Watkins on Latin Square Puzzles

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Abstract

Mathematics professor, John J. Watkins, brings up fundamental mathematics such as factorization, multiplication, and summation in his 2012 Article, “Triangular Numbers, Gaussian Integers, and KenKen,” on well-known Latin square puzzles. I take this opportunity to elaborate on Watkins’ insights while critiquing what worked and what fell short in the eyes of an undergraduate reader.

Keywords: KenKen, Latin Square, Triangular Number, Gaussian Integer

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Sparking reader Interest, Watkins begins his article with a prelude introducing his beginnings in mathematical readings. Either due to the title's ring, or the underlying principle, Watkins' cites Martin Gardner's 1959 column, "Euler's Spoilers," praising it as one of his favorites while introducing the concept of Latin squares. Their fundamentals are described in conjunction with their application in common puzzle application in papers through Sudokus and KenKens. After the setting is places, Watkins takes the opportunity to dive into some of the mathematics behind these structured puzzles. Solution techniques, such as the powerful concept of triangular numbers, and introduced along side puzzles that would be much more challenging without their aid. Some puzzles are started to examine technique application before leaving the ease of finishing to the readers. The concept of Gaussian integers is now plundered while extending the new numbers to the same puzzles. The puzzles become increasingly harder while hints are given along the way. Elaborating on some of the tricks and nuances Watkins leaves 3 puzzles to be solved by the reader while providing solutions to those only using Real numbers. A final summary is put in place to touch on how these same concepts are required in multiple concepts while studying mathematics. This article is directed towards to puzzle solving and math enthusiasts. Written for the Mathematics Association of America, the intended audience is for those studying or working in the field of mathematics. The purpose of this article is to spark interest in day-to-day mathematics used in problem solving. Watkins subtly introduces the beauty of mathematics and the application of mathematical fundamentals.

Triangular Numbers introduced new terminology to both my mathematical dictionary and my problem-solving strategy. Form a summation perspective I am quite familiar with triangular numbers yet had never heard the specific terminology. The application of this mathematics introduced a new strategy to solving Latin squares. When utilizing this tool, the key factor is that all rows and columns must sum to the triangular number defined by the size of the puzzle grid. In short, when given cages are known in a puzzle triangular numbers can be used to deduce the sum of nearby cages and cells. Figure 1 demonstrates the strategy within a single row. This same strategy can be used when solving sudoku cages, a new and interesting idea to me. Following the discussion on triangular numbers, Watkins delves into the use of Gaussian integers within Latin square puzzles. This sequence led me to the idea of the application of triangular numbers within a Gaussian integer based puzzle. By the same principal, the set of available Gaussian integers can be summed to form a new Gaussian triangular number. Again this concept can be used as a strategy to confirm and find solutions to Latin squares with Gaussian integers as place holders. (See Figure 2 for summation example.)

One of the more challenging ideas Watkins proposed is the idea of unique based solutions, one he got from Barry Cipra. This idea left me asking the question, “why can't the same puzzle have multiple solutions?” Implied that each puzzle has one unique key is the crux of this solving strategy. As a result I drove out to do something the Watkins did not by exploring KenKens that do not have a single unique solution. As shown in Figure 3, this is possible and relatively straight forward. Although I did not follow typical convention by allowing negative numbers, the concept is doable and stems in my original confusion. Additionally, this concept could allow for any 2 real numbers when changing the – operator shown to a +. Continuing on, the Gaussian numbers could also be replaced with real numbers that always multiply to the same N x cage. In reasoning out why unique solutions are often a required subtlety I came to the conclusion that it is to add complexity. To be frank, puzzles with multiple keys are often easier to solve. Playing devils advocate, by allowing multiple keys the solver is allowed additional ambiguity. Two different subjects presented with the same puzzle can arrive at differing solutions.

In conclusion, Watkins has successfully reached me, an undergraduate in mathematics, as well as a puzzle solving and mathematics advocate. His article inspired me to find beauty in mathematics and to dissect such application in everyday scenarios. Walking through techniques step by step was very clear and easy to follow. The ideas on uniqueness mention the ability to flip flop numbers within a given cage and I utilized this strategy to expand upon an example of the circumstance. I would recommend this article to any friends or colleagues who enjoy problem solving.

