MTH 375
Fall 2020
Hw 8 – due 3/24/2022

Key Concepts: Bayesian credible intervals, hypothesis testing, size = level of significance, power curve, likelihood ratio, Neyman-Pearson most powerful test.

Read Sec 6.1-2.

#1. Let $X_1, \cdots, X_{50}$ be iid Poisson(θ) random variables, where θ has the prior distribution exponential(5). Determine a 98% Bayesian credible interval for θ when $\sum_{i=1}^{50} X_i = 170$.
Hints: For this problem and for #4,
1. Use the version of the exponential distribution with pdf $f(x|\theta) = \frac{1}{\theta}e^{-x/\theta}$, $x > 0$.
2. The sum of n independent exponential(θ) RV's is gamma(n, θ).

#2. Let $X_1, \ldots X_{25}$ be a sample of iid Bernoulli(p) random variables, with p uniform on $[0, 1]$. Determine a 96% credible interval for p when $\sum_{i=1}^{25} X_i = 10$.

#3. Let $X_1, \ldots, X_{25}$ be a sample of iid Binomial$(1, p)$ random variables. Suppose we test the hypotheses $H_0 : p = 0.6$ vs. $H_A : p > 0.6$ using the test
"Reject H_0 if $\sum_{i=1}^{25} x_i \geq 20$." Determine
a) the level of significance, α.
b) the power of this test for p=.7, .8, .9, and 1.0.

#4. Let $X_1, \ldots, X_{10}$ be iid exponential(θ) random variables. For testing $H_0 : \theta = 5$ vs. $H_A : \theta = 8$, determine
a) the form of the Neyman-Pearson most powerful test.
b) the actual test at level $\alpha = .05$. (Your answer will be "Reject H_0 if ... .")
c) The power of this test.

#5. Let $X_1, \ldots, X_{15}$ be of iid Normal ($\mu = 10$,$\sigma^2 = \theta$) random variables. For testing $H_0 : \theta = 6$ vs. $H_A : \theta = 2$, determine
a) the form of the Neyman-Pearson most powerful test at level of significance α.
b) the actual test at level $\alpha = .05$.
c) The power of your test.