MTH 372
Hw 8
Due Thursday, 11/11/2021.

1. Assume that Y_1, Y_2, Y_3 are random variables with

$$\begin{array}{ccc} \mathrm{E}(Y_1) = 2, & \mathrm{E}(Y_2) = -1, & \mathrm{E}(Y_3) = 4, \\ \mathrm{V}(Y_1) = 3, & \mathrm{V}(Y_2) = 6, & \mathrm{V}(Y_3) = 8, \\ \mathrm{Cov}(Y_1, Y_2) = 2, & \mathrm{Cov}(Y_1, Y_3) = -1, & \mathrm{Cov}(Y_2, Y_3) = 0. \end{array}$$

Let $Z = 3Y_1 - 4Y_2 + 6Y_3$. Evaluate $\mathrm{E}(Z)$ and $\mathrm{V}(Z)$.

2. Suppose that $X_1, X_2, \ldots, X_n$ are independent and have common mean μ and common variance σ^2. Let $\overline{X} = \dfrac{X_1 + X_2 + \cdots + X_n}{n}$. Evaluate $\mathrm{E}(\overline{X})$ and $\mathrm{V}(\overline{X})$.

3. A random variable Y has moment-generating function $M_Y(t) = (1 - 2t)^{-3}$.
Evaluate $\mathrm{E}(Y)$, $\mathrm{V}(Y)$ and $\mathrm{E}(Y^3)$.

4, (a) Let the random variable Z have pmf $p_Z(z) = \begin{cases} .4 & \text{if } z = 2 \\ .5 & \text{if } z = 3 \\ .1 & \text{if } z = 7 \end{cases}$.
Find its moment-generating function $M_Z(t)$.

(b) A random variable W has mgf $M_W(t) = .6e^{5t} + .4e^{-2t}$. Find the pmf of W.
Hint for (b): Look carefully at your answer to (a).

5. (a) Let $X_1, X_2, \ldots, X_n$ be independent and have common mgf $M(t)$. Show that $S = X_1 + X_2 + \cdots + X_n$ has mgf $M_S(t) = (M(t))^n$.

(b) Show that $\overline{X} = \dfrac{X_1 + X_2 + \cdots + X_n}{n}$ has mgf $M_{\overline{X}}(t) = (M(t/n))^n$.