Cason Konzer ECN 370 Homework #1, Fall 2022

Thursday, September 8, 2022   8:03 PM

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ECN 370
Homework 1
Due: Thursday, September 15th by the end of the day.

Directions: Please complete this homework on a separate piece of paper and email your answers to me at ccdougla@umich.edu. A .pdf file is preferred, though any file works in practice.

1.   Suppose a consumer has income of $3,000 that he only spends on wine and cheese. Suppose that wine costs $3 a glass and cheese costs $6 per pound.

   a.   Draw the consumer's budget constraint. Put wine on the y-axis and cheese on the x-axis. What is the slope of his budget constraint and what does it tell us?

   b.   Draw a consumer's indifference curves for wine and cheese. Put wine on the y-axis and cheese on the x-axis. Describe and explain the four properties of these indifference curves.

   c.   Pick a point on the indifference curve for wine and cheese and show the marginal rate of substitution. What does the marginal rate of substitution tell us? (NOTE: you don't have to pick actual numbers on the indifference curve for the marginal rate of substitution. Just identify where the marginal rate of substitution is and what it tells us).

   d.   Show a consumer's budget constraint and indifference curves for wine and cheese. Pick a point on the budget constraint and show the optimal consumption choice using indifference curves (again, you don't have to use actual numbers). If the price of wine is $3 a glass and the price of cheese is $6 a pound, what is the marginal rate of substitution at this optimum? (HINT: what does the marginal rate of substitution equal at the optimum?)

   e.   What would happen to the consumer's budget constraint if the price of cheese increased to $10 per pound (maybe because ethanol subsidies are increasing the price of corn used to feed cattle), but her income remains at $3,000 and the price of wine remains at $3 per glass? What would happen to her budget constraint if the price of wine remained at $3 per glass and the price of cheese remained at $6 per pound, but she got a big raise at work and her income is now $6000? Draw both budget constraints to answer the question.

1

2. Consider two citizens, Bill and Ted, who live in the fictional country of "Econoland". Bill works as a lawyer and earns $50 per hour. Ted works at Meijer and earns $6 per hour. Both Bill and Ted have identical utility functions equal to $U = Y^{1/2}$ (which is the same as $U = \sqrt{Y}$) where Y is take-home pay (which also equals consumption, if neither Bill nor Ted save).
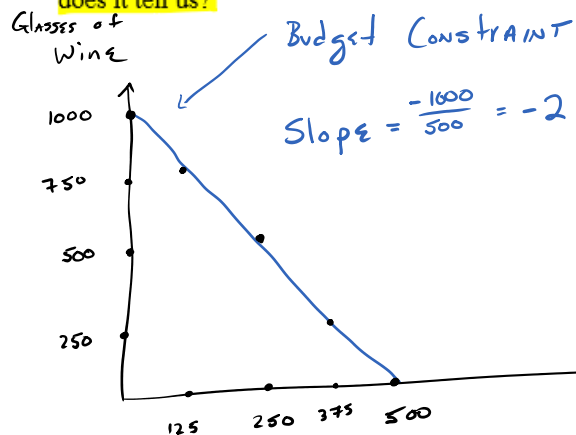
The government of Econoland is concerned about inequality. As a result, the President of Econoland wants you to evaluate three policy options that tax Bill and redistribute the proceeds to Ted. If the current policy of taxing Bill at 0% and redistributing nothing to Ted is left in place, Bill will choose to work 20 hours. If the government taxes Bill at a rate of 25%, Bill will chose to work 16 hours. If the government taxes Bill at a rate of 40%, Bill will choose to work 12 hours. Ted will choose to work 20 hours a week no matter what the tax rate is.

Note: Parts c. and d. may require a scientific calculator as a lot of four-function calculators do not allow you to raise something to anything but a 1/2 power. But, you do not need to run to a store to buy a scientific calculator if you don't have one. Just log onto windows, click on "Start" - > "All Programs" -> "Accessories" -> "Calculator". Then click on "view" and select "scientific calculator". The button "x^y" allows you to raise a number to any power you want. For example, 1/5 is the same as 0.2. So, to raise 50 to the 1/5 power (that is, calculate $50^{1/5}$), you just type 50, then click on x^y, and then type 0.2 and hit enter. If you do this correctly, you will see that $50^{1/5} =$ 2.19.

a. What is Bill's take-home pay and utility at each possible tax rate? What is Ted's?

b. Rank the three policy proposals using a Utilitarian social welfare function. Rank them using a Rawlsian social welfare function.

c. How would your answer to part b. change if the utility function was $U = Y^{1/5}$?

d. Suppose a more realistic representation of the problem is to assume a utility function of $U = Y^{1/3}$ for Bill and $U = Y^{1/2}$ for Ted. This is because Bill has to support a family with his income, and thus needs a higher level of income to get on the same level of utility. However, Ted is just a high school student living at home and working at Meijer as a summer job for some extra spending money. Thus, he needs less income than Bill to get on the same level of utility. How does the Rawlsian social welfare function rank the three policy proposals now?

