where t denotes fictitious time and $\eta(x,t)$ the Gaussian white noise with the properties

$$\begin{aligned} &\langle \eta(x,t)\rangle = 0 , \\ &\langle \eta(x,t)\eta(x',t')\rangle = 2\delta^4(x-x')\delta(t-t') . \end{aligned} \tag{4}$$

From (3), replacing $-\bar{S}$ with iS, we obtain the following complex Langevin equation corresponding to Minkowski field theory:

$$\frac{\partial}{\partial t}\phi(x,t) = i\left.\frac{\delta S[\phi]}{\delta\phi(x)}\right|_{\phi(x)=\phi(x,t)} + \eta(x,t) . \tag{5}$$

In fact, some people have attempted the numerical simulations of (5) or other complex Langevin equations.[2]

As for Eq. (3), it is already known that there exists a unique equilibrium state $\exp(-\bar{S})$ irrelevant to the initial condition unless any symmetry causing random walks, such as gauge symmetry, exists. This fact proves the equivalence of stochastic quantization to conventional ones in Euclidean space. Consider the complex Langevin Eq. (5). In this case, we have two problems: (i) whether the system corresponding to (5) has any unique equilibrium and (ii) if it exists, what the equilibrium state is. To these questions, one may answer naively that there exists the unique equilibrium state $\exp(iS)$ which is obtained from the Euclidean state $\exp(-\bar{S})$ through the replacement of $-\bar{S}$ with iS. It should be remarked then that the complex-valued "probability"[3] has been introduced into the theory. But the situation is not so simple, as discussed below.

The purpose of this paper is to answer the above two questions. We can indeed show that even in Minkowski space, if we add an infinitesimal damping term to the system, there exists a unique equilibrium probability distribution which is real positive and different from the naively expected distribution $\exp(iS)$, and that expectation values over this probability are nothing but the values derived from the formal path-integral form (1). We can demonstrate these properties explicitly in the case of free scalar fields.

To discuss the behavior in the limit of $t\to\infty$, it is necessary to go into the Fokker-Planck formalism. Before entering into the Fokker-Planck formalism, a few remarks are needed regarding the Langevin equation. For simplicity and definiteness, we consider a free-scalar-field case characterized by an action

$$S_0 = \int d^4x \tfrac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2) \tag{6a}$$

or, in momentum space,

$$S_0 = \int d^4k \tfrac{1}{2} q\, |\phi(k)|^2, \quad q = k^2 - m^2 . \tag{6b}$$

Then the Langevin equation takes the form

$$\frac{\partial}{\partial t}\phi(x,t) = i(-\Box - m^2 + i\epsilon)\phi(x,t) + \eta(x,t) . \tag{7}$$

Here we have explicitly introduced an infinitesimal positive number ϵ to lead the system into equilibrium as $t\to\infty$. Note that after all the calculations have been done, ϵ should be set equal to zero. Without this procedure, no equilibrium state could be attained and only a very formal discussion would be allowed. Clearly, the complex Langevin equation (7) brings the field $\phi(x,t)$, which is originally thought as real, an imaginary part as time t develops. That is, based on the complex Langevin equation, we have to deal with the duplicated fields inevitably. This is a very interesting and important point for us to derive the Fokker-Planck equation from the Langevin equation (7).

First let us consider the case where η is real in analogy with the Euclidean case (3). Dividing (7) into real and imaginary parts, we obtain

$$\begin{aligned}\dot{\phi}_R(k,t) &= -q\phi_I(k,t)-\epsilon\phi_R(k,t)+\eta(k,t)\ ,\\ \dot{\phi}_I(k,t) &= q\phi_R(k,t)-\epsilon\phi_I(k,t)\ ,\end{aligned} \tag{8}$$

where $\phi(x,t)=\phi_R(x,t)+i\phi_I(x,t)$, and $\phi_R(k,t)$ and $\phi_I(k,t)$ are Fourier components of $\phi_R(x,t)$ and $\phi_I(x,t)$, respectively. As long as the random source η is real, the second equation in (8) is a constraint equation connecting ϕ_R and ϕ_I.

