

# HW 8

Wednesday, November 3, 2021 5:53 AM



372hw8

Cason Kongas

MTH 372

Hw 8

Due Thursday, 11/11/2021.

1. Assume that  $Y_1, Y_2, Y_3$  are random variables with

$$\begin{aligned} E(Y_1) &= 2, & E(Y_2) &= -1, & E(Y_3) &= 4, \\ V(Y_1) &= 3, & V(Y_2) &= 6, & V(Y_3) &= 8, \\ \text{Cov}(Y_1, Y_2) &= 2, & \text{Cov}(Y_1, Y_3) &= -1, & \text{Cov}(Y_2, Y_3) &= 0. \end{aligned}$$

Let  $Z = 3Y_1 - 4Y_2 + 6Y_3$ . Evaluate  $E(Z)$  and  $V(Z)$ .

2. Suppose that  $X_1, X_2, \dots, X_n$  are independent and have common mean  $\mu$  and common variance  $\sigma^2$ . Let  $\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n}$ . Evaluate  $E(\bar{X})$  and  $V(\bar{X})$ .

3. A random variable  $Y$  has moment-generating function  $M_Y(t) = (1 - 2t)^{-3}$ . Evaluate  $E(Y)$ ,  $V(Y)$  and  $E(Y^3)$ .

4. (a) Let the random variable  $Z$  have pmf  $p_Z(z) = \begin{cases} .4 & \text{if } z = 2 \\ .5 & \text{if } z = 3 \\ .1 & \text{if } z = 7 \end{cases}$

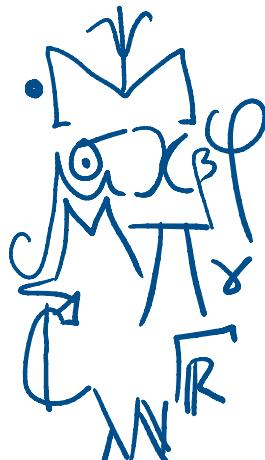
Find its moment-generating function  $M_Z(t)$ .

- (b) A random variable  $W$  has mgf  $M_W(t) = .6e^{5t} + .4e^{-2t}$ . Find the pmf of  $W$ .

**Hint** for (b): Look carefully at your answer to (a).

5. (a) Let  $X_1, X_2, \dots, X_n$  be independent and have common mgf  $M(t)$ . Show that  $S = X_1 + X_2 + \dots + X_n$  has mgf  $M_S(t) = (M(t))^n$ .

- (b) Show that  $\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n}$  has mgf  $M_{\bar{X}}(t) = (M(t/n))^n$ .



1. Assume that  $Y_1, Y_2, Y_3$  are random variables with

$$\begin{aligned} E(Y_1) &= 2, & E(Y_2) &= -1, & E(Y_3) &= 4, \\ V(Y_1) &= 3, & V(Y_2) &= 6, & V(Y_3) &= 8, \\ \text{Cov}(Y_1, Y_2) &= 2, & \text{Cov}(Y_1, Y_3) &= -1, & \text{Cov}(Y_2, Y_3) &= 0. \end{aligned}$$

Let  $Z = 3Y_1 - 4Y_2 + 6Y_3$ . Evaluate  $E(Z)$  and  $V(Z)$ .

$$\begin{aligned} M_Z &= 3\mu_{Y_1} - 4\mu_{Y_2} + 6\mu_{Y_3} = 3(2) - 4(-1) + 6(4) \\ &= 6 + 4 + 24 = 34 \end{aligned}$$

$$\begin{aligned} \sigma_Z^2 &= 3\sigma_{Y_1}^2 - 4\sigma_{Y_2}^2 + 6\sigma_{Y_3}^2 + 2(\text{cov}(Y_1, Y_2) + \text{cov}(Y_1, Y_3) + \text{cov}(Y_2, Y_3)) \\ &= 3(3) - 4(6) + 6(8) + 2(2 - 1 + 0) \\ &= 9 - 24 + 48 + 2(1) = 34 \end{aligned}$$