References

Watkins, J. J. (2012). Triangular Numbers, Gaussian Integers, and KenKen. *The College Mathematics Journal*, Pages 37 - 42.

Figures

Figure 1: Applying Triangular Numbers

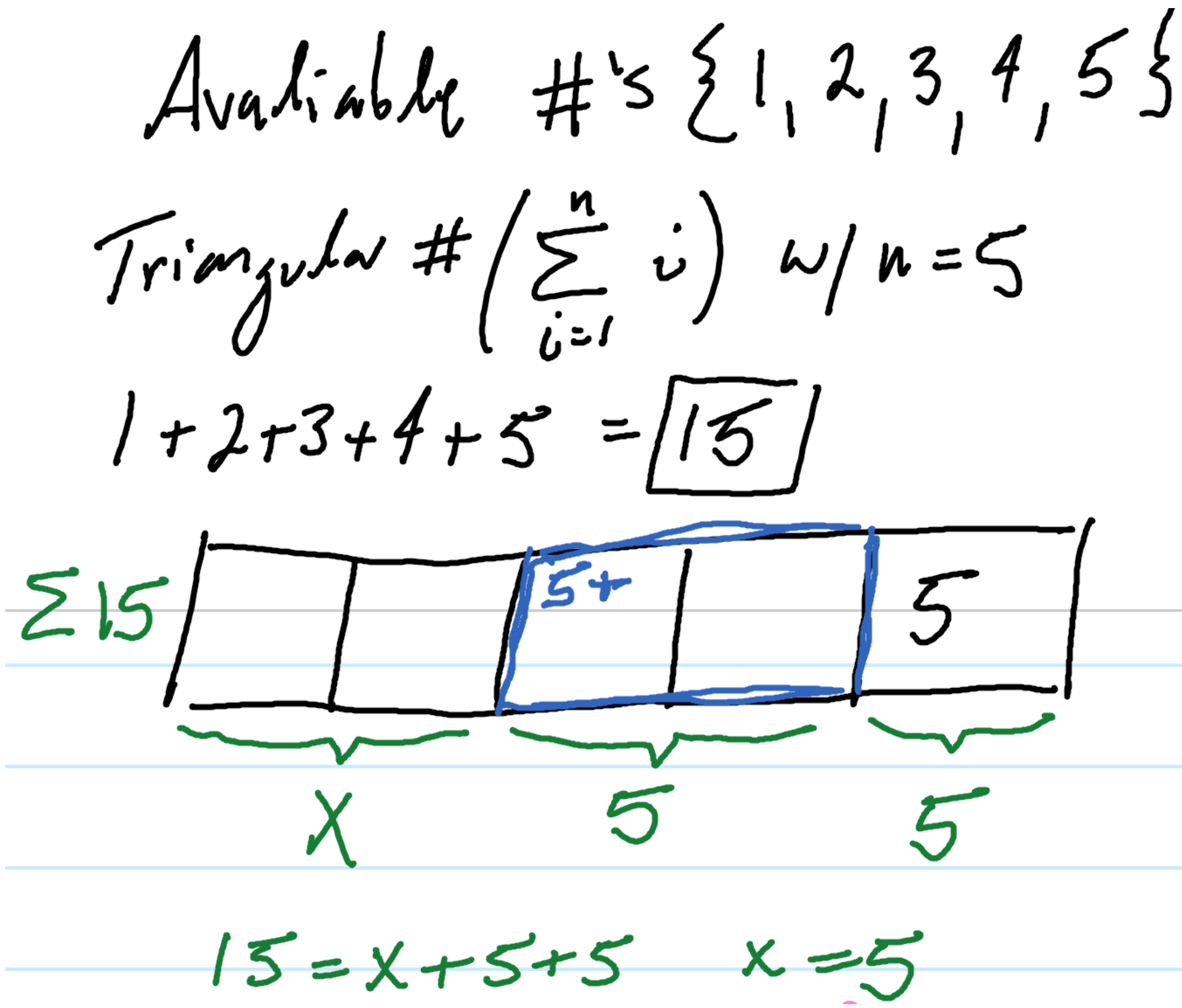

Figure 1. Illustration of row summation's relationship to triangular numbers.

