CHAPTER-1

INTRODUCTION

1.1 INTRODUCTION

Nucleus is many body quantum system in which nucleons interacting mainly by strong nuclear forces. The complete theory of atomic nuclei must describe structure of nucleus (distribution and properties of nuclear levels) and mechanism involved in nuclear reactions (dynamical properties of nuclei). Development of theory of nucleus needs to overcome following three main problems:

1. The nucleon-nucleon interaction acting inside the nucleus as an effective force is not well understood as yet. Actually, the nucleon-nucleon interaction is not one of nature's basic forces, but it is the result of the strong force that acts between the quarks of which the nucleon is composed. At low nuclear excitation energies however, quarks do not seem to play an explicit role in determining the nuclear structure properties. Nucleons can thus be thought to be the basic nuclear constituent particles. Usually, the nucleon-nucleon interaction is represented by well-constructed effective interactions that approximate the attractive long-range character inside the nucleus.
2. The atomic nucleus is a typical example of a many-body system. On the one hand, such systems have too many constituents to allow for an exact calculation of all of their properties. The most massive nuclei can count up to almost 300 nucleons. Ab-initio calculations start from the free nucleon-nucleon interaction in order to explain properties of the nucleus. At present, such calculations can be carried out only for the very light nuclei, up to 12 nucleons [*Carlson and Sciavilla, 1998, Pieper 2002*]. On the other hand, nuclei have far too few constituents to allow for statistical methods to be used successfully.
3. Equations describing motion of nucleons in the nucleus are very complicated – problem of mathematical description.

1.2 NUCLEAR MODELS

To describe the properties and state of a nucleus, we need the wave function of the nuclear system (nucleus). It is possible to solve the Schrodinger equation only for the simplest nuclei. We thus need models to describe the complex nuclear system. The models make use of similarities between the nucleus and simpler systems, which are well understood mathematically. There is a variety of nuclear models, considering that the nucleons interact with various coupling constants.

1.2.1 Liquid Drop Model

The Liquid Drop Model (LDM) was historically the first nuclear model that was developed. It describes very accurately the ”bulk” properties of nuclei, i.e. the properties to which all the nucleons of the nucleus contribute, such as the nuclear mass, or equivalently, nuclear binding energies, or the derived nucleon-separation energies. The LDM describes these bulk properties, in terms of volume energy, surface energy, coulomb energy, symmetry energy, very much like the description of a charged liquid drop that is suspended in space. This was the first model to successfully describe the nuclear fission process by calculating the deformation of a nucleus until it breaks up in two fragments. The Collective Model can be viewed as an extension of the Liquid Drop Model [*Bohr and Mottelson, 1969, 1975*]. It emphasizes the coherent behavior in the dynamics of all of the nucleons. Among the kinds of collective motion that can occur in nuclei are, rotations or vibrations that involve the entire nucleus. In heavier nuclei, far from the magic numbers, this results in rather low-lying nuclear excited states that are connected with strong electromagnetic quadrupole transitions.

1.2.2 Nuclear Shell Model

The Nuclear Shell Model (NSM) studies nuclear properties starting from the individual nucleons, moving independently from each other to a first approximation [*Hyde, 1994*]. It incorporates a series of approximations to minimize the number of nucleon degrees of freedom.

A first approximation is to assume that the motion of each nucleon is governed by an average attractive force of all the other nucleons. Usually, a schematic Harmonic Oscillator (HO) potential is used to approximate this average spherical force field, because one can derive analytic solutions. For the rest, independently from each other, nucleons move through the nucleus along orbits that are grouped into shells. Each orbit can contain only a limited number of nucleons because of the Pauli principle. In the nuclear ground state, and using the most simple shell-model considerations, nucleons will fill orbits from the bottom of the potential up to a certain level, known as the Fermi level. When a nucleus becomes excited, it rearranges some of its nucleons from below the Fermi level up to some higher-lying free orbits. Nuclei with a *magic* number of protons or neutrons (Z, N = 2, 8, 20, 28, 50, 82, 126…) that fill a shell completely are extra stable because its nucleons have to bridge a large energy gap between the filled shell and the higher lying unfilled orbitals in the next shell.

