---

1. (1 point)
Determine whether the vector field is conservative and, if so, find the general potential function.

$$\mathbf{F} = \left\langle \cos z, 2y^{17}, -x\sin z \right\rangle$$

$\varphi =$ __________ $+c$

Note: if the vector field is not conservative, write "DNE".

Solution:

Solution: We examine whether $\mathbf{F}$ satisfies the cross partials condition:

$$\begin{array}{l} \frac{\partial F_1}{\partial y} = \frac{\partial}{\partial y}(\cos z) = 0 \\ \frac{\partial F_2}{\partial x} = \frac{\partial}{\partial x}(2y^{17}) = 0 \end{array} \quad \Rightarrow \quad \frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}$$

$$\begin{array}{l} \frac{\partial F_2}{\partial z} = \frac{\partial}{\partial z}(2y^{17}) = 0 \\ \frac{\partial F_3}{\partial y} = \frac{\partial}{\partial y}(-x\sin z) = 0 \end{array} \quad \Rightarrow \quad \frac{\partial F_2}{\partial z} = \frac{\partial F_3}{\partial y}$$

$$\begin{array}{l} \frac{\partial F_3}{\partial x} = \frac{\partial}{\partial x}(-x\sin z) = -\sin z \\ \frac{\partial F_1}{\partial z} = \frac{\partial}{\partial z}(\cos z) = -\sin z \end{array} \quad \Rightarrow \quad \frac{\partial F_3}{\partial x} = \frac{\partial F_1}{\partial z}$$

We see that the conditions are satisfied, therefore $\mathbf{F}$ is conservative. We find the general potential function for $\mathbf{F}$.

Step 1. Use the condition $\frac{\partial\varphi}{\partial x} = F_1$.

$\varphi(x,y,z)$ is an antiderivative of $F_1 = \cos z$ when y and z are fixed, therefore:

$$\varphi(x,y,z) = \int \cos z\,dx = x\cos z + g(y,z) \quad (\mathbf{1})$$

Step 2. Use the condition $\frac{\partial\varphi}{\partial y} = F_2$.

Using $(\mathbf{1})$ we get:

$$\frac{\partial}{\partial y}\left(x\cos z + g(y,z)\right) = 2y^{17}$$

$$g_y(y,z) = 2y^{17}$$

We integrate with respect to y, holding z fixed:

$$g(y,z) = \int 2y^{17}\,dy = \frac{1}{9}\cdot y^{18} + g(z)$$

Substituting in $(\mathbf{1})$ gives

$$\varphi(x,y,z) = x\cos z + \frac{y^{18}}{9} + g(z) \quad (\mathbf{2})$$

Step 3. Use the condition $\frac{\partial\varphi}{\partial z} = F_3$.

By $(\mathbf{2})$ we have

$$\frac{\partial}{\partial z}\left(x\cos z + \frac{y^{18}}{9} + g(z)\right) = -x\sin z$$

$$-x\sin z + g'(z) = -x\sin z$$

$$g'(z) = 0 \quad \Rightarrow \quad g(z) = c$$

Substituting in $(\mathbf{2})$ we obtain the general potential function:

$$\varphi(x,y,z) = x\cos z + \frac{y^{18}}{9} + c$$

Answer(s) submitted:

- `xcos(z) + y^18/9`

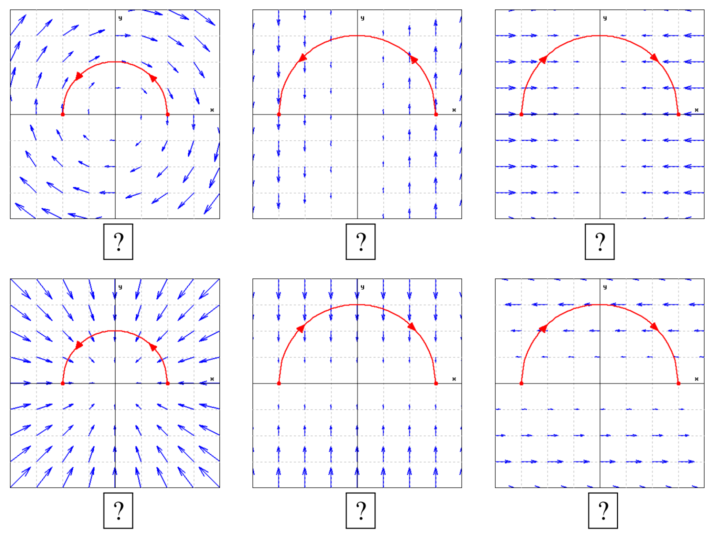
(correct)

Correct Answers:

- `x*cos(z)+y^18/9`

---

2. (1 point) Determine whether the line integral of each vector field (in blue) along the semicircular, oriented path (in red) is positive, negative, or zero.



