HW 10

Wednesday, November 3, 2021 5:53 AM

372hw10

MTH 372
Hw 10
Due Thursday, 12/2/2021.

CASON KONZER

Read Chapters 20, 21, 22, 28. of *Huber*.

p.134 #20.4 Suppose I roll a fair six-sided die over and over until I get a 5. Let T be the number of rolls that I make. What is $E[T]$?

#20.8 Let B_i be a Bernoulli process with parameter 0.2.
(a) Find $P(\min\{i : B_i = 1\} = 4)$. (In words: What is the probability that the first 1 occurs on the fourth trial?)
(b) Find $P(\min\{i : B_i = 0\} = 4)$. (In words: What is the probability that the first 0 occurs on the fourth trial?)

p.142 #21.2 For a Poisson process with a rate of 3.2 occurrences per hour, what is the expected time to the first occurrence?

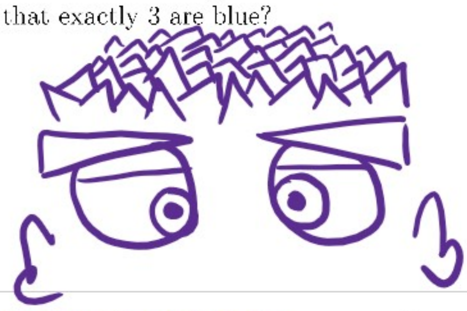
#21.4 Suppose $T_1, T_2, \cdots$ are an iid sequence of Exp(2) random variables. Let $N = \max\{n : T_1 + \cdots + T_n\} \leq 4.1\}$. What is $P(N = 8)$?

#21.6 Requests for information at Honnold library during finals week arrive according to a Poisson process at rate 4.2 per hour.
(a) What is the expected number of requests seen during a six hour shift?
(b) What is the chance that the third request arrives before the end of the first hour?
(c) What is the covariance between the time of the third request and the time of the fourth request? (Hint: Define T_k to be the time of the k^{th} request. Then $T_4 = T_3 + (T_4 - T_3)$, and $T_3, T_4 - T_3$ are independent random variables.)
(d) Each request (independently) has a 5% chance of being unsolvable. What is the chance that at least one unsolvable request comes in during a six hour shift?

p.146 #22.2 Say $N_1, \ldots, N_{10}$ are Poisson random variables with mean 0.5. What is the chance that their sum is greater than 1?

p.178 #28.1 (modified) A small plastic bucket contains tiles with the letters MISSISSIPPI.
(a) Four of these tiles are drawn out of the bucket at random without replacement. Let X be the number or drawn tiles that are either M or I. Write out the values of the pmf $p_X(x)$.
(b) Four of these tiles are drawn out of the bucket at random **with** replacement. Let Y be the number or drawn tiles that are either M or I. Write out the values of the pmf $p_Y(y)$.

#28.2 A jar contains five blue and ten green marbles. Seven marbles are drawn from the jar, what is the chance that exactly 3 are blue?



p.134 #20.4 Suppose I roll a fair six-sided die over and over until I get a 5. Let T be the number of rolls that I make. What is $E[T]$?

$T \sim$ GEOMETRIC $(p = 1/6)$; $\mu_T = \frac{1}{p} = 1 \cdot \frac{6}{1} = \boxed{6}$

#20.8 Let B_i be a Bernoulli process with parameter 0.2

#20.8 Let B_i be a Bernoulli process with parameter 0.2.

B_i ; $p_i = 1/5$; $q_i = 4/5$

(a) Find $P(\min\{i : B_i = 1\} = 4)$. (In words: What is the probability that the first 1 occurs on the fourth trial?)

$W \sim$ NEGATIVE BINOMIAL $(r = 1, p = 1/5)$

$$P_W(\omega = 4) = \binom{4-1}{1-1}(1/5)^1(4/5)^3 = \binom{3}{0}(1/5)(64/125) = \frac{1}{5} \times \frac{64}{125}$$

$$\boxed{= \frac{64}{625} = 0.1024}$$

(b) Find $P(\min\{i : B_i = 0\} = 4)$. (In words: What is the probability that the first 0 occurs on the fourth trial?)

