WS 3: Tautologies

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Tautologies

Section 8. Logic of Statements

Problem 1. Use truth tables to prove the *distributive law for conjunction (/or)*:

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Predict the analogous *distributive law for disjunction (/and)*.

P	Q	-Q	R	Q ^ R	P v (Q ^ R)	P v Q	P v R	(P v Q) ^ (P v R)	Q v R	P -> (Q v R)	P ^ -Q	(P ^ -Q) -> R	(P v Q) -> R
T	T	F	T	T	T	T	T	T	T	T	F	T	T
T	T	F	F	F	T	T	T	T	T	T	F	T	F
T	F	T	T	F	T	T	T	T	T	T	T	T	T
T	F	T	F	F	T	T	T	T	F	F	T	F	F
F	F	T	F	F	F	F	F	F	F	T	F	T	T
F	F	T	T	F	F	F	T	F	T	T	F	T	T
F	T	F	F	F	F	T	F	F	T	T	F	T	F
F	T	F	T	T	T	T	T	T	T	T	F	T	T

Problem 2. Use truth tables to prove the first *conditional rule for disjunction*:

$$p \to (q \vee r) \equiv (p \wedge \neg q) \to r$$

Predict the equivalence given by the second conditional rule for disjunction:

$(P \vee Q) \to R \equiv (P \to R) \wedge (Q \to R)$ $(p \vee q) \to r \equiv$ ________

Conditional Statement	$p \to q$	If p, then q
Converse	$q \to p$	If q, then p
Inverse	$\neg p \to \neg q$	If not p, then not q
Contrapositive	$\neg q \to \neg p$	If not q, then not p

Problem 3. Consider the statement

If an object is a triangle, then it is a polygon.

Write down its inverse, converse, and contrapositive, and decide which are true.

Inverse: If an object is not a triangle, then it is not a polygon F
converse: " " " " " a polygon, " " " a triangle F
Contrapositive: " " " " " not a polygon, " " " not a triangle T

Problem 4. Consider the statement

If a number is even, then it is a multiple of four.

Write down its inverse, converse, and contrapositive, and decide which are true.

Inverse: If a number is not even, then it is not a multiple of 4. T
Converse: " " " " a multiple of 4, " " " even T
Contrapositive: " " " " not a multiple of 4, " " " not even F

Problem 5. Conjecture some relationships between the truth values of a statement, its inverse, its converse, and its contrapositive. Prove your conjectures using truth tables.

Both the inverse & convers, as well as the conditional & contrapositive are logically equivalent.

P	-P	Q	-Q	P -> Q	-P -> -Q	Q -> P	-Q -> -P
T	F	T	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	F	T	T	T	T	T
F	T	T	F	T	F	F	T

Problem 6. Write the negation of each of the following statements:

- If you score 70%, then you have done well in this course.

  you score 70% & have done horrible in the course

- If it rains, then I will stay at home.

  it rains & I go out from home

- If $x^2 + 2x + 1 = 0$, then $x = -1$.

  $x^2+2x+1=0 \wedge x \neq -1$

- $x^2 + x - 2 = 0$ implies $x = -1$ or $x = -2$.

  $x^2+x-2=0 \wedge (x \neq -1 \wedge x \neq -2)$

Note that for the remainder of the course, to negate an English sentence means more than just writing "It is not true that...." at the start. It means to unpack the meaning of that sentence's negation and re-write it as a natural English sentence.

Problem 7. The following are tautologies. Figure out what they're saying in English:

a) • $(p \wedge q) \to p$
b) • $((p \to q) \wedge (q \to r)) \to (p \to r)$.
c) • $(p \wedge (p \to q)) \to q$
d) • $((p \to q) \wedge \neg q) \to \neg p$
e) • $((\neg p \to q) \wedge (\neg p \to \neg q)) \to p$
f) • $(p \wedge q \wedge \neg p) \to q$

Note that it is these kinds of statements that allow us to progress from step to step in a formal proof.

d) if this is true that must be true. That is false thus this cannot be true.
e.) if this is false then that is true and that is false. that cannot be both true & false so this must be true

a) given this and that are true, proven by fact, then this is true

f.) if we know that is true while also knowing this & that are simultaneously true & false, we can infer that is still true.

b) we know fact 1 implies fact 2, we also know that fact 2 implies fact 3. Knowing these statements simultaneously, we can say fact 1 implies fact 3.

c) This is true, when this is true we know that is true. We know this is true so that is true.