BLOCK INDIRECT

BRIEF

Modern processors obtain their power increasing the number of “cores” or HW threads, which permit them to execute several processes simultaneously, with a shared memory structure.

SPEED OR LOW MEMORY

In the parallel sorting algorithms, we can find two categories .

SUBDIVISION ALGORITHMS
Filter the data and generate two or more parts. Each part obtained is filtered and divided by other threads, until the size of the data to sort is smaller than a predefined size, then it is sorted by a single thread. The algorithm most frequently used in the filter and sort is quick sort.

These algorithms are fast with a small number of threads, but inefficient with a large number of HW threads. Examples of this category are :

- Intel Threading Building Blocks (TBB)
- Microsoft PPL Parallel Sort.

MERGING ALGORITHMS
Divide the data into many parts at the beginning, and sort each part with a separate thread. When the parts are sorted, merge them to obtain the final result. These algorithms need additional memory for the merge, usually an amount equal to the size of the input data.

With a small number of threads, these algorithms usually have similar speed to the subdivision algorithms, but with many threads they are much faster . Examples of this category are :

- GCC Parallel Sort (based on OpenMP)
- Microsoft PPL Parallel Buffered Sort

SPEED AND LOW MEMORY

This new algorithm is an unstable parallel sort algorithm, created for processors connected with shared memory. This provides excellent performance in machines with many HW threads, similar to the GCC Parallel Sort, and better than TBB, with the additional advantage of lower memory consumption.

This algorithm uses as auxiliary memory a block_size elements buffer for each thread. The block_size is an internal parameter of the algorithm, which, in order to achieve the highest speed, change according the size of the objects to sort according the next table. The strings use a block_size of 128.

object size	1 - 15	16 - 31	32 - 63	64 - 127	128 - 255	256 - 511	512 -
block_size	4096	2048	1024	768	512	256	128

The worst case memory usage for the algorithm is when elements are large and there are many threads. With big elements (512 bytes), and 12 threads, the memory measured was:

- GCC Parallel Sort 1565 MB
- Threading Building Blocks (TBB) 783 MB
- Block Indirect Sort 812 MB

Among the unstable parallel sorting algorithms, there are basically two types:

1.- SUBDIVISION ALGORITHMS

As Parallel Quick Sort. One thread divides the problem in two parts. Each part obtained is divided by other threads, until the subdivision generates sufficient parts to keep all the threads buys. The below example shows that this means with a 32 HW threads processor, with N elements to sort.

Step	Threads working	Threads waiting	Elements to process by each thread
1	1	31	N
2	2	30	N / 2
3	4	28	N / 4
4	8	24	N / 8
5	16	16	N / 16
6	32	0	N / 32

Very even splitting would be unusual in reality, where most subdivisions are uneven

This algorithm is very fast and don't need additional memory, but the performance is not good when the number of threads grows. In the table before, until the 6th division, don't have work for to have busy all the HW threads, with the additional problem that the first division is the hardest, because the number of elements is very large.

2.- MERGING ALGORITHMS,

Divide the data into many parts at the beginning, and sort each part with a separate thread. When the parts are sorted, merge them to obtain the final result. These algorithms need additional memory for the merge, usually an amount equal to the size of the input data.

These algorithms provide the best performance with many threads, but their performance with a low number of threads is worse than the subdivision algorithms.

This new algorithm (Block Indirect), is a merging algorithm. It has similar performance to GCC Parallel Sort With many threads, but using a low amount of additional memory, close to that used by subdivision algorithms.

Internally, the algorithm, manage blocks of elements. The number of elements of the block (block_size), change according the size of the objects to sort according the next table. The strings use a block_size of 128.

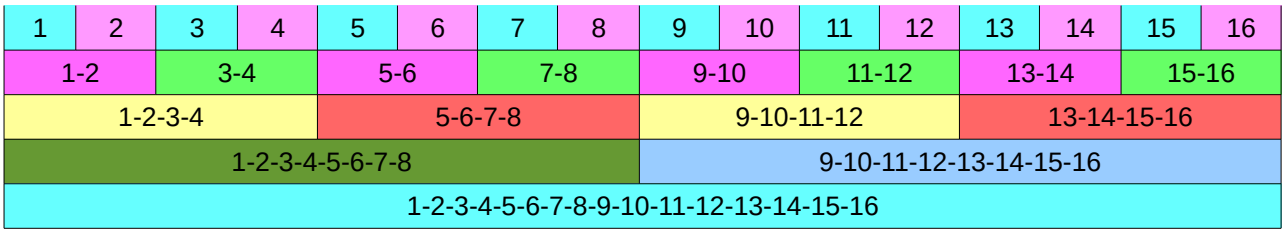
object size	1 - 15	16 - 31	32 - 63	64 - 127	128 - 255	256 - 511	512 -
block_size	4096	2048	1024	768	512	256	128

This new algorithm only need an auxiliary memory of one block of elements for each HW thread. The worst case memory usage for the algorithm is when elements are large and there are many threads. With big elements (512 bytes), and 12 threads, the memory measured was:

- GCC Parallel Sort 1565 MB
- Threading Building Blocks (TBB) 783 MB
- Block Indirect Sort 812 MB

These algorithms are not optimal when using just 1 thread, but are easily parallelized, and need only a small amount of auxiliary memory.

