Sample Quiz 0

Q1 Linear Algebra Which of the followings sets of vectors are bases for $\mathbb{R}^2$?
A, D

(A) $\{(1,0),(1,-1)\}$

(B) $\{(1,\sqrt{2}),(\sqrt{2},2)\}$

(C) $\{(1,0),(1,1),(1,-1)\}$

(D) $\{(1,1),(1,1.1)\}$

Q2 Probability and Statistics Suppose X and Y are two jointly Gaussian random variables. Which of the followings are true? A, B, D

(A) $X+1$ and $X+Y$ are also jointly Gaussian

(B) $Z = X + \frac{1}{2}Y$ is also Gaussian

(C) $Z = \sqrt{XY}$ is also Gaussian

(D) If X and Y are uncorrelated, then X and Y are independent

Q3 Functional Analysis and Calculus Suppose $\mathbf{x}$ and $\mathbf{a}$ are two arbitrary vectors in $\mathbb{R}^n$ with $n \geq 3$. Which of the following functions are convex? A, B

(A) $f(\mathbf{x}) = \max_i a_i x_i$

(B) $f(\mathbf{x}) = \sum_i \exp(a_i x_i)$

(C) $f(\mathbf{x}) = (\sum_i (a_i x_i)^3)^{1/3}$

(D) $f(\mathbf{x}) = x_1 + \min\{x_2, x_3\}$

Q4 Let $f(x) = \ln(1+x) - \frac{x}{1+x}$. Prove that $f(x) \geq 0$ for $x > -1$.

Note that $f'(x) = \frac{1}{1+x} - \frac{1}{(1+x)^2} = \frac{x}{(1+x)^2}$. Since $f'(x) < 0$ when $x \in (-1,0)$ and $f'(x) \geq 0$ when $x \geq 0$, we have $f(x)$ is non-increasing when $x \in (-1,0)$ and non-decreasing when $x \geq 0$. Combining with $f(0) = 0$ completes the proof.