QUESTION 5

EIGENVALUES:

$$\det(\overset{n}{\xi} - \lambda I) = \det\begin{pmatrix} 8-\lambda & 34/3 \\ 34/3 & \frac{146}{9}-\lambda \end{pmatrix} = 0$$

$$(8-\lambda)(\frac{146}{9}-\lambda) - \frac{1156}{9} = 0$$

$$\frac{1168}{9} - 8\lambda - \frac{146}{9}\lambda + \lambda^2 - \frac{1156}{9} = 0$$

$$\lambda^2 - \frac{218}{9}\lambda + \frac{4}{3} = 0$$

$$\lambda = \frac{\frac{218}{9} \pm \sqrt{(-\frac{218}{9})^2 - 4(1)(4/3)}}{2}$$

$$\lambda = \frac{\frac{218}{9} \pm \sqrt{\frac{47,524}{81} - \frac{16}{3}}}{2}$$

$$\lambda = \frac{\frac{218}{9} \pm \sqrt{\frac{47,092}{81}}}{2}$$

$$\lambda = \left(\frac{218}{9} \pm \frac{\sqrt{47,092}}{9}\right)\frac{1}{2}$$

$$\lambda_1 = \left(\frac{218 + \sqrt{47,092}}{9}\right)\frac{1}{2} = \frac{218 + \sqrt{47,092}}{18} \quad \text{or } 24.22$$

$$\lambda_2 = \left(\frac{218 - \sqrt{47,092}}{9}\right)\frac{1}{2} = \frac{218 - \sqrt{47,092}}{18} \quad \text{or } 0.06$$

QUESTION 5

EIGENVECTORS

$$\lambda_1 = \frac{218 + \sqrt{47,092}}{18} = 24.22$$

$$\begin{pmatrix} 8-24.22 & 11.33 \\ 11.33 & 16.22-24.22 \end{pmatrix} \begin{pmatrix} V_{11} \\ V_{12} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -16.22 & 11.33 \\ 11.33 & -8 \end{pmatrix} \begin{pmatrix} V_{11} \\ V_{12} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-16.22 V_{11} + 11.33 V_{12} = 0$$

$$V_{12} = \frac{-16.22}{-11.33} V_{11}$$

$$V_{12} = 1.43 V_{11}$$

NORMALIZE:

$$\|v\| = \sqrt{1^2 + (1.43)^2} = \sqrt{1 + 2.04} = \sqrt{3.04} = 1.74$$

$$\begin{aligned} \text{NORMALIZED} \\ \text{EIGENVECTOR} = \vec{V_1} \end{aligned} = \begin{bmatrix} 1 \\ 1.43 \end{bmatrix} \frac{1}{1.74} = \begin{bmatrix} \frac{1}{1.74} \\ \frac{1.43}{1.74} \end{bmatrix} = \begin{bmatrix} 0.57 \\ 0.82 \end{bmatrix}$$

QUESTION 5

EIGENVECTORS

$$\lambda_2 = \frac{218 - \sqrt{47,092}}{18} = 0.06$$

$$\begin{pmatrix} 8-0.06 & 11.33 \\ 11.33 & 16.22-0.06 \end{pmatrix} \begin{pmatrix} v_{21} \\ v_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 7.94 & 11.33 \\ 11.33 & 16.16 \end{pmatrix} \begin{pmatrix} v_{21} \\ v_{22} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$7.94 v_{21} + 11.33 v_{22} = 0$$

$$v_{22} = -\frac{7.94}{11.33} v_{21}$$

$$v_{22} = -0.70 v_{21}$$

NORMALIZE:

$$||V|| = \sqrt{1^2 + (-0.70)^2} = \sqrt{1 + 0.49} = \sqrt{1.49} = 1.22$$

$$\begin{matrix} \text{NORMALIZED} \\ \text{EIGENVECTORS} \end{matrix} = \vec{V} = \begin{bmatrix} 1 \\ -0.70 \end{bmatrix} \frac{1}{1.22} = \begin{bmatrix} \frac{1}{1.22} \\ \frac{-0.70}{1.22} \end{bmatrix} = \begin{bmatrix} 0.82 \\ -0.57 \end{bmatrix}$$