Statistical averages over η can be realized by the following probability distribution:

$$\exp\left[-\frac{1}{4}\int |\eta(k,t)|^2 d^4k\,dt\right]\mathscr{D}\eta\ . \tag{9}$$

Then the usual procedure leads us to the Fokker-Planck equation for a probability distribution $P[\phi;t]$:

$$\frac{\partial}{\partial t}P[\phi;t]=H[\phi]P[\phi;t]\ , \tag{10a}$$

$$\begin{aligned}H[\phi]=\int d^4k\,\Bigg[&\frac{\delta}{\delta\phi_R(k)}\frac{\delta}{\delta\phi_R(-k)}\\ &+\frac{\delta}{\delta\phi_R(k)}[q\phi_I(k)+\epsilon\phi_R(k)]\\ &+\frac{\delta}{\delta\phi_I(k)}[-q\phi_R(k)+\epsilon\phi_I(k)]\Bigg]\ .\end{aligned} \tag{10b}$$

This equation can be solved exactly. To see this, we introduce a kernel K by the relation

$$P[\phi;t]=\int \mathscr{D}\phi' K[\phi,t;\phi',t_0]P_0[\phi']\ , \tag{11}$$

where

$$\mathscr{D}\phi'=\prod_k d\phi'_R(k)d\phi'_I(k)$$

and P_0 is an initial distribution at $t=t_0$. (We put $t_0=0$ in the following.) With the initial condition $K[\phi,0;\phi',0]=\delta[\phi-\phi']$, we obtain the following form of K:

$$K[\phi,t;\phi',0]=\theta(t)\delta[\phi_R-\Phi_R(\phi',t)]\delta[\phi_I-\Phi_I(\phi',t)]\ , \tag{12}$$

with

$$\begin{aligned}\Phi_R(\phi',t)&=e^{tH^\dagger[\phi']}\phi'_R e^{-tH^\dagger[\phi']}\ ,\\ \Phi_I(\phi',t)&=e^{tH^\dagger[\phi']}\phi'_I e^{-tH^\dagger[\phi']}\ ,\end{aligned} \tag{13}$$

where $H^\dagger$ is an adjoint operator of H:

$$\begin{aligned}H^\dagger[\phi']=\int d^4k\,\Bigg[&\frac{\delta}{\delta\phi'_R(k)}\frac{\delta}{\delta\phi'_R(-k)}\\ &-[q\phi'_I(k)+\epsilon\phi'_R(k)]\frac{\delta}{\delta\phi'_R(k)}\\ &-[-q\phi'_R(k)+\epsilon\phi'_I(k)]\frac{\delta}{\delta\phi'_I(k)}\Bigg]\ .\end{aligned} \tag{14}$$

Introducing the two operators $D_R(\phi',t)$ and $D_I(\phi',t)$ by

$$\begin{aligned}D_R(\phi',t)&=e^{tH^\dagger[\phi']}\frac{\delta}{\delta\phi'_R}e^{-tH^\dagger[\phi']}\ ,\\ D_I(\phi',t)&=e^{tH^\dagger[\phi']}\frac{\delta}{\delta\phi'_I}e^{-tH^\dagger[\phi']}\ ,\end{aligned} \tag{15}$$

we get a set of differential equations for the Φ's and D's:

$$\begin{aligned}\dot{\Phi}_R+\epsilon\Phi_R&=2D_R-q\Phi_I\ ,\\ \dot{\Phi}_I+\epsilon\Phi_I&=q\Phi_R\ ,\\ \dot{D}_R-\epsilon D_R&=-qD_I\ ,\\ \dot{D}_I-\epsilon D_I&=qD_R\ .\end{aligned} \tag{16}$$

After some elementary calculations, solutions of (16) turn out to be

$$\begin{aligned}\Phi_R=\ &\phi'_R(k)e^{-\epsilon t}\cos(qt)-\phi'_I(k)e^{-\epsilon t}\sin(qt)\\ &+\frac{1}{\epsilon(\epsilon^2+q^2)}\Bigg[[\epsilon q\sin(qt)\cosh(\epsilon t)+(2\epsilon^2+q^2)\cos(qt)\sinh(\epsilon t)]\frac{\delta}{\delta\phi'_R(k)}\\ &\qquad+\{-\epsilon^2\sin(qt)\cosh(\epsilon t)+[-(\epsilon^2+q^2)\sin(qt)+\epsilon q\cos(qt)]\sinh(\epsilon t)\}\frac{\delta}{\delta\phi'_I(k)}\Bigg]\ ,\end{aligned} \tag{17a}$$

$$\begin{aligned}\Phi_I=\ &\phi'_I(k)e^{-\epsilon t}\cos(qt)+\phi'_R(k)e^{-\epsilon t}\sin(qt)\\ &+\frac{1}{\epsilon(\epsilon^2+q^2)}\Bigg[[-\epsilon q\sin(qt)\cosh(\epsilon t)+q^2\cos(qt)\sinh(\epsilon t)]\frac{\delta}{\delta\phi'_I(k)}\\ &\qquad+\{-\epsilon^2\sin(qt)\cosh(\epsilon t)+[-(\epsilon^2+q^2)\sin(qt)+\epsilon q\cos(qt)]\sinh(\epsilon t)\}\frac{\delta}{\delta\phi'_R(k)}\Bigg]\ .\end{aligned} \tag{17b}$$

