QUESTION 2

2.1 GRAM-SCHMIDT OTHOGONALIZATION

$$A_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 1.2 \\ 2.0 \\ 3.0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 3.0 \\ 2.5 \\ 4.0 \end{bmatrix}$$

$$U_0 = A_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$P_{10} = \frac{A_1^T U_0}{\|U_0\|^2} = \frac{6.2}{3} = 2.07$$

$$U_1 = A_1 - P_{10} U_0 = \begin{bmatrix} 1.2 \\ 2.0 \\ 3.0 \end{bmatrix} - 2.07 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.87 \\ -0.07 \\ 0.93 \end{bmatrix}$$

$$P_{20} = \frac{A_2^T U_0}{\|U_0\|^2} = \frac{9.5}{3} = 3.16$$

$$U_2 = (A_2 - P_{20} U_0) - P_{21} U_1$$

$$A_2 - P_{20} U_0 = \begin{bmatrix} 3.0 \\ 2.5 \\ 4.0 \end{bmatrix} - 3.16 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.16 \\ -0.66 \\ 0.84 \end{bmatrix}$$

$$P_{21} = \frac{0.14 + 0.05 + 0.78}{0.75 + 0.0049 + 0.86} = \frac{0.97}{1.61} = 0.60$$

$$U_2 = \begin{bmatrix} -0.16 \\ -0.66 \\ 0.84 \end{bmatrix} - 0.60 \begin{bmatrix} -0.87 \\ -0.07 \\ 0.93 \end{bmatrix} = \begin{bmatrix} 0.36 \\ -0.62 \\ 0.28 \end{bmatrix}$$

2.2 QR DECOMPOSITION

$\tilde{D} = QR$

$$Q = \begin{bmatrix} 0.58 & -0.68 & 0.46 \\ 0.58 & -0.05 & -0.81 \\ 0.58 & 0.73 & 0.36 \end{bmatrix} \quad \text{\# NORMALIZED}$$

$[v_0, v_1, v_2]$

$R = Q^T A$

$$R = \begin{bmatrix} 0.58 & 0.58 & 0.58 \\ -0.68 & -0.05 & +0.73 \\ 0.46 & -0.81 & 0.36 \end{bmatrix} \begin{bmatrix} 1 & 1.2 & 3.0 \\ 1 & 2.0 & 2.5 \\ 1 & 3.0 & 4.0 \end{bmatrix}$$

$$= \begin{bmatrix} 1.74 & 3.6 & 5.51 \\ 0.00 & 1.27 & 0.76 \\ 0.01 & 0.01 & 0.79 \end{bmatrix}$$

$$= \begin{bmatrix} 1.74 & 3.6 & 5.51 \\ 0 & 1.27 & 0.76 \\ 0 & 0 & 0.79 \end{bmatrix}$$

2.3 SOLUTION

SOLVE $R\tilde{w} = Q^T Y$

$$Q^T Y = \begin{bmatrix} 0.58 & 0.58 & 0.58 \\ -0.68 & -0.05 & 0.73 \\ 0.46 & -0.81 & 0.36 \end{bmatrix} \begin{bmatrix} 180 \\ 210 \\ 290 \end{bmatrix}$$

$$= \begin{bmatrix} 394.4 \\ 78.8 \\ 17.1 \end{bmatrix}$$

$$\begin{bmatrix} 1.74 & 3.6 & 5.51 \\ 0 & 1.27 & 0.76 \\ 0 & 0 & 0.79 \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 394.4 \\ 78.8 \\ 17.1 \end{bmatrix}$$

$.79 w_2 = 17.1$

$$\boxed{w_2 = 21.65}$$

$1.27 w_1 + 0.76 w_2 = 78.8$

$1.27 w_1 + 0.76(21.65) = 78.8$

$1.27 w_1 + 16.45 = 78.8$

$1.27 w_1 = 62.35$

$$\boxed{w_1 = 49.09}$$

$$1.74 w_0 + 3.6 w_1 + 5.51 w_2 = 394.4$$

$$1.74 w_0 + 3.6(49.09) + 5.51(21.65) = 394.4$$

$$1.74 w_0 + 176.72 + 119.29 = 394.4$$

$$1.74 w_0 = 98.39$$

$$\boxed{w_0 = 56.55}$$

$$\tilde{w} = \begin{bmatrix} 56.55 \\ 49.09 \\ 21.65 \end{bmatrix}$$

NOTE: AFTER CHECKING MY WORK I BELIEVE THE DIFFERENCES IN THE "$\tilde{w}$" VALUES FROM QUESTIONS 1 AND 2 CAN BE EXPLAINED BY ROUNDING ERRORS / PRECISION.