HW 3

Sunday, September 5, 2021 2:14 AM

372hw3

Carson Konzer

MTH 372 - Hw 3 Due 9/23/2021

Read Chapters 6,7 of *Huber*.

Solutions are required, not just answers. Unsupported answers will recieve little or no credit.

p.41 #6.2 The chance of rain on Tuesday is 40%. Given that it rains on Tuesday, the chance of rain on Wednesday is 50%. What is the chance that it rains on both Tuesday and Wednesday?

#6.4 Suppose A and B are independent with $\mathrm{P}(A) = 0.35$ and $\mathrm{P}(B) = 0.21$. $\mathrm{P}(A|B) =$?

#6.8 Suppose that (U_1, U_2) is uniform over $[0,1] \times [0,1]$. Find $\mathrm{P}(U_1 \geq 0.5 \mid U_1 \geq 3\,U_2)$.

#6.10 Suppose $\mathrm{P}(X \in A) = 0.2$, $\mathrm{P}(X \in B) = 0.7$, $P(X \in C) = 0.4$, and $\mathrm{P}(X \in A \cap C) = 0.15$. Then $\mathrm{P}(X \in A \mid X \in C) = ?$

#6.12 (modified) Let X_1, X_2 be iid Unif$(\{1, 2, \ldots, 6\})$. Let $R = \min\{X_1, X_2\}$. What is $\mathrm{P}(R - 6 \mid R \geq 3)$?

p.48 #7.2 Suppose $X \sim$ Bin(10, 0.23). What is $\mathrm{P}(X \leq 2)$?

#7.4 Each letter in a DNA sequence is equally likely to be any one of $\{A, G, C, T\}$. What is the chance that exactly 10 out of 40 letters in a sequence are A?

#7.10 Dimer Pharmaceuticals creates 3 types of drugs for a particular illness. The first is effective in 50% of patients, the second in 37%, and the third in 5%.

(a) If a patient is equally likely to receive any of the three drugs, what is the probability that the drug is effective on their illness?

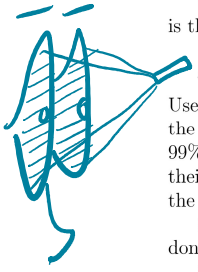
(b) If the drug is effective for the patient, what is the probability that the drug was of the third type?

#7.12 Archytas Manufacturing has four factories for their new laptops. Each laptop manufactured has a small chance of failure. Factory 1 has a 0.03% chance of failure, Factory 2 has a 0.02% chance, Factory 3 has a 0.07% chance, and Factory 4 has a 0.01% chance.

(a) If a laptop is equally likely to come from each of the four factories, what is the overall chance that it is defective?

(b) If a laptop is defective, what is the chance that it came from Factory 1?

(c) Investigation reveals that the defective laptop came from Factory 1 or 2. Now what is the probability that it came from Factory 1?



7.14 The Happy Eyes LASIK medical center owns three machines for performing surgery. Use of the first machine in surgery results in a successful operation with 95% of patients, the second is successful 97% of the time, and the third machine results in successful surgery 99% of the time. Incoming patients are randomly assigned a machine for surgery: 50% have their surgery done on the first machine, while 20% have it done on the second, and 30% on the third.

(a) Given that the surgery is not a success for a patient, what is the chance that it was done using the first machine?

(b) Given that the surgery is not a success, and either the first or second machine was used, what is the chance that it was the second machine that was used?

p.41 #6.2 The chance of rain on Tuesday is 40%. Given that it rains on Tuesday, the chance of rain on Wednesday is 50%. What is the chance that it rains on both Tuesday and Wednesday?