Figure 2: Triangular Gaussian Numbers

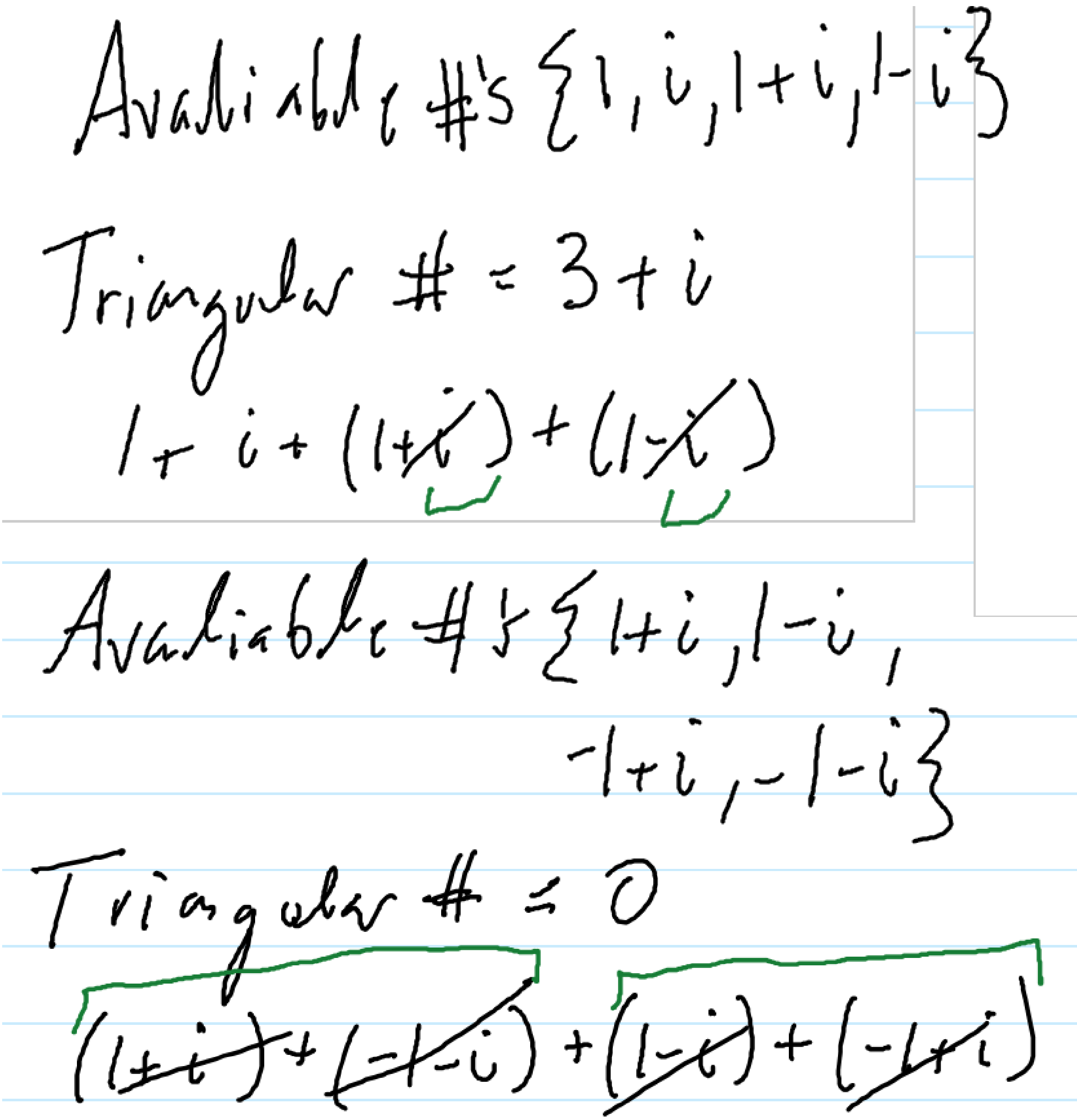

Figure 2: Application of triangular numbers to Gaussian integers.

