MTH 372
Hw 4
Due Thursday, 9/30/2021.

Read Chapters 8,9 of *Huber*.

8.1 Suppose $X = \sqrt[3]{U}$, where $U \sim$Unif($[0, 1]$). Find the density of X.

8.2(Corrected) Suppose that X has density $f_X(s) = (4x^3)1(x \in [0, 1])$.
(a) Find $P(X \in [0, 0.3])$.
(b) Find a value m such that $P(X \le m) = 0.5$. (Such a value m is called a *median* of the distribution of X or more simply a median of X.)

8.4 The average weight of chickens (in kg) on a poultry farm is modeled as having density

$$f(s) = \begin{cases} 25(x - 1.8) & \text{if } x \in [1.8, 2]) \\ 25(2.2 - x) & \text{if } x \in [2, 2.2]) \end{cases}$$

(a) What is the probability that a chicken weighs more than 2.1 kilos?
(b) What is the probability that a chicken weighs more than 2.5 kilos?

#8.6 Suppose U has distribution Unif($[-1, 1]$).
(a) Find the density of U. (b) Find the density of $W = -2U + 1$.

#8.12 Suppose $U \sim$ Unif($[-1, 1]$) and $X = \arctan(U)$. Find the density of X.

p.63, #9.2 Suppose $p_X(i) = 0.3 \, 1(i = 2) + 0.2 \, 1(i = 4) + 0.5 \, 1(i = 5)$.
(a) What is $P(X \ge 2.5)$?
(b) Graph the cdf of X.

9.4 Let U_1, U_2, U_3 be iid Unif($\{1, 2, 3, 4, 5, 6\}$), and $X = \max\{U_1, U_2, U_3\}$.
(a) Find the cdf $F_X(a)$.
(b) What is $P(X = 4)$?

Note that in problem 9.4, U_1, U_2, U_3 are discrete random variables. In problem 9.10, W_1, W_2, W_3 are continuous random variables.

9.10 (modified) Let W_1, W_2, W_3 be independent and Unif($[0, 1]$).
(a) Find the cdf of $M = \max\{W_1, W_2, W_3\}$.
(b) Find the pdf of $M = \max\{W_1, W_2, W_3\}$
(Hint: What must be true about W_1, W_2, W_3 in order for $M \le a$ to be true?)