$P(R_T) = .4$; $P(R_W|R_T) = .5$; $\boxed{0.2} = (.4)(.5)$

$P(R_W|R_T) = \frac{P(R_W \cap R_T)}{P(R_T)} \Rightarrow P(R_W \cap R_T) = P(R_W|R_T) \cdot P(R_T)$

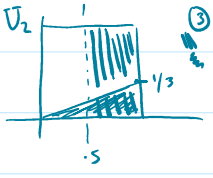
#6.4 Suppose A and B are independent with $\mathrm{P}(A) = 0.35$ and $\mathrm{P}(B) = 0.21$. $\mathrm{P}(A|B) = ?$

IF A & B independent; then $P(A|B) = P(A)$

$\therefore P(A|B) = \boxed{0.35} = P(A)$ B has nothing to do with A :)

#6.8 Suppose that (U_1, U_2) is uniform over $[0,1] \times [0,1]$. Find $\mathrm{P}(U_1 \geq 0.5 \mid U_1 \geq 3U_2)$.

$3U_2 : [0,3]$; $U_1 : [0,1]$ ① $P(U_1 \geq .5) = \frac{1-.5}{1-0} = .5 = \frac{1}{2}$ $U_2 = U_1/3$



③ $P(U_1 \geq \frac{1}{2} \cap U_1 \geq 3U_2) = \frac{1}{6} - [\frac{1}{2} \times \frac{1}{2} \times \frac{1}{6}] = \frac{4}{24} - \frac{1}{24} = \frac{3}{24}$

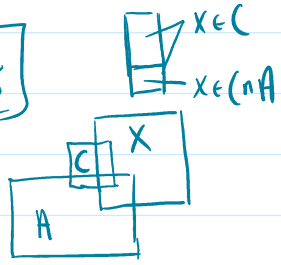
② $P(U_1 \geq 3U_2) = \frac{\frac{1}{2} \times 1 \times \frac{1}{3}}{1 \times 1} = \frac{1}{6} = \frac{4}{24}$

④ $P(U_1 \geq \frac{1}{2} \mid U_1 \geq 3U_2) = \frac{③}{②} = \frac{3/24}{4/24} = \boxed{3/4}$

#6.10 Suppose $\mathrm{P}(X \in A) = 0.2$, $\mathrm{P}(X \in B) = 0.7$, $P(X \in C) = 0.4$, and $\mathrm{P}(X \in A \cap C) = 0.15$. Then $\mathrm{P}(X \in A \mid X \in C) = ?$

$P(X \in A \mid X \in C) = \frac{P(X \in A \cap C)}{P(X \in C)} = \frac{3/20}{8/20} = \boxed{3/8}$

$* \; 0.15 = 3/20 \neq 0.4 = 8/20$



#6.12 (modified) Let X_1, X_2 be iid Unif$(\{1, 2, \ldots, 6\})$. Let $R = \min\{X_1, X_2\}$. What is $\mathrm{P}(R = 6 \mid R \geq 3)$?

Total Outcomes For $\{X_1, X_2\} = 6^2 = 36$

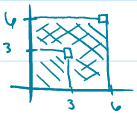
$R \geq 3 \Rightarrow \{X_1 \geq 3, X_2 \geq 3\}$

$R \in \{1,2,3,4,5,6\}$ $P(R=6)$; only $\{6,6\}$; $= 1/36$

$P(X_i \geq 3) = \frac{\#\{3,4,5,6\}}{\#\Omega} = \frac{4}{6}$

$P(R=6 \mid R \geq 3) = \frac{P(R=6) \cap P(R \geq 3)}{P(R \geq 3)} = \frac{1/36}{16/36} = \boxed{1/16}$

$P(R \geq 3) = (\frac{4}{6})^2 = \frac{16}{36}$



p.48 #7.2 Suppose $X \sim \mathrm{Bin}(10, 0.23)$. What is $\mathrm{P}(X \leq 2)$?

$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$; $1 - .23 = .77$

$= \binom{10}{0}(.77)^{10} + \binom{10}{1}(.23)(.77)^9 + \binom{10}{2}(.23)^2(.77)^8 = \boxed{0.5863}$

$\binom{10}{0} = 1$; $\binom{10}{1} = 10$; $\binom{10}{2} = 45$

#7.4 Each letter in a DNA sequence is equally likely to be any one of $\{A, G, C, T\}$. What is the chance that exactly 10 out of 40 letters in a sequence are A?

