

HW 9

Wednesday, November 3, 2021 5:53 AM

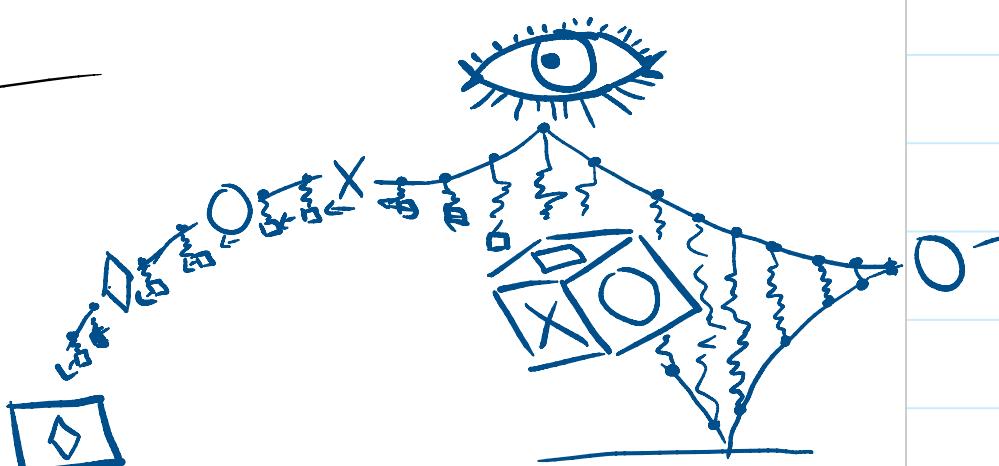


372hw9

CASON KONZER

MTH 372
Hw 9
Due Thursday, 11/18/2021.

Read Chapters 18,19 of Huber.



18.1 (Modified) For Z a standard normal random variable, find
 (a) $P(-1 \leq Z < 2)$ (b) $P(1.5 < Z \leq 2)$ (c) $P(-1.5 \leq Z \leq -1)$.

18.1.5. For $X \sim N(\mu = -2, \sigma^2 = 4)$, find
 (a) $P(0 < X \leq 2)$ (b) $P(-4 \leq X < 1)$ (c) $P(-6 < X < -5)$.

18.2 (Modified) For Z_1, Z_2 iid $N(0, 1)$, find $P(Z_1 \leq 2Z_2)$.

18.3 (Modified) The Digital Life conference draws a number of attendees each year that is normally distributed with mean 65,000 and standard deviation 12,000. Independently, E3 draws a number of attendees that is normally distributed with mean 70,000 and standard deviation 4,000.

- (a) Suppose I average the two numbers. What is the distribution of the average?
- (b) What is the chance that the average of the two conferences is greater than 70,000?
- (c) What is the distribution of the number attending Digital Life minus the number attending E3?
- (d) What is the chance that more people attend Digital Life than E3?

p.128 #19.2 Let A_1, \dots, A_{10} be iid $\text{Exp}(2)$. Approximate $P(A_1 + \dots + A_{10} \geq 7)$ using the CLT.

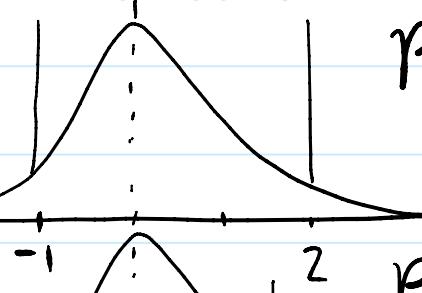
19.4 Suppose X has density $f_X(x) = (3/2)x(2-x)$ for $x \in [0, 1]$.

- (a) What is $E[X]$?
- (b) What is $SD(X)$?
- (c) For X_1, X_2, \dots, X_{20} iid with pdf $f_X(x)$ above, approximate with the CLT
 $P(X_1 + \dots + X_{20} \geq 13.4)$.