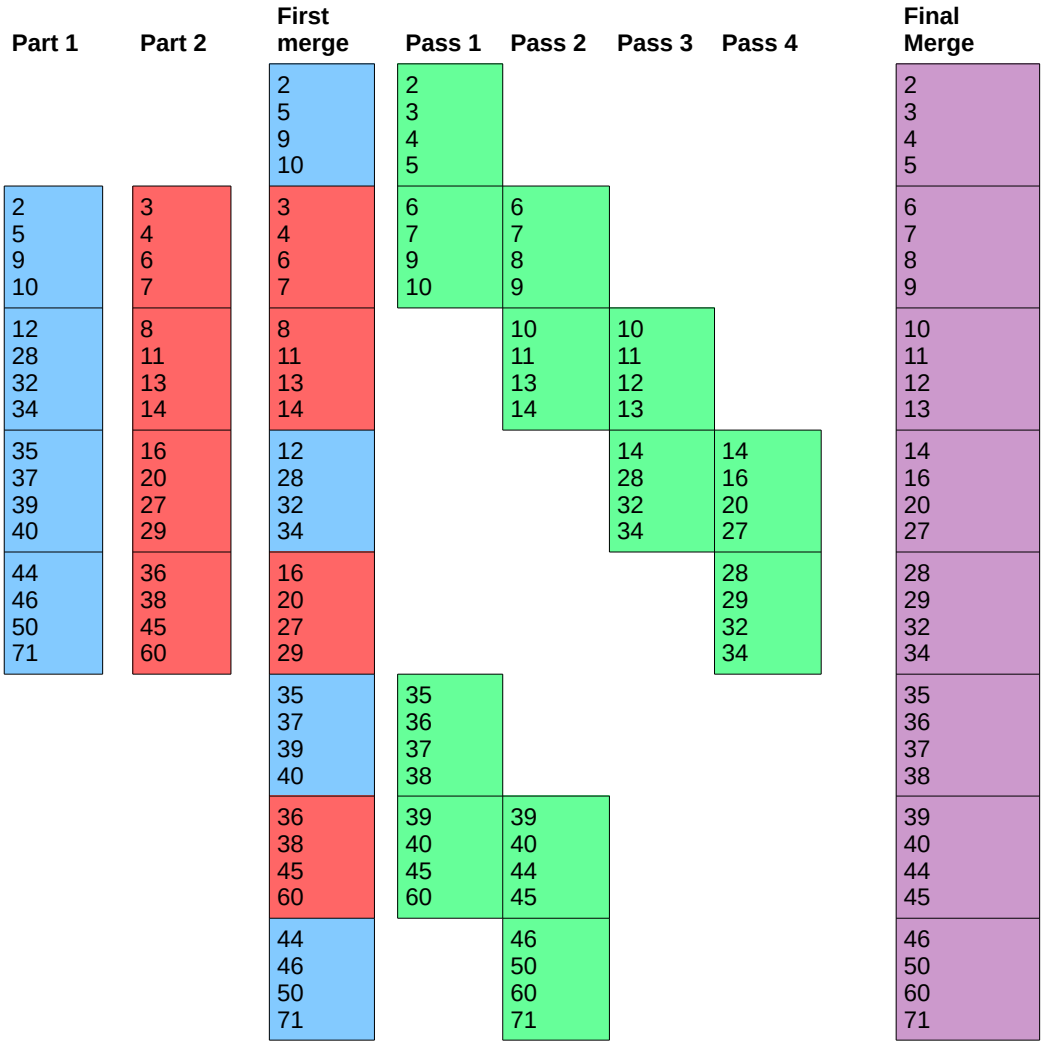
The algorithm divide the number of elements in a number of parts that is the first power of two greater than or equal to the number of threads to use. (For example: with 3 threads, make 4 parts, for 9 threads make 16 parts…) Each part obtained is sorted by the parallel intro sort algorithm.



