

MTH 375
Fall 2022
Midterm Exam

Please take no more than about 2 hours to complete this exam. It is due at midnight tonight (Thursday, 2/24). Class does not meet today.

You may use your textbook, and any notes or documents you have from class. You may use the result of any theorem or computation from any of those sources, provided you cite it for me. As usual, I grade solutions, not just answers.

1. Let X_1, \dots, X_n be beta with α unknown and $\beta = 3$. Find the Method of Moments estimator of α .

2. Let Y_1, \dots, Y_n have common pdf $f(y) = \begin{cases} \frac{2y}{a^2}, & 0 \leq y \leq a \\ 0 & \text{otherwise} \end{cases}$.

(a) Find a sufficient statistic for a .

(b) Find an UMVUE for $\frac{1}{a}$.

3. Let Y_1, \dots, Y_n have common pmf $p(y) = \begin{cases} 2p & \text{if } y = -1 \\ 1/2 - p & \text{if } y = 0 \\ 1/2 - p & \text{if } y = 1 \end{cases}$, $0 < p < \frac{1}{2}$

(a) Find a sufficient statistic for p .

(b) Find the MOM estimator of p . Is it unbiased?

(c) Find the MLE estimator of p . Is it unbiased?

(d) Find the MSE of both estimators. For which values of p is the MSE of the MLE the smaller of the two?

4. Let X_1, \dots, X_n be exponential with parameter β . Find UMVUE's for β , β^2 and β^3 .

Hint: The exponential distribution has pdf $p(x) = \frac{1}{\beta}e^{-x/\beta}$ ($x > 0$). The sum of n independent exponential(β) random variables has pdf Gamma($\alpha = n, \beta$).

5. Let X_1, \dots, X_{10} be iid normal with unknown μ, σ^2 .

Let $\bar{X} = \frac{X_1 + \dots + X_{10}}{10}$ and $S^2 = \frac{1}{9} \sum_{i=1}^{10} (X_i - \bar{X})^2$.

(a) Find a number a such that $P\left(-a < \frac{\bar{X} - \mu}{S} < a\right) = .95$.

(b) Find positive numbers a, b such that $P\left(a < \frac{S^2}{\sigma^2} < b\right) = .95$.