

QUESTION 3

DATA POINT : 1

ITERATION : 1

$$\hat{x}_1 = \begin{bmatrix} 1 \\ 1.2 \\ 3 \end{bmatrix} \quad \tilde{w}^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \hat{y}_1 = (\tilde{w}^0)^T \hat{x}_1 = 0$$
$$y_1 = 180$$

$$\begin{aligned} \nabla J(w) &= (w^T x_i - y_i) x_i \\ &= (0 - 180) \cdot \begin{bmatrix} 1 \\ 1.2 \\ 3 \end{bmatrix} \\ &= -180 \cdot \begin{bmatrix} 1 \\ 1.2 \\ 3 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} -180 \\ -216 \\ -540 \end{bmatrix}$$

$$\tilde{w}' = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - 0.01 \cdot \begin{bmatrix} -180 \\ -216 \\ -540 \end{bmatrix}$$

$$\tilde{w}' = \begin{bmatrix} 1.8 \\ 2.16 \\ 5.40 \end{bmatrix}$$

DATA POINT: 1

ITERATION: 2

$$\tilde{x}_1 = \begin{bmatrix} 1 \\ 1.2 \\ 3 \end{bmatrix} \quad \tilde{w}' = \begin{bmatrix} 1.8 \\ 2.16 \\ 5.40 \end{bmatrix} \quad y_1 = 180$$

$$\hat{y}_2 = (\tilde{w}')^T \tilde{x}_1 = 1.8 + 2.59 + 16.2 = \underline{20.59}$$

$$\begin{aligned} \nabla J(w) &= (w^T x_1 - y_1) x_1 \\ &= (20.59 - 180) \cdot \begin{bmatrix} 1 \\ 1.2 \\ 3 \end{bmatrix} \\ &= -159.41 \cdot \begin{bmatrix} 1 \\ 1.2 \\ 3 \end{bmatrix} \\ &= \begin{bmatrix} -159.41 \\ -191.29 \\ -478.23 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \tilde{w}^2 &= \begin{bmatrix} 1.8 \\ 2.16 \\ 5.40 \end{bmatrix} - 0.01 \cdot \begin{bmatrix} -159.41 \\ -191.29 \\ -478.23 \end{bmatrix} \\ &= \begin{bmatrix} 1.8 \\ 2.16 \\ 5.40 \end{bmatrix} - \begin{bmatrix} -1.59 \\ -1.91 \\ -4.78 \end{bmatrix} \\ &= \begin{bmatrix} 3.39 \\ 4.07 \\ 10.18 \end{bmatrix} \end{aligned}$$

DATA POINT: 1

ITERATION: 3

$$\hat{x}_1 = \begin{bmatrix} 1 \\ 1.2 \\ 3 \end{bmatrix} \quad \tilde{w}^2 = \begin{bmatrix} 3.39 \\ 4.07 \\ 10.18 \end{bmatrix} \quad y_1 = 180$$

$$\hat{y}_2 = (\tilde{w}^2)^T x_1 = 3.39 + 4.88 + 30.54 = \underline{38.81}$$

$$\begin{aligned} \nabla J(w) &= (w^T x_1 - y_1) x_1 \\ &= (38.81 - 180) \cdot \begin{bmatrix} 1 \\ 1.2 \\ 3 \end{bmatrix} \\ &= -141.19 \cdot \begin{bmatrix} 1 \\ 1.2 \\ 3 \end{bmatrix} \\ &= \begin{bmatrix} -141.19 \\ -169.43 \\ -423.57 \end{bmatrix} \end{aligned}$$

$$\tilde{w}^3 = \begin{bmatrix} 3.39 \\ 4.07 \\ 10.18 \end{bmatrix} - 0.01 \begin{bmatrix} -141.19 \\ -169.43 \\ -423.57 \end{bmatrix}$$

$$= \begin{bmatrix} 3.39 \\ 4.07 \\ 10.18 \end{bmatrix} - \begin{bmatrix} -1.41 \\ -1.69 \\ -4.24 \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} 4.8 \\ 5.76 \\ 14.42 \end{bmatrix}}}$$