$M \sim$ NEGATIVE BINOMIAL $(r = 1, p = 4/5)$

$$P_M(m = 4) = \binom{4-1}{1-1}(4/5)^1(1/5)^3 = \binom{3}{0}(4/5)(1/125) = \frac{4}{5} \times \frac{1}{125}$$

$$\boxed{= \frac{4}{125} = 0.032}$$

p.142 #21.2 For a Poisson process with a rate of 3.2 occurrences per hour, what is the expected time to the first occurrence? Time to First Arrival = T

Poisson process $(\lambda = \frac{3.2}{\text{hour}})$ $\quad T \sim Exp(\lambda) \therefore \mu_T = 1/\lambda$

$$\boxed{\square \quad \mu_T = 1/3.2/\text{hour} = 0.3125 \text{ hours}}$$

#21.4 Suppose $T_1, T_2, \cdots$ are an iid sequence of Exp(2) random variables. Let $N = \max\{n : T_1 + \cdots + T_n\} \leq 4.1\}$. What is $P(N = 8)$?

8 ARRIVALS IN ≤ 4.1 UNITS ; $(T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7 + T_8) \leq 4.1$

$\mu = \lambda l$; here $\lambda = 2$, $l = 4.1 - 0$ $\therefore$ $\mu = 2 \times 4.1 = 8.2$

$\therefore N \sim Poisson(8.2)$ $\quad \boxed{\Sigma \; P(N=8) = e^{-8.2} \times \frac{(8.2)^8}{8!}}$

$$= e^{-8.2} \times 506.9794 \; \boxed{= 0.1392}$$

#21.6 Requests for information at Honnold library during finals week arrive according to a Poisson process at rate 4.2 per hour.

$\lambda = 4.2$ ARRIVALS / hour

(a) What is the expected number of requests seen during a six hour shift?

$\mu = \lambda l = 4.2$ ARRIVALS / hour $\times$ 6 hours $\boxed{= 25.2 \text{ ARRIVALS}}$

(b) What is the chance that the third request arrives before the end of the first hour?

$R \sim Poisson(4.2)$ $\quad P_R(r \geq 3) = 1 - P_R(r \in \{0, 1, 2\})$

$P_R(r = 0) = e^{-4.2} \times \frac{4.2^0}{0!} = 0.014996$

$$P_R(r=0) = e^{-4.2} \times \frac{4.2^0}{0!} = 0.014996$$
$$P_R(r=1) = e^{-4.2} \times \frac{4.2^1}{1!} = 0.062981 \quad \Sigma = 0.210238$$
$$P_R(r=2) = e^{-4.2} \times \frac{4.2^2}{2!} = 0.132261$$

$$\therefore P_R(r \geq 3) = 1 - 0.210238 = \boxed{0.789762}$$

(c) What is the covariance between the time of the third request and the time of the fourth request? (Hint: Define T_k to be the time of the k^{th} request. Then $T_4 = T_3 + (T_4 - T_3)$, and $T_3, T_4 - T_3$ are independent random variables.)

$T_3 \sim \text{Gamma}(3, 4.2)$

$$\text{Cov}(T_3, T_4) = \text{Cov}(T_3, T_3 + (T_4 - T_3))$$
$$= \text{Cov}(T_3, T_3) + \text{Cov}(T_3, (T_4 - T_3))$$
$$= \sigma^2_{T_3} + 0 \text{ as } T_3, T_4 - T_3 \text{ are iid}$$

$$\boxed{\sigma^2_{T_3} = 3/4.2^2 = 0.17007}$$

(d) Each request (independently) has a 5% chance of being unsolvable. What is the chance that at least one unsolvable request comes in during a six hour shift?

$U \sim \text{Poisson}(1.26)$ $\quad \lambda p l = 4.2 \times 0.05 \times 6$

$$p(u \geq 1) = 1 - p(u=0)$$

$$p(u=0) = e^{-1.26} \times \frac{1.26^0}{0!} = e^{-1.26} = 0.28365$$

$$p(u \geq 1) = 1 - 0.28365 = \boxed{0.71635}$$

p.146 #22.2 Say $N_1, \ldots, N_{10}$ are Poisson random variables with mean 0.5. What is the chance that their sum is greater than 1?

