HW 1

Tuesday, August 31, 2021 6:29 PM

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MTH 372
Hw 1
Due Thursday, 9/9/2021.

Read Chapters 1-3 of *Huber*.

p.15, #2.2 Suppose A_1, A_2 and A_3 are disjoint sets, $P(A_1) = P(A_2) = P(A_3) = 0.3$.
(a) What is $P(A_1 \cap A_2 \cap A_3)$?
(b) What is $P(A_1 \cup A_2)$?

#2.9 (revised) Suppose a fair 8-sided die with sides labeled $\{1, 2, \cdots, 8\}$ is rolled 4 times. There are many possible outcomes, for instance, $(2, 3, 3, 1)$ is one possible outcome.
(a) How many possible outcomes are there?
(b) If each outcome is equally likely, what must the probability of each outcome be?
(c) What is the probability of getting 7's on all four rolls?
(d) What is the probability of not getting all 7's on the four rolls.
(e) What is the probability of not getting a 7 on any of the four rolls?
(f) What is the probability of getting at least one 7? (Hint: Use your answer to part (e).)

#2.10 A department store models every person entering the store as either no spend, mid spend, or high spend. If the probability a person is no spend is 0.15 and mid spend is 0.4. What is the probability a person is high spend?

p.21 #3.2(a) Let X be uniform over the positive even numbers that are at most 100. What is the chance that X is a multiple of 4?
#3.2(b) Let Y be uniform over the positive even numbers that are at most 101. What is the chance that X is a multiple of 4? (Hint: The answer is not the same as the answer to (a).)

#3.6 Let $W \sim$ Unif $(\{a, b\} \times \{c, d\})$. What is P$(W = (a, c))$?

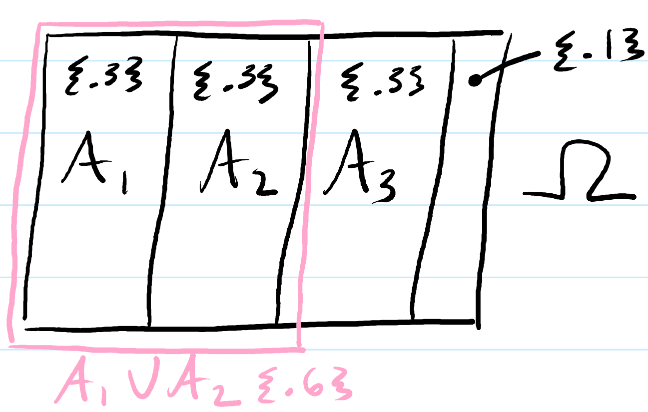
#3.7 (revised) Let $X_1 \sim$ Unif$(\{2, 3, 4\})$ and $X_2 \sim$ Unif$(\{3, \ldots, 7\})$ be independent. Evaluate P$(X_1 + X_2 = k)$ for all of the possible values of k.

#3.8 Suppose that (X, Y) is drawn uniformly from $\{1, 2, 3\} \times \{1, 2\}$. What is the chance of picking $X = Y = 2$?

p.15, #2.2 Suppose A_1, A_2 and A_3 are disjoint sets, $P(A_1) = P(A_2) = P(A_3) = 0.3.$

(a) What is $P(A_1 \cap A_2 \cap A_3)$?

$\mathbb{P}(A_1 \cap A_2 \cap A_3) = 0$



(b) What is $P(A_1 \cup A_2)$?

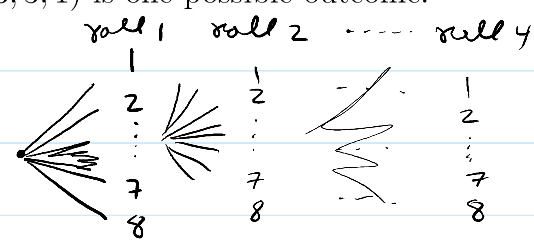
$\mathbb{P}(A_1 \cup A_2) = 0.6$

$= \mathbb{P}(A_1) + \mathbb{P}(A_2) = 0.3 + 0.3$

#2.9 (revised) Suppose a fair 8-sided die with sides labeled $\{1, 2, \cdots, 8\}$ is rolled 4 times. There are many possible outcomes, for instance, $(2, 3, 3, 1)$ is one possible outcome.

roll 1 roll 2 ----- roll 4

#2.9 (revised) Suppose a fair 8-sided die with sides labeled $\{1, 2, \cdots, 8\}$ is rolled 4 times. There are many possible outcomes, for instance, $(2, 3, 3, 1)$ is one possible outcome.



