MTH 375
Fall 2022
Hw 9 – due Thursday, 4/14

Key Concepts: χ^2 test for goodness of fit, linear models, estimators of β, σ^2 and their distributions.

Read Sec. 7.4.

1. A six-sided die is rolled 100 times. The numbers that come up are

number	1	2	3	4	5	6
frequency	20	15	17	11	18	19

We want to use the χ^2 text decide whether there is sufficient evidence at level of significance $\alpha = .05$ to conclude that the die is unfair, i.e., that some numbers are more likely than others.

State H_0 and H_A in statistical language, describe the test you will use, carry out the test, and state the conclusion.

2. Use the χ^2 test to decide, at level of significance $\alpha = .05$, whether the data supports or refutes the claim that the 60 data points below came from an exponentially distributed population.

0.105, 0.183, 0.219, 0.313, 0.326, 0.345, 0.454, 0.461, 0.467, 0.551,
0.603, 0.757, 0.802, 0.824, 0.826, 0.844, 0.987, 1.087, 1.159, 1.180,
1.249, 1.252, 1.317, 1.326, 1.390, 1.398, 1.580, 1.618, 1.653, 1.660,
1.759, 1.850, 1.875, 2.638, 2.691, 2.811, 2.823, 2.828, 2.924, 3.108,
3.323, 3.671, 3.792, 3.797, 4.574, 4.855, 4.924, 5.098, 5.287, 5.346,
6.000, 6.335, 6.491, 6.625, 7.125, 7.586, 8.028, 9.071, 10.783, 11.034

Hints: The data was put in order from least to greatest and sorted into 6 groups of 10 only to make it easier to read. The sum of these numbers is 176.0.

Divide the positive real numbers into 5 intervals in such a way that the probability of landing in a given interval is 0.2, and use those intervals to perform the test.

Since you will be estimating a parameter, you will substract one additional degree to freedom when performing the χ^2 test.

3. Find the equation of the line that best fits the points

$$\{(-4,5),(-4,4),(-3,2),(-2,3),(-2,1),(0,1),(0,2),(1,3),(1,0),(2,0),(2,-1),(3,1)\}$$

(in the sense of least squares). Sketch a graph containing these points and your line.

4. Suppose the data in #3 came from a population satisfying the model in which $Y_1, \ldots, Y_{12}$ are independent normal($\mu_k = \beta_0 + \beta_1 x_k, \sigma^2$). Test the hypotheses at level of significance $\alpha = .01$

$$H_0 : \beta_1 = 0 \ \text{ vs. } \ H_A : \beta_1 \neq 0.$$