With the sorted parts, merge pairs of parts. We consider the elements are in blocks of fixed size. We initially restrict the number of elements to sort (N) to a multiple of the block size. For to explain the algorithm, I use several examples with a block size of 4. Each thread receives a number of blocks to sort, and when complete we have a succession of blocks sorted.

For the merge, we have two successions of block sorted. We sort the blocks of the two parts the first element of the block. This merge is not fully sorted, is sorted only by the first element. We call this First Merge

But if we merge the first block with the second, the second with the third and this successively, we will obtain a new list fully sorted with a size of the sum of the blocks of the first part plus the blocks of the second part. This merge algorithm, only need an auxiliary memory of the size of a block.

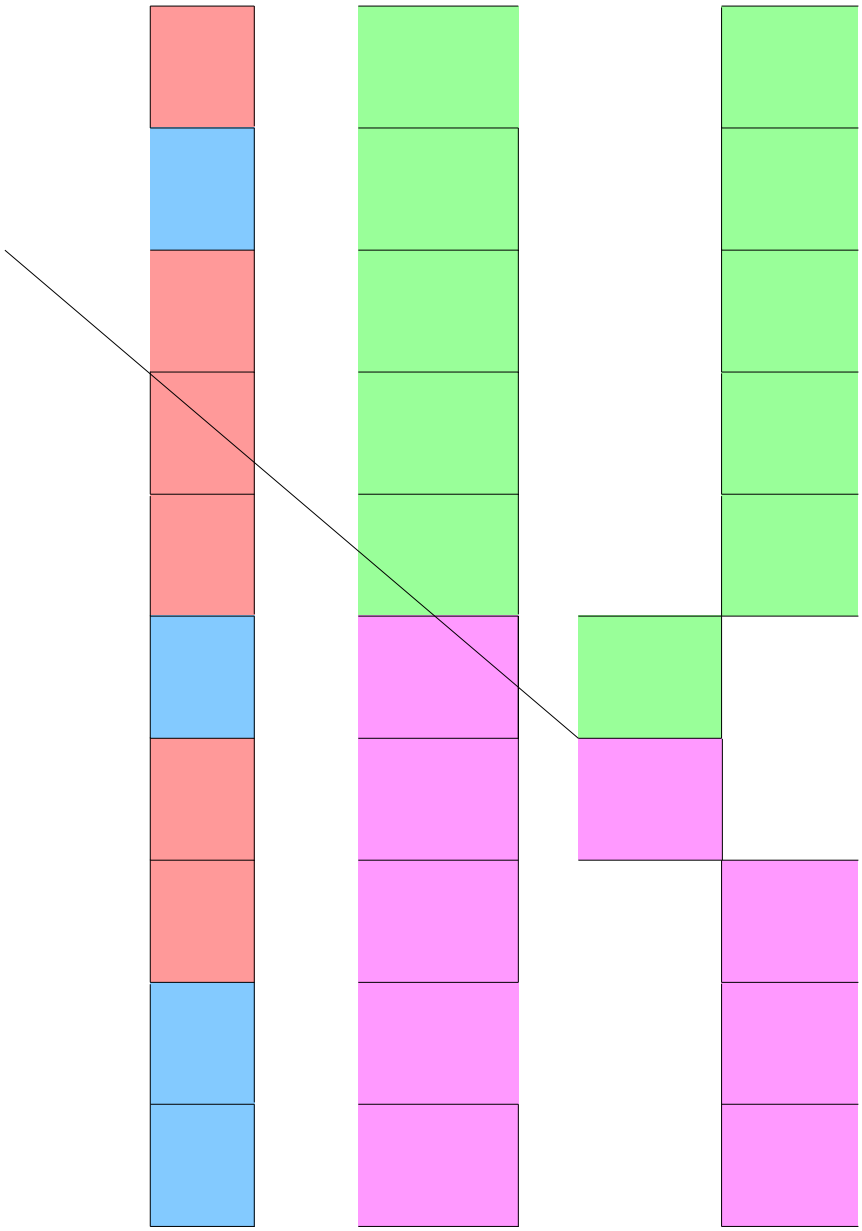


The idea which make interesting this algorithm, is that you can divide, in an easy way, the total number of blocks to merge, in several parts, obtaining several groups of blocks, independents between them, which can be merged in parallel.

2.1.- MERGE SUBDIVISION

Suppose, we have the next first merge, with block size of 4, and want to divide in two independent parts, to be executed by different threads.

To divide, we must looking for a border between two blocks of different color. And make the merge between them. In the example, to obtain parts of similar size , we can cut in the frontier 4-5, or in the border 5 -6, being the two options.



firstH with the blocks 1-4, and the second with the last two paefts, whicel can be merged in parallel.