Figure 3: Non-Unique Solutions

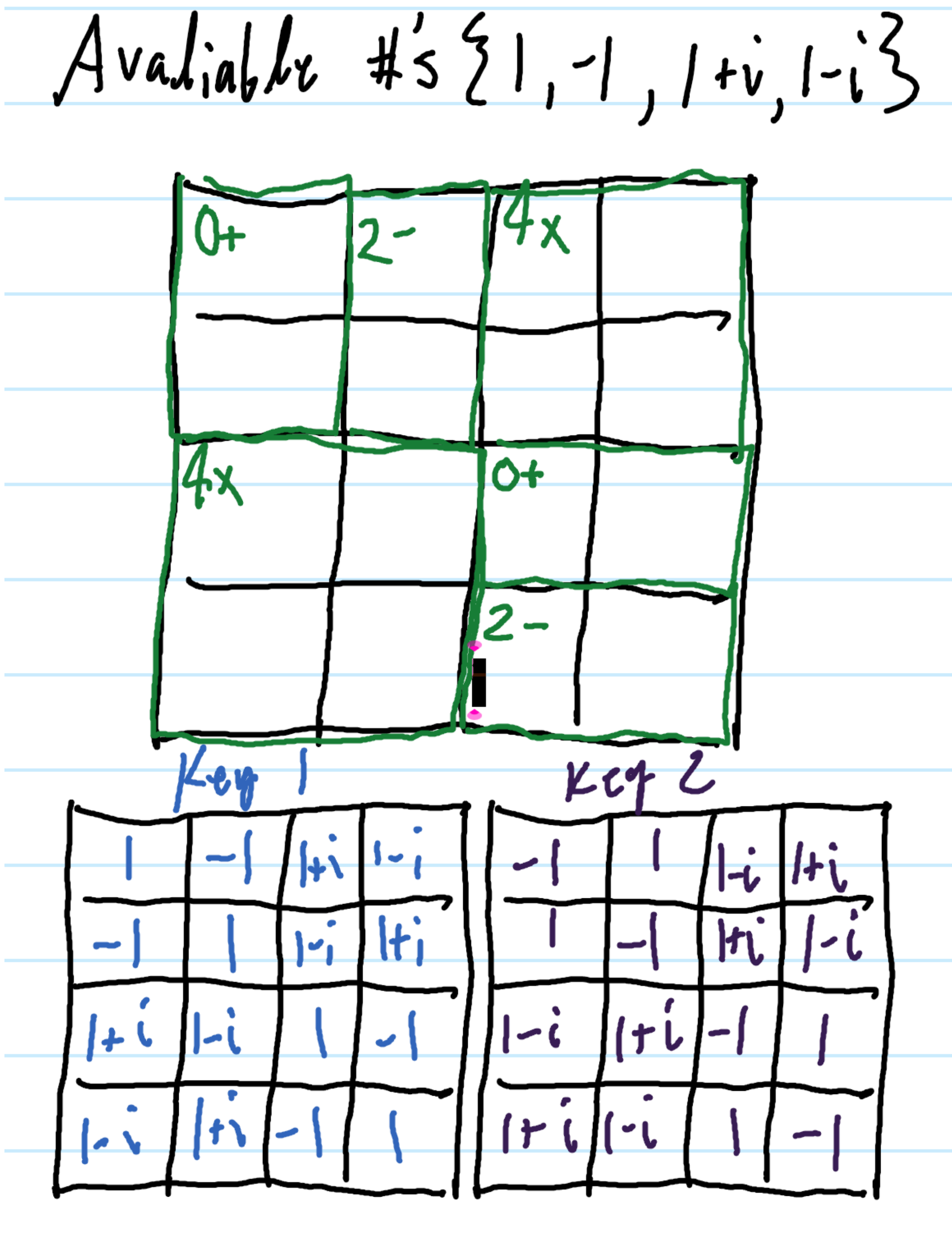

Figure 3: An example of a KenKen using Gaussian integers while having multiple keys.