(Click on a graph to enlarge it)

Solution:
SOLUTION

1. The vector field points in the opposite direction as the curve. Thus the line integral is negative.

2. The vector field points in the same direction as the curve. Thus the line integral is positive.

3. The line integral in the first quadrant counterbalances the line integral in the second quadrant. Thus the line integral along the semicircle is zero.

4. The vectors of the radial vector field are perpendicular to the curve. Thus the line integral is zero.

5. The line integral in the first quadrant counterbalances the line integral in the second quadrant. Thus the line integral along the semicircle is zero.

6. The vectors point generally in the opposite direction as the curve. Thus the line integral is negative.

Answer(s) submitted:

- `Negative`
- `Positive`
- `Zero`
- `Zero`
- `Zero`
- `Negative`

(correct)
Correct Answers:

- `NEGATIVE`
- `POSITIVE`
- `ZERO`
- `ZERO`
- `ZERO`
- `NEGATIVE`

---

3. (1 point) Consider the vector field $\mathbf{F}(x,y,z) = (4z+4y)\,\mathbf{i} + (2z+4x)\,\mathbf{j} + (2y+4x)\,\mathbf{k}$.

a) Find a function f such that $\mathbf{F} = \nabla f$ and $f(0,0,0) = 0$.
$f(x,y,z) =$ ____________

b) Suppose C is any curve from $(0,0,0)$ to $(1,1,1)$. Use part a) to compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

____

Answer(s) submitted:

- `4xy + 4xz + 2yz`
- `10`

(correct)
Correct Answers:

- `4*z*x +4*y*x +2*z*y`
- `10`

---

4. (1 point) Suppose $\vec{F}(x,y) = (3x-y)\vec{i} + x\vec{j}$.

(a) Find a vector parametric equation for the line segment from the origin to the point $(4,16)$ using t as a parameter.
$\vec{r}(t) =$ ______

(b) Find the line integral of $\vec{F}$ along the line segment from the origin to $(4,16)$. ______

(c) Find a vector parametric equation for the parabola $y = x^2$ from the origin to the point $(4,16)$ using t as a parameter.
$\vec{r}(t) =$ ______

(d) Find the line integral of $\vec{F}$ along the parabola $y = x^2$ from the origin to $(4,16)$. ______

(e) True or False: The line integral of any vector field $\vec{F}$ from point P to point Q is the same no matter what path is taken from P to Q. [?/True/False]

Answer(s) submitted:

- `ti + 4tj`
- `24`
- `ti + t^2j`
- `136/3`
- `False`

(correct)
Correct Answers:

- `<t,4*t>`
- `24`
- `<t,t^2>`
- `45.3333`
- `False`

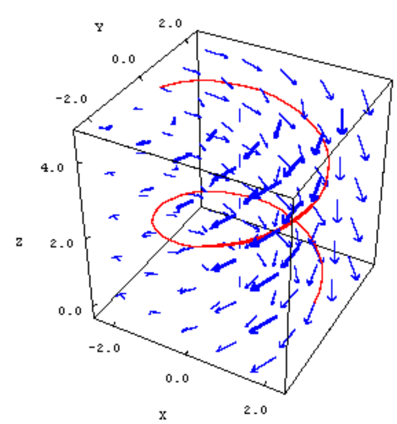
---

5. (1 point)

Suppose $\vec{F}(x,y,z) = y\vec{i} - x\vec{j} - 0.5\vec{k}$ and C is the helix given by $x(t) = 2\cos(t)$, $y(t) = 2\sin(t)$, $z(t) = t/2$ for $0 \le t \le 5\pi$.

(a) From the graph, do you expect the line integral of $\vec{F}$ along the helix C to be positive, negative, or zero? [?/positive/negative/zero]

(b) Find the line integral of $\vec{F}$ along the helix C.
$\int_C \vec{F} \cdot d\vec{r} =$ __________



Answer(s) submitted:

- `negative`
- `-(85pi/4)`

(correct)
Correct Answers:

- `negative`
- `-1*5*pi*(2^2+0.5/2)`

6. (1 point) Let $\mathbf{F}(x,y) = \frac{-y\mathbf{i}+x\mathbf{j}}{x^2+y^2}$ and let C be the circle $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j}$, $0 \le t \le 2\pi$.

A. Compute $\frac{\partial Q}{\partial x}$

____

Note: Your answer should be an expression of x and y; e.g. "3xy - y"

B. Compute $\frac{\partial P}{\partial y}$

____

Note: Your answer should be an expression of x and y; e.g. "3xy - y"

C. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$

____

Note: Your answer should be a number

D. Is $\mathbf{F}$ conservative in the whole xy-plane? $\boxed{?}$

Answer(s) submitted:

- `(y^2-x^2)/(x^2+y^2)^2`
- `(y^2-x^2)/(x^2+y^2)^2`
- `2pi`
- `n`

(correct)

Correct Answers:

- `(y^2 - x^2) / (x^2 + y^2)^2`
- `(y^2 - x^2) / (x^2 + y^2)^2`
- `2*pi`
- `N`

7. (1 point) Suppose $\vec{F}(x,y) = \langle x^2+4y, 2x-8y^2\rangle$. Use Green's Theorem to calculate the circulation of $\vec{F}$ around the perimeter of the triangle C oriented counter-clockwise with vertices $(8,0)$, $(0,4)$, and $(-8,0)$.

