x_i, x_j

$\|x_1\|_2 = \|x_2\|_2 = 1$

$$E(x_i, x_j)$$

$$= \sqrt{\|x_i - x_j\|^2}$$

$$= \sqrt{\|x_i\|^2 + \|x_j\|^2 - 2\langle x_i, x_j\rangle}$$

$$= \sqrt{2(\underbrace{1 - \langle x_i, x_j\rangle}_{C(x_i, x_j)})}$$

$$C(x_i, x_j) = 1 - \frac{\langle x_i, x_j\rangle}{\|x_i\|_2 \|x_j\|}$$

$$= 1 - \langle x_i, x_j\rangle$$

$$E(x_i, x_j) = \sqrt{2 \cdot C(x_i, x_j)}$$

$$C_1 > C_2 \Rightarrow E_1 > E_2$$

P2.

$$\tilde{X} = N \times (D+1)$$

$$\tilde{X}^T \tilde{X} = (D+1) \times (D+1)$$

$$r(\tilde{X}^T \tilde{X}) < D+1$$

$$r(\tilde{X}) \le \min(N, D+1)$$

$$r(X) \le \min(n, p+1)$$

$$r(X^\top X) \le \min(n, p+1)$$

$$\le n < p+1$$

$$w_0^* = \operatorname{argmin} \| y - w_0 1_n - Xw \|^2$$

$$(n,1)$$

$$\underset{\triangle}{1_N^T}(y - \underset{\triangle}{w_0} 1_N - Xw) = 0$$

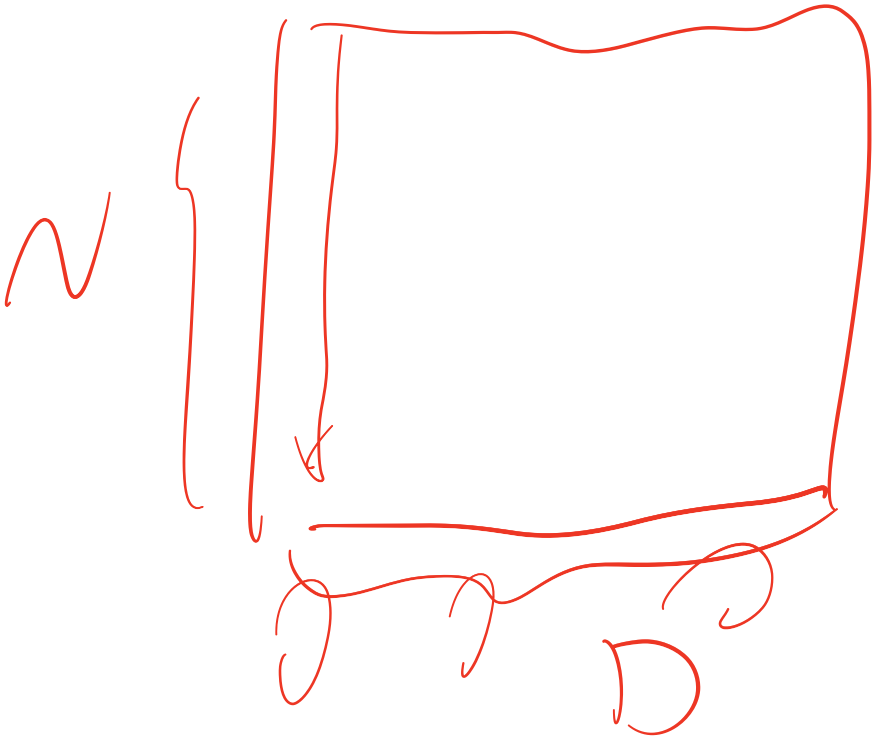
$$1_N^T y - N w_0$$

$$- 1^T X w = 0$$

$$w_0^* = \frac{1}{N} 1^T (y - Xw)$$

$$\frac{1}{N} \sum x_{nd} = 0$$

$\forall d, \sim D$



$(1,1,1) \times X$

$1^T X = 0$

$$w_0^* = \frac{1}{n} 1^T y$$

$$\frac{d\|y - w_0 1_n\|_2^2}{dw_0}$$

$$= \sum (y - w_0 1_n)^T$$

$$(-1)$$

$$\frac{\partial (y - w_0 1_n)}{\partial w_0}$$

$$(y_i - w_0 \cdot 1)$$

$$(-1, -1, -1, \dots)$$

