MTH 375
Fall 2022
Hw 4 – due 2/10/2022

Key Concepts: Rao-Blackwell Theorem UMVUE, Lehman-Sheffé Theorem.

Read Sec 4.3. Do

#1. Let $X_1, \ldots, X_n$ be iid Binomial$(1, p)$ random variables.
(a) Write out the likelihood function $L(p|x_1, \ldots, x_n)$, and find a sufficient statistic for p.
(b) Use your answer to (a) to find an UMVUE for p.
(c) Evaluate $E(\overline{X}^2)$. **Hint**: Use $E(\overline{X})$ and $\mathrm{V}(\overline{X})$.
(d) Use your answer to (c) to find an UMVUE for p^2.

#2. Let $X_1, \ldots, X_n$ be iid Normal$(\mu, 1)$. We know that $\overline{X}$ is an UMVUE for μ.
(a) Find an UMVUE for μ^2
(b) We know that $\overline{X}$ is $\mathrm{N}(\mu, \sigma^2)$, and with the same μ as X_i, and we know σ^2. (It's not 1.) Use the MGF of $\overline{X}$ to compute $\mathrm{E}(\overline{X}^3)$.
(c) Use (b) to find an UMVUE for μ^3.

#3. Let $X_1 \ldots, X_n$ be iid with common pdf

$$f(x; \theta) = \frac{3x^2}{\theta^3} \quad \text{for } 0 \leq x \leq \theta.$$

(a) Find the likelihood function $L(\theta|x_1, \cdots x_n)$, and find a sufficient statistic T for θ.
(b) Find the pdf of T and $E(T)$, and find an UMVUE for θ.

#4. Let $X_1 \ldots, X_n$ be iid with common pdf

$$f(x; \theta_1, \theta_2) = \frac{1}{\theta_1} e^{-(x-\theta_2)/\theta_1} \quad \text{for } x > \theta_2.$$

(a) Show that the pair $(\sum_{k=1}^n X_k \ , \ X_{(1)})$ is sufficient for (θ_1, θ_2). We use the notation $X_{(1)} = \min\{X_1, \ldots, X_n\}$.
(b) Find a pair (T_1, T_2) which is an UMVUE for (θ_1, θ_2).