18.1 (Modified) For Z a standard normal random variable, find

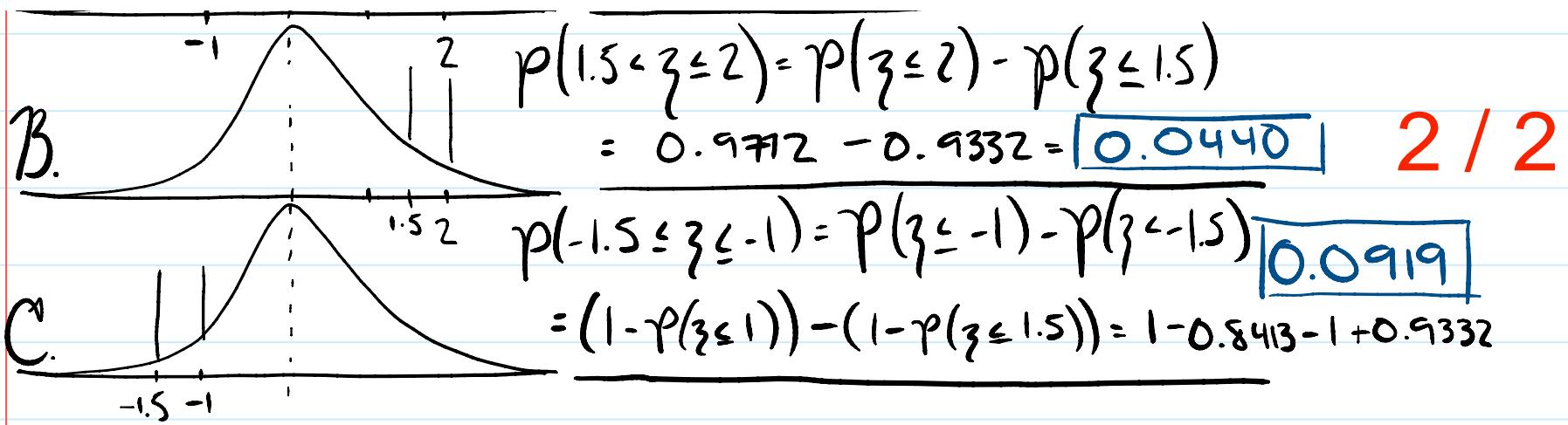
- (a) $P(-1 \leq Z < 2)$ (b) $P(1.5 < Z \leq 2)$ (c) $P(-1.5 \leq Z \leq -1)$.

A.



$$\begin{aligned} P(-1 \leq Z < 2) &= P(Z < 2) - P(Z \leq -1) & 0.8185 \\ &= P(Z < 2) - (1 - P(Z \leq 1)) & 0.9772 - 1 + 0.8413 \end{aligned}$$

$$P(1.5 < Z \leq 2) = P(Z \leq 2) - P(Z \leq 1.5)$$



18.1.5. For $X \sim N(\mu = -2, \sigma^2 = 4)$, find $\sigma = \sqrt{4} = 2$

- (a) $P(0 < X \leq 2)$ (b) $P(-4 \leq X < 1)$ (c) $P(-6 < X < -5)$.

A. $z = \frac{x+2}{2}$; $z(0) = 1$; $z(2) = 2$

$$P(0 < X \leq 2) = P(1 < z \leq 2) = P(z \leq 2) - P(z \leq 1)$$

$$= 0.9772 - 0.8413 = \boxed{0.1359}$$

B. $z(-4) = -1$; $z(1) = 3/2$

$$P(-1 \leq z \leq 3/2) = P(z \leq 3/2) - P(z \leq -1)$$

$$= P(z \leq 1.5) - (1 - P(z \leq 1)) = 0.9332 - 1 + 0.8413 = \boxed{0.7745}$$

C. $z(-6) = -2$; $z(-5) = -3/2$

$$P(-2 < z < -3/2) = P(z \leq -3/2) - P(z \leq -2)$$

$$= (1 - P(z \leq 3/2)) - (1 - P(z \leq 2)) = 1 - 0.9332 - 1 + 0.9772 = \boxed{0.0440}$$

2/2

18.2 (Modified) For Z_1, Z_2 iid $N(0, 1)$, find $P(Z_1 \leq 2Z_2)$.

$$P(z_1 \leq 2z_2) = P(z_1 - 2z_2 \leq 0)$$

$$\begin{aligned} \mu_1 &= \mu_2 = 0 \\ \sigma_1^2 &= \sigma_2^2 = 1 \end{aligned}$$

$$[z_1 - 2z_2] \sim N\left(\mu_1 - 2\mu_2, \sigma_1^2 + 2^2\sigma_2^2\right) = N(0, 5)$$

2/2

$$z = \frac{z_1 - 2z_2}{\sqrt{5}} = \frac{z_1 - 2z_2}{\sqrt{5}}$$

$$P(z_1 - 2z_2 \leq 0) = P\left(z \leq \frac{0}{\sqrt{5}}\right) = P(z \leq 0) = \boxed{0.5000}$$

18.3 (Modified) The Digital Life conference draws a number of attendees each year that is normally distributed with mean 65,000 and standard deviation 12,000. Independently, E3 draws a number of attendees that is normally distributed with mean 70,000 and standard deviation 4,000.

$$DLC \sim N(65,000, 12,000^2); E3 \sim N(70,000, 4,000^2)$$

(a) Suppose I average the two numbers. What is the distribution of the average.

$$\bar{X} = \frac{DLC + E3}{2}; \bar{X} \sim N\left(\frac{65,000 + 70,000}{2}, \frac{12,000^2 + 4,000^2}{4}\right) = N(67,500, 40,000,000)$$

$$\sigma = \sqrt{40,000,000} = 6,324.55 \quad \bar{\mu} = 67,500$$

(b) What is the chance that the average of the two conferences is greater than 70,000?

