Residual Reduction Algorithm (RRA)

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1 Introduction

2 Reducing Residuals

Throughout this paper, when we use the term "reduce a residual", or "reduce" a DC offset, it really means "try to eliminate" a residual or DC offset. That is, our strategy is to compute parameter alterations that would, in theory, completely eliminate the DC offset of a particular residual. But since the theoretical solutions we compute are not entirely true in practice since there are many factors affecting a particular residual other than just the one parameter we are altering. However, this failure to completely eliminate the DC offset is actually a good thing: we know that there are unmodeled forces in our system (for example, our model has no arms), so we actually do want some small DC offsets to remain for our residuals just to make us feel like we haven't eliminated these unmodeled forces.

To reduce a particular residual R with DC offset $d_R \in \mathbb{R}$ by altering a particular parameter p by an amount Δp, we must compute Δp using the following equation:

$$R_{old} - R_{new} = d_R$$

We require that R_{new} and R_{old} be expressible in terms of inertial parameters and possibly joint variables, and that R_{new} also be expressed in terms of Δp. Then the above equation can be solved for Δp in terms of the inertial parameters and the DC offset d_R. It is easier to see why this equation is true if we look at it this way:

$$R_{new} = R_{old} - d_R$$

Here, R_{old} is the original residual with the DC offset d_R. If we *remove the DC offset* from R_{old}, i.e. if we subtract the DC offset from R_{old}, we get R_{new}.

2.1 Reducing Forward-Backward Rocking

We will reduce the residual MZ by independently altering two parameters: the torso center of mass x-coordinate by an amount Δt_x and the lumbar extension angle by an amount Δl_e.

2.1.1 Altering the Torso Center of Mass

Here we will compute an amount Δt_x by which to alter the x-coordinate of the torso center of mass in order to balance the DC offset of the MZ residual. Let m be the mass of the torso and let $\mathbf{g}$ denote acceleration due to gravity. Let d_{MZ} be the DC offset of the MZ residual. Let $\mathbf{r_0}$ be the moment arm (lever arm) of the torso, which we define to be the vector pointing from the pelvis center of mass to the torso center of mass. Note that $\mathbf{r_0}$ varies as the torso position varies, but its magnitude stays fixed. Then we have that the original value of MZ at any torso position is:

$$MZ_{old} = \mathbf{r_0} \times m\mathbf{g}$$

Let $\mathbf{r_1}$ be the torso moment arm after the center of mass has been displaced in the x direction by Δt_x. Note that $\mathbf{r_1}$ may not have the same magnitude as $\mathbf{r_0}$. Then the new value of MZ is:

$$\begin{aligned} MZ_{new} &= \mathbf{r_1} \times m\mathbf{g} \\ &= (\mathbf{r_0} + (\Delta t_x, 0, 0)) \times m\mathbf{g} \\ &= \mathbf{r_0} \times m\mathbf{g} + (\Delta t_x, 0, 0) \times m\mathbf{g} \end{aligned}$$

The last step is correct since the cross product distributes over addition. Let $\mathbf{d_{MZ}} = (0, 0, d_{MZ})$, i.e. $\mathbf{d_{MZ}}$ is a vector representation of the DC offset. Now we plus the above expressions into the equation $MZ_{old} - MZ_{new} = \mathbf{d_{MZ}}$:

$$\mathbf{r_0} \times m\mathbf{g} - (\mathbf{r_0} \times m\mathbf{g} + (\Delta t_x, 0, 0) \times m\mathbf{g}) = \mathbf{d_{MZ}}$$

The $\mathbf{r_0} \times m\mathbf{g}$ expressions cancel out on both sides, and the value of the remaining cross product is

$$(\Delta t_x, 0, 0) \times m\mathbf{g} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \Delta t_x & 0 & 0 \\ 0 & -mg & 0 \end{vmatrix} = (0, 0, -mg\Delta t_x).$$

So we are left with

$$-(0, 0, -mg\Delta t_x) = (0, 0, d_{MZ})$$

or looking at just the z-coordinates

$$\begin{aligned} mg\Delta t_x &= d_{MZ} \\ \Delta t_x &= \frac{d_{MZ}}{mg}. \end{aligned} \tag{1}$$

So, in order to reduce the DC offset of the MZ residual, i.e. to reduce the average forward-backward rocking motions of a walking model, our computation suggests that we should alter the torso center of mass x-coordinate by an amount d_{MZ}/mg.

2.1.2 Altering the Lumbar Extension Angle

Now we wish to compute an amount Δl_e by which to alter the lumbar extension angle (throughout the entire time interval, not just at the initial time) so that the DC offset for MZ is reduced. We can represent the alteration of the lumbar extension angle with the following geometry: consider the triangle consisting of two vectors $\mathbf{r_0}$ and $\mathbf{r_1}$ with equal length r_0 and with a common starting point with an angle Δl_e between them. Suppose the vectors are oriented so that Δl_e is drawn in a positive sense (counterclockwise) when it is drawn from $\mathbf{r_0}$ to $\mathbf{r_1}$. Let $\mathbf{\Delta l} = \mathbf{r_1} - \mathbf{r_0}$. Assuming Δl_e is small, we can apply an approximation from biomechanics which states that the moment arm (lever arm) of a muscle is equal to $\delta l/\delta\theta$ where δl is the change in length of the muscle when the joint spanned by the muscle rotates by a small angle $\delta\theta$. Applying this approximation to our triangle, we have that

$$r_0 = \Delta l/\Delta l_e$$

$$\Delta l = r_0 \Delta l_e,$$

where $\Delta l = \|\mathbf{\Delta l}\|$. We will show how to compute the (direction of) the vector $\mathbf{\Delta l}$ later. As before, we define $MZ_{new} = \mathbf{r_1} \times m\mathbf{g}$ and $MZ_{old} = \mathbf{r_0} \times m\mathbf{g}$. From the definition of $\mathbf{\Delta l}$, we know that $\mathbf{r_1} = \mathbf{r_0} + \mathbf{\Delta l}$. Plugging into the equation $MZ_{old} - MZ_{new} = \mathbf{d_{MZ}}$, we have

$$\mathbf{r_0} \times m\mathbf{g} - (\mathbf{r_0} \times m\mathbf{g} + \mathbf{\Delta l} \times m\mathbf{g}) = (0, 0, d_{MZ})$$

$$-\mathbf{\Delta l} \times m\mathbf{g} = (0, 0, d_{MZ}).$$

If we write $\mathbf{\Delta l} = (\Delta l_x, \Delta l_y, 0)$, then we have

$$\mathbf{\Delta l} \times m\mathbf{g} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \Delta l_x & \Delta l_y & 0 \\ 0 & -mg & 0 \end{vmatrix} = (0, 0, -mg\Delta l_x).$$