Then the kernel K is expressed as

$$K[\phi,t;\phi',0]=\theta(t)\int \mathscr{D}\lambda\mathscr{D}\xi \exp\left[\int d^4k(-a_1\lambda^2-a_2\xi^2+iR\lambda+iI\xi-a_3\lambda\xi)\right] \tag{18a}$$

with

$$\begin{aligned}
a_1&=\frac{e^{-\epsilon t}}{2\epsilon(\epsilon^2+q^2)}(\epsilon\sin(qt)[\epsilon\sin(qt)+q\cos(qt)]\cosh(\epsilon t)+\{\epsilon^2+q^2+\epsilon\cos(qt)[\epsilon\cos(qt)-q\sin(qt)]\}\sinh(\epsilon t))\ ,\\
a_2&=\frac{e^{-\epsilon t}}{2\epsilon(\epsilon^2+q^2)}(-\epsilon\sin(qt)[\epsilon\sin(qt)+q\cos(qt)]\cosh(\epsilon t)+\{\epsilon^2+q^2-\epsilon\cos(qt)[\epsilon\cos(qt)-q\sin(qt)]\}\sinh(\epsilon t))\ ,\\
a_3&=\frac{e^{-\epsilon t}}{\epsilon(\epsilon^2+q^2)}(\{-\epsilon\sin(qt)[\epsilon\cos(qt)-q\sin(qt)]\cosh(\epsilon t)+\epsilon\cos(qt)[\epsilon\sin(qt)+q\cos(qt)\sinh(\epsilon t)\})\ ,\\
R&=\phi_R(k)-\phi'_R(k)e^{-\epsilon t}\cos(qt)+\phi'_I(k)e^{-\epsilon t}\sin(qt)\ ,\\
I&=\phi_I(k)-\phi'_R(k)e^{-\epsilon t}\sin(qt)-\phi'_I(k)e^{-\epsilon t}\cos(qt)\ .
\end{aligned} \tag{18b}$$

After performing the integrations over λ and ξ, we finally obtain the explicit form of the probability distribution $P[\phi;t]$:

$$\begin{aligned}
P[\phi;t]&=\int \mathscr{D}\phi' K[\phi,t;\phi',0]P_0[\phi']\\
&=N\theta(t)\int \mathscr{D}\phi' \exp\left[-\int d^4k\left[\frac{1}{4c_+}|-\beta_2R+\beta_1I|^2+\frac{1}{4c_-}|\alpha_2R-\alpha_1I|^2\right]\right]P_0[\phi']\ ,
\end{aligned} \tag{19}$$

where real numbers α_i, β_i $(i=1,2)$, and $c_\pm$ are given by

$$c_\pm=\{a_1+a_2\pm[(a_1-a_2)^2+a_3{}^2]^{1/2}\}/2\ ,$$

$$\begin{bmatrix} a_1 & a_3/2\\ a_3/2 & a_2\end{bmatrix}=\begin{bmatrix}\alpha_1 & \beta_2\\ \alpha_2 & \beta_2\end{bmatrix}\begin{bmatrix} c_+ & 0\\ 0 & c_-\end{bmatrix}\begin{bmatrix}\alpha_1 & \alpha_2\\ \beta_1 & \beta_2\end{bmatrix},\quad \alpha_1{}^2+\alpha_2{}^2=\beta_1{}^2+\beta_2{}^2=1\ .$$

The above explicit expressions show that in the $t\to\infty$ limit there exists a unique equilibrium state irrelevant to the initial distribution P_0 owing to the damping factor $\exp(-\epsilon t)$. So we can safely take the limit $t\to\infty$ in (19) to get

$$P_{\rm eq}[\phi]=\lim_{t\to\infty}P[\phi;t]=N'\exp\left[-\epsilon\int d^4k\{|\phi_R(k)|^2+[1+\epsilon^2/(\epsilon^2+q^2)]|\phi_I(k)|^2-2(\epsilon/q)\phi_R(k)\phi_I(-k)\}\right]\ . \tag{20}$$