$$z = \frac{\bar{X} - 70,000}{6,324.55}; z(70,000) = \frac{70,000 - 67,500}{6,324.55} = 0.3953$$

$$\bar{z} = \frac{\bar{x} - 6324.55}{6324.55} = 0.3953$$

$$P(\bar{x} > 70,000) = P(z > 0.3953) \\ = 1 - P(z \leq 0.3953) = 1 - \frac{(0.6554 + 0.6517)}{2} \\ = 1 - 0.65355 = \underline{\underline{0.34645}}$$

(c) What is the distribution of the number attending Digital Life minus the number attending E3?

$$DL - E3 = S \quad S \sim N(65000 - 70000, 12000^2 + 4000^2) \\ \sim N(-5,000, 160,000,000)$$

$$\sigma = \sqrt{160000000} = 12,649.11 \quad \mu = -5,000$$

(d) What is the chance that more people attend Digital Life than E3?

$$P(DL > E3) = P(DL - E3 > 0) = P(S > 0) \quad 2/2 \\ = P(z > 0.3953) \\ = 1 - P(z \leq 0.3953) \\ = 1 - \frac{(0.6554 + 0.6517)}{2} \\ = 1 - 0.65355 = \underline{\underline{0.34645}}$$

p.128 #19.2 Let A_1, \dots, A_{10} be iid $\text{Exp}(2)$. Approximate $P(A_1 + \dots + A_{10} \geq 7)$ using the CLT.

$$A = A_1 + \dots + A_{10}$$

$$\mu = \frac{1}{\beta} = \frac{1}{2} \quad \sigma^2 = \frac{1}{\beta^2} = \frac{1}{4} \quad A_i \sim N\left(\frac{1}{2}, \frac{1}{4}\right)$$

$$\bar{A} = \frac{A_1 + \dots + A_{10}}{10} \quad z = \frac{\bar{A} - \frac{1}{2}}{\sqrt{\frac{1}{10}}} = (\bar{A} - \frac{1}{2})\sqrt{10} \quad 1.5/2$$

$$z(7) = \left(\frac{7}{10} - \frac{1}{2}\right)\sqrt{10} = 0.6325$$

$$Z = (\bar{A} - 1/2) / (\sigma / \sqrt{10})$$

$$P(A \geq 7) = P(\bar{A} \geq 7/10) = P(z \geq 0.6325) \\ = (\bar{A} - 1/2) / ((1/2) / \sqrt{10}) \\ = 1 - P(z \leq 0.6325) \\ = 1 - 0.7357 = \underline{\underline{0.2643}}$$

19.4 Suppose X has density $f_X(x) = (3/2)x(2-x)$ for $x \in [0, 1]$.

$$3x - \frac{3x^2}{2}$$

(a) What is $E[X]?$

$$E[X] = \int_0^1 x f_X(x) dx = \int_0^1 x(3/2)x(2-x) dx$$

(a) What is $E[X]$?

$$3 \int_0^1 x^2 dx - \frac{3}{2} \int_0^1 x^3 dx = 3 \left[\frac{x^3}{3} \Big|_0^1 \right] - \frac{3}{2} \left[\frac{x^4}{4} \Big|_0^1 \right]$$
$$= 3 \left(\frac{1}{3} \right) - \frac{3}{2} \left(\frac{1}{4} \right) = 1 - \frac{3}{8} = \boxed{5/8 = M_x}$$

(b) What is $SD(X)$?

$$\mu_x^2 = \left(\frac{5}{8} \right)^2 = \frac{25}{64}$$

$$M_{x^2} = 3 \int_0^1 x^3 dx - \frac{3}{2} \int_0^1 x^4 dx$$
$$= 3 \left[\frac{x^4}{4} \Big|_0^1 \right] - \frac{3}{2} \left[\frac{x^5}{5} \Big|_0^1 \right]$$
$$= 3 \left(\frac{1}{4} \right) - \frac{3}{2} \left(\frac{1}{5} \right) = \frac{3}{4} - \frac{3}{10}$$
$$= \frac{15}{20} - \frac{6}{20} = \boxed{\frac{9}{20}}$$

9/20, so the rest is off. 1.5 / 2

$$\sigma_x^2 = M_{x^2} - \mu_x^2 = \frac{9}{20} - \frac{25}{64} = \frac{163}{320}$$

$$\sigma_x = \boxed{\sigma_x = 0.7137}$$

(c) For X_1, X_2, \dots, X_{20} iid with pdf $f_X(x)$ above, approximate with the CLT
 $P(X_1 + \dots + X_{20} \geq 13.4)$. $\bar{X} = \frac{X_1 + \dots + X_{20}}{20}$

$$X \sim N(5/8, 0.7137)$$

$$\bar{X} = \frac{X_1 + \dots + X_{20}}{20}$$

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{(\bar{X} - 5/8)}{0.7137} \sqrt{20}$$

$$P(13.4) = \frac{\left(\frac{13.4}{20} - \frac{5}{8} \right) \sqrt{20}}{0.7137} = 0.28198$$

$$P(X \geq 13.4) = P(Z \geq 0.28198) = 1 - P(Z < 0.28198)$$

$$= 1 - 0.6183 = \boxed{0.3897}$$