MTH 375
Fall 2022
Hw 7 – due 3/17/2020

Key Concepts: Pivotal quantities, confidence intervals.
Read Sec 5.3.

#1. Let $X_1, \cdots, X_{10}$ be a sample of iid N(0, θ) random variables. (θ is the variance, not the standard deviation.)

a) Show that $\mathrm{T} = \frac{1}{\theta}\sum_{i=1}^{n} X_i^2$ is a pivotal quantity and find its distribution.

b) Determine a 99% confidence interval for θ based on T.

c) Suppose that $\sum_{i=1}^{10} x_i^2 = 27.3$. What is the 99% confidence interval for θ?

d) Determine a 99% confidence interval for $\mathrm{SD}(X) = \sqrt{\theta}$. (Hint: You have already done almost all of the necessary work.)

#2. Let $X_1, \cdots, X_6$ be a sample of iid $U[\theta, 1]$ random variables.

a) Show that $T = \frac{1 - X_{(1)}}{1 - \theta}$ is a pivotal quantity and find its cdf.

b) Determine a 85% confidence interval for θ based on T.

c) Suppose $\vec{x} = \{0.527, 0.803, 0.842, 0.880, 0.474, 0.558\}$. Find the 85% confidence interval for θ.

#3. Let $X_1, \cdots, X_8$ be a sample of iid Gamma(4, θ) random variables.

a) Show that $\frac{2}{\theta}\sum_{i=1}^{n} X_i$ is a pivotal quantity and find its distribution.

b) Determine a 95% confidence interval for θ based on T.

c) Suppose that $\vec{x} = \{17.40, 15.95, 15.17, 7.92, 11.54, 10.46, 8.47, 15.54\}$. Find a 95% confidence interval for θ.

#4. Let $X_1, \cdots, X_6$ be a sample of iid N(μ, θ) random variables. (μ, θ are both unknown. θ is the variance, not the standard deviation.)

a) Why is $T = (n-1)S^2/\theta$ a pivotal quantity? What is the distribution of $(n-1)S^2/\theta$?

b) Determine a 93% confidence interval for θ based on T.

c) Suppose that $\vec{x} = \{0.71, 0.62, 2.28, 1.01, 10.01, 7.64\}$. Find a 95% confidence interval for θ.

#5. A pollster chooses 500 citizens at random, and asks whether each plans to vote 'yes' or 'no' on proposition X. In the sample, 375 indicate a 'yes' vote.

Find a 95% confidence interval for the proportion of the total population who plan to vote 'yes.' (Hint: Use the Central Limit Theorem. Where you need a variance, replace it by its greatest possible value.)