$$= -2 \langle \quad \rangle$$

$$-2 1^T (y - w_0 1_n)$$

1

$$\underline{w_{k+1} = w_k + y_i x_i}$$

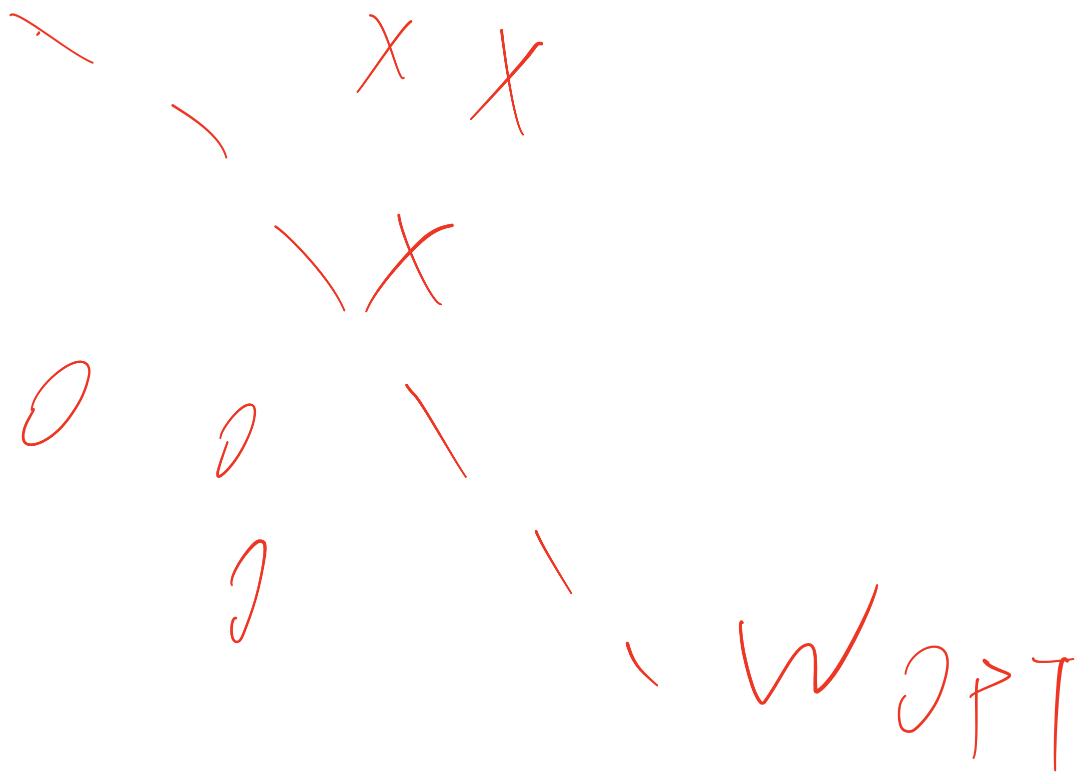
$$\Downarrow$$

$$\underline{w_{k+1}}^T = w_k^T + y_i x_i^T$$

$$w_{k+1}^T w_{opt}$$

$= w_k^T w_{opt}$

$+ y_i x_i^T w_{opt}$



$\text{sign}(w_{opt}^T x_i) = 1$

$$y_i x_i^T w_{opt}$$

$$= x_i^T w_{opt}$$

$$= |x_i^T w_{opt}|$$

$$w_{k+1}^T w_{opt}$$

$$= w_k^T w_{opt} +$$

$$|x_i^T w_{opt}|$$

$$\frac{|x_i^T w_{opt}|}{} \geq \gamma \|w_{opt}\|$$

$$\gamma \leq \frac{|w_{opt}^T x_i|}{\|w_{opt}\|}$$

$$w_{k+1} = w_k + \underline{y_i x_i}$$

$$\|w_{k+1}\|^2$$

$$= (w_k + y_i x_i)^T (w_k$$

$$+ y_i x_i)$$

$$= \|w_k\|^2 + \underbrace{2 y_i w_k^T x_i}_{<0} + \underbrace{y_i^2 x_i^T x_i}_{= 1 \times 1 = 1}$$

$$\le \|w_k\|^2 + 1$$

$$W_{k+1}^{T} W_{OPT}$$

$$\geq W_k^T W_{OPT}$$

$$+ r \| W_{OPT} \|$$

$W_0 = 0$

$\downarrow$

W_1

$\downarrow$

W_2

$\downarrow$

$\left.\begin{matrix} W_0 = 0 \\ \downarrow \\ W_1 \\ \downarrow \\ W_2 \\ \downarrow \end{matrix}\right\} M$

$$\cancel{w_{k+1}} \leq \|w_{k+1}^T\| \, \|w_{opt}\|$$

$$w_{k+1}^T w_{opt} \geq$$

$$\gamma M \|w_{opt}\|$$

$$\|w_{k+1}\| \geq \gamma M$$

$$\|w_{k+1}\|^2 \leq M$$

$$n \leq \frac{t}{\gamma^2}$$