Cason Konzer **202210-MTH357-01**
Assignment hw3 due 09/21/2021 at 11:59pm EDT

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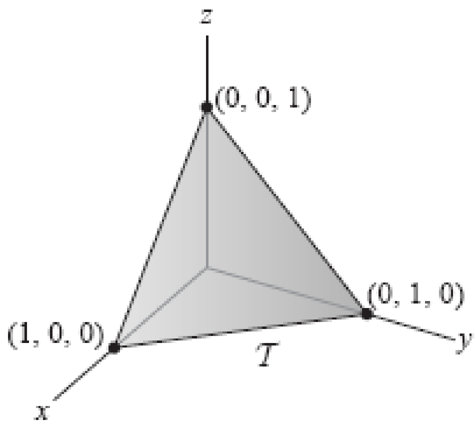
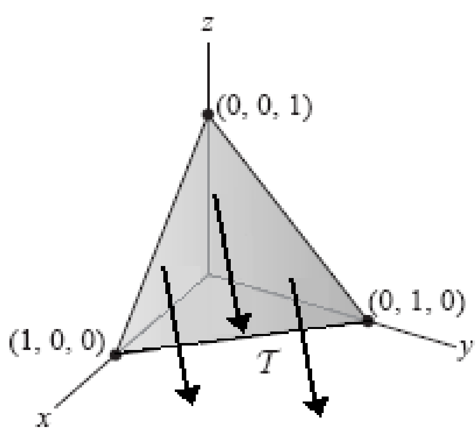
1. (1 point)
Let $\mathcal{T}$ be the triangular region with vertices $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$ oriented with upward-pointing normal vector.



A fluid flows with constant velocity field $\mathbf{v} = 9\mathbf{i} + 7\mathbf{j}$ m/s. Calculate:
(a) The flow rate through $\mathcal{T}$
(b) The flow rate through the projection of $\mathcal{T}$ onto the xy-plane [the triangle with vertices $(0,0,0)$, $(1,0,0)$, and $(0,1,0)$]
Assume distances are in meters.
(a) $\iint_S \mathbf{v} \cdot d\mathbf{S} =$ __________
(b) $\iint_S \mathbf{v} \cdot d\mathbf{S} =$ __________
Solution:
Solution: (a)



We compute the flow rate through $\mathcal{T}$. Since the unit normal vector is $\mathbf{e_n} = \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$ we have,

$$\mathbf{v} \cdot \mathbf{e_n} = \langle 9,7,0 \rangle \cdot \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle = \frac{16}{\sqrt{3}}$$

Therefore, the flow rate through $\mathcal{T}$ is the following flux:

$$\iint_S \mathbf{v} \cdot d\mathbf{S} = \iint_S (\mathbf{v} \cdot \mathbf{e_n})\, dS =$$
$$\iint_S \frac{16}{\sqrt{3}}\, dS = \frac{16}{\sqrt{3}} \cdot \text{Area}(S) =$$
$$\frac{16}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} = \frac{16}{2} = 8$$

(b) The upward pointing normal to the projection $\mathcal{D}$ of $\mathcal{T}$ onto the xy-plane is $\mathbf{n} = \langle 0,0,1 \rangle$.
Since $\mathbf{v} = \langle 9,7,0 \rangle$ is orthogonal to $\mathbf{n}$, the flux of $\mathbf{v}$ through $\mathcal{D}$ is zero.

Answer(s) submitted:
- `8`
- `0`

(correct)
Correct Answers:
- `8`
- `0`

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2. (2 points) A fluid has density 5 and velocity field $\mathbf{v} = -y\mathbf{i} + x\mathbf{j} + 5z\mathbf{k}$.
Find the rate of flow outward through the sphere $x^2 + y^2 + z^2 = 16$

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Answer(s) submitted:
- `6400pi/3`

(correct)
Correct Answers:
- `6702.05866666667`

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3. (2 points) (a) Set up a double integral for calculating the flux of the vector field $\vec{F}(\vec{r}) = \vec{r}$, where $\vec{r} = \langle x,y,z \rangle$, through the part of the upward oriented surface $z = 3(x^2+y^2)$ that lies above the disk $x^2 + y^2 \leq 16$.

Flux = $\iint_{Disk}$ ____________________ $dx\,dy$

(b) Evaluate the integral.
Flux = $\iint_S \vec{F} \cdot d\vec{A} =$ __________

Hint: Change to polar coordinates to evaluate the integral.
Answer(s) submitted:
- `-3x^2-3y^2`
- `-384pi`

(correct)
Correct Answers:
- `-3*(x^2+y^2)`
- `-3*pi/2*4^4`

4. (2 points) (a) Set up a double integral for calculating the flux of the vector field $\vec{F}(x,y,z) = x\vec{i} + y\vec{j}$ through the open-ended circular cylinder of radius 3 and height 4 with its base on the xy-plane and centered about the positive z-axis, oriented away from the z-axis. If necessary, enter θ as *theta.*

Flux = $\int_A^B \int_C^D$ ________________ $dz\,d\theta$

A = _____
B = _____
C = _____
D = _____

(b) Evaluate the integral.
Flux = $\iint_S \vec{F} \cdot d\vec{A}$ = __________

Answer(s) submitted:

- `9`
- `0`
- `2pi`
- `0`
- `4`
- `72pi`

(correct)
Correct Answers:

- `3^2`
- `0`
- `2*pi`
- `0`
- `4`
- `2*pi*3^2*4`

5. (2 points)
A rectangular channel of width 3 and depth 3 meters lies in the $\vec{j}$ direction. At a point d_1 meters from one side and d_2 meters from the other side, the velocity vector of fluid in the channel is $\vec{v} = d_1 d_2 \vec{j}$ meters/sec. Find the flux through a rectangle stretching the full width and depth of the channel, and perpendicular to the flow.
flux = ________________
(Include **units**.*)*
Solution:
SOLUTION
Let x be the distance d_1. Since the total width of the channel is 3, we have $d_2 = 3 - x$. The flux through a rectangle with dimensions $A = 3 \times 3$, is given by

$$\text{Flux } = \int_A \vec{v} \cdot d\vec{A}.$$

For a thin section of the channel of width dx, we have $d\vec{A} = (3\,dx)\vec{j}$. Thus

$$\text{Flux } = \int_0^3 \vec{v} \cdot (3\,dx)\vec{j} = \int_0^w x(3-x)\vec{j} \cdot (3 \cdot dx)\vec{j} = 3\int_0^3 x(3-x)\,dx$$
$$= 3\int_0^3 (3x - x^2)\,dx = 3\left(3\frac{x^2}{2} - \frac{x^3}{3}\right)\Bigg|_{x=0}^{x=3} = \frac{27}{2}\text{m}^3/\text{s}$$

Answer(s) submitted:

- `13.5m^3/s`

(correct)
Correct Answers:

- `13.5 m^3/s`

6. (2 points) The temperature u in a star of conductivity 6 is inversely proportional to the distance from the center: $u = \frac{2}{\sqrt{x^2+y^2+z^2}}$.
If the star is a sphere of radius 7, find the rate of heat flow outward across the surface of the star.

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Answer(s) submitted:

- `48pi`

(correct)
Correct Answers:

- `150.79644737231`