HW 9

Wednesday, November 3, 2021 5:53 AM

372hw9

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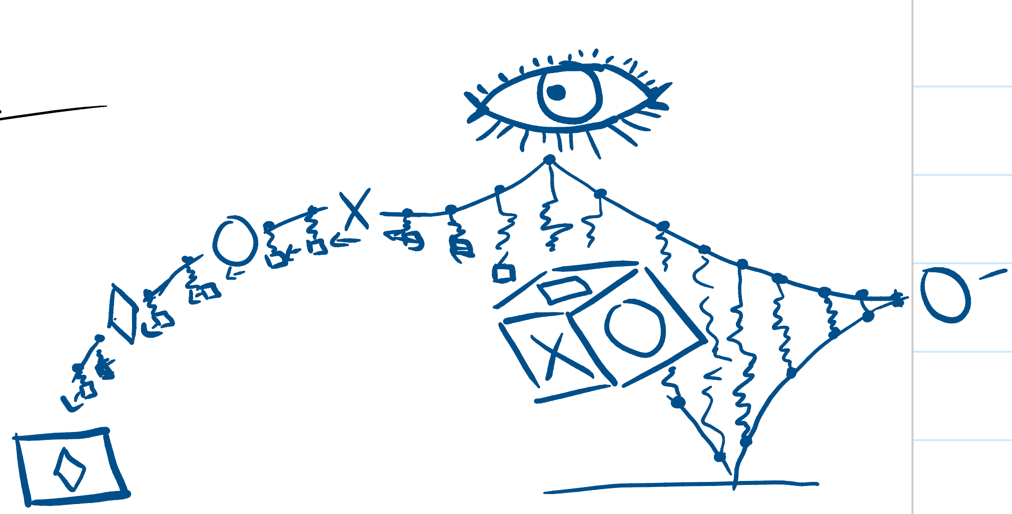


MTH 372
Hw 9
Due Thursday, 11/18/2021.

Read Chapters 18,19 of *Huber*.

18.1 (Modified) For Z a standard normal random variable, find
(a) $P(-1 \le Z < 2)$ (b) $P(1.5 < Z \le 2)$ (c) $P(-1.5 \le Z \le -1)$.

18.1.5. For $X \sim N(\mu = -2, \sigma^2 = 4)$, find
(a) $P(0 < X \le 2)$ (b) $P(-4 \le X < 1)$ (c) $P(-6 < X < -5)$.

18.2 (Modified) For Z_1, Z_2 iid $N(0, 1)$, find $P(Z_1 \le 2Z_2)$.

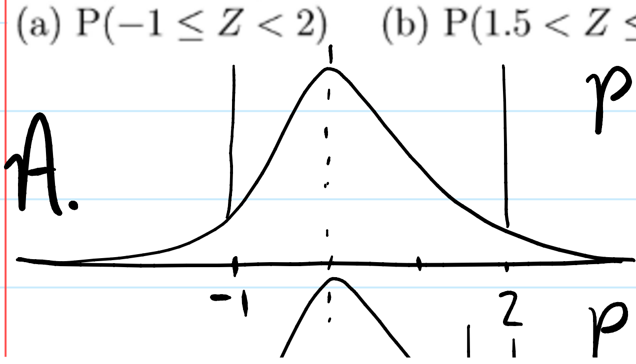
18.3 (Modified) The Digital Life conference draws a number of attendees each year that is normally distributed with mean 65,000 and standard deviation 12,000. Independently, E3 draws a number of attendees that is normally distributed with mean 70,000 and standard deviation 4,000.
(a) Suppose I average the two numbers. What is the distribution of the average.
(b) What is the chance that the average of the two conferences is greater than 70,000?
(c) What is the distribution of the number attending Digital Life minus the number attending E3?
(d) What is the chance that more people attend Digital Life than E3?

p.128 #19.2 Let $A_1, \ldots, A_{10}$ be iid Exp(2). Approximate $P(A_1 + \ldots + A_{10} \ge 7)$ using the CLT.

19.4 Suppose X has density $f_X(x) = (3/2)x(2 - x)$ for $x \in [0, 1]$).
(a) What is E[X]?
(b) What is SD(X)?
(c) For $X_1, X_2, \ldots, X_{20}$ iid with pdf $f_X(x)$ above, approximate with the CLT $P(X_1 + \cdots + X_{20} \ge 13.4)$.

18.1 (Modified) For Z a standard normal random variable, find
(a) $P(-1 \le Z < 2)$ (b) $P(1.5 < Z \le 2)$ (c) $P(-1.5 \le Z \le -1)$.

A.

