HW 8

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372hw8

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MTH 372
Hw 8
Due Thursday, 11/11/2021.

1. Assume that Y_1, Y_2, Y_3 are random variables with

$$\begin{array}{ccc} \mathrm{E}(Y_1) = 2, & \mathrm{E}(Y_2) = -1, & \mathrm{E}(Y_3) = 4, \\ \mathrm{V}(Y_1) = 3, & \mathrm{V}(Y_2) = 6, & \mathrm{V}(Y_3) = 8, \\ \mathrm{Cov}(Y_1, Y_2) = 2, & \mathrm{Cov}(Y_1, Y_3) = -1, & \mathrm{Cov}(Y_2, Y_3) = 0. \end{array}$$

Let $Z = 3Y_1 - 4Y_2 + 6Y_3$. Evaluate $\mathrm{E}(Z)$ and $\mathrm{V}(Z)$.

2. Suppose that $X_1, X_2, \ldots, X_n$ are independent and have common mean μ and common variance σ^2. Let $\overline{X} = \dfrac{X_1 + X_2 + \cdots + X_n}{n}$. Evaluate $\mathrm{E}(\overline{X})$ and $\mathrm{V}(\overline{X})$.

3. A random variable Y has moment-generating function $M_Y(t) = (1 - 2t)^{-3}$. Evaluate $\mathrm{E}(Y)$, $\mathrm{V}(Y)$ and $\mathrm{E}(Y^3)$.

4, (a) Let the random variable Z have pmf $p_Z(z) = \begin{cases} .4 & \text{if } z = 2 \\ .5 & \text{if } z = 3 \\ .1 & \text{if } z = 7 \end{cases}$.

Find its moment-generating function $M_Z(t)$.

(b) A random variable W has mgf $M_W(t) = .6e^{5t} + .4e^{-2t}$. Find the pmf of W.
Hint for (b): Look carefully at your answer to (a).

5. (a) Let $X_1, X_2, \ldots, X_n$ be independent and have common mgf $M(t)$. Show that $S = X_1 + X_2 + \cdots + X_n$ has mgf $M_S(t) = (M(t))^n$.

(b) Show that $\overline{X} = \dfrac{X_1 + X_2 + \cdots + X_n}{n}$ has mgf $M_{\overline{X}}(t) = (M(t/n))^n$.

1. Assume that Y_1, Y_2, Y_3 are random variables with

$$\begin{array}{ccc} E(Y_1) = 2, & E(Y_2) = -1, & E(Y_3) = 4, \\ V(Y_1) = 3, & V(Y_2) = 6, & V(Y_3) = 8, \\ \mathrm{Cov}(Y_1, Y_2) = 2, & \mathrm{Cov}(Y_1, Y_3) = -1, & \mathrm{Cov}(Y_2, Y_3) = 0. \end{array}$$

Let $Z = 3Y_1 - 4Y_2 + 6Y_3$. Evaluate $E(Z)$ and $V(Z)$.

$$\boxed{\mu_z} = 3\mu_{Y_1} - 4\mu_{Y_2} + 6\mu_{Y_3} = 3(2) - 4(-1) + 6(4)$$

$$= 6 + 4 + 24 = \boxed{34}$$

$$\boxed{\sigma_z^2} = 3\sigma_{Y_1}^2 - 4\sigma_{Y_2}^2 + 6\sigma_{Y_3}^2 + 2(\mathrm{cov}(Y_1,Y_2) + \mathrm{cov}(Y_1,Y_3) + \mathrm{cov}(Y_2,Y_3))$$

$$= 3(3) - 4(6) + 6(8) + 2(2 - 1 + 0)$$

$$= 9 - 24 + 48 + 2(1) = \boxed{34}$$

2. Suppose that $X_1, X_2, \ldots, X_n$ are independent and have common mean μ and common variance σ^2. Let $\overline{X} = \dfrac{X_1 + X_2 + \cdots + X_n}{n}$. Evaluate $E(\overline{X})$ and $V(\overline{X})$.

$$\boxed{\mu_{\bar{x}}} = \frac{1}{n}\sum_{i=1}^{n} \mu_{x_i} = \frac{1}{n}\sum_{i=1}^{n} \mu = \frac{n\mu}{n} = \boxed{\mu}$$

$$\boxed{\sigma_{\bar{x}}^2} = \frac{1}{n^2}\sum_{i=1}^{n} \sigma_{x_i}^2 = \frac{1}{n^2}\sum_{i=1}^{n} \sigma^2 = \frac{n\sigma^2}{n^2} = \boxed{\frac{\sigma^2}{n}}$$

3. A random variable X has moment-generating function $M_X(t) = (1 - 2t)^{-3}$. Evaluate $E(X)$, $V(X)$ and $E(X^3)$.

$$\overset{k}{m}_0 = \mu_{x^k}$$

$$\dot{m} = 6(1-2t)^{-4}$$

$$\ddot{m} = 48(1-2t)^{-5}$$

$$\dddot{m} = 480(1-2t)^{-6}$$

$$\boxed{\mu_x} = \dot{m}_0 = \boxed{6}$$

$$\mu_{x^2} = \ddot{m}_0 = 48$$

$$\boxed{\mu_{x^3}} = \dddot{m}_0 = \boxed{480}$$

$$\mu_x^2 = 36$$

$$\boxed{\sigma_x^2} = 48 - 36 = \boxed{12}$$

4, (a) Let the random variable Z have pmf $p_Z(z) = \begin{cases} .4 & \text{if } z = 2 \\ .5 & \text{if } z = 3 \\ .1 & \text{if } z = 7 \end{cases}$.

Find its moment-generating function $M_Z(t)$.

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$\boxed{M_Z} = \mu_{e^{Zt}} = \sum_z \mu_{e^{zt}} P_Z(z)$

$= \boxed{\mu_{e^{2t}}(.4) + \mu_{e^{3t}}(.5) + \mu_{e^{7t}}(.1)}$

(b) A random variable W has mgf $M_W(t) = .6e^{5t} + .4e^{-2t}$. Find the pmf of W.
Hint for (b): Look carefully at your answer to (a).

$$\boxed{P_W = \begin{cases} .6 & w = 5 \\ .4 & w = -2 \end{cases}}$$

5. (a) Let $X_1, X_2, \ldots, X_n$ be independent and have common mgf $M(t)$. Show that $S = X_1 + X_2 + \cdots + X_n$ has mgf $M_S(t) = (M(t))^n$.

$\boxed{M_S} = \prod_{i=1}^{n} M_{X_i} = \prod_{i=1}^{n} M = \boxed{M^n}$

(b) Show that $\overline{X} = \dfrac{X_1 + X_2 + \cdots + X_n}{n}$ has mgf $M_{\overline{X}}(t) = (M(t/n))^n$.

$\overline{X} = \frac{S}{n}$

$\boxed{M_{\overline{X}}} = \mu_{e^{t\overline{X}}} = \mu_{e^{\frac{tS}{n}}} = M_S\left(\frac{t}{n}\right) = \boxed{M^n(t/n)}$