HW 6 - Cason Konzer

Thursday, March 18, 2021 1:04 PM

This week's homework builds off some comments in class about how to prove two sets are equals. Since two sets are equal if and only if they are subsets of each other, to prove that A=B once can prove that every element of A is also an element of B, and vice versa. Writing down proofs of that type takes a bit of practice, and Section 5.2 of the book does a really good job of talking through all the ways that can look like (and introduces a few new set words while we're at it, e.g., the "choose an element method" and "disjoint sets").
So the homework this week is relatively short with the hope that you'll *really* read Section 5.2 carefully, and then focus any extra time on your next papers.

0) Read Section 5.2 (Proving Set Relationships)

1) 5.2, # 8, 9, 14, 17.

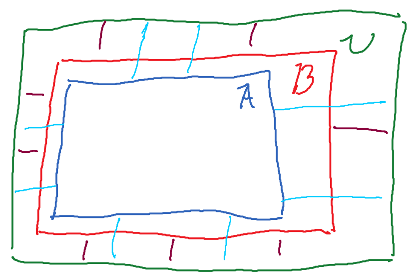
8. Let A and B be subsets of some universal set U. From Proposition 5.10, we know that if $A \subseteq B$, then $B^c \subseteq A^c$. Now prove the following proposition:

For all sets A and B that are subsets of some universal set U, $A \subseteq B$ if and only if $B^c \subseteq A^c$.

We must prove both that, $\forall (A,B) \subseteq U$,

Both $(A \subseteq B) \rightarrow (B^c \subseteq A^c)$ &

$(B^c \subseteq A^c) \rightarrow (A \subseteq B)$



$U^c = \emptyset$

$B^c = U - B$

$A^c = U - A$

The First statement is given.

Consider $B^c \subseteq A^c$; Let $\dot{B} = B^c$ & $\dot{A} = A^c$

Now $\dot{B} \subseteq \dot{A}$. From the given statement, $\dot{A}^c \subseteq \dot{B}^c$

resubstituting; $(A^c)^c \subseteq (B^c)^c$. Any $(X^c)^c = X$

Therefore $A \subseteq B$ proving $(B^c \subseteq A^c) \rightarrow (A \subseteq B)$

Because it is given $(A \subseteq B) \rightarrow (B^c \subseteq A^c)$; we have proven $(A \subseteq B) \leftrightarrow (B^c \subseteq A^c)$ ■

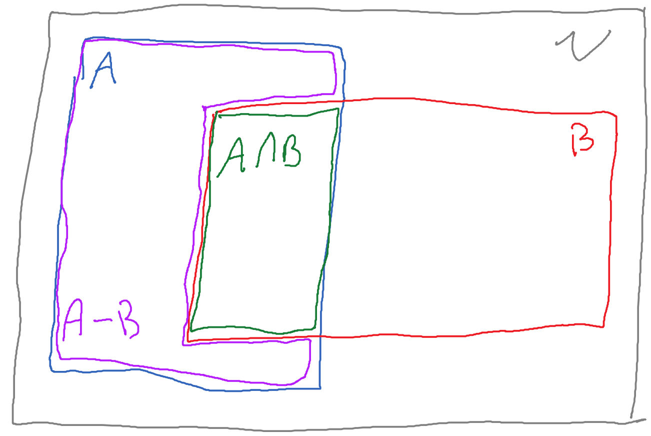
9. Is the following proposition true or false? Justify your conclusion with a proof or a counterexample.

