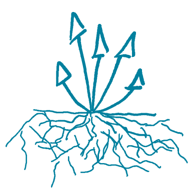
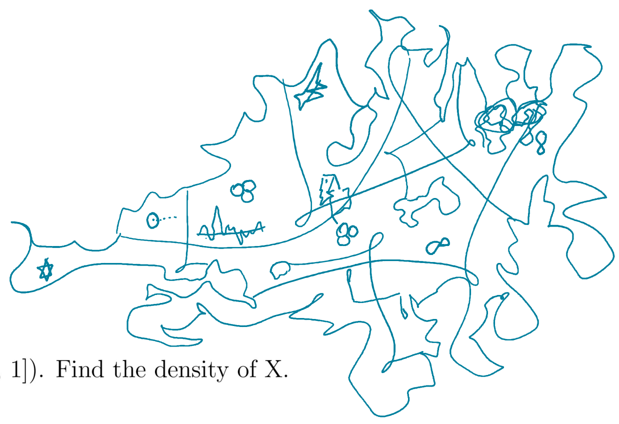
# HW 4

Sunday, September 5, 2021 2:15 AM

372hw4

Jason Konzer





MTH 372
Hw 4
Due Thursday, 9/30/2021.

Read Chapters 8,9 of *Huber*.

8.1 Suppose $X = \sqrt[3]{U}$, where $U \sim$Unif([0, 1]). Find the density of X.

8.2(Corrected) Suppose that X has density $f_X(s) = (4x^3)1(x \in [0, 1])$.
(a) Find P($X \in [0, 0.3]$).
(b) Find a value m such that P($X \leq m$) = 0.5 . (Such a value $m$ is called a *median* of the distribution of $X$ or more simply a median of $X$.)

8.4 The average weight of chickens (in kg) on a poultry farm is modeled as having density

$$f(s) = \begin{cases} 25(x - 1.8) & \text{if } x \in [1.8, 2]) \\ 25(2.2 - x) & \text{if } x \in [2, 2.2]) \end{cases}$$

(a) What is the probability that a chicken weighs more than 2.1 kilos?
(b) What is the probability that a chicken weighs more than 2.5 kilos?

#8.6 Suppose $U$ has distribution Unif([-1, 1]).
(a) Find the density of $U$. (b) Find the density of $W = -2U + 1$.

#8.12 Suppose $U \sim$ Unif($[-1, 1]$) and $X = \arctan(U)$. Find the density of $X$.

p.63, #9.2 Suppose $p_X(i) = 0.3\ 1(i = 2) + 0.2\ 1(i = 4) + 0.5\ 1(i = 5)$.
(a) What is P($X \geq 2.5$)?
(b) Graph the cdf of $X$.

# 9.4 Let $U_1, U_2, U_3$ be iid Unif({1, 2, 3, 4, 5, 6}), and $X = \max\{U_1, U_2, U_3\}$.
(a) Find the cdf $F_X(a)$.
(b) What is P($X = 4$)?

**Note that in problem 9.4, $U_1, U_2, U_3$ are discrete random variables. In problem 9.10, $W_1, W_2, W_3$ are continuous random variables.**

9.10 (modified) Let $W_1, W_2, W_3$ be independent and Unif([0, 1]).
(a) Find the cdf of $M = \max\{W_1, W_2, W_3\}$.
(b) Find the pdf of $M = \max\{W_1, W_2, W_3\}$
(Hint: What must be true about $W_1, W_2, W_3$ in order for $M \leq a$ to be true?)

8.1 Suppose $X = \sqrt[3]{U}$, where $U \sim$Unif([0, 1]). Find the density of X.

$f_u = \frac{1}{1-0} = 1$ ; $u = x^3$ ; $u_x = 3x^2$

$f_x = f_u u_x = 1 \cdot 3x^2 = 3x^2$

8.2(Corrected) Suppose that X has density $f_X(s) = (4x^3)1(x \in [0, 1])$.

(a) Find P($X \in [0, 0.3]$).

a=0 b

$b^4$

$\int_0^{0.3} 4x^3\,dx = \frac{\cancel{4}x^4}{\cancel{4}} \Big|_0^{0.3} = 0.3^4 - 0 = 0.3^4$

(b) Find a value m such that P($X \leq m$) = 0.5 . (Such a value $m$ is called a *median* of the distribution of $X$ or more simply a median of $X$.)

* From Above

$P(X \in [0,m]) = m^4$ let $m^4 = 0.5$ thus $m = \sqrt[4]{.5}$

8.4 The average weight of chickens (in kg) on a poultry farm is modeled as having density

$$f(s) = \begin{cases} 25(x-1.8) & \text{if } x \in [1.8, 2]) \\ 25(2.2-x) & \text{if } x \in [2, 2.2]) \end{cases}$$

(a) What is the probability that a chicken weighs more than 2.1 kilos?

$P(X \in [2.1, 2.2])$ ; $25\int_{2.1}^{2.2} 2.2 - x \, dx = 25\left[2.2x - \frac{x^2}{2} \Big|_{2.1}^{2.2}\right]$

$= 25\left[\left(\frac{2.2^2}{2} \cancel{-\frac{2.2^2}{2}}\right) - \left((2.2)(2.1) - \frac{(2.1)^2}{2}\right)\right] = \boxed{1/8}$

(b) What is the probability that a chicken weighs more than 2.5 kilos?