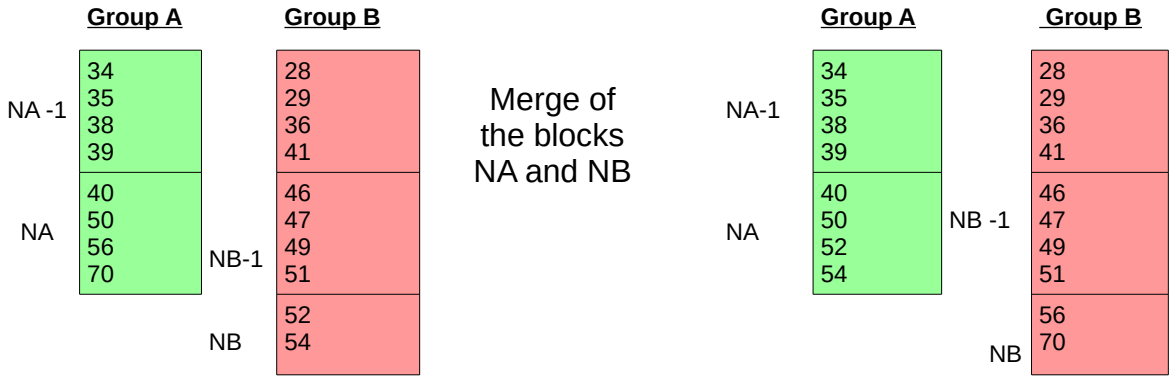
In the option 2, the frontier is between the blocks 5 and 6, and the last of the block 5 is greater than the first of the block 6, and we must do the merge of the two blocks, and appear new values for the blocks 5 and 6. Then we have two parts, which can be merged in a parallel way. The first with the blocks from 0 to 5, and the second with the 6 to 9.

2.2.- NUMBER OF ELEMENTS NOT MULTIPLE OF THE BLOCK SIZE

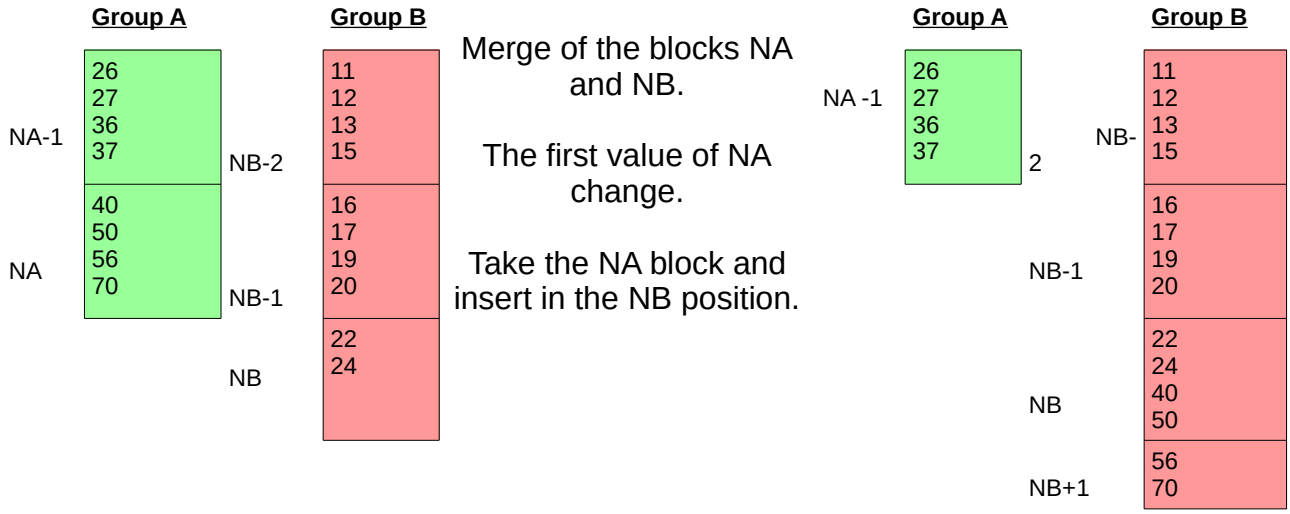
Until now, we assumed the number of elements is a multiple of the block size. When this is not true, we configure the blocks, beginning for the position 0, and at end we have an incomplete block called tail. The tail block always is in the last part to merge. We use a special operation just for this block, described in the next example :

We have two groups of blocks A and B, with NA and NB blocks respectively. The tail block, if exist, always is in the group B. Merge the tail block with the last block of the group A. With the merge two cases appear shown in the next examples:

Case 1: The first value of the NA block don't change, and don't do nothing with the blocks.