Notice that in spite of the fact that the equilibrium distribution (20) never takes the form of $\exp(iS)$, expectation values over this real probability distribution coincide with those obtained from the formal path-integral form (1) which are, in general, complex quantities. In fact, for example, the two-point correlation function $\langle\phi(k)\phi(k')\rangle$ takes the usual form, i.e.,

$$\begin{aligned}
\langle\phi(k)\phi(k')\rangle&=\langle\phi_R(k)\phi_R(k')-\phi_I(k)\phi_I(k')\rangle+i\langle\phi_R(k)\phi_I(k')+\phi_I(k)\phi_R(k')\rangle\\
&=\delta^4(k+k')\left[\frac{\epsilon}{q^2+\epsilon^2}+i\frac{q}{q^2+\epsilon^2}\right]=\delta^4(k+k')\frac{i}{q+i\epsilon}=\delta^4(k+k')\frac{i}{k^2-m^2+i\epsilon}\ .
\end{aligned} \tag{21}$$

This example shows that in Minkowski-space stochastic quantization the complex property of expectation values is traced back to that of field variables themselves, while in the path-integral quantization the formal measure $\exp(iS)$ makes them complex. In this sense, the complex Langevin equation (6) can bring an interesting method of simulating the system directly in Minkowski space,[2] because no complex probability such as $\exp(iS)$ arises.

Next, let us generalize the complex Langevin equation (6). In the above discussions we have assumed the real η. We can also consider the case where the random noise η is complex and has two degrees of freedom, η_R and η_I $(\eta=\eta_R+i\eta_I)$. In this case, the Langevin equation corresponding to Eq. (8) becomes

$$\begin{aligned}
\dot\phi_R(k,t)&=-q\phi_I(k,t)-\epsilon\phi_R(k,t)+\eta_R(k,t)\ ,\\
\dot\phi_I(k,t)&=q\phi_R(k,t)-\epsilon\phi_I(k,t)+\eta_I(k,t)\ .
\end{aligned} \tag{22}$$

Two independent sources η_R and η_I have the Gaussian white-noise properties, respectively,

$$\begin{aligned}
\langle\eta_R(k,t)\eta_R(k',t')\rangle&=2\alpha\delta^4(k+k')\delta(t-t')\ ,\\
\langle\eta_I(k,t)\eta_I(k',t')\rangle&=2\beta\delta^4(k+k')\delta(t-t')\ ,\\
\langle\eta_R(k,t)\eta_I(k',t')\rangle&=0\ ,
\end{aligned} \tag{23}$$

where α and β are real positive parameters with $\alpha-\beta=1$. In this case, the Fokker-Planck equation

$$\frac{\partial}{\partial t}P[\phi;t]=\tilde{H}[\phi]P[\phi;t]\,,$$
$$\tilde{H}[\phi]=\int d^4k\left[\alpha\frac{\delta}{\delta\phi_R(k)}\frac{\delta}{\delta\phi_R(-k)}+\beta\frac{\delta}{\delta\phi_I(k)}\frac{\delta}{\delta\phi_I(-k)}+\frac{\delta}{\delta\phi_R(k)}[q\phi_I(k)+\epsilon\phi_R(k)]+\frac{\delta}{\delta\phi_I(k)}[-q\phi_R(k)+\epsilon\phi_I(k)]\right] \tag{24}$$

can also be solved to have a similar solution to (19). It is easily shown that the unique equilibrium state takes the Gaussian form

$$P_{\rm eq}[\phi]=N\exp\left[-\int d^4k\left[\frac{1}{4c_+}\,|\bar{\phi}_R\,|^2+\frac{1}{4c_-}\,|\bar{\phi}_I\,|^2\right]\right], \tag{25}$$

where $\bar{\phi}_R$ and $\bar{\phi}_I$ are linear combinations of ϕ_R and ϕ_I:

$$\begin{bmatrix}\bar{\phi}_R\\ \bar{\phi}_I\end{bmatrix}=\begin{bmatrix}-\beta_2 & \beta_1\\ \alpha_2 & -\alpha_1\end{bmatrix}\begin{bmatrix}\phi_R\\ \phi_I\end{bmatrix}. \tag{26}$$