For all sets A and B that are subsets of some universal set U, the sets $A \cap B$ and $A - B$ are disjoint.

disjoint $:= X \cap Y = \emptyset$

Let $(A \cap B) = X$ & $(A - B) = Y$

we will prove by Contradiction



Assume $X \cap Y \neq \emptyset$ thus, some

$\zeta \in X, Y, X \cap Y, (A \cap B), (A - B)$

if $\zeta \in (A \cap B)$, $\zeta \in A, B, (A \cap B)$

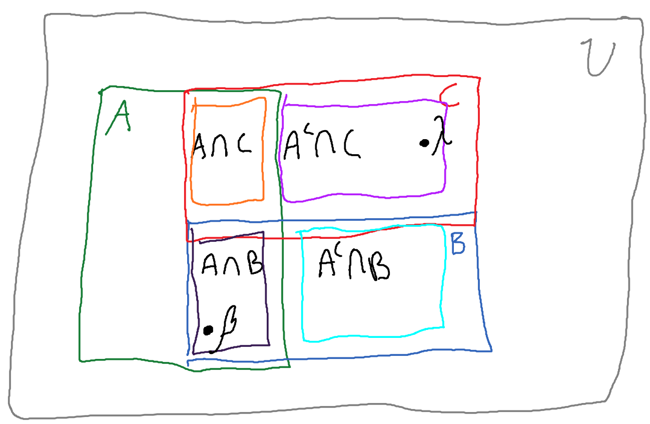
But as $\zeta \in A, B$ $\zeta \notin (A - B)$. This is a Contradiction, implying that $\neg(X \cap Y \neq \emptyset)$; Therefor $X \cap Y = \emptyset$, meaning they are disjoint!

As $X = A \cap B$ & $Y = A - B$, $A \cap B$ & $A - B$ are disjoint $\blacksquare$

14. Prove the following proposition:

For all sets A, B, and C that are subsets of some universal set, if $A \cap B = A \cap C$ and $A^c \cap B = A^c \cap C$, then $B = C$.

assume $B \neq C$



we will prove by Contrapositive

There fore as $B \neq C$, either

$(A \cap B) \neq (A \cap C)$ or $(A^c \cap B) \neq (A^c \cap C)$

since $B \neq C$, $\exists \beta \in B \mid \beta \notin C$

Letting $\beta \in A$, it is clear $(A \cap B) \neq (A \cap C)$ as $\beta \in (A \cap B)$ & $\beta \notin (A \cap C)$. we also know $\beta \notin A^c$ as $\beta \in A$.

Since $B \neq C$, $\exists \lambda \in C \mid \lambda \in B$. Letting $\lambda \in A^c$, we now have $\lambda \in (A^c \cap C)$ & $\lambda \in (A^c \cap B)$, there fore $(A^c \cap B) \neq (A^c \cap C)$

It is now proven that there exists some sets $A, B, C, \subseteq U$ such that

$(B \neq C) \rightarrow ((A \cap B) \neq (A \cap C) \vee (A^c \cap B) \neq (A^c \cap C))$

$$(B \neq C) \Rightarrow \big((A \cap B) \neq (A \cap C) \vee (A^c \cap B) \neq (A^c \cap C)\big)$$

This is logically equivalent to the original proposition

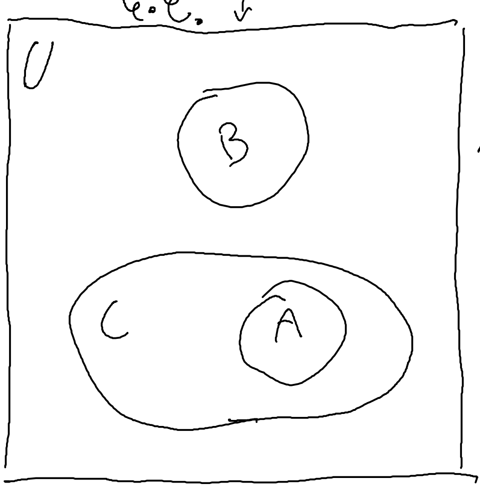
17. Evaluation of Proofs

See the instructions for Exercise (19) on page 100 from Section 3.1.

(a) Let A, B, and C be subsets of some universal set. If $A \not\subseteq B$ and $B \not\subseteq C$, then $A \not\subseteq C$.

***Proof*.** We assume that A, B, and C are subsets of some universal set and that $A \not\subseteq B$ and $B \not\subseteq C$. This means that there exists an element x in A that is not in B and there exists an element x that is in B and not in C. Therefore, $x \in A$ and $x \notin C$, and we have proved that $A \not\subseteq C$. ■

C.E. ?