Case 2: The first value of the block A, changes, and we delete the block of the group A and insert in the group B, immediately before the tail block. With this operation we guarantee the tail block is the last



2.3.- INDIRECT SORT

In today's computers, the main bottleneck is the data bus. When the process needs to manage memory in memory different locations, as in the parallel sorting algorithms, the data bus limits the speed of the algorithm.

In the benchmarks done on a 32 HW threads, sorting N elements of 64 Bytes of size requires the same time as sorting N / 2 elements 128 bytes of size. The comparison is the same with the two sizes.

In the algorithm described, in each merge, the blocks are moved, to be merged in the next step. This slows down the algorithm, due to the data bus bottleneck.

To avoid this, do an indirect merge. We have an index with the relative position of the blocks. This implies the blocks to merge are not contiguous, but that doesn't change the validity of the algorithm.

When all the merges are complete, with this index we move the blocks, and have all the data sorted. This block movement is done with a simple and parallel algorithm,

2.4.- IN PLACE REARRANGEMENT FROM A INDEX (BLOCK SORTING)

The tail block, if one exists, is always in the last position, and doesn't need to be moved. We move blocks with the same size using an index. The best way is to see with an example :

We have an unsorted data vector (D) with numbers. We also have, an index (I), which is a vector with the relative position of the elements of D

200	500	600	900	100	400	700	800

4	0	5	1	2	6	7	3

To move the data, we need an auxiliary variable (Aux) of the same size as the data. In this example the data is a number, but in the case of blocks, the variable must have the size of a block.

This can be a bit confusing. Here are the steps of the process :

- Aux = D[0], and after this we must find which element must be copied in the position D[0], we find it in the position 0 of the index, and it is the position 4.
- The next step is D[0] = D[4], and find the position to move to the position 4, in the position 4 of the index, and this is the position 2.
- When doing this successively, once the new position obtained is the first position used, (in this example is the 0), move to this position from the Aux variable, and the cycle is closed.

In this example the steps are:

Aux ← D[0]
D[0] ← D[4]
D[4] ← D[2]
D[2] ← D[5]
D[5] ← D[6]
D[6] ← D[7]
D[7] ← D[3]
D[3] ← D[1]
D[1] ← Aux

If we follow the arrows, we see a closed cycle. This cycle has a sequence formed by the position of the elements passed. In this example the sequence is 0, 4, 2, 5, 6, 7, 3, 1.

With small elements the sequence is useless, because instead of extracting the sequence, we can move the data, and all is done. But with big elements, as the blocks used in this algorithm, the sequence are very useful, because from the sequences, we generate the parallel work for the threads.

There can be several cycles In an index, with their corresponding sequences. To extract the sequences, begin with the index.

- If in a position, the content is the position, indicate this element is sorted, and don't need to be moved.
- If it's different, this imply it's the beginning of a cycle, and must extract the sequence, as described before, and determine the positions visited in the index.

We can see with an example of an index with several cycles.

Data vector (D)

24	23	20	15	12	10	21	17	19	13	22	18	14	11	16

Index (I)

5	13	4	9	12	3	14	7	11	8	2	6	10	1	0

Doing the procedure previously described, find 3 sequences

- 5, 3, 9, 8, 11, 6, 14, 0
- 4, 12, 10, 2
- 13, 1

In the real problems, usually appear a few long sequences, and many small sequences. This permits parallel execution, but it's not very efficient, because a long sequence can keep one thread busy while the other threads are waiting, because they are finished with the sort sequences. Or even worse, there can be just one sequence.

To deal with this issue, the long sequences can be easily divided and done in parallel. This permit an optimal parallelization.

2.5- SEQUENCE PARALLELIZATION

The procedure can be a bit confusing, so here's an example:

Data vector (D)

100	140	70	60	90	00	160	80	50	130	150	20	170	10	110	30	120	40

Index vector (I)

5	13	11	15	17	8	3	2	7	4	0	14	16	9	1	10	6	12

If we extract the sequence, as described before, we find only one loop, and one sequence. This sequence is;

5	8	7	2	11	14	1	13	9	4	17	12	16	6	3	15	10	0

We want to divide in 3 sequences of 6 elements. Each sequence obtained are independent between them and can be done in parallel. The procedure is :

Generate a fourth sequence with the contents on the last position of each sequence. In this example is

14	12	0

Now, consider the 3 sequences as independent and can be applied in parallel. We apply the sequences over the vector of data (D), and when all are finished, apply the sequence obtained with the last position of the sub sequences. And all is done

See this example:

