

Lect 8 – Recursion

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INLS 490-172

Recursion

- A method of solving problems by breaking them down into smaller and smaller subproblems until you get a small enough problem that it can be solved trivially.
- Usually involves a function calling itself

Example #1

- Print a countdown from 10 to 0

```
def countdown(n):  
    for i in range(n, 0, -1):  
        print i  
    print "Blastoff!"
```

```
countdown(10)
```

Example #1

- Print a countdown from 10 to 0

```
def countdown(n):  
    for i in range(n, 0, -1):  
        print i  
    print "Blastoff!"
```

```
countdown(10)
```

```
def countdown2(n):  
    if n <= 0:  
        print "Blastoff!"  
    else:  
        print n  
        countdown(n-1)
```

```
countdown2(10)
```

Essential Elements of Recursion

- A recursive algorithm must
 - have a base case
 - change its state and move toward the base case
 - call itself, recursively

Base Case

- The base case is what allows the algorithm to stop recursing
- It should represent a case that is *trivial*
 - Meaning that it cannot be decomposed further and that the solution (to the base case) is simple
- Examples:
 - A list of length 1 is always in sorted order
 - The sum of elements in a list of length 1 is just the value of the one element.

Change state to move toward base case

- The algorithm should, for each recursive call, move closer to the base case
- Examples
 - A list shrinks by one
 - A number is divided by some factor

Function should call itself, recursively

- If the function is not in the base case
 - It will typically perform some operation
 - And call itself with an argument that moves things closer to the base case.

Recursion is a form of iteration

Be careful of infinite recursion!

```
def neverending(t) :  
    return neverending(t)
```

Example #2

- Definition of factorial

$$0! = 1$$

$$n! = n * (n-1)!$$

```
print fac(4)          # ans = 4 * 3 * 2 * 1
24
```

- Think:

- What should be the base case?
- How do we progress toward the base case?
- How will we call the function recursively?

Example #3

- Fibonacci

$$\text{fib}(0) = 0$$

$$\text{Fib}(1) = 1$$

$$\text{fib}(n) = \text{fib}(n-1) + \text{fib}(n-2)$$

```
print fib(4)
```

3

$$\begin{aligned}\text{fib}(4) &= \text{fib}(3) + \text{fib}(2) \\ &= (\text{fib}(2) + \text{fib}(1)) + (\text{fib}(1) + \text{fib}(0)) \\ &= ((\text{fib}(1) + \text{fib}(0)) + \text{fib}(1)) + (\text{fib}(1) + \text{fib}(0)) \\ &= 1 + 0 + 1 + 1 + 0 \\ &= 3\end{aligned}$$

Example #4

- Sum a list of numbers

`t = [1, 3, 5, 7, 9]`

- First, let's do this with a loop
 - (collaboratively write code to do this)

Example #4

- Sum a list of numbers

`t = [1, 3, 5, 7, 9]`

- Now let's try to write a recursive solution
- Think:
 - What should be the base case?
 - How do we progress toward the base case?
 - How will we call the function recursively?

Example #4

- Sum a list of numbers

`t = [1, 3, 5, 7, 9]`

- Idea:

`listsum(t) = first(t) + listsum(all_except_first(t))`

`total = (1 + (3 + (5 + (7 + 9))))`

`= (1 + (3 + (5 + 16)))`

`= (1 + (3 + 21))`

`= (1 + 24)`

`= 25`

Stack Frames

```
def listsum(t):  
    if len(t) == 1:  
        return t[0]  
    else:  
        return t[0] + listsum(t[1:])
```

```
listsum([1, 3, 5, 7])  
  1 + listsum([3, 5, 7])  
    1 + 3 + listsum([5, 7])  
      1 + 3 + 5 + listsum([7])  
        1 + 3 + 5 + 7  
      1 + 3 + 12  
    1 + 15
```


Illustration of Recursive Calls

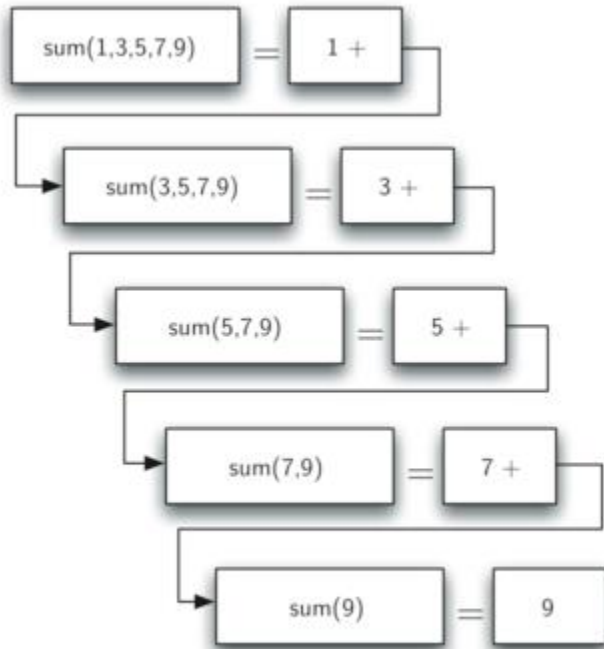


Figure 1: Series of Recursive Calls Adding a List of Numbers

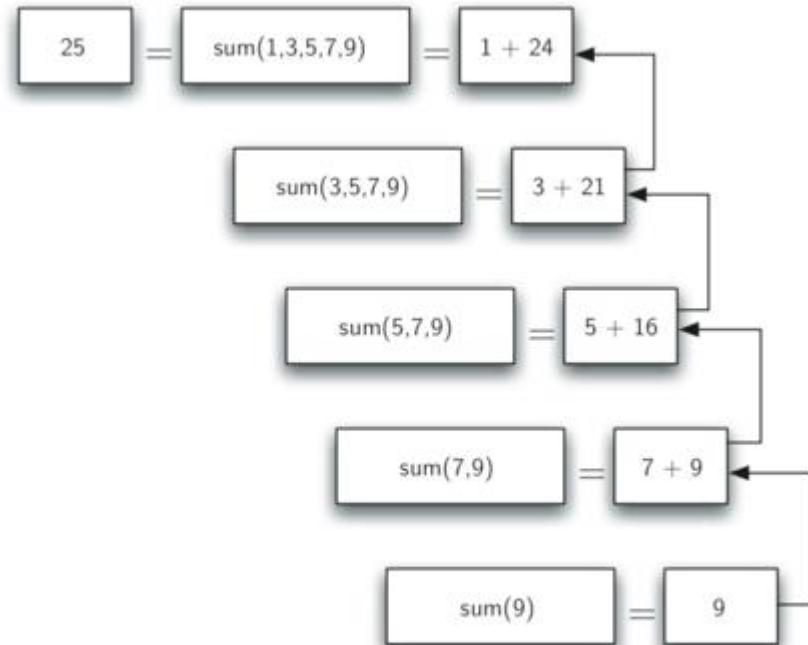


Figure2: Series of Recursive Returns from Adding a List of Numbers

Example #5

- Reverse a string

```
print str_reverse('python')  
nohtyp
```

- First, let's do this with a loop
 - (collaboratively write code to do this)