MTH 375
Winter 2022
Hw 3 - due Thursday, 2/3/2022

Key Concepts: Likelihood function, sufficient statistic, Fisher-Neyman Lemma.

Read Sec. 4.2 (including 4.2.1).

#1. Do 4.2.2 on p.200.

#2. Do 4.2.4 A Beta(a,b) random variable X has pdf

$$f_X(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1} \text{ for } 0 \le x \le 1.$$

#3. Do 4.2.8.

#4. Do 4.2.10. Add the the word 'sufficient' to the statement of the problem.

#5. Consider the family of distributions with pmf $p_X(x) = \begin{cases} p & \text{if } x = -1 \\ 2p & \text{if } x = 0 \\ 1-3p & \text{if } x = 1 \end{cases}$.

Here p is an unknown parameter, and $0 \le p \le 1/3$.

Let $X_1, X_2, \ldots, X_n$ be iid with common pmf a member of this family. Consider the statistics

$$\begin{aligned} A &= \text{the number of } i \text{ with } X_i = -1, \\ B &= \text{the number of } i \text{ with } X_i = 0, \\ C &= \text{the number of } i \text{ with } X_i = 1. \end{aligned}$$

(i) Write down the joint pmf of $X_1, X_2, \cdots, X_n$. This is most easily done using the statistics A, B, C.

(ii) Use the Fisher-Neyman lemma to show that C is a sufficient statistic for p. (Hint: Use the fact that $A + B + C = n$.)

(iii) Does it appear that A is also sufficient for p? Explain why or why not.

#6. Let X_1, X_2, X_3 be iid, binomial$(1,p)$. Let $T = X_1 + X_2 + 2X_3$. The purpose of this problem is to determine whether T is a sufficient statistic for p. Recall that the definition says that T is sufficient for p if for all $p \in [0,1]$

$$p_{X_1,X_2,X_3|T}(x_1, x_2, x_3 \mid T = x_1 + x_2 + 2x_3) \quad \text{does not depend on } p.$$

Let's examine one particular instance of this definition.

Compute $p_{X_1,X_2,X_3|T}(0,0,1 \mid T = 2)$. Does it depend on p? Is T a sufficient statistic for p?