Now a second approximation comes in. An ordinary non-magic nucleus can be thought to have an inert core of magic numbers of protons and neutrons, and only a limited number of valence nucleons over which the nucleus distributes its excitation energy. These valence nucleons have either a particle or a hole nature, depending on whether they are situated outside a closed shell, or inside. The nucleon-nucleon interaction was approximated by an average attractive field, but there will be some residual interactions that are not contained in this average mean-field. Incorporating these residual interactions between the valence nucleons produces the nuclear energy spectra as observed experimentally, and also permits the detailed calculation of a whole range of nuclear properties. Typical residual interactions exhibit a pairing property, that expresses the tendency of the nucleon-nucleon interaction to couple nucleons to pairs with angular momentum $j^{\pi} = 0^{+}$, as well as the long-range proton-neutron quadrupole interaction that favors nuclear deformation. As the number of valence nucleons grows, the core becomes polarized and collective nuclear motion comes in, in a natural way.

1.2.3 Collective Model

The Collective model of nuclear structure thus succeeds in combining the seemingly irreconcilable points of view of the liquid drop and independent particle models. Spherical and deformed shell model view the nucleus as a collection of fermions occupying single-particle states in the potential well. These models can successfully predict properties of nuclear states with configuration, dominated by a single nucleon or to some extent by relatively small number of nucleons. These models are useful in predicting spins, parities, magnetic, and quadrupole moments of states with a single nucleon, or a pair of nucleons outside an even-even core. Nuclear properties which are determined by a single nucleon are often referred to as the single-particle properties. In numerous cases nuclear behavior can be described in terms of the single particle properties, however, in equally numerous examples the single particle description is far from being adequate. Experimental data suggest that in many nuclei ground states or low-energy excitations involve a coordinated, large-amplitude motion of many nucleons. Nuclear properties which are determined by such a coordinated, large-amplitude motion of many nucleons are often referred to as collective properties. Surprisingly, collective properties are often quite simple to describe in terms of deformation of nuclear surface. Examples are provided by nuclear vibrations around spherical shape or nuclear rotation of a deformed shape.

1.2.4 Nilsson Model

Most nuclei near closed shell are spherical in shape and for these nuclei, an independent particle approximation in a spherically symmetric potential is found to work admirably well. In a similar way, it is reasonable to expect that an independent particle wave function generated using a deformed potential should work for the nuclei which are found to be deformed. In spherical shell model, each single particle state with a specific j value has a degeneracy of $2j+1$. With respect to any axis, all the $(2j+1)$ orientations are equal. But this is no longer true in the case of deformed nucleus, since the energy levels in the deformed potential depend upon the spatial orientation of the orbits. Precisely the energy depends on the projection component of j on the symmetry axis and is denoted as Ω. The axially symmetric nuclei have a reflection symmetry which makes the energy levels corresponding to $\pm\Omega$ coincide. In the Nilsson Model [*Nilsson et al., 1969*], the potential in the Hamiltonian comprises the anisotropic harmonic oscillator potential plus the spin-orbit coupling term and the centrifugal potential. The oscillator part is given by

$$\mathrm{H}_0 = -\frac{\hbar^2}{2m}\nabla^2 + \frac{1}{2}m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_x^2 z^2)$$

Considering the case of cylindrical symmetry, and introducing a single parameter of deformation δ, we can write

$\omega_x^2 = \omega_y^2 = \omega_\rho^2 = \omega_0^2\left[1+(2/3)\delta\right];\ \omega_z^2 = \omega_0^2\left[1-\frac{4}{3}\delta\right]$, with $\omega_x\omega_y\omega_z = \omega_0^3$ = constant, which is the condition for the constant volume of the nucleus. The dependence of ω_0 on δ is given by

$$\omega_0(\delta) = \omega_0^0\left[1+\frac{2}{3}\delta\right]^2\left[1-\frac{4}{3}\delta\right]^{-1/6} = \omega_0^0\left[f(\delta)\right]$$

where $\hbar\omega_0^0 = 41A^{-1/3} f(\delta) MeV$. The total Hamiltonian in terms of stretched coordinates can been written as:

$$\mathrm{H} = \frac{1}{2}\hbar\omega_0\left[-\nabla^2 + r^2\right] - \delta\hbar\omega_0\frac{4}{3}\sqrt{\frac{\pi}{5}}r^2Y_{20} + C\vec{l}.\vec{s} + Dl^2$$

In realistic calculations of single particle energies in well deformed nuclei, Nilsson used strength parameters κ and μ in place of C and D. The energy Eigen value and mixed amplitude of wave functions can be obtained by diagonlization of the above total Hamiltonian.

