MTH 372
Hw 6
Due Thursday, 10/28/2021.

Read Chapters 12,13 of *Huber.*

p.83 #12.2 Suppose I choose $N \sim$ Unif($\{1,2\}$). Then I roll N dice independently and identically distributed as Unif($\{1,2,3,4,5,6\}$) and sum them to get S. That is,
$[S|N=1]=X_1, \quad [S|N=2]=X_1+X_2.$ Or more compactly, $[S|N]=\sum_{i=1}^{N} X_i$.
(a) What is the probability $S=4$?
(b) What is the probability $S=7$?
(c) Find the pmf of S, $p_S(i)$ for $i \in \{1,2,\ldots,12\}$.
(d) Find $E[S]$ from $p_S(i)$.
(e) Find $E[S]$ from the Fundamental Theorem of Probability.

#12.4 Lisa and Bart go spelunking in a cave, and unfortunately, soon get lost. Each time they try to find the exit, they have a 20% chance of finding the exit in an hour, a 45% of returning back to where they started after an hour, and a 35% of returning back to where they started after three hours.
(a) What is the chance that they find their way out after exactly four hours?
(b) What is the chance that they find their way out after exactly eight hours?
(c) What is the expected amount of time they spend in the cave? (Hint: Use the "Fundamental Theorem".)

#12.6 The probability p of success for an experiment is modeled as uniform over $[0.4,0.5]$. Then 27 independent trials of the experiment are run. What is the expected number of successes?

p.90 #13.2 (modified) Suppose (X,Y) has pmf $p_{X,Y}(x,y)=(x^3+y^2)/150$ for $X \in \{1,2,3\}$ and $Y \in \{1,2,3\}$.
(a) Find the marginal pmf's $p_X(x)$ and $p_Y(y)$.
(b) Find E[XY].
(c) True or false?: $E(XY)=E(X)\cdot E(Y)$.

13.3 (modified) Suppose (X,Y) has joint pmf

$$p_{X,Y}(x,y)=\frac{x}{9\sqrt{y}}\mathbf{1}(x \in \{1,2,3\})\mathbf{1}(y \in \{1,4\})$$

(a) Prove that X and Y are independent.
(b) What is $P(X=2)$? What is $P(Y=4)$?

13.K Let $X \sim$Unif($\{1,2,3,4\}$) and $Y \sim$Unif($\{1,..,X\}$).
(In words, X is chosen uniformly from 1,2,3,4, then Y is chosen uniformly from the numbers from 1 up to Y).
(a) Write in table form the joint pmf $p_{X,Y}(x,y)$.
(b) Find the marginal pmf $p_Y(y)$, and use it to compute $E(Y)$.
(c) Find $E(Y)$ by another method, using the "Fundamental Theorem of Probablity."