$\int_C \vec{F}\cdot d\vec{r} =$ __________________

Answer(s) submitted:

- `-64`

(correct)

Correct Answers:

- `-64`

8. (1 point) Suppose $\vec{F}(x,y) = 4y\vec{i} + 3xy\vec{j}$. Use Green's Theorem to calculate the circulation of $\vec{F}$ around the perimeter of a circle C of radius 4 centered at the origin and oriented counter-clockwise.

$\int_C \vec{F}\cdot d\vec{r} =$ __________________

Answer(s) submitted:

- `-64pi`

(correct)

Correct Answers:

- `-201.062`

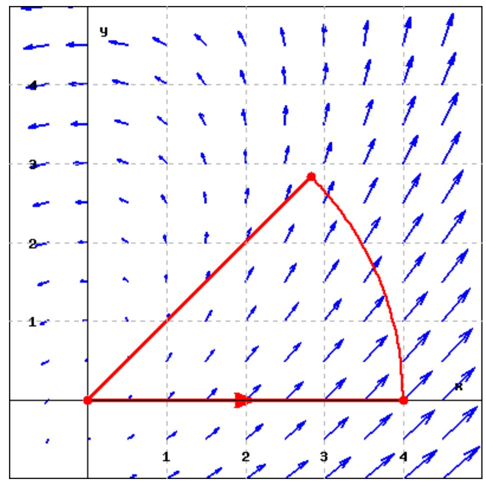
9. (1 point)

Suppose

$$\vec{F}(x,y) = (3x-2y)\vec{i} + 3x\vec{j}$$

and C is the counter-clockwise oriented sector of a circle centered at the origin with radius 4 and central angle $\pi/4$. Use Green's theorem to calculate the circulation of $\vec{F}$ around C.

Circulation = __________________



(Click on graph to enlarge)

Solution:
SOLUTION

Let D be the region enclosed by the curve C.
First note that the curve has a positive orientation. We will use Green's Theorem. We have $P = 3x-2y$, $Q = 3x$, therefore

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 3+2 = 5.$$

Hence, Green's Theorem implies

$$\begin{aligned}
\int_C \vec{F}\cdot d\vec{r} &= \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA \\
&= \iint_D 5\,dA \\
&= 5\int_0^{\pi/4}\int_0^4 r\,dr\,d\theta \quad \text{[Using polar coordinates]} \\
&= 5\left(\frac{\pi}{4}\right)\left(\frac{4^2}{2}\right) \\
&= 10\pi
\end{aligned}$$

Answer(s) submitted:

- `10pi`

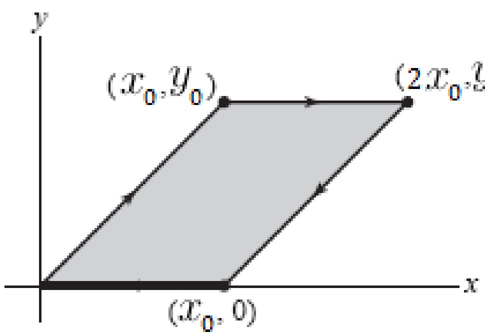
(correct)

Correct Answers:

- `(2+3)*pi*16/8`

10. (1 point)

Use Green's Theorem to evaluate the line integral of $\mathbf{F} = \langle x^9, 2x \rangle$
around the boundary of the parallelogram in the following figure (note the orientation).



With $x_0 = 2$ and $y_0 = 2$.

$\int_C x^9\,dx + 2x\,dy =$ __________

Solution:

Solution: First note that the orientation of the boundary curve is clockwise. We will use Green's Theorem remembering that the boundary curve must be oriented counterclockwise. We have $P = x^9$ and $Q = 2x$, therefore

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2 - 0 = 2$$

Hence, Green's Theorem implies

$$\int_{\partial\mathcal{D}} x^9\,dx + 2x\,dy = \iint_{\mathcal{D}} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA =$$

$$\iint_{\mathcal{D}} 2\,dA = 2\iint_{\mathcal{D}} dA = 2\,\mathrm{Area}(\mathcal{D}) = 2 \cdot 4 = 8$$

So now accounting for the orientation,

$$\int_C x^9\,dx + 2x\,dy = -\int_{\partial\mathcal{D}} x^9\,dx + 2x\,dy = -8$$

Answer(s) submitted:

- -8

(correct)

Correct Answers:

- -8