MTH 375
Fall 2022
Hw 9 – due 3/31/2022

Key Concepts: Uniformly Most Powerful (UMP) test of hypotheses.
Read Sec. 6.3.

#1. Let $X_1, \cdots X_n$ be of iid Exponential(θ) random variables. (The common pdf is $f(x;\theta) = (1/\theta)e^{-x/\theta}, x > 0.$) Determine
a) the form of the UMP test of the hypotheses $H_0 : \theta = \theta_0$ vs. $H_A : \theta > \theta_0$.
b) the form of the UMP test of the hypotheses $H_0 : \theta = \theta_0$ vs. $H_A : \theta < \theta_0$.
c) the sampling distribution (i.e., the pdf) of the statistic defining the UMP test.

#2. Let $X_1, \cdots, X_{10}$ be iid binomial$(1, \theta)$ random variables.
a) Determine the form of the UMP test of the hypotheses $H_0 : \theta = .6$ vs $H_A : \theta < .6$.
b) Find the UMP test at level of significance $\alpha = .05$.
c) In one run of this experiment, the data came out to be $\vec{x} = \{0, 0, 1, 0, 1, 0, 1, 0, 1, 0\}$.
Do we reject H_0?

#3. Let $X_1, \cdots, X_{10}$ be iid normal$(\mu = 0, \sigma^2 = \theta)$ random variables.
a) Determine the form of the UMP test of the hypotheses $H_0 : \theta = 5$ vs $H_A : \theta < 5$.
b) Find the UMP test at level of significance $\alpha = .01$.
c) In one run of this experiment, the data came out to be
$\vec{x} = \{5.98, 1.94, 1.19, -3.28, -0.28, 3.43, -2.25, 0.39, 1.02, -2.19\}$.
Do we reject H_0?

#4. Let $X_1, \cdots, X_{10}$ be iid random variables with pdf $f(x;\theta) = \theta x^{\theta-1}$ for $x \in (0, 1)$.
a) Determine the form of the UMP test of the hypotheses $H_0 : \theta = 2$ vs $H_A : \theta > 2$.
b) Given that the statistic $T = -\sum_{k=1}^{10} \ln(X_i)$ has pdf gamma$(10, 1/\theta)$, determine the UMP test of the hypotheses in part(a) at level of significance $\alpha = 0.04$.
c) In one run of this experiment, the data came out to be
$\vec{x} = \{0.912, 0.839, 0.978, 0.789, 0.690, 0.502, 0.862, 0.691, 0.587, 0.557\}$.
Do we reject H_0?

Extra credit: Show that, in problem 4, the pdf of T really is gamma$(10, 1/\theta)$.