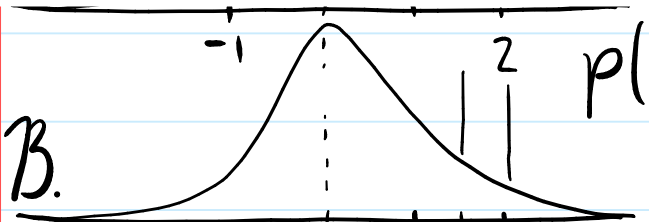


$P(-1 \le z < 2) = P(z < 2) - P(z < -1)$ $\boxed{0.8185}$

$= P(z < 2) - (1 - P(z < 1)) = 0.9772 - 1 + 0.8413$

$P(1.5 < z \le 2) = P(z \le 2) - P(z \le 1.5)$

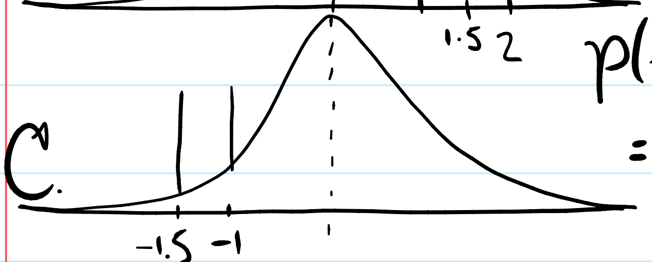
B.



$P(1.5<z\le 2)=P(z\le 2)-P(z\le 1.5)$

$=0.9772-0.9332=\boxed{0.0440}$

C.



$P(-1.5\le z\le -1)=P(z\le -1)-P(z<-1.5)$ $\boxed{0.0919}$

$=(1-P(z\le 1))-(1-P(z\le 1.5))=1-0.8413-1+0.9332$

18.1.5. For $X \sim N(\mu=-2, \sigma^2=4)$, find $\sigma=\sqrt{4}=2$

(a) $P(0<X\le 2)$ (b) $P(-4\le X<1)$ (c) $P(-6<X<-5)$.

A. $z=\frac{X+2}{2}$; $z(0)=1$; $z(2)=2$

$P(0<X\le 2)=P(1<z\le 2)=P(z\le 2)-P(z\le 1)$

$=0.9772-0.8413=\boxed{0.1359}$

B. $z(-4)=-1$; $z(1)=3/2$

$P(-1\le z<3/2)=P(z<3/2)-P(z<-1)$ $\boxed{0.7745}$

$=P(z\le 1.5)-(1-P(z\le 1))=0.9332-1+0.8413$

C. $z(-6)=-2$; $z(-5)=-3/2$

$P(-2<z<-3/2)=P(z<-3/2)-P(z\le -2)$ $\boxed{0.0440}$

$=(1-P(z<3/2))-(1-P(z\le 2))=1-0.9332-1+0.9772$

18.2 (Modified) For Z_1, Z_2 iid N(0, 1), find $P(Z_1 \le 2Z_2)$.

$\mu_1=\mu_2=0$
$\sigma_1^2=\sigma_2^2=1$

$P(z_1\le 2z_2)=P(z_1-2z_2\le 0)$

$[z_1-2z_2]\sim N(\overset{(0-0)}{\mu_1-2\mu_2},\ \overset{(1+4)}{\sigma_1^2+2^2\sigma_2^2})=N(0,5)$

$z=\frac{[z_1-2z_2]-0}{\sqrt{5}}=\frac{z_1-2z_2}{\sqrt{5}}$

$P(z_1-2z_2\le 0)=P(z\le \frac{0}{\sqrt{5}})=P(z\le 0)=\boxed{0.5000}$

18.3 (Modified) The Digital Life conference draws a number of attendees each year that is normally distributed with mean 65,000 and standard deviation 12,000. Independently, E3 draws a number of attendees that is normally distributed with mean 70,000 and standard deviation 4,000.

$DLC\sim N(65{,}000, 12{,}000^2)$; $E3\sim N(70{,}000, 4{,}000^2)$

(a) Suppose I average the two numbers. What is the distribution of the average.

$\bar{X}=\frac{DLC+E3}{2}$; $\bar{X}\sim N\left(\frac{65000}{2}+\frac{70000}{2}, \frac{12000^2}{4}+\frac{4000^2}{4}\right)=N(67{,}500, 40{,}000{,}000)$

$\boxed{\sigma=\sqrt{40000000}=6{,}324.55\ \&\ \mu=67{,}500}$

(b) What is the chance that the average of the two conferences is greater than 70,000?

$z=\frac{\bar{X}-67500}{6{,}324.55}$ $z(70{,}000)=\frac{70000-67500}{6{,}324.55}=0.3953$

$z = \frac{\bar{x} - 67{,}500}{6{,}324.55}$ $\quad z(70{,}000) = \frac{70{,}000 - 67{,}500}{6{,}324.55} = 0.3953$

$$p(\bar{x} > 70{,}000) = p(z > 0.3953)$$
$$= 1 - p(z \le 0.3953) = 1 - \left(\frac{0.6554 + 0.6517}{2}\right)$$
$$= 1 - 0.65355 = \boxed{0.34645}$$

(c) What is the distribution of the number attending Digital Life minus the number attending E3?

