MTH 375
Fall 2022
Hw1 – due 1/20/2020

Key Concepts: Multivariate normal($\boldsymbol{\mu}, \Sigma$), χ^2 with n df, t with n df, F with m, n df

1. Suppose that Z_1, Z_2 are independent standard normal random variables.
Let $Y_1 = Z_1 - 2Z_2$, $Y_2 = Z_1 - Z_2$.

(a) Find the joint pdf $f_{Y_1,Y_2}(y_1, y_2)$.

You may do this either of two ways. Either (i) use the change of variables theorem from MTH 372, OR (ii) evaluate the matrices Σ and Σ^{-1} from class, then multiply the necessary matrices and vectors to obtain a formula for $f_{Y_1,Y_2}(y_1, y_2)$. In either case, obtain a formula containing no matrices and no vectors.

(b) Find the marginal pdf $f_{Y_2}(y_2)$. Don't use integration – you can derive the needed pdf doing only the simplest arithmetic using some facts from MTH 372.

(c) Find the conditional pdf of Y_1 given $Y_2 = 1$, that is, $f_{Y_1|Y_2}(y_1|1)$. Use (a) and (b), and do the necessary division. By completing the square, identify by name – *including parameters* – the required conditional pdf.

2. Suppose $Z_1, Z_2, \ldots, Z_6$ are independent standard normal random variables.

(a) Find a number a such that $P(-a < 3Z_1 + 2Z_2 - 4Z_3 < a) = .99$.

(b) Find numbers a, b such that $P(a < Z_1^2 + Z_2^2 + \cdots + Z_6^2 < b) = .99$.

(c) Find a numbers a such that $P(-a < \dfrac{Z_1}{\sqrt{Z_2^2 + Z_3^2 + \cdots + Z_6^2}} < a) = .99$.

(d) Find numbers a, b such that: $P(a < \dfrac{Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2}{Z_5^2 + Z_6^2} < b) = .99$.

3. Let $X_1, X_2, \ldots, X_8$ be independent normal(μ, σ^2) random variables,
$\overline{X} = \frac{1}{8}(X_1 + X_2 + \ldots + X_8)$, and $S^2 = \frac{1}{7}\sum_{i=1}^{8}(X_i - \overline{X})^2$.

Find a number a such that $P\left(-a < \dfrac{\overline{X} - \mu}{S} < a\right) = .99$.