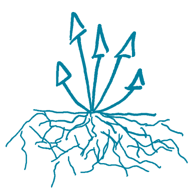
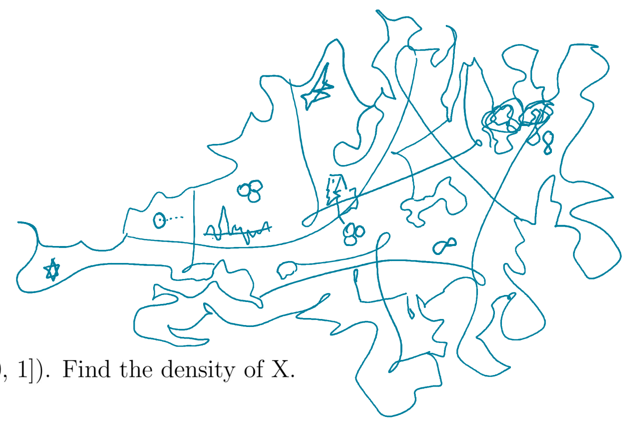
HW 4

Sunday, September 5, 2021 2:15 AM

372hw4

Jason Konzer





MTH 372
Hw 4
Due Thursday, 9/30/2021.

Read Chapters 8,9 of *Huber*.

8.1 Suppose $X = \sqrt[3]{U}$, where $U \sim$Unif([0, 1]). Find the density of X.

8.2(Corrected) Suppose that X has density $f_X(s) = (4x^3)1(x \in [0, 1])$.
(a) Find P($X \in [0, 0.3]$).
(b) Find a value m such that P($X \le m$) = 0.5 . (Such a value m is called a *median* of the distribution of X or more simply a median of X.)

8.4 The average weight of chickens (in kg) on a poultry farm is modeled as having density

$$f(s) = \begin{cases} 25(x - 1.8) & \text{if } x \in [1.8, 2]) \\ 25(2.2 - x) & \text{if } x \in [2, 2.2]) \end{cases}$$

(a) What is the probability that a chicken weighs more than 2.1 kilos?
(b) What is the probability that a chicken weighs more than 2.5 kilos?

#8.6 Suppose U has distribution Unif([-1, 1]).
(a) Find the density of U. (b) Find the density of $W = -2U + 1$.

#8.12 Suppose $U \sim$ Unif($[-1, 1]$) and $X = \arctan(U)$. Find the density of X.

p.63, #9.2 Suppose $p_X(i) = 0.3\ 1(i = 2) + 0.2\ 1(i = 4) + 0.5\ 1(i = 5)$.
(a) What is P($X \ge 2.5$)?
(b) Graph the cdf of X.

9.4 Let U_1, U_2, U_3 be iid Unif({1, 2, 3, 4, 5, 6}), and $X = \max\{U_1, U_2, U_3\}$.
(a) Find the cdf $F_X(a)$.
(b) What is P($X = 4$)?

Note that in problem 9.4, U_1, U_2, U_3 are discrete random variables. In problem 9.10, W_1, W_2, W_3 are continuous random variables.

9.10 (modified) Let W_1, W_2, W_3 be independent and Unif([0, 1]).
(a) Find the cdf of $M = \max\{W_1, W_2, W_3\}$.
(b) Find the pdf of $M = \max\{W_1, W_2, W_3\}$
(Hint: What must be true about W_1, W_2, W_3 in order for $M \le a$ to be true?)

8.1 Suppose $X = \sqrt[3]{U}$, where $U \sim$Unif([0, 1]). Find the density of X.

$f_u = \frac{1}{1-0} = 1$; $u = x^3$; $u_x = 3x^2$

$f_x = f_u u_x = 1 \cdot 3x^2 = 3x^2$ for x in [0,1]

8.2(Corrected) Suppose that X has density $f_X(s) = (4x^3)1(x \in [0, 1])$.
(a) Find P($X \in [0, 0.3]$).

$a=0 \quad b \to b^4$

$$\int_0^{0.3} 4x^3\,dx = \frac{4x^4}{4}\Big|_0^{0.3} = 0.3^4 - 0 = 0.3^4$$

(b) Find a value m such that P($X \le m$) = 0.5 . (Such a value m is called a *median* of the distribution of X or more simply a median of X.)

* From Above ?

$P(X \in [0,m]) = m^4$ let $m^4 = 0.5$ thus $m = \sqrt[4]{.5}$

8.4 The average weight of chickens (in kg) on a poultry farm is modeled as having density

$$f(s) = \begin{cases} 25(x-1.8) & \text{if } x\in[1.8,2]) \\ 25(2.2-x) & \text{if } x \in [2,2.2]) \end{cases}$$

(a) What is the probability that a chicken weighs more than 2.1 kilos?

$P(X\in[2.1,2.2])\ ;\ 25\int_{2.1}^{2.2} 2.2-x\,dx = 25\left[2.2x - \frac{x^2}{2}\Big|_{2.1}^{2.2}\right]$

$= 25\left[\left(2.2^2 - \frac{2.2^2}{2}\right) - \left((2.2)(2.1) - \frac{(2.1)^2}{2}\right)\right] = \boxed{1/8}$

(b) What is the probability that a chicken weighs more than 2.5 kilos?