$p(A) = 1/4$; $1 - p(A) = 3/4$; $X \sim Bin(40, .25)$

$P(X=10) = \binom{40}{10}\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^{30} = \boxed{0.1444}$

#7.10 Dimer Pharmaceuticals creates 3 types of drugs for a particular illness. The first is effective in 50% of patients, the second in 37%, and the third in 5%.

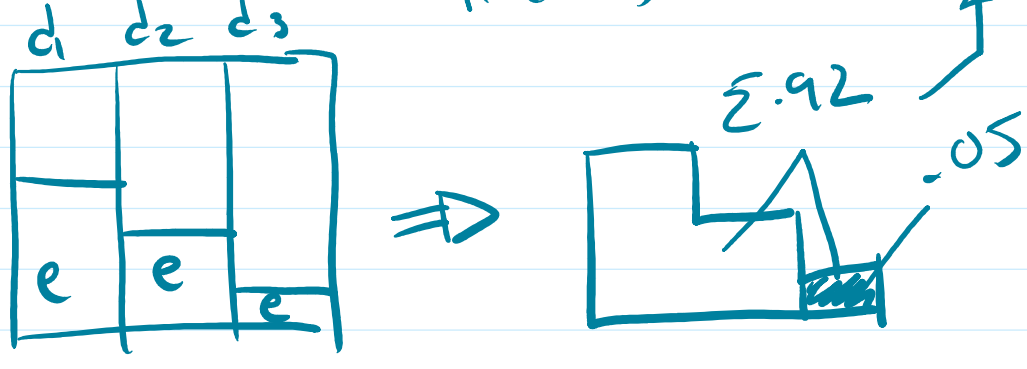
$P(e|d_1) = .5$; $P(e|d_2) = .37$; $P(e|d_3) = 0.05$

(a) If a patient is equally likely to receive any of the three drugs, what is the probability that the drug is effective on their illness?

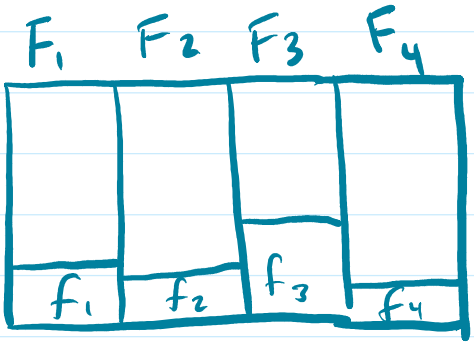
Average effectiveness $= P(e) = \frac{\sum_d e}{\#(d)} = \frac{.5 + .37 + .05}{3} = \boxed{\frac{.92}{3}}$

(b) If the drug is effective for the patient, what is the probability that the drug was of the third type?

$P(d_3|e) = \frac{P(d_3 \cap e)}{P(e)} = \frac{.05/3}{.92/3} = \boxed{.0543}$



#7.12 Archytas Manufacturing has four factories for their new laptops. Each laptop manufactured has a small chance of failure. Factory 1 has a 0.03% chance of failure, Factory 2 has a 0.02% chance, Factory 3 has a 0.07% chance, and Factory 4 has a 0.01% chance.



$\frac{100\%}{100\%} = 1$

$f_1 = \frac{.03\%}{100\%} = 0.0003$

$f_2 = 0.0002 \quad f_3 = 0.0007$

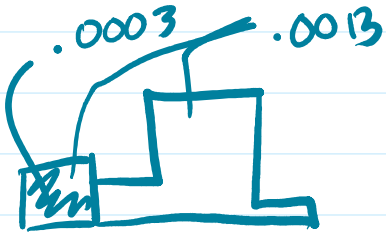
$f_4 = 0.0001$

(a) If a laptop is equally likely to come from each of the four factories, what is the overall chance that it is defective?