Note that four numbers, α_i and β_i ($i=1,2$), are independent of α and β while $c_\pm$ is expressed as

$$c_\pm=[(\alpha+\beta)/\epsilon\pm(\alpha-\beta)(\epsilon^2+q^2)^{-1/2}]/4$$

and that if we put $\alpha=1$, $\beta=0$ in Eq. (25), the probability distribution (19) can be reproduced. The Gaussian form in (25) is convenient to discuss the expectation values of the functionals of ϕ. Inverting (26) and noting $\alpha_1{}^2+\alpha_2{}^2=\beta_1{}^2+\beta_2{}^2=1$, we can express the complex-field variable ϕ as

$$\begin{aligned}\phi(k)&=\phi_R(k)+i\phi_I(k)\\ &=e^{i\zeta(k)}[\bar{\phi}_R(k)-i\bar{\phi}_I(k)]\end{aligned} \tag{27}$$

with

$$\tan\zeta(k)=[(\epsilon^2+q^2)^{1/2}-\epsilon]/q\ .$$

Then the expectation value of $F[\phi]$ becomes

$$\begin{aligned}\langle F[\phi]\rangle&=\int\mathscr{D}\phi F[\phi]P_{\rm eq}[\phi]\\ &=\int\mathscr{D}\bar{\phi}_R\mathscr{D}\bar{\phi}_I F[e^{i\zeta}(\bar{\phi}_R-i\bar{\phi}_I)]P_{\rm eq}[\bar{\phi}_R,\bar{\phi}_I]\\ &=\int\mathscr{D}\Phi F[\Phi]\exp\{iS_0[\Phi]/(\alpha-\beta)\}\ .\end{aligned} \tag{28}$$

In the last stage of the above equation, we have changed the integration variable from $\bar{\phi}_R$ to $\phi=(\bar{\phi}_R-i\bar{\phi}_I)\exp(i\zeta)$ and performed the $\bar{\phi}_I$ integration. The above equation proves the equivalence of the stochastic quantization in Minkowski space to the path-integral quantization because $\alpha-\beta=1$.

In summary, the complex Langevin equation describing the quantum system in Minkowski space can be reasonably formulated in the sense of a real positive probability. This probability measure may be considered as an alternative definition of the Feynman measure $\exp(iS)$.

In this paper we have discussed only the free-scalar-field case and finally obtain a unique equilibrium distribution (19) or (25). In interacting cases, the Fokker-Planck equation becomes so complicated that we have not found any equilibrium yet. If we introduce a damping factor, such as the ϵ term in (7), however, the same method based on a hidden supersymmetry[4] can work well even in Minkowski space to show the equivalence of the stochastic quantization method to the conventional method perturbatively.[5] By using the supersymmetry of the generating functional of the Langevin equation, the proof of the equivalence to the conventional quantization method has been given nonperturbatively but only in a very formal manner.[6] Unfortunately, in the interacting case, it is not clear now whether the unique equilibrium state exists and if it exists what the equilibrium state is. Further applications to vector and spinor fields are now in progress.

Finally, we comment on the relation between the stochastic quantization in Minkowski space and thermo field dynamics[7] (TFD) with respect to the duplication of field variables. In thermo field dynamics, degrees of freedom are duplicated by the introduction of the tilde field $\tilde{\phi}$ and the two-point Green's functions at zero temperature are given by

$$\begin{aligned}&\langle\phi(k)\phi(k')\rangle_{\rm TFD}=\delta^4(k+k')\frac{i}{q+i\epsilon}\,,\\ &\langle\tilde{\phi}(k)\tilde{\phi}(k')\rangle_{\rm TFD}=\delta^4(k+k')\frac{-i}{q-i\epsilon}\,,\\ &\langle\phi(k)\tilde{\phi}(k')\rangle_{\rm TFD}=0\ .\end{aligned} \tag{29}$$

On the other hand, remembering that the probability distribution (19) or (25) is real and the two-point correlation function $\langle\phi(k)\phi(k')\rangle$ coincides with $\langle\phi(k)\phi(k')\rangle_{\rm TFD}$, we get similar quantities as in (29),

$$\begin{aligned}&\langle\phi(k)\phi(k')\rangle=\delta^4(k+k')\frac{i}{q+i\epsilon}\,,\\ &\langle\phi^*(k)\phi^*(k')\rangle=\delta^4(k+k')\frac{-i}{q-i\epsilon}\,,\end{aligned} \tag{30a}$$

with one exception:

$$\langle\phi(k)\phi^*(k')\rangle=\delta^4(k-k')(\alpha+\beta)/\epsilon\ . \tag{30b}$$

This correspondence is an interesting implication of the relationship between two apparently different approaches and through it we may obtain a deeper understanding of stochastic quantization.

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*Present address: Department of Physics, University of Alberta, Edmonton, Alberta T6G 2J1, Canada.

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