The data vector is

100	140	70	60	90	00	160	80	50	130	150	20	170	10	110	30	120	40

Apply this sequence

5	8	7	2	11	14

The new data vector is

100	140	20	60	90	50	160	70	80	130	150	110	170	10	00	30	120	40

Apply this sequence

1	13	9	4	17	12

The new data vector is

100	10	20	60	40	50	160	70	80	90	150	110	140	130	00	30	120	170

Apply this sequence

16	6	3	15	10	0

The new data vector is

120	10	20	30	40	50	60	70	80	90	100	110	140	130	00	150	160	170

These 3 sub sequences can be done in parallel, because don't have any shared element.
Finally, when the subsequences are been applied, we apply the last sequence, obtained with the last positions of the sub sequences

14	12	0

The new data vector is fully sorted

00	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170

3.-

3.1.- INTRODUCTION

To benchmark this we use the implementation proposed for the Boost Sort Parallel Library. It's pending of the final approval, due this can suffer some changes until the final version and definitive approval in the boost library. You can find in https://github.com/fjtapia/sort_parallel.

If you want run the benchmarks in your machine, you can find the code, instructions and procedures in https://github.com/fjtapia/sort_parallel_benchmark

For the comparison, we use these parallel algorithms:

1. GCC Parallel Sort
2. Intel TBB Parallel Sort
3. Block Indirect Sort

3.2.- DESCRIPTION

The benchmark are running in a machine with a I7 5820 3.3 GHz 6 cores, 12 threads, quad channel memory (2133 MHz) with Ubuntu and the GCC 5.2 compiler

The compiler used was the GCC 5.2 64 bits

The benchmark have 3 parts:

1.- Sort of 100000000 uint64_t numbers randomly generated. The utility of this benchmark is to see the speed with small elements with a very fast comparison.

2.- Sort of 10000000 of strings randomly filled. The comparison is no so easy as the integers.

3.- Sort of objects of several sizes. The objects are arrays of 64 bits numbers, randomly filled. We will check with arrays of 1 , 2 , 4, 8, 16, 32 and 64 numbers.

Definition of the object	Bytes	Number of elements to sort
uint64_t [1]	8	100 000 000
uint64_t [2]	16	50 000 000
uint64_t [4]	32	25 000 000
uint64_t [8]	64	12 500 000
uint64_t [16]	128	6 250 000
uint64_t [32]	256	3 125 000
uint64_t [64]	512	1 562 500

The C++ definition of the objects is

```
template <uint32_t NN>
struct int_array
{   uint64_t M[NN];
};
```

The comparison between objects can be of two ways:

- Heavy comparison : The comparison is done with the sum of all the numbers of the array. In each comparison, make the sum.
- Light comparison : It's done using only the first number of the array, as a key in a register.

3.3.- LINUX 64 GCC 5.2 Benchmarks

The benchmark are running in a I7 5820 3.3 GHz 6 cores, 12 threads, quad channel memory (2133 MHz) with Ubuntu and the GCC 5.2 compiler.

3.3.1.-SINGLE THREAD ALGORITHMS

The algorithms involved in this benchmark are :

	Stable	Memory used	Comments
GCC sort	no	N + Log N	
boost sort	no	N + Log N	
GCC stable_sort	yes	N + N / 2	
Boost stable_sort	yes	N + N / 2	
Boost spreadsort	yes	N + Log N	Extremely fast algorithm, only for integers, floats and strings

INTEGER BENCHMARKS Sort of 100000000 64 bits numbers, randomly filled

	Time	Memory
GCC sort	8.33 secs	784 MB
Boost sort	8.11 secs	784 MB
GCC stable sort	8.69 secs	1176 MB
Boost stable sort	8.75 secs	1175 MB
Boost Spreadsort	4.33 secs	784 MB

STRINGS BENCHMARKS Sort of 10 000 000 strings randomly filled

	Time	Memory
GCC sort	6.39 secs	820 MB
Boost sort	7.01 secs	820 MB
GCC stable sort	12.99 secs	1132 MB
Boost stable sort	9.17 secs	976 MB
Boost Spreadsort	2.44 secs	820 MB

OBJECTS BENCHMARKS Sorting of objects of different sizes. The objects are arrays of 64 bits numbers. This benchmark is done using two kinds of comparison.

Heavy comparison : The comparison is done with the sum of all the numbers of the array. In each comparison, make the sum.

	8 bytes	16 bytes	32 bytes	64 bytes	128 bytes	256 bytes	512 bytes	Memory used
GCC sort	8.75	4.49	3.03	1.97	1.71	1.37	1.17	783 MB
Boost sort	8.19	4.42	2.65	1.91	1.67	1.35	1.09	783 MB
GCC stable_sort	10.23	5.67	3.67	2.94	2.6	2.49	2.34	1174 MB
Boost stable_sort	8.85	5.11	3.18	2.41	2.01	1.86	1.60	1174 MB

Light comparison : It's done using only the first number of the array, as a key in a register.