$\frac{\sum_i f_i}{\#_i} \Rightarrow \frac{0.0001 + 0.0002 + 0.0003 + 0.0007}{4}$

$P(f) = \frac{0.0013}{4}$ or $\frac{0.13\%}{4}$

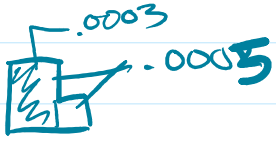
(b) If a laptop is defective, what is the chance that it came from Factory 1?



$P(F_1 | f) = \frac{P(F_1 \cap f)}{P(f)} = \frac{.0003/4}{.0013/4}$

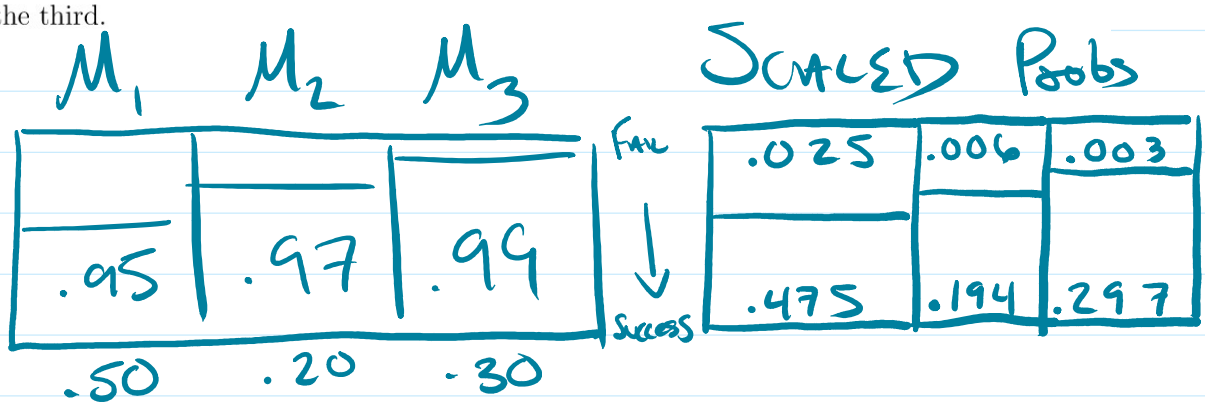
$= \frac{.0003}{.0013} = \frac{3}{13}$

(c) Investigation reveals that the defective laptop came from Factory 1 or 2. Now what is the probability that it came from Factory 1?

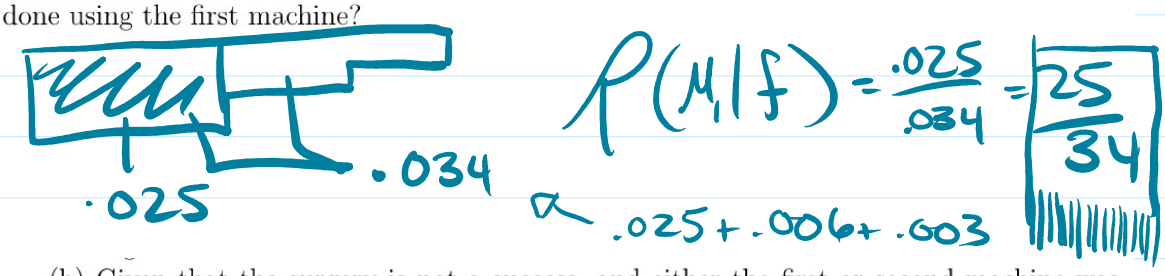


$\Rightarrow \frac{.0003}{.0005} = \frac{3}{5}$

7.14 The Happy Eyes LASIK medical center owns three machines for performing surgery. Use of the first machine in surgery results in a successful operation with 95% of patients, the second is successful 97% of the time, and the third machine results in successful surgery 99% of the time. Incoming patients are randomly assigned a machine for surgery: 50% have their surgery done on the first machine, while 20% have it done on the second, and 30% on the third.



(a) Given that the surgery is not a success for a patient, what is the chance that it was done using the first machine?



(b) Given that the surgery is not a success, and either the first or second machine was used, what is the chance that it was the second machine that was used?