1.3 PRESENT FOCUS

1.3.1 One Particle plus Rotor Model and Signature Effects in one quasiparticle bands

The Particle plus Rotor Model (PRM) approach being presented in the present thesis is in terms of an angular momentum, a physical observable in the experiments, so direct comparison with the experimental results can be made. This is the main motivation behind the use of PRM for present study. Depending on the shape of core and number of valence particles treated, the different versions of the one particle plus rotor model approach exists in literature. But the present study being presented in this thesis revolves only around the theoretical understanding of signature effects in one quasiparticle rotational bands observed in deformed axially symmetric nuclei.

Signature is a quantum number related to the invariance of the wave function of a axially symmetric deformed nucleus with respect to a rotation by π about an axis (say x-axis) perpendicular to the symmetry axis. If R_x is the corresponding rotation operator then

$$R_x(\pi)\Psi_\alpha = e^{-i\pi J_x}\Psi_\alpha = e^{-i\pi\alpha}\Psi_\alpha$$

where Ψ_α denotes the wave function with signature α and the Eigen values of R_x are denoted by r are given as:

$r = e^{-i\pi\alpha}$ and one obtains for integer I

$$r = (-1)^I$$

where I is total nuclear spin. We thus have the following classification of wave function for even nucleon number systems

$$r = +1(\alpha = 0),\ I = 0,2,4,6...$$
$$r = -1(\alpha = 1),\ I = 1,3,5,7...$$

while for odd nucleon number systems [*Bengtsson and Frauendorf 1979; Singh et al. 2007*]

$$r = -i(\alpha = +\frac{1}{2}), I = \frac{1}{2}, \frac{5}{2}, \frac{9}{2}......$$
$$r = +i(\alpha = -\frac{1}{2}), I = \frac{3}{2}, \frac{7}{2}, \frac{11}{2}......$$

Odd–even staggering in the rotational bands of odd-A, nuclei is the most significant characteristic linked to the signature quantum number. When the rotation axis coincides with one of the principal axes perpendicular to the symmetry axis, the signature remains a good quantum number. Under suitable conditions, the odd-I (i.e. odd integer plus half) members of a band get shifted in energy with respect to the even-I (i.e. even integer plus half) members of a band. In such situations, odd-even staggering is identified as the signature splitting of a band because the signature is a good quantum number. In order to understand the cause of this shift in energy levels (signature splitting) let us consider signature quantum number in more detail. We know that signature quantum no. is related to the invariance of the wave function under rotation by an angle π about an axis perpendicular to symmetry axis. This invariance introduces a phase factor $(-1)^{I+K}$ (called signature) in the wave function and hence in the energy expression. This term alternates the sign for successive values of I, as a result of which the odd I (i.e. odd integer plus half) members of a band get shifted in energy with respect to even I (i.e. even integer plus half) members of a band. Therefore, there are two ΔI=2 bands instead of one ΔI=1 band and alternative spin states having the same signature are connected by E2 transitions. These two ΔI=2 bands are distinguished from each other by the signature quantum number α or r. In ΔI=1 rotational bands, the two signature branches are usually not equivalent energetically. Due to the Coriolis force acting on the valence particles, one of them called favoured, lies lower in energy than the other branch, called unfavoured. Whenever an expected favoured signature branch becomes unfavoured at higher spins i.e. a signature branch, which is expected to be lower in energy, becomes higher in energy, then it is called signature inversion.

1.4 CURRENT STATUS OF THE PRESENT STUDY

Hjorth et al; 1972**,** presented Coriolis mixing calculations for ^{161}Dy, but their main focus was the calculation of transition probabilities for this nuclide. In these calculations, they considered the active role of pairing and ΔN=2 interactions. *Lovhoiden and Tjom, 1972,* studied positive parity states in ^{169}Er using these Coriolis mixing calculations. *Hornshoj et al, 1975,* successfully explains the positive parity rotational bands of ^{231}Pa using Coriolis coupling of $i_{13/2}$ orbitals and two N=4 orbitals. *Jain 1984*, presented the systematic band mixing calculations for strongly coupled bands appearing ion in A=180 mass region. *Jain and Jain, 1984* and presented effective decoupling from particle rotor model. In these calculations they used schematic one-BCS-quasiparticle plus rotor band mixing calculation. *Sun et al. 1994* presented varied signature splitting phenomena in the frame work of angular momentum projection theory. Recently *Bi et al, 2009*, presented signature splitting in ^{173}W using Tri-axial Rotor Model. Although Coriolis mixing is assumed to be the main cause for the existence of signature effects on odd-A nuclei. But there are no any quantitative calculations exists in literature which incorporate the Angular Momentum Dependence (AMD) of inertia parameter and Rotational Correction Term (RCT) for explanation of signature effects observed in one quasiparticle rotational bands observed in A=180 mass region, which is the main objective of present study.