DATA POINT: 2

ITERATION: 1

$$\hat{x}_2 = \begin{bmatrix} 1 \\ 2.0 \\ 2.5 \end{bmatrix} \quad \tilde{w}^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \hat{y}_1 = (\tilde{w}^0)^T \hat{x}_2 = 0$$
$$y_2 = \cancel{+80} \quad 210$$

$$\begin{aligned} \nabla J(w) &= (w^T x_2 - y_2) x_2 \\ &= (0 - 210) \cdot \begin{bmatrix} 1 \\ 2.0 \\ 2.5 \end{bmatrix} \\ &= -210 \cdot \begin{bmatrix} 1 \\ 2.0 \\ 2.5 \end{bmatrix} \\ &= \begin{bmatrix} -210 \\ -420 \\ -525 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \tilde{w}_1 &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - 0.01 \cdot \begin{bmatrix} -210 \\ -420 \\ -525 \end{bmatrix} \\ &= \begin{bmatrix} 2.1 \\ 4.2 \\ 5.25 \end{bmatrix} \end{aligned}$$

DATA POINT : 2

ITERATION : 2

$$\tilde{x}_2 = \begin{bmatrix} 1 \\ 2.0 \\ 2.5 \end{bmatrix} \quad \tilde{w}^1 = \begin{bmatrix} 2.1 \\ 4.2 \\ 5.25 \end{bmatrix} \quad y_2 = 210$$

$$\hat{y}_2 = (\tilde{w}^1)^T \tilde{x}_2 = 2.1 + 8.4 + 13.13 = \underline{23.63}$$

$$\begin{aligned} \nabla J(w) &= (w^T x_2 - y_2) x_2 \\ &= (23.63 - 210) \cdot \begin{bmatrix} 1 \\ 2.0 \\ 2.5 \end{bmatrix} \\ &= (-186.37) \cdot \begin{bmatrix} 1 \\ 2.0 \\ 2.5 \end{bmatrix} \\ &= \begin{bmatrix} -186.37 \\ -372.74 \\ -465.93 \end{bmatrix} \end{aligned}$$

$$\tilde{w}^2 = \begin{bmatrix} 2.1 \\ 4.2 \\ 5.25 \end{bmatrix} - 0.01 \cdot \begin{bmatrix} -186.37 \\ -372.74 \\ -465.93 \end{bmatrix}$$

$$= \begin{bmatrix} 2.1 \\ 4.2 \\ 5.25 \end{bmatrix} - \begin{bmatrix} -1.86 \\ -3.73 \\ -4.66 \end{bmatrix}$$

$$= \begin{bmatrix} 3.96 \\ 7.93 \\ 9.91 \end{bmatrix}$$

DATA POINT: 2

ITERATION: 3

$$\hat{x}_2 = \begin{bmatrix} 1 \\ 2.0 \\ 2.5 \end{bmatrix} \quad \tilde{w}^2 = \begin{bmatrix} 3.96 \\ 7.93 \\ 9.91 \end{bmatrix} \quad y_2 = 210$$

$$\hat{y}_3 = (\tilde{w}^2)^T x_2 = 3.96 + 15.86 + 24.78 = \underline{44.6}$$

$$\begin{aligned} \nabla J(w) &= (w^T x_2 - y_2) x_2 \\ &= (44.6 - 210) \cdot \begin{bmatrix} 1 \\ 2.0 \\ 2.5 \end{bmatrix} \\ &= -165.4 \cdot \begin{bmatrix} 1 \\ 2.0 \\ 2.5 \end{bmatrix} \\ &= \begin{bmatrix} -165.4 \\ -330.8 \\ -413.5 \end{bmatrix} \end{aligned}$$

$$\hat{w}^3 = \begin{bmatrix} 3.96 \\ 7.93 \\ 9.91 \end{bmatrix} - 0.01 \begin{bmatrix} -165.4 \\ -330.8 \\ -413.5 \end{bmatrix}$$

$$= \begin{bmatrix} 3.96 \\ 7.93 \\ 9.91 \end{bmatrix} - \begin{bmatrix} -1.65 \\ -3.31 \\ -4.14 \end{bmatrix}$$