Let $N = \sum_{n=1}^{10} N_i \quad \therefore \quad N \sim \text{Poisson}(5)$

$$P_N(n > 1) = 1 - P_N(n=0) - P_N(n=1)$$

$$P_N(n=0) = e^{-5} \times \frac{5^0}{0!} = 0.0067 \quad \Sigma = 0.0404$$
$$P_N(n=1) = e^{-5} \times \frac{5^1}{1!} = 0.0337$$

$$\therefore \boxed{P_N(n > 1) = 1 - 0.0404 = 0.9596}$$

p.178 #28.1 (modified) A small plastic bucket contains tiles with the letters MISSISSIPPI.

1m 1i 1s 2s 2i 3s 4s 3i 1p 2p 4i 1 m; 4 i's; 4 s's; 2 p's p.178 #28.1 (modified) A small plastic bucket contains tiles with the letters MISSISSIPPI.

1m 1i 1s 2s 2i 3s 4s 3i 1p 2p 4i
M I S S I S S I P P I
1 2 3 4 5 6 7 8 9 10 11

1 m ; 4 I's ; 4 S's ; 2 P's
11 TOTAL LETTERS

(a) Four of these tiles are drawn out of the bucket at random without replacement. Let X be the number or drawn tiles that are either M or I. Write out the values of the pmf $p_X(x)$.

M ~~7~~ I's = 5 $\quad X \sim$ HyperGeometric(11, 5, 4)

$$p_X(0) = \frac{\binom{5}{0}\binom{6}{4}}{\binom{11}{4}} = \frac{1\times 15}{330} = \frac{1.5}{33} \quad X=0$$

$$p_X(1) = \frac{\binom{5}{1}\binom{6}{3}}{\binom{11}{4}} = \frac{5\times 20}{330} = \frac{10}{33} \quad X=1$$

$$p_X(2) = \frac{\binom{5}{2}\binom{6}{2}}{\binom{11}{4}} = \frac{10\times 15}{330} = \frac{15}{33} \quad X=2$$

$$p_X(3) = \frac{\binom{5}{3}\binom{6}{1}}{\binom{11}{4}} = \frac{10\times 6}{330} = \frac{6}{33} \quad X=3$$

$$p_X(4) = \frac{\binom{5}{4}\binom{6}{0}}{\binom{11}{4}} = \frac{5\times 1}{330} = \frac{0.5}{33} \quad Y=4$$

(b) Four of these tiles are drawn out of the bucket at random **with** replacement. Let Y be the number or drawn tiles that are either M or I. Write out the values of the pmf $p_Y(y)$.

$Y \sim$ Binomial(4, 5/11)

$$p_Y(0) = \binom{4}{0}(5/11)^0(6/11)^4 = 1\times 1\times\left(\frac{6}{11}\right)^4 = \frac{1296}{14641} \quad Y=0$$

$$p_Y(1) = \binom{4}{1}(5/11)^1(6/11)^3 = 4\times\frac{5}{11}\times\left(\frac{6}{11}\right)^3 = \frac{4320}{14641} \quad Y=1$$

$$p_Y(2) = \binom{4}{2}(5/11)^2(6/11)^2 = 6\times\left(\frac{5}{11}\right)^2\times\left(\frac{6}{11}\right)^2 = \frac{5400}{14641} \quad Y=2$$

$$p_Y(3) = \binom{4}{3}(5/11)^3(6/11)^1 = 4\times\left(\frac{5}{11}\right)^3\times\frac{6}{11} = \frac{3000}{14641} \quad Y=3$$

$$p_Y(4) = \binom{4}{4}(5/11)^4(6/11)^0 = 1\times\left(\frac{5}{11}\right)^4\times 1 = \frac{625}{14641} \quad Y=4$$

#28.2 A jar contains five blue and ten green marbles. Seven marbles are drawn from the jar, what is the chance that exactly 3 are blue?

5B + 10G = 15[B or G]

$B \sim$ HyperGeometric($N=15$, $K=5$, $n=7$)

$B \sim \text{HyperGeometric}(N=15, K=5, n=7)$

$$P_B(b=3) = \frac{\binom{5}{3}\binom{10}{4}}{\binom{15}{7}} = \frac{10 \times 210}{6435} = \boxed{\frac{2100}{6435}}$$

or 32.63%

* Surprisingly high chance