HW 5

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372hw5

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MTH 372
Hw 5

Read Chapters 10,11 of *Huber*.

p.70, #10.2 Say that $P(R = 0) = 0.3$, $P(R = 2) = 0.45$, and $P(R = 3) = 0.25$. What is $E[R]$?

#10.4 Suppose $p_W(w) = \begin{cases} 1/10 & \text{if } w \in \{1, 2, 3, 4\} \\ 2/10 & \text{if } w \in \{5, 6, 7\} \end{cases}$.
What is $E[W]$?.

#10.6 Let $E[X] = 2$. What is $E[15 - 5X]$?

p.76 #11.2 Let $X \sim \text{Unif}([3, 6])$. Find $E[X]$.

#11.8 Suppose $Y = 1/U$ where $U \sim \text{Unif}([0, 1])$. Show that $E[Y]$ does not exist.

#11.10 Let $U \sim \text{Unif}([0, 1])$. Find the expected value of $W = \sqrt{U}$.

11.14 For a random variable A, the mean absolute deviation of A is defined as

$$\text{MAD}(A) = E[\,|\,A - E[A]\,|\,].$$

For $U \sim \text{Unif}([0, 1])$, find $\text{MAD}(U)$.

11.16 Two birds are flying with speed (independently of each other) uniform between 21.1 and 32.3 mph. What is the expected speed of the faster bird?

#11.20 A random variable X has the Beta distribution with parameters a and b if it has density $f_X(s) = s^{a-1}(1 - s)^{b-1}$ ($s \in [0, 1]$).
(a) For X Beta with parameters 3 and 1, find $E[X]$.
(b) Find $E[3X + 6]$.
(c) Find $E[X^2]$.

$$\frac{(a+b-1)!}{(a-1)!(b-1)!}$$

p.70, #10.2 Say that $P(R = 0) = 0.3$, $P(R = 2) = 0.45$, and $P(R = 3) = 0.25$. What is $E[R]$?

$$E(R) = \sum_r r\, p_r(r) = (0 \cdot 0.3) + (2 \cdot 0.45) + 3(0.25)$$

$$E(R) = \sum_{r} r\, p_x(r) = (0 \cdot 0.3) + (2 \cdot 0.45) + 3(0.25)$$
$$= 0 + 0.9 + 0.75 = 1.65$$

#10.4 Suppose $p_W(w) = \begin{cases} 1/10 & \text{if } w \in \{1,2,3,4\} \\ 2/10 & \text{if } w \in \{5,6,7\} \end{cases}$

What is E[W]?.

$$E(w) = \sum_{w} w\, p_w(w) = \frac{1}{10}(1+2+3+4) + \frac{2}{10}(5+6+7)$$
$$= \frac{10}{10} + \frac{\cancel{2 \cdot 18}\ 32}{10} = \frac{42}{10} = 4.2$$

#10.6 Let E[X] = 2. What is E[15 − 5X]?

$$E[15-5X] = E[15] - 5E[X]$$
$$= 15 - 5(2) = 15 - 10 = 5$$

p.76 #11.2 Let $X \sim$ Unif([3, 6]). Find E[X].

$$f_x = \frac{1}{6-3} = \frac{1}{3}\ ; \quad E(x) = \int_3^6 \frac{t\, dt}{3} = \frac{t^2}{6}\Big|_3^6$$

$$= \frac{36}{6} - \frac{9}{6} = \frac{25}{6}$$

#11.8 Suppose Y = 1/U where $U \sim$ Unif([0, 1]). Show that E[Y] does not exist.

$$f_u = \frac{1}{1-0} = 1 \qquad Y_u = \ln(u) \qquad f_y = Y_u f_u$$

$$f_y = \ln(u) \qquad E(Y) = \int_0^1 u \ln(u)\, du$$

$$a = u \qquad b = u\ln(u) - u$$
$$da = du \qquad db = \ln(u)$$

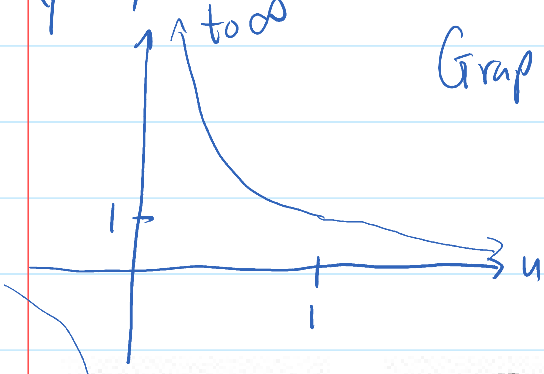
$$= u^2(\ln(u) - 1)\Big|_0^1 - \int_0^1 u\ln(u) - u\, du$$

$a = u \quad b = u\ln(u) - u \qquad u\,(u\ln(u)-u)\Big|_0^1 - \int_0^1 u\ln(u) - u \, du$

$da = du \quad db = \ln(u)$

$\ln(0)$ dne thus $\mathbb{E}(Y)$ dne

$y = 1/u$



Graphically the function y is not defined @ 0

* simple solution (: *

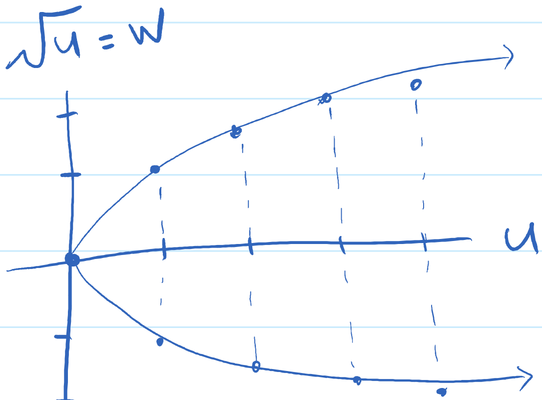
#11.10 Let $U \sim \mathrm{Unif}([0,1])$. Find the expected value of $W = \sqrt{U}$.