	8 bytes	16 bytes	32 bytes	64 bytes	128 bytes	256 bytes	512 bytes	Memory used
GCC sort	8.69	4.31	2.35	1.50	1.23	0.86	0.79	783 MB
Boost sort	8.18	4.04	2.25	1.45	1.24	0.88	0.76	783 MB
GCC stable_sort	10.34	5.26	3.20	2.57	2.47	2.41	2.30	1174 MB
Boost stable_sort	8.92	4.59	2.51	1.94	1.68	1.68	1.50	1174 MB

3.3.2.-PARALLEL ALGORITHMS

The algorithms involved in this benchmark are :

	Stable	Memory used	Comments
GCC parallel sort	No	2N	Based on OpenMP
TBB parallel sort	No	N + LogN	
Boost parallel sort	No	N +block_size*num threads	New parallel algorithm
GCC parallel stable sort	Yes	2 N	Based on OpenMP
Boost parallel stable sort	Yes	N / 2	
Boost sample sort	Yes	N	
TBB parallel stable sort	Yes	N	Experimental code, not in the TBB official

The block_size is an internal parameter of the algorithm, which in order to achieve the highest speed, change according the size of the objects to sort according to the next table. The strings use a block_size of 128.

object size (bytes)	1 - 15	16 - 31	32 - 63	64 - 127	128 - 255	256 - 511	512 -
block_size	4096	2048	1024	768	512	256	128

For the benchmark I use the next additional code:

- Threading Building Blocks (TBB)
- OpenMP
- Threading Building Block experimental code (https://software.intel.com/sites/default/files/managed/48/9b/parallel_stable_sort.zip)

The most significant of this parallel benchmark is the comparison between the Parallel Sort algorithms. GCC parallel sort is extremely fast with many cores, but need an auxiliary memory of the same size then the data. In the other side Threading Building Blocks (TBB), is not so fast with many cores , but the auxiliary memory is LogN.

The Boost Parallel Sort (internally named Block Indirect Sort), is a new algorithm created and implemented by the author for this library, which combine the speed of GCC Parallel sort, with a small memory consumption (block_size elements for each thread). The worst case for this algorithm is when have very big elements and many threads. With big elements (512 bytes), and 12 threads, The memory measured was:

GCC Parallel Sort (OpenMP)	1565 MB
Threading Building Blocks (TBB)	783 MB
Block Indirect Sort	812 MB

In machines with a small number of HW threads, TBB is faster than GCC, but with a great number of HW threads GCC is more faster than TBB. Boost Parallel Sort have similar speed than GCC Parallel Sort with a great number of HW threads, and similar speed to TBB with a small number.

INTEGER BENCHMARKS Sort of 100 000 000 64 bits numbers, randomly filled

	time (secs)	**memory (MB)**
OMP parallel_sort	1,25	1560
TBB parallel_sort	1,64	783
Boost parallel_sort	1,08	786
OMP parallel_stable_sort	1,56	1948
TBB parallel_stable_sort	1,56	1561
Boost sample_sort	1,19	1565
Boost parallel_stable_sort	1,54	1174

STRING BENCHMARK Sort of 10000000 strings randomly filled

	time (secs)	**memory (MB)**
OMP parallel_sort	1,49	2040
TBB parallel_sort	1,84	820
Boost parallel_sort	1,3	822
OMP parallel_stable_sort	2,25	2040
TBB parallel_stable_sort	2,1	1131
Boost sample_sort	1,51	1134
Boost parallel_stable_sort	2,1	977

OBJECT BENCHMARKS Sorting of objects of different sizes. The objects are arrays of 64 bits number. This benchmark is done using two kinds of comparison.

Heavy comparison : The comparison is done with the sum of all the numbers of the array. In each comparison, make the sum.

	8 bytes	**16 bytes**	**32 bytes**	**64 bytes**	**128 bytes**	**256 bytes**	**512 bytes**	**Memory Used**
OMP parallel_sort	1,27	0,72	0,56	0,45	0,41	0,39	0,32	1565
TBB parallel_sort	1,63	0,8	0,56	0,5	0,44	0,39	0,32	783
Boost parallel_sort	1,13	0,67	0,53	0,47	0,43	0,41	0,34	812
OMP parallel_stable_sort	1,62	1,38	1,23	1,19	1,09	1,07	0,97	1954
TBB parallel_stable_sort	1,58	1,02	0,81	0,76	0,73	0,73	0,71	1566
Boost sample_sort	1,15	0,79	0,63	0,62	0,62	0,61	0,6	1566
Boost parallel_stable_sort	1,58	1,02	0,8	0,76	0,73	0,73	0,71	1175

Light comparison : It's done using only the first number of the array, as a key in a register.