$P(X\in[2.5,\infty]) = \boxed{0}$ | outcome space is only $\omega\in[1.8,2.2]$

#8.6 Suppose U has distribution Unif([-1, 1]).

$1 - -1 = 2$

(a) Find the density of U.

$P(U\in[a,b]) = \frac{b-a}{2} \Leftarrow \boxed{f_u=\frac12}$

As $\frac{b-a}{2} = \int_a^b \frac{du}{2}$

(b) Find the density of $W = -2U + 1$.

$W-1=-2u \ \therefore\ u = \frac12 - \frac{w}{2}$

$\boxed{f_w = -\frac14}$

$u_w = -\frac12$ & $f_u=\frac12$

$f_w = u_w f_u = -\frac12\cdot\frac12 = -\frac14$

Correct as far as it goes.
For which values of x is
f_W(x) = 1/4 ? 1.5 / 2

#8.12 Suppose $U\sim$ Unif([−1,1]) and $X = \arctan(U)$. Find the density of X.

$f_u = \frac{1}{1+1} = \frac12$ $\quad f_x = f_u\cdot u_x$ $\quad$ ↳ $u=\tan(x)$ $\quad u_x = 1/\cos^2(x)$

$f_x = \frac12\cdot\frac{1}{\cos^2(x)}$ $\quad \boxed{f_x = \frac{1}{2\cos^2(x)}}$

Again -- for which values of x?

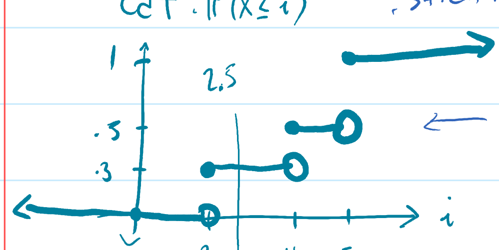
$\tan = \frac{\sin}{\cos} \ \therefore\ \tan' = \frac{\cos(\cos)-\sin(-\sin)}{\cos^2} = \frac{\cos^2+\sin^2}{\cos^2} = \frac{1}{\cos^2}$

p.63, #9.2 Suppose $p_X(i) = 0.3\,1(i=2)+0.2\,1(i=4)+0.5\,1(i=5)$.

(a) What is P($X \geq 2.5$)? $= p_X(4)+p_X(5) = .2+.5 = \boxed{0.7}$

$\sum_i p_X(i) \mid i \ge 2.5$

cdf: $P(X\le i)$ $\quad$ .3+.2+.5



.3+.2

(b) Graph the cdf of X.

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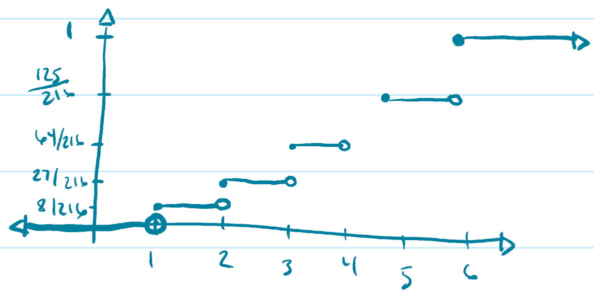
Note that in problem 9.4, U_1, U_2, U_3 are discrete random variables. In problem 9.10, W_1, W_2, W_3 are continuous random variables. Pmf $u_i = 1/6$

9.4 Let U_1, U_2, U_3 be iid Unif({1, 2, 3, 4, 5, 6}), and $X = \max\{U_1, U_2, U_3\}$.

(a) Find the cdf $F_X(a)$. $P(X \le a) = \left(\frac{a}{6}\right)^3 = \frac{a^3}{216}$ * as each u is iid

$$F_x = \begin{cases} a^3/216 & a \in \{1,2,3,4,5,6\} \\ 0 & \text{else} \end{cases}$$



(b) What is P($X = 4$)? $F_{x_a} = 3a^2/216 = a^2/72 = f_x$ $\qquad \frac{\#\{4\}^3}{\#\Omega}$

u_1 or u_2 or u_3

$f_x(4) = 4^2/72 = 16/72 = \boxed{2/9} \leftarrow 3 \cdot \left[\frac{4}{6} \cdot \frac{4}{6} \cdot \frac{1}{6}\right]$ $\quad \frac{\#\{1,2,3,4\}}{\#\Omega}$

No. 1.5 / 2

(Hint: What must be true about W_1, W_2, W_3 in order for $M \le a$ to be true?)

9.10 (modified) Let W_1, W_2, W_3 be independent and Unif([0, 1]). $\frac{1}{1-0} = 1$

(a) Find the cdf of $M = \max\{W_1, W_2, W_3\}$.

$W \sim \text{Unif}([0,1])$ ∃ $M = \sqrt[3]{w}$ $\qquad$ w/ $b = r$ ∃ $a = 0$

$\int_a^b f_x \, dx = F_x$ ∴ For W, $F_x = b - a$ ∴ for M $\boxed{F_x = r^3}$

$F_x = \left(\int_0^r dx\right)^3 = (r)^3$

* For independent cdf, we can multiply

As $F_x = \int_0^r 3r^2 d$

$f_x = F_{x_r} = 3r^2$

(b) Find the pdf of $M = \max\{W_1, W_2, W_3\}$

* Same as 8.1 *