$f_u = \frac{1}{1-0} = 1 \quad ; \; W = u^{1/2} \quad ; \; W_u = \frac{1}{2u^{1/2}}$

$f_w = f_u \cdot W_u = \frac{1}{2u^{1/2}} \qquad \frac{1}{2}\int_0^1 u^{1/2} du = 3u^{3/2}\Big|_0^1$

$= \pm 3 - 0 = 3 - 3 = 0$

$\sqrt{u} = w$



$\mathbb{E}(W) = 0$ *

11.14 For a random variable A, the mean absolute deviation of A is defined as

$$\mathrm{MAD}(A) = E[\,|\,A \;-\; E[A]\,|\,].$$

For $U \sim \mathrm{Unif}([0,1])$, find $\mathrm{MAD}(U)$.

$f_u = \frac{1}{1-0} = 1 \qquad \mathbb{E}(U) = \int_0^1 u\,du = \frac{u^2}{2}\Big|_0^1 = \frac{1^2}{2} - 0 = 1/2$

$\mathrm{MAD}(U) = \mathbb{E}(|U - \tfrac{1}{2}|) \qquad = \left\{ \begin{matrix} U - 1/2 & | & 1/2 < U \le 1 \\ U + 1/2 & | & 0 \le U < 1/2 \end{matrix} \right\}$

$\mathrm{MAD}(U) = \frac{1}{2} + \frac{1}{2} \quad + \int_0^{1/2} \frac{u\,du}{2} - \int_{1/2}^1 \frac{u\,du}{2} = 1 + u^2\Big|_0^{1/2} - u^2\Big|_{1/2}^1$

$= 1 + \frac{1}{4} - (1 - \frac{1}{4}) = \frac{2}{4} = \frac{1}{2}$

11.16 Two birds are flying with speed (independently of each other) uniform between 21.1 and 32.3 mph. What is the expected speed of the faster bird?

B_1 & $B_2 \sim Unif([21.1, 32.3])$ ∴ $f_X = \frac{1}{32.3-21.1} = \frac{1}{11.2}$

$M = \max(B_1, B_2)$ $P(M \le r) = P(B_1 \le r)P(B_2 \le r)$

$P(B_1 \le r)$ & $P(B_2 \le r) = \begin{cases} \frac{r-21.1}{11.2} & 21.1 \le r \le 32.3 \\ 0 & \text{else} \end{cases} = \begin{cases} \frac{r}{11.2} & 0 \le r + 21.1 \le 11.2 \\ 0 & \text{else} \end{cases}$

$F_M = F_B^2$; $E(M) = \int_{-\infty}^{\infty} r F_B^2 \, dr = r F_B \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} F_B^2 \, dr + 21.1$

$= \frac{r^2}{11.2}\Big|_0^{11.2} - \int_0^{11.2} \frac{r^2}{11.2^2} = 11.2 + 21.1 - \left[\frac{r^3}{3(11.2)^2}\Big|_0^{11.2}\right]$

$= 32.3 - \frac{11.2}{3} = \boxed{28.5\overline{6}}$

#11.20 A random variable X has the Beta distribution with parameters a and b if it has density ~~$f_X(s) = s^{a-1}(1-s)^{b-1}$ $(s \in [0,1])$~~

$f_X(s) = \frac{(a+b-1)!}{(a-1)!(b-1)!} s^{a-1}(1-s)^{b-1} \quad (s \in [0,1])$ $a=3$ & $b=1$

(a) For X Beta with parameters 3 and 1, find $E[X]$.

$E(X) = \int_0^1 \frac{(3+1-1)!}{(3-1)!(1-1)!} s^{3-1}(1-s)^{1-1} \cdot s \, ds = \frac{3!}{2!0!} \int_0^1 s^2 \cdot s (1-s)^0 \, ds$

$= \frac{3 \times \cancel{2} \times \cancel{1}}{\cancel{2} \times \cancel{1} \times \cancel{1}} \int_0^1 s^3 \cdot 1 \, ds = 3 \int_0^1 s^3 \, ds = \frac{3s^4}{4}\Big|_0^1 = \boxed{3/4}$ 1st MOMENT

(b) Find $E[3X + 6]$.

$E(3X+6) = E(3X) + E(6) = 3E(X) + 6 = \frac{3 \times 3}{4} + 6$

$= 9/4 + 24/4 = \boxed{33/4}$

(c) Find $E[X^2]$.

$E(X^2) = 3\int_0^1 s^2 s^2 \, ds = 3\int_0^1 s^4 \, ds = \frac{3s^5}{5}\Big|_0^1 = \boxed{\frac{3}{5}}$

$\mathbb{E}(X\;) = [illegible] \int_0 [illegible] = [illegible] \int_0 [illegible] = [illegible]$

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2nd MOMENT