$$= \begin{bmatrix} 5.61 \\ 11.24 \\ 14.05 \end{bmatrix}$$

DATA POINT: 3

ITERATION: 1

$$\tilde{x}_3 = \begin{bmatrix} 1 \\ 3.0 \\ 4.0 \end{bmatrix} \quad \tilde{\omega}^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \hat{y}_1 = (\tilde{\omega}^0)^T \tilde{x}_3 = 0$$
$$y_3 = 290$$

$$\begin{aligned} \nabla J(\omega) &= (\omega^T x_i - y_i) x_i \\ &= (0 - 290) \cdot \begin{bmatrix} 1 \\ 3.0 \\ 4.0 \end{bmatrix} \\ &= -290 \cdot \begin{bmatrix} 1 \\ 3.0 \\ 4.0 \end{bmatrix} \\ &= \begin{bmatrix} -290 \\ -870 \\ -1160 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \tilde{\omega}^1 &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - 0.01 \begin{bmatrix} -290 \\ -870 \\ -1160 \end{bmatrix} \\ &= \begin{bmatrix} 2.9 \\ 8.7 \\ 11.6 \end{bmatrix} \end{aligned}$$

DATA POINT: 3

ITERATION: 2

$$\hat{x}_3 = \begin{bmatrix} 1 \\ 3.0 \\ 4.0 \end{bmatrix} \quad \tilde{w}^1 = \begin{bmatrix} 2.9 \\ 8.7 \\ 11.6 \end{bmatrix} \quad y_3 = 290$$

$$\hat{y}_2 = (\tilde{w}^1)^T \hat{x}_3 = 2.9 + 26.1 + 46.4 = \underline{75.4}$$

$$\begin{aligned} \nabla J(w) &= (w^T x_3 - y_3) x_3 \\ &= (75.4 - 290) \cdot \begin{bmatrix} 1 \\ 3.0 \\ 4.0 \end{bmatrix} \\ &= -214.6 \cdot \begin{bmatrix} 1 \\ 3.0 \\ 4.0 \end{bmatrix} \\ &= \begin{bmatrix} -214.6 \\ -643.8 \\ -858.4 \end{bmatrix} \end{aligned}$$

$$\tilde{w}^2 = \begin{bmatrix} 2.9 \\ 8.7 \\ 11.6 \end{bmatrix} - 0.01 \cdot \begin{bmatrix} -214.6 \\ -643.8 \\ -858.4 \end{bmatrix}$$

$$= \begin{bmatrix} 2.9 \\ 8.7 \\ 11.6 \end{bmatrix} - \cancel{0.01} \begin{bmatrix} -2.15 \\ -6.44 \\ -8.58 \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} 5.05 \\ 15.14 \\ 20.18 \end{bmatrix}}}$$

DATA POINT : 3

ITERATION : 3

$$\tilde{x}_3 = \begin{bmatrix} 1 \\ 3.0 \\ 4.0 \end{bmatrix} \quad \tilde{w}^2 = \begin{bmatrix} 5.05 \\ 15.14 \\ 20.18 \end{bmatrix} \quad y_3 = 290$$

$$\hat{y}_3 = (\tilde{w}^2)^T x_3 = 5.05 + 45.42 + 80.72 = \underline{131.19}$$

$$\begin{aligned} \nabla J(w) &= (w^T x_3 - y_3) x_3 \\ &= (131.19 - 290) \cdot \begin{bmatrix} 1 \\ 3.0 \\ 4.0 \end{bmatrix} \\ &= (-158.81) \cdot \begin{bmatrix} 1 \\ 3.0 \\ 4.0 \end{bmatrix} \\ &= \begin{bmatrix} -158.81 \\ -476.43 \\ -635.24 \end{bmatrix} \end{aligned}$$

$$\tilde{w}^3 = \begin{bmatrix} 5.05 \\ 15.14 \\ 20.18 \end{bmatrix} - 0.01 \begin{bmatrix} -158.81 \\ -476.43 \\ -635.24 \end{bmatrix}$$

$$= \begin{bmatrix} 5.05 \\ 15.14 \\ 20.18 \end{bmatrix} - \begin{bmatrix} -1.59 \\ -4.76 \\ -6.35 \end{bmatrix}$$

$$= \begin{bmatrix} 6.64 \\ 19.9 \\ 26.53 \end{bmatrix}$$