2

1. Suppose a consumer has income of $3,000 that he only spends on wine and cheese. Suppose that wine costs $3 a glass and cheese costs $6 per pound.

a. Draw the consumer's budget constraint. Put wine on the y-axis and cheese on the x-axis. What is the slope of his budget constraint and what does it tell us?

Glasses of Wine

1000
750
500
250

125 250 375 500

Pounds of Cheese

Budget Constraint

$$Slope = \frac{-1000}{500} = -2$$

The slope tells us the Optimal (MRS) MARGINAL RATE of Substitution, eg. 2 glasses of wine will substitute for 1 pound of cheese. This is as 1 pound of cheese costs twice that of 1 glass of wine.

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b. Draw a consumer's indifference curves for wine and cheese. Put wine on the y-axis and cheese on the x-axis. Describe and explain the four properties of these indifference curves.

Glasses of Wine

1000
500

I₄ I₃ I₂ I₁

250 500

Pounds of Cheese
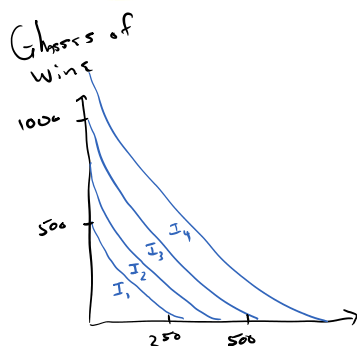
1. Consumers prefer higher indifference curves (more is preferred to less)

2. indifference curves are concave (bowed inward) → there is decreasing marginal utility

3. indifference curves slope downward. → goods must be substituted if one is removed.

4. indifference curves do not cross → This follows as more is preferred to less.

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c. Pick a point on the indifference curve for wine and cheese and show the marginal rate of substitution. What does the marginal rate of substitution tell us? (NOTE: you don't have to pick actual numbers on the indifference curve for the marginal rate of substitution. Just identify where the marginal rate of substitution is and what it tells us).

Glasses of Wine

1000

500

Pounds of Cheese

$$MRS = \frac{2 \ Glasses \ of \ Wine}{1 \ Pound \ of \ Cheese}$$
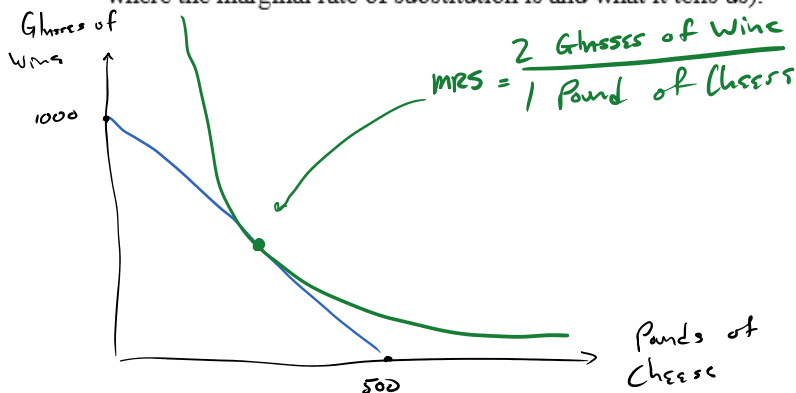
The MRS tells us how much wine the consumer is willing to give up in exchange for cheese.

d. Show a consumer's budget constraint and indifference curves for wine and cheese. Pick a point on the budget constraint and show the optimal consumption choice using indifference curves (a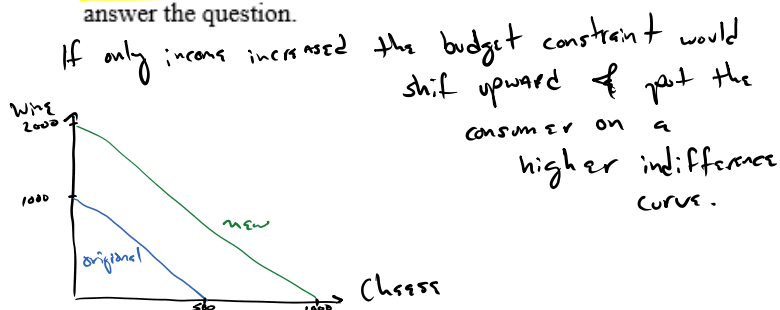gain, you don't have to use actual numbers). If the price of wine is $3 a glass and the price of cheese is $6 a pound, what is the marginal rate of substitution at this optimum? (HINT: what does the marginal rate of substitution equal at the optimum?)

See the graph above :) The marginal Rate of Substitution is Equal to the Slope of the Budget Constraint At Optimal Consumption Choice.

e. What would happen to the consumer's budget constraint if the price of cheese increased to $10 per pound (maybe because ethanol subsidies are increasing the price of corn used to feed cattle), but her income remains at $3,000 and the price of wine remains at $3 per glass? What would happen to her budget constraint if the price of wine remained at $3 per glass and the price of cheese remained at $6 per pound, but she got a big raise at work and her income is now $6000? Draw both budget constraints to answer the question.

