Deriving the action for the lattice

Continuum action:

$r_\perp^2 = (x^2+y^2)$

$$S[\phi] = \int d^dx\, d\tau \left[\phi^*\left(\partial_\tau - \mu - \frac{\nabla^2}{2m} - \frac{m}{2}\omega_{tr}^2 r_\perp^2 - i\omega_z(x\partial_y - y\partial_x)\right)\phi + \lambda(\phi^*\phi)^2\right]$$

Discrete derivatives:

$$\partial_j \phi = \frac{1}{a}(\phi_{r-\hat{j}} - \phi_r) \quad \text{and} \quad \nabla^2\phi = \sum_{j=1}^{d} \frac{1}{a^2}\left(\phi_{r-\hat{j}} - 2\phi_r + \phi_{r+\hat{j}}\right)$$

$$\phi^*\partial_\tau\phi = \frac{1}{d\tau}\left(\phi^*_r \phi_{r-\hat{\tau}} - \phi^*_r\phi_r\right)$$

$$\phi^*\nabla^2\phi = \frac{1}{a^2}\sum_{j=1}^{d}\left(\phi^*_r\phi_{r-\hat{j}} - 2\phi^*_r\phi_r + \phi^*_r\phi_{r+\hat{j}}\right)$$

$$\phi^*\partial_x\phi = \frac{1}{a}\left(\phi^*_r\phi_{r-\hat{x}} - \phi^*_r\phi_r\right)$$

$$\phi^*\partial_y\phi = \frac{1}{a}\left(\phi^*_r\phi_{r-\hat{y}} - \phi^*_r\phi_r\right)$$

$$\int d^dx\, d\tau \rightarrow \sum_{\vec{x},\tau} a^d d\tau$$ (a is spatial lattice spacing. $d\tau$ is temp.)

So our discrete (lattice) action is:

$$S = \sum_{\vec{x},\tau}\Big[a^d\phi^*_r\phi_{r-\tau} - a^d\phi^*_r\phi_r - a^d d\tau\,\mu\,\phi^*_r\phi_r - \frac{a^{d-2}d\tau}{2m}\sum_{j=1}^{d}\left(\phi^*_r\phi_{r-\hat{j}} - 2\phi^*_r\phi_r + \phi^*_r\phi_{r+\hat{j}}\right)$$
$$- \frac{m a^d d\tau}{2}\omega_{tr}^2 r_\perp^2\phi^*_r\phi_r - i\omega_z a^{d-1}d\tau\left(x\phi^*_r\phi_{r-\hat{y}} - x\phi^*_r\phi_r - y\phi^*_r\phi_{r-\hat{x}}\right.$$
$$\left. + y\phi^*_r\phi_r\right) + a^d d\tau\,\lambda(\phi^*_r\phi_r)^2\Big]$$

Let's update our parameters to incorporate $d\tau$ and a

$x = a\tilde{x}$ $\quad y = a\tilde{y}$ $\quad r^2 = a^2\tilde{r}^2$ $\quad \mu = \frac{\tilde{\mu}}{d\tau}$ $\quad m = \frac{\tilde{m}\,d\tau}{a^2}$ $\quad \omega_{tr} = \tilde{\omega}_{tr}\frac{a^2}{d\tau}$

$\omega_z = \frac{\tilde{\omega}_z}{d\tau}$ $\quad \lambda = \frac{\tilde{\lambda}}{d\tau}$

$$S = \sum_{\vec{x},\tau} \Big[a^d \phi^*_r \phi_{r-\tau} - a^d \phi^*_r \phi_r - a^d \tilde{\mu} \phi^*_r \phi_r - \frac{a^d}{2\tilde{m}} \sum_{j=1}^{d} \left(\phi^*_r \phi_{r-\hat{j}} - 2\phi^*_r \phi_r + \phi^*_r \phi_{r+\hat{j}} \right)$$

$$- a^d \frac{\tilde{m}}{2} \tilde{\omega}_{tr}^2 \tilde{r}_\perp^2 \phi^*_r \phi_r - i\omega_z a^d \left(\tilde{x} \phi^*_r \phi_{r-\hat{y}} - \tilde{x} \phi^*_r \phi_r - \tilde{y} \phi^*_r \phi_{r-\hat{x}} + \tilde{y} \phi^*_r \phi_r \right)$$

$$+ a^d \tilde{\lambda} (\phi^*_r \phi_r)^2 \Big]$$

we can pull out a constant factor of a^d now:

$$S = a^d \sum_{\vec{x},\tau} \Big[\phi^*_r \phi_{r-\tau} - \phi^*_r \phi_r - \tilde{\mu} \phi^*_r \phi_r - \frac{1}{2\tilde{m}} \sum_{j=1}^{d} \left(\phi^*_r \phi_{r-\hat{j}} - 2\phi^*_r \phi_r + \phi^*_r \phi_{r+\hat{j}} \right)$$

$$- \frac{\tilde{m}}{2} \tilde{\omega}_{tr}^2 \tilde{r}_\perp^2 \phi^*_r \phi_r - i\omega_z \left(\tilde{x} \phi^*_r \phi_{r-\hat{y}} - \tilde{x} \phi^*_r \phi_r - \tilde{y} \phi^*_r \phi_{r-\hat{x}} + \tilde{y} \phi^*_r \phi_r \right)$$

$$+ \tilde{\lambda} (\phi^*_r \phi_r)^2 \Big]$$