$P(X \in [2.5, \infty)) = 0$ | outcome space is only $\omega \in [1.8, 2.2]$

#8.6 Suppose $U$ has distribution Unif([-1, 1]).

$1 - -1 = 2$

(a) Find the density of $U$.

$P(u \in [a,b]) = \frac{b-a}{2} \Leftarrow f_u = \frac{1}{2}$

As $\frac{b-a}{2} = \int_a^b \frac{du}{2}$

(b) Find the density of $W = -2U + 1$.

$W - 1 = -2u$ ∴ $u = \frac{1}{2} - \frac{w}{2}$

$u_w = -\frac{1}{2}$ & $f_u = \frac{1}{2}$

$f_w = -\frac{1}{4}$

$f_w = u_w f_u = -\frac{1}{2} \cdot \frac{1}{2} = -\frac{1}{4}$

#8.12 Suppose $U \sim$ Unif($[-1, 1]$) and $X = \arctan(U)$. Find the density of $X$.

$\hookrightarrow u = \tan(x)$ $u_x = 1/\cos^2(x)$

$f_u = \frac{1}{1+1} = \frac{1}{2}$ $f_x = f_u \cdot u_x$

$f_x = \frac{1}{2} \cdot \frac{1}{\cos^2(x)}$

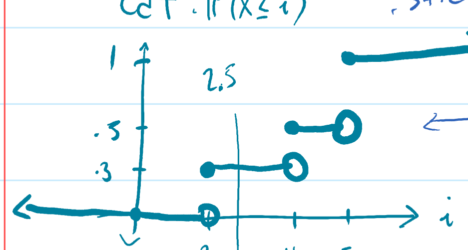
$\boxed{f_x = \frac{1}{2\cos^2(x)}}$

$\tan = \frac{\sin}{\cos}$ ∴ $\tan' = \frac{\cos(\cos) - \sin(-\sin)}{\cos^2} = \frac{\cos^2 + \sin^2}{\cos^2} = \frac{1}{\cos^2}$

p.63, #9.2 Suppose $p_X(i) = 0.3\ 1(i = 2) + 0.2\ 1(i = 4) + 0.5\ 1(i = 5)$.

(a) What is P($X \geq 2.5$)? $= p_X(4) + p_X(5) = .2 + .5 = 0.7$

cdf : $P(X \le i)$ $.3 + .2 + .5$




$.3 + .2$

$\sum_i p_x(i) \mid i \ge 2.5$

(b) Graph the cdf of $X$.

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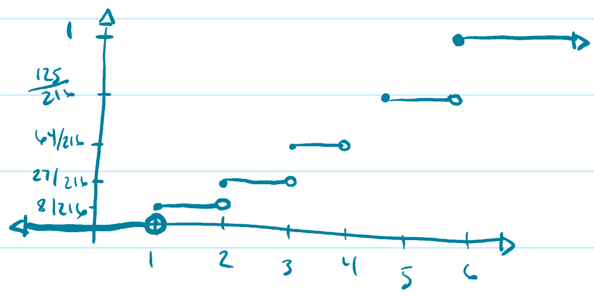
**Note that in problem 9.4, $U_1, U_2, U_3$ are discrete random variables. In problem 9.10, $W_1, W_2, W_3$ are continuous random variables.** pmf $u_i = 1/6$

# 9.4 Let $U_1, U_2, U_3$ be iid Unif({1, 2, 3, 4, 5, 6}), and $X = \max\{U_1, U_2, U_3\}$.

(a) Find the cdf $F_X(a)$. $P(X \le a) = \left(\frac{a}{6}\right)^3 = \frac{a^3}{216}$ * as each $u$ is iid

$$F_X = \begin{cases} a^3/216 & a \in \{1,2,3,4,5,6\} \\ 0 & \text{else} \end{cases}$$



(b) What is P($X = 4$)? $F_{X_a} = 3a^2/216 = a^2/72 = f_X$ $\quad \frac{\#\{4\}3}{\#\Omega}$

$f_x(4) = 4^2/72 = 16/72 = \boxed{2/9} \leftarrow 3 \cdot \left[\frac{4}{6} \cdot \frac{4}{6} \cdot \frac{1}{6}\right]$ $\quad$ $u_1$ or $u_2$ or $u_3$ $\quad \frac{\#\{1,2,3,4\}}{\#\Omega}$

(Hint: What must be true about $W_1, W_2, W_3$ in order for $M \le a$ to be true?)

9.10 (modified) Let $W_1, W_2, W_3$ be independent and Unif([0, 1]). $\frac{1}{1-0} = 1$

(a) Find the cdf of $M = \max\{W_1, W_2, W_3\}$.

$W \sim \text{Unif}([0,1])$ ∃ $M = \sqrt[3]{w}$ $\quad$ w/ $b = r$ ∃ $a = 0$

$\int_a^b f_x \, dx = F_x$ ∴ For $W$, $F_x = b - a$ ∴ for $M$ $\boxed{F_x = r^3}$

$F_x = \left(\int_0^r dx\right)^3 = (r)^3$

* For independent cdf, we can multiply

As $F_x = \int_0^r 3r^2 \, d$

$f_x = F_{x_r}' = 3r^2$

(b) Find the pdf of $M = \max\{W_1, W_2, W_3\}$

* Same as 8.1 *