MTH 372
Hw 11
Due Thursday, 12/9/2021.

Read Chapters 23 of *Huber*.

p.153 #23.2 (Modified) Suppose (X_1, X_2, X_3) has joint density

$$f(x_1, x_2, x_3) = k(x_1 + x_2 + x_3)\mathbf{1}(x_1, x_2, x_3 \in [0, 1]).$$

(a) Evaluate k.
(b) Find the joint marginal density of X_1, X_2.
(c) Find the marginal density of X_1.
(d) Find $\text{Cov}(X_1, X_3)$.

#23.4. Suppose Z_1, Z_2, Z_3 are iid standard normal random variables. Find their joint density.

A. Let Y_1, Y_2 have joint pdf $f_{Y_1,Y_2}(y_1, y_2) = \begin{cases} ky_1(1 - y_2), & 0 \le y_1 \le y_2 \le 1 \\ 0, & \text{otherwise.} \end{cases}$
(a) Determine k.
(b) Evaluate $P(Y_1 \le \frac{3}{4}, Y_2 \ge \frac{1}{2})$.
(c) Find the marginal pdf's $f_{Y_1}(y_1)$ and $f_{Y_2}(y_2)$
(d) Evaluate $P(Y_2 \le \frac{1}{2} \, |Y_1 \le \frac{3}{4})$.
(e) Find the conditional density of Y_1 given $Y_2 = 2/3$.
(f) Find the conditional density of Y_2 given Y_1.
(g) Find $P(Y_2 \ge \frac{3}{4} | Y_1 = \frac{1}{2})$.

B. Let $f_Y(y) = 2(1 - y)$ for $0 \le y \le 1$.
(a) Find the pdf of $T = 2Y - 1$.
(b) Find the pdf of $U = 1 - 3Y$.

C. Let X, Y be iid Exp[1]. Let $R = \dfrac{X}{Y}$ and $S = Y$.
(a) Find the joint pdf $f_{R,S}(r, s)$.
(b) Find the marginal pdf $f_R(r)$.
Hint for (b): You can avoid integration by carefully using the fact that when $Z \sim$Exp[β], we know that $E[Z] = \dfrac{1}{\beta} = \displaystyle\int_0^\infty z \cdot \beta e^{-\beta z}\, dz$.