If cheese increased in price while wine stayed constant, then the budget constraint would become steeper, e.g. The maximum amount of cheese available would decrease.

If only income increased the budget constraint would shift upward & put the consumer on a higher indifference curve.

2. Consider two citizens, Bill and Ted, who live in the fictional country of "Econoland". Bill works as a lawyer and earns $50 per hour. Ted works at Meijer and earns $6 per hour. Both Bill and Ted have identical utility functions equal to $U = Y^{1/2}$ (which is the same as $U = \sqrt{Y}$) where Y is take-home pay (which also equals consumption, if neither Bill nor Ted save).

The government of Econoland is concerned about inequality. As a result, the President of Econoland wants you to evaluate three policy options that tax Bill and redistribute the proceeds to Ted. If the current policy of taxing Bill at 0% and redistributing nothing to Ted is left in place, Bill will choose to work 20 hours. If the government taxes Bill at a rate of 25%, Bill will chose to work 16 hours. If the government taxes Bill at a rate of 40%, Bill will choose to work 12 hours. Ted will choose to work 20 hours a week no matter what the tax rate is.

a. What is Bill's take-home pay and utility at each possible tax rate? What is Ted's?

Bill's Tax Rate	Hourly Rate	0 %				25 %				40 %			
		hours	Earned ($)	Taxes Take Home	Utility	hours	Earned ($)	Taxes Home $	Utility	hours	Earned ($)	Take $ Home	Utility
Bill	$50	20	1000	1000	31.62	16	800	600	24.49	12	600	360	18.97
Ted	$6	20	120	120	10.95	20	120	120+200 =320	17.88	20	120	120+240 =360	18.97

Answers

M...	$50	20	1000	10..	3...	16	...	...	24...	12...	...		Answers
Ted	$6	20	120	120	10.95	20	120	120+200 =320	17.88	20	120	120+240 =360	18.97

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b. Rank the three policy proposals using a Utilitarian social welfare function. Rank them using a Rawlsian social welfare function.

SWF \ Tax Rate	0%	25%	40%	
Utilitarian	31.62+10.95 = 42.57	24.49+17.88 = 42.37	18.97+18.97 = 37.94	SWF OUTPUT
	#1	#2	#3	RANK
Rawlsian	min(31.62,10.95)= 10.95	min(24.49,17.88)= 17.88	min(18.97,18.97)= 18.97	
	#3	#2	#1	

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c. How would your answer to part b. change if the utility function was $U = Y^{1/5}$?

	0%		25%		40%		New Util
	Take Home	Util	Take Home	Util	Take Home	Util	
Bill	1000	3.981	600	3.554	360	3.245	
Ted	120	2.605	320	3.169	360	3.245	

SWF \ Tax Rate	0%	25%	40%	
Utilitarian	3.981+2.605 = 6.586	3.554+3.169 = 6.763	3.245+3.245 = 6.49	SWF OUTPUT
	#2	#1	#3	RANK
Rawlsian	min(3.981, 2.605) = 2.605	min(3.554,3.169) = 3.169	min(3.245, 3.245) = 3.245	
	#3	#2	#1	

A slight taxation now increases the Utilitarian SWF rather than decreasing it.
→ the optimal taxation rate shifts right

Note: The Rawlsian SWF always prefers taxation to a level such that the poorest has equal take home pay & utility as the richest.

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d. Suppose a more realistic representation of the problem is to assume a utility function of $U = Y^{1/3}$ for Bill and $U = Y^{1/2}$ for Ted. This is because Bill has to support a family with his income, and thus needs a higher level of income to get on the same level of utility. However, Ted is just a high school student living at home and working at Meijer as a summer job for some extra spending money. Thus, he needs less income than Bill to get on the same level of utility. How does the Rawlsian social welfare function rank the three policy proposals now?

	0%		25%		40%		New Util
	Take Home	Util	Take Home	Util	Take Home	Util	
Bill	1000	10	600	8.43	360	7.11	
Ted	120	10.95	320	17.88	360	18.97	

Tax	0%	25%	40%

The RANK of TAXATION policies for a Rawlsian SWF is now flipped as with Bills lifestyle even with no taxation he has less utility given his take home pay thus the lower take home pay.

SWF \ Tax Rate	0%	25%	40%
Utilitarian	10 + 10.95 = 20.95 #3	8.43 + 17.88 = 26.31 #1	7.11 + 18.97 = 26.08 #2
Rawlsian	min(10, 10.95) = 10 #1	min(8.43, 17.88) = 8.43 #2	min(7.11, 18.97) = 7.11 #3

SWF OUTPUT

RANK

given his take home pay that is
the lower take home pay
for the teenage lifestyle
of Ted.
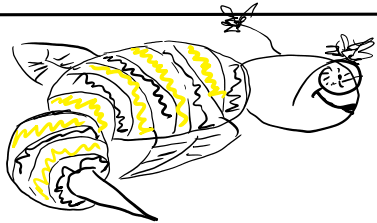
atch out for the
Killer Beez