2. Suppose that  $X_1, X_2, \dots, X_n$  are independent and have common mean  $\mu$  and common variance  $\sigma^2$ . Let  $\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n}$ . Evaluate  $E(\bar{X})$  and  $V(\bar{X})$ .

$$M_{\bar{X}} = \frac{1}{n} \sum_{i=1}^n M_{X_i} = \frac{1}{n} \sum_{i=1}^n \mu = \frac{n\mu}{n} = \mu$$

$$\sigma_{\bar{X}}^2 = \frac{1}{n^2} \sum_{i=1}^n \sigma_{X_i}^2 = \frac{1}{n^2} \sum_{i=1}^n \sigma^2 = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

3. A random variable  $X$  has moment-generating function  $M_X(t) = (1 - 2t)^{-3}$ . Evaluate  $E(X)$ ,  $V(X)$  and  $E(X^3)$ .

$$\begin{aligned} M_X &= m_0 = 6 \\ M_X^2 &= m_2 = 48 \\ M_X^3 &= m_3 = 480 \end{aligned}$$

$$M_X^2 = 36$$

$$\sigma_X^2 = 48 - 36 = 12$$

$$m_k = M_{X^k}$$

$$\begin{aligned} m_1 &= (1 - 2t)^{-4} \\ m_2 &= 48(1 - 2t)^{-5} \\ m_3 &= 480(1 - 2t)^{-6} \end{aligned}$$

4, (a) Let the random variable  $Z$  have pmf  $p_Z(z) = \begin{cases} .4 & \text{if } z = 2 \\ .5 & \text{if } z = 3 \\ .1 & \text{if } z = 7 \end{cases}$

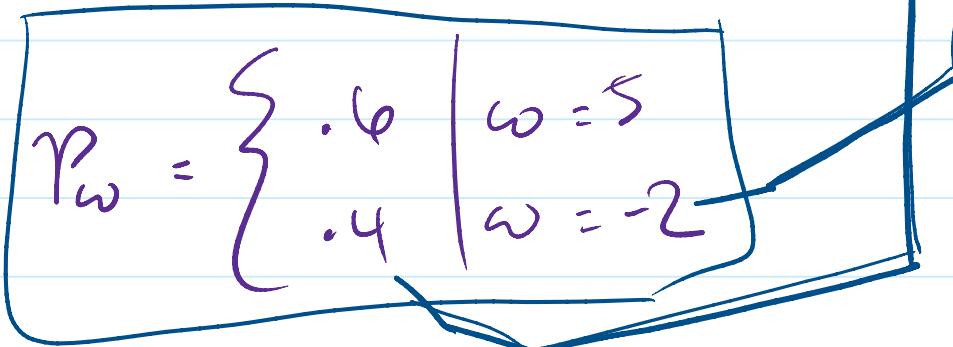
Find its moment-generating function  $M_Z(t)$ .

Find its moment-generating function  $M_Z(t)$ .

$$\boxed{m_z} = M_{e^{zt}} = \sum_{z} M_{e^{zt}} p_z(z) \\ = \boxed{M_{e^{2t}}(.4) + M_{e^{3t}}(.5) + M_{e^{7t}}(.1)}$$

(b) A random variable  $W$  has mgf  $M_W(t) = .6e^{5t} + .4e^{-2t}$ . Find the pmf of  $W$ .

**Hint** for (b): Look carefully at your answer to (a).



5. (a) Let  $X_1, X_2, \dots, X_n$  be independent and have common mgf  $M(t)$ . Show that  $S = X_1 + X_2 + \dots + X_n$  has mgf  $M_S(t) = (M(t))^n$ .

$$\boxed{m_S} = \prod_{i=1}^n m_{X_i} = \prod_{i=1}^n m = \boxed{m^n}$$

(b) Show that  $\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n}$  has mgf  $M_{\bar{X}}(t) = (M(t/n))^n$ .

$$\boxed{m_{\bar{X}}} = M_{e^{t\bar{X}}} = M_{e^{\frac{tS}{n}}} = m_S\left(\frac{t}{n}\right) = \boxed{m^n(t/n)}$$