MTH 375
Fall 2020
Hw 5 – due 2/22/2020

Key Concepts: MOM and MLE estimators.
Read Sec 5.1. Do the following problems.

Some of these problems refer to distributions we studied in class and/or on previous assignments . You are encouraged to make use of anything we learned then.

#1. (5.1.2 modified) Let $X_1, \ldots, X_n$ be a sample of iid Negative Binomial (4,θ) random variables with $\theta \in [0, 1]$. Determine the MLE and the MOM estimators of θ.

#2. (5.1.4 modified) Let $X_1, \ldots, X_n$ be a sample of iid N(0,θ) random variables with $\theta > 0$. (θ should be the variance of X_k.) Determine
a) the MLE $\hat{\theta}$ of θ.
b) E($\hat{\theta}$) and V($\hat{\theta}$).
c) the MLE of SD(X_i) $= \sqrt{\theta}$. (Hint: There is almost no work to do.)

#3. Recall the family of distributions with pmf $p_X(x; p) = \begin{cases} p & \text{if } x = -1 \\ 2p & \text{if } x = 0 \\ 1 - 3p & \text{if } x = 1 \end{cases}$.
Here p is an unknown parameter, and $0 \le p \le 1/3$.
Let $X_1, X_2, \ldots, X_n$ be iid with common pmf a member of this family.
(i) Find the MOM estimator of p.
(ii) Find the MLE estimator of p. (Hint: The statistics A, B, C from assignment #3 will be useful.)
(iii) A random sample of size 100 from this distribution produced the values
{ 0, -1, -1, 0, 0, -1, 1, 0, 0, 0, -1, 1, -1, 1, 0, -1, 1, -1, 1, 0, 1, -1, 0, 1, 0, -1, 0, 0, 1, -1, 0, 1, 0, 0, 0, 1, 0, 1, -1, 1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, -1, 1, 0, 1, 0, 1, 1, -1, -1, 1, 1, 0, -1, -1, -1, 0, 1, 1, 0, 1, 1, 0, 1, -1, 1, 0, 1, 0, -1, 0, 1, 0, 1, 0, -1, -1, 1, 0, -1, 0, 0, 1, 1, 1, 0, -1, 1, 1, 0, 1 }
(There are 23 -1's, 38 0's and 39 1's.)
Evaluate the MOM and MLE estimates of p for this data set.

#4. (5.1.8 modified) Let $X_1, \ldots, X_n$ be a sample of iid Gamma(α, θ) random variables with α known and $\theta > 0$. Determine
a) the MLE $\hat{\theta}$ of θ.
b) E($\hat{\theta}$).
e) whether or not $\hat{\theta}$ is a UMVUE of θ.

#5. (5.1.30 modified) Let $X_1, \ldots, X_n$ be a sample of iid random variables with pdf

$$f(x; \theta_1, \theta_2) = \frac{1}{\theta_1} e^{-(x-\theta_2)/\theta_1}, \text{ for } x > \theta_2.$$

Here $\theta_1 > 0$, and θ_2 can be any real number.
Find the MOM and MLE estimators of (θ_1, θ_2).