

Read Sec. 5.2

Key Concepts: Bayesian statistics, prior and posterior distributions, Bayesian estimators for squared error loss function and absolute error loss function.

#1. Let  $X_1, \dots, X_n$  be iid binomial(1,  $\theta$ ) random variables.

If the prior pdf of  $\theta$  is Beta(1, 2), determine

a) the posterior distribution of  $\theta|x_1, \dots, x_n$ .

b) the Bayesian estimator of  $\theta$  for the squared error loss.

c) Suppose that  $n = 10$ ,  $\sum_{k=1}^{10} x_i = 7$ . Compute the Bayesian estimate of  $\theta$  for square error loss, and the Bayesian estimate for  $\theta$  for absolute error loss.

#2. Let  $X_1, \dots, X_{50}$  be iid geometric( $\theta$ ), and suppose that  $\theta$  has prior pdf Beta(5,10). Say  $\sum_{i=1}^{50} x_i = 149$ .

a) Find the posterior pdf of  $\theta$ .

b) Find the value of the Bayesian estimator of  $\theta$  for the squared error loss function.

c) Find the value of the Bayesian estimator of  $\theta$  for the absolute error loss function.

#3. Let  $X_1, \dots, X_{60}$  be iid random variables with pdf  $f(x|\theta)$  exponential( $\theta$ ). If the prior pdf of  $\theta$  is Gamma( $\alpha, \beta$ ), determine

a) the posterior distribution of  $\theta|x_1, \dots, x_n$ .

b) the Bayesian estimator of  $\theta$  for the squared error loss function.

c) the Bayesian estimate of  $\theta$  for the squared error loss function when  $\sum_{i=1}^{60} x_i = 143.1$ ,  $\alpha = 3.5$ , and  $\beta = 6$ .

d) the Bayesian estimate of  $\theta$  for the absolute error loss function when  $\sum_{i=1}^{60} x_i = 143.1$ ,  $\alpha = 3.5$ , and  $\beta = 6$ .

#4. Let  $X_1, \dots, X_n$  be iid binomial(2,  $\theta$ ) random variables. If the prior pdf of  $\theta$  is Uniform[0, 1], find

a) the posterior distribution of  $\theta|x_1, \dots, x_n$ ,

b) the Bayesian estimator of  $\theta$  for the squared error loss function,

c) the Bayesian estimate for the squared error loss function when when  $n = 10$  and  $\sum_{i=1}^{10} x_i = 17$ ,

d) the Bayesian estimate of  $\theta$  for the absolute error loss function.

#5. Let  $X_1, \dots, X_{10}$  be iid binomial(1,  $\theta$ ) random variables, with prior pdf

$$\pi(\theta) = \begin{cases} 1/3 & \text{if } \theta = .5 \\ 2/3 & \text{if } \theta = .8 \end{cases}.$$

(a) Find the posterior distribution of  $\theta|x_1, \dots, x_{10}$ .

(Hint: Since  $\theta$  can take only two values, you need only compute  $p(\theta = .5 | x_1, \dots, x_{10})$  and  $p(\theta = .8 | x_1, \dots, x_{10})$ .)

(b) Suppose  $\sum_{k=1}^{10} x_i = 6$ . Find the Bayesian estimate of  $\theta$  for the squared error loss function, and for the absolute error loss function.