

Discrete vs. Continuous Probability

Discrete

Continuous

Single variable X

cdf: $F_X(x) = P(X \leq x)$

pmf: $p_X(x) = P(X = x)$

pdf: $f_X(x) = F'(x)$

$$P(X \in A) = \sum_{x \in A} p_X(x)$$

$$P(X \in A) = \int_A f_X(x) \, dx$$

$$E(X) = \sum_x x \, p_X(x)$$

$$E(X) = \int_{-\infty}^{\infty} x \, f_X(x) \, dx$$

$$E(g(X)) = \sum_x g(x) \, p_X(x)$$

$$E(g(X)) = \int_{-\infty}^{\infty} g(x) \, f_X(x) \, dx$$

$$V(X) = \sigma_X^2 = E(X^2) - (E(X))^2$$

$$\sigma_X = \sqrt{V(X)}$$

Bivariate X, Y

cdf: $F_{X,Y}(x, y) = P(X \leq x \text{ \& } Y \leq y)$

joint pmf: $p_{X,Y}(x, y) = P(X = x \text{ \& } Y = y)$ joint pdf: $f_{X,Y}(x, y) = \frac{\partial^2 F(x, y)}{\partial x \partial y}$

$$P((X, Y) \in A) = \sum_{(x, y) \in A} p_{X,Y}(x, y)$$

$$P((X, Y) \in A) = \int \int_A f_{X,Y}(x, y) \, dx \, dy$$

$$\text{marginal pmf: } p_X(x) = \sum_y p_{X,Y}(x, y)$$

$$\text{marginal pdf: } f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x, y) \, dy$$

$$\text{conditional pmf: } p_{Y|X}(y|x) = \frac{p_{X,Y}(x, y)}{p_X(x)}$$

$$\text{conditional pdf: } f_{Y|X}(y|x) = \frac{f_{X,Y}(x, y)}{f_X(x)}$$

$$E(g(X, Y)) = \sum_{x, y} g(x, y) \, p_{X,Y}(x, y)$$

$$E(g(X, Y)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) \, f_{X,Y}(x, y) \, dx \, dy$$

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y)$$

$$\rho_{X,Y} = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$