

MTH 375
Fall 2022
Hw 2 – due 1/27/2020

Key Concepts: Estimator, bias, unbiased estimator, standard error, MSE, δ -method.

Read Sec 4.1. Do

1. Let X_1, \dots, X_n and Y_1, \dots, Y_n be two random samples with the same mean μ and variance σ^2 . (The pdf of X_i and Y_j are not specified.)
(a) Show that $T = \frac{1}{2}\bar{X} + \frac{1}{2}\bar{Y}$ and $U = \frac{1}{3}\bar{X} + \frac{2}{3}\bar{Y}$ are both unbiased estimators of μ .
(b) Evaluate $\text{MSE}(T; \mu)$ and $\text{MSE}(U; \mu)$. According to the MSE criterion, is T or U the better estimator of μ ?

2. Let X_1, \dots, X_n be a random sample uniform on $[0, \theta]$. (Hint: We know f_{X_i} , $E(X_i)$ and $V(X_i)$; use them all.)

(a) Show that $T = 2\bar{X}$ is an unbiased estimator of θ , and evaluate $\text{MSE}(T; \theta)$.

(b) Let $M = \max\{X_1, \dots, X_n\}$. Find the pdf of M .

(Hint: Use the fact that $F_M(m) = P(M \leq m) = P(X_1 \leq m \& \dots \& X_n \leq m)$.)

(c) Compute $E(M)$ and $V(M)$.

(d) Using your answers to (c), find an unbiased estimator M^* of θ based on M , and evaluate $\text{MSE}(M^*; \theta)$. According to the MSE criterion, which is a better estimator of θ , T or M^* ?

3. Do problem #4.1.10. There are typos in this problem: θ and p are two names for the same parameter. Use either one, but consistently. (Hint: We know f_{X_i} , $E(X_i)$ and $V(X_i)$; use them all.)

4. Let X_1, \dots, X_n be a sample of i.i.d $N(0, \sigma^2)$ random variables. Let $T = \frac{1}{n} \sum_{i=1}^n X_i^2$.

(a) Is T an unbiased estimator of σ^2 ?

(b) Find $\text{MSE}(T, \sigma^2)$. **Hint:** The mgf of X_i is $M(t) = e^{\frac{1}{2}\sigma^2 t^2}$.

5. Do #4.1.20.