Important families of Sampling Distributions

1. Multivariate Normal (AKA Gaussian) $(\boldsymbol{\mu}, \Sigma)$

$$f_{\mathbf{Y}}(\boldsymbol{y}) = \frac{1}{(2\pi)^{n/2}\sqrt{|\det(\Sigma)|}} \exp\left[-\frac{1}{2}(\boldsymbol{y}-\boldsymbol{\mu})^T\Sigma^{-1}(\boldsymbol{y}-\boldsymbol{\mu})\right].$$

$\mathrm{E}(\boldsymbol{Y}) = \boldsymbol{\mu}$. $\quad \mathrm{Cov}(\boldsymbol{Y}) = \Sigma$. $\;M_{\mathbf{Y}}(\mathbf{t}) = \exp\left[\boldsymbol{t}^T\boldsymbol{\mu} + \frac{1}{2}\mathbf{t}^T\Sigma\boldsymbol{t}\right]$

2. Gamma(α, β)

Definition: When α is a positive integer, X is the sum of α independent exponential RV's with mean β.

pdf: $f_X(x) = \frac{1}{\Gamma(\alpha)\ \beta^\alpha} x^{\alpha-1} e^{-x/\beta}$, $x > 0$.

$\mathrm{E}(X) = \alpha\beta$. $\quad \mathrm{V}(X) = \alpha\beta^2$. $\quad M_X(t) = (1-\beta t)^{-\alpha}$.

3. $\chi^2(n)$ $\qquad$ ($\chi^2(n)$ is the same as gamma$(\alpha = n/2,\ \beta = 2)$.)

Definition: $X = Z_1^2 + Z_2^2 + \cdots Z_n^2$, where Z_i are independent, standard normal.

$f_X(x) = \frac{1}{\Gamma(n/2)2^{n/2}} x^{n/2-1} e^{-x/2}$, $x > 0$.

$E(Y) = n$. $\mathrm{V}(Y) = 2n$. $M_X(t) = (1-2t)^{-n/2}$.

4. Student's $t(n)$

Definition: $T = Z/\sqrt{U/n}$, where Z, U are independent, Z is standard normal, U is χ^2 with n df.

$$f_T(t) = \frac{\Gamma((n+1)/2)}{\sqrt{\pi n}\Gamma(n/2)} \cdot \frac{1}{(1+t^2/n)^{(n+1)/2}}.$$

$E(T) = 0$. $\mathrm{V}(T) = n/(n-2)$.

5.$F(n_1, n_2)$

Definition: $W = \frac{U_1/n_1}{U_2/n_2}$, where U_1, U_2 are independent, U_i is χ^2 with n_i df.

$$f_W(w) = \frac{(n_1/n_2)^{n_1/2}\Gamma[(n_1+n_2)/2]w^{n_1/2-1}}{\Gamma(n_1/2)\Gamma(n_2/2)[1+(n_1 w/n_2)]^{(n_1+n_2)/2}},\ w > 0\ .$$

$E(W) = n_2/(n_2 - 2)$, $\mathrm{Var}(W) = 2n_2^2(n_1 + n_2 - 2)/((n_1(n_2 - 2)^2(n_2 - 4)$.