FALSE PROPOSITION / The Error in this proof is assuming that because an element is not in B, then contradicting and saying it is.

$x \in A \Rightarrow x \notin B$ & $x \in B \Rightarrow x \notin C$.

But $x \notin C$ does not imply $x \in B$.

Counter Example Aside

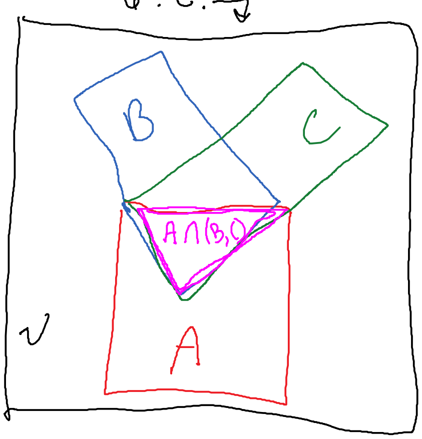
(b) Let A, B, and C be subsets of some universal set. If $A \cap B = A \cap C$, then $B = C$.

Proof. We assume that $A \cap B = A \cap C$ and will prove that $B = C$. We will first prove that $B \subseteq C$.

So let $x \in B$. If $x \in A$, then $x \in A \cap B$, and hence, $x \in A \cap C$. From this we can conclude that $x \in C$. If $x \notin A$, then $x \notin A \cap B$, and hence, $x \notin A \cap C$. However, since $x \notin A$, we may conclude that $x \in C$. Therefore, $B \subseteq C$.

The proof that $C \subseteq B$ may be done in a similar manner. Hence, $B = C$. ■

C.E. ↴



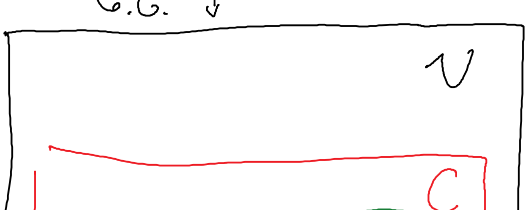
FALSE PROPOSITION | This proof is assuming $B \subseteq C$ in order to Claim $x \in A \cap C$, you could not know this unless you knew $x \in A, C$.

Counter Example Aside

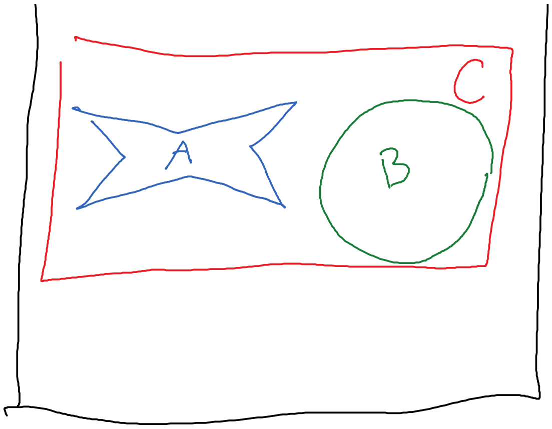
(c) Let A, B, and C be subsets of some universal set. If $A \not\subseteq B$ and $B \subseteq C$, then $A \not\subseteq C$.

Proof. Assume that $A \not\subseteq B$ and $B \subseteq C$. Since $A \not\subseteq B$, there exists an element x such that $x \in A$ and $x \notin B$. Since $B \subseteq C$, we may conclude that $x \notin C$. Hence, $x \in A$ and $x \notin C$, and we have proved that $A \not\subseteq C$. ■

C.E. ↴



FALSE PROPOSITION | The Error in this proof is that the author forgot a set B can be smaller than another set while still being a subset. In



set B can be smaller than another set C while still being a subset. In order to claim $x \notin C$ as they did, B must be equal to C, or $A \not\subseteq C$ must be given.

Counter Example Aside