	8 bytes	16 bytes	32 bytes	64 bytes	128 bytes	256 bytes	512 bytes	Memory used
OMP parallel_sort	1,24	0,71	0,48	0,41	0,38	0,35	0,32	1565
TBB parallel_sort	1,66	0,8	0,52	0,43	0,4	0,35	0,32	783
Boost parallel_sort	1,11	0,65	0,49	0,43	0,41	0,37	0,34	812
OMP parallel_stable_sort	1,55	1,36	1,23	1,18	1,09	1,07	0,97	1954
TBB parallel_stable_sort	1,58	0,91	0,75	0,72	0,71	0,72	0,71	1566
Boost parallel_stable_sort	1,16	0,74	0,63	0,62	0,61	0,61	0,6	1566
Boost sample_sort	1,56	0,91	0,75	0,72	0,72	0,72	0,71	1175

3.4.- WINDOWS 10 VISUAL STUDIO 2015 x64 Benchmarks

The benchmark are running in a virtual machine with Windows 10 and 10 threads over a I7 5820 3.3 GHz with Visual Studio 2015 C++ compiler.

3.4.1.-SINGLE THREAD ALGORITHMS

The algorithms involved in this benchmark are :

	Stable	Memory used	Comments
std::sort	no	N + Log N	
boost sort	no	N + Log N	
std::stable_sort	yes	N + N / 2	
Boost stable_sort	yes	N + N / 2	
Boost spreadsort	yes	N + Log N	Extremely fast algorithm, only for integers, floats and strings

INTEGER BENCHMARKS Sort of 100000000 64 bits numbers, randomly filled

	Time (secs)	Memory (MB)
std::sort	13	763
Boost sort	10,74	763
std::stable_sort	14,94	1144
Boost stable_sort	13,37	1144
Boost spreadsort	9,58	763

STRING BENCHMARKS Sort of 10 000 000 strings randomly filled

	Time (secs)	Memory (MB)
std::sort	13,3	862
Boost sort	13,6	862
std::stable_sort	26,99	1015
Boost stable_sort	20,64	1015
Boost spreadsort	5,7	862

cts of different sizes.
comparison.

with the sum of

STRINGS BENCHMARK Sort of 10000000 strings randomly filled

	Time (secs)	**Memory (MB)**
PPL parallel sort	3,76	864
PPL parallel buffered sort	3,77	1169
Boost parallel sort	3,41	866
Boost sample sort	3,74	1168
Boost parallel stable sort	5,7	1015

OBJECTS BENCHMARKS Sorting of objects of different sizes. The objects are arrays of 64 bits number. This benchmark is done using two kinds of comparison.

Heavy comparison : The comparison is done with the sum of all the numbers of the array. In each comparison, make the sum.

	8 bytes	***16 bytes***	***32 bytes***	***64 bytes***	***128 bytes***	***256 bytes***	***512 bytes***	***Memory used***
PPL parallel sort	2,84	1,71	1,01	0,84	0,89	0,77	0,65	764
PPL parallel buffered sort	2,2	1,29	2	0,88	0,98	1,32	0,82	1527
Boost parallel sort	1,93	0,82	0,9	0,72	0,77	0,68	0,69	764
Boost sample sort	3,02	2,03	2,15	1,41	1,55	1,82	1,39	1526
Boost parallel stable sort	3,36	2,67	1,62	1,45	1,38	1,19	1,37	1145

Light comparison : It's done using only the first number of the array, as a key in a register.

	8 bytes	***16 bytes***	***32 bytes***	***64 bytes***	***128 bytes***	***256 bytes***	***512 bytes***	***Memory used***
PPL parallel sort	3,1	1,37	0,97	0,7	0,61	0,58	0,57	764
PPL parallel buffered sort	2,31	1,39	0,9	0,88	1,1	0,89	1,44	1527
Boost parallel sort	2,15	1,21	0,7	0,72	0,41	0,51	0,54	764
Boost sample sort	3,4	1,94	1,56	1,41	2	1,41	1,96	1526
Boost parallel stable sort	3,56	2,37	1,79	1,45	1,72	1,34	1,44	1145

- **Introduction to Algorithms,** 3rd Edition (Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein)
- **Structured Parallel Programming: Patterns for Efficient Computation** (Michael McCool, James Reinders, Arch Robison)
- **Algorithms + Data Structures = Programs** (Niklaus Wirth)

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