(a) How many possible outcomes are there?

$$8^4 = 4096$$

(b) If each outcome is equally likely, what must the probability of each outcome be?

$$\forall \omega_i \in \Omega ; P(\omega_i) = 1/4096$$

(c) What is the probability of getting 7's on all four rolls?

$$P(\{7,7,7,7\}) = 1/4096$$

* only 1 way to roll quad 7

(d) What is the probability of not getting all 7's on the four rolls.

$$P(A^c) = 1 - P(A) \therefore P(\neg\{7,7,7,7\}) = 1 - 1/4096 = 4095/4096$$

(e) What is the probability of not getting a 7 on any of the four rolls?

$$P(\text{No } 7\text{'s}) = (7/8)^4 = \frac{2401}{4096}$$

$$\frac{\#\{1,2,3,4,5,6,8\}}{\#\Omega} = \frac{7}{8}$$

All 4 events are independent

(f) What is the probability of getting at least one 7? (Hint: Use your answer to part (e).)

$$P(\geq 1 \text{ } 7\text{'s}) = P(\text{'No } 7\text{'s'}^C) = 1 - P(\text{no } 7\text{'s})$$

$$\therefore P(\geq 1 \text{ } 7\text{'s}) = 1 - \frac{2401}{4096} = \frac{1695}{4096}$$

#2.10 A department store models every person entering the store as either no spend, mid spend, or high spend. If the probability a person is no spend is 0.15 and mid spend is 0.4. What is the probability a person is high spend?

$$P(\Omega) = 1 = P(\text{no}) + P(\text{mid}) + P(\text{high}) = 0.15 + 0.4 + P(\text{high})$$

$$\therefore P(\text{high}) = 1 - 0.15 - 0.4 = \boxed{0.45}$$

p.21 #3.2(a) Let X be uniform over the positive even numbers that are at most 100. What is the chance that X is a multiple of 4?

$$\{2,4,6,8,10,12,14,16,18,20,\cdots\cdots,80,82,84,86,88,90,92,94,96,98,100\}$$

$$\frac{\# \overset{1}{\{4},\overset{2}{8},\overset{3}{12},\overset{4}{16},\overset{5}{20},\overset{6}{24},\overset{7}{28},\overset{8}{32},\overset{9}{36},\overset{10}{40},\overset{11}{44},\overset{12}{48},\overset{13}{52},\overset{14}{56},\overset{15}{60},\overset{16}{64},\overset{17}{68},\overset{18}{72},\overset{19}{76},\overset{20}{80},\overset{21}{84},\overset{22}{88},\overset{23}{92},\overset{24}{96},\overset{25}{100\}}}{\#\Omega} = \boxed{\frac{1}{2}}$$

$25/50$

#3.2(b) Let Y be uniform over the positive even numbers that are at most 101. What is the chance that Y is a multiple of 4? [redacted]

$$\{2,4,6,8,10,12,14,16,18,20,\cdots\cdots,80,82,84,86,88,90,92,94,96,98,100\}$$

SAME AS 3.2(b) : $\frac{25}{50} = \frac{1}{2}$

#3.6 Let $W \sim \text{Unif}(\{a,b\} \times \{c,d\})$. What is $P(W = (a,c))$?

$\Omega = \{(a,c)_1, (a,d)_2, (b,c)_3, (b,d)_4\}$

$P(a,c) = 1/4 \leftarrow \frac{\#\{(a,c)\}}{\#\{\Omega\}}$

#3.7 (revised) Let $X_1 \sim \text{Unif}(\{2,3,4\})$ and $X_2 \sim \text{Unif}(\{3,\ldots,7\})$ be independent. Evaluate $P(X_1 + X_2 = k)$ for all of the possible values of k.

$\Omega_1 = \{2,3,4\}$ $\Omega_2 = \{3,4,5,6,7\}$ $3 \times 15 = 15$ outcomes

$\Omega_{1+2} = \{5, 6, 7, 8, 9, 6, 7, 8, 9, 10, 7, 8, 9, 10, 11\}$

$$\boxed{P(5) = P(11) = 1/15 \,;\, P(6) = P(10) = 2/15 \,;\, P(7) = P(8) = P(9) = 3/15}$$

#3.8 Suppose that (X, Y) is drawn uniformly from $\{1,2,3\} \times \{1,2\}$. What is the chance of picking $X = Y = 2$?

$\Omega = \{(1,1)_1, (1,2)_2, (2,1)_3, (2,2)_4, (3,1)_5, (3,2)_6\}$

$P(2,2) = 1/6 \leftarrow \frac{\#\{(2,2)\}}{\#\{\Omega\}}$