$DLC - E3 = \delta \quad \delta \sim N(65000 - 70000,\ 12000^2 + 4000^2)$
$\sim N(-5{,}000,\ 160{,}000{,}000)$

$$\boxed{\sigma = \sqrt{160000000} = 12{,}649.11 \quad \mu = -5{,}000}$$

(d) What is the chance that more people attend Digital Life than E3?

$$p(DLC > E3) = p(DLC - E3 > 0) = p(\delta > 0)$$

$z = \frac{\delta + 5000}{12{,}649.11}$ $\quad = p(z > 0.3953)$

$= 1 - p(z \le 0.3953)$

$z(0) = \frac{5000}{12{,}649.11} = 0.3953$ $\quad = 1 - \left(\frac{0.6554 + 0.6517}{2}\right)$

$$= 1 - 0.65355 = \boxed{0.34645}$$

p.128 #19.2 Let $A_1, \ldots, A_{10}$ be iid Exp(2). Approximate $P(A_1 + \ldots + A_{10} \ge 7)$ using the CLT.

$$A = A_1 + \ldots + A_{10}$$

$\mu = \frac{1}{\beta} = \frac{1}{2} \quad \sigma^2 = \frac{1}{\beta^2} = \frac{1}{4} \quad A_i \sim N\left(\frac{1}{2}, \frac{1}{4}\right)$

$\bar{A} = \frac{A_1 + \cdots + A_n}{10} \quad z = \frac{\bar{A} - \frac{1}{2}}{\frac{1}{\sqrt{10}}} = \left(\bar{A} - \frac{1}{2}\right)\sqrt{10}$

$$z(7) = \left(\frac{7}{10} - \frac{1}{2}\right)\sqrt{10} = 0.6325$$

$$p(A \ge 7) = p(\bar{A} \ge 7/10) = p(z \ge 0.6325)$$
$$= 1 - p(z < 0.6325)$$
$$= 1 - 0.7357 = \boxed{0.2643}$$

19.4 Suppose X has density $f_X(x) = (3/2)x(2 - x)$ for $x \in [0, 1]$).

$3x - \frac{3x^2}{2}$

(a) What is E[X]?

(a) What is E[X]?

$$3\int_0^1 x^2 \delta x - 3/2\int_0^1 x^3 \delta x = 3\left[\frac{x^3}{3}\Big|_0^1\right] - \frac{3}{2}\left[\frac{x^4}{4}\Big|_0^1\right]$$

$$= 3\left(\frac{1}{3}\right) - \frac{3}{2}\left(\frac{1}{4}\right) = 1 - \frac{3}{8} = \boxed{5/8 = \mu_x}$$

(b) What is SD(X)?

$$\mu_x^2 = \left(\frac{5}{8}\right)^2 = \frac{25}{64}$$

$$\mu_{x^2} = 3\int_0^1 x^3 \delta x - \frac{3}{2}\int_0^1 x^4 \delta x$$

$$= 3\left[\frac{x^4}{4}\Big|_0^1\right] - \frac{3}{2}\left[\frac{x^5}{5}\Big|_0^1\right]$$

$$= 3\left(\frac{1}{4}\right) - \frac{3}{2}\left(\frac{1}{5}\right) = \frac{3}{4} - \frac{3}{10}$$

$$= \frac{15}{20} - \frac{6}{20} = \frac{9}{10}$$

$$\sigma_x^2 = \mu_{x^2} - \mu_x^2 = \frac{9}{10} - \frac{25}{64} = \frac{163}{320}$$

$$\sqrt{\sigma_x^2} = \boxed{\sigma_x = 0.7137}$$

(c) For $X_1, X_2, \ldots, X_{20}$ iid with pdf $f_X(x)$ above, approximate with the CLT $P(X_1 + \cdots + X_{20} \geq 13.4)$. $X = X_1 + \cdots + X_{20}$

$$X \sim N(5/8, 0.7137) \qquad \bar{X} = \frac{X_1 + \cdots + X_{20}}{20}$$

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{(\bar{x} - 5/8)\sqrt{20}}{0.7137}$$

$$z(13.4) = \frac{\left(\frac{13.4}{20} - \frac{5}{8}\right)\sqrt{20}}{0.7137} = 0.28198$$

$$P(X \geq 13.4) = P(z \geq 0.28198) = 1 - P(z < 0.28198)$$

$$= 1 - 0.6103 = \boxed{0.3897}$$