MTH 385 Homework due 2022-02-21

Exercise 1 (3.3.2). *Show that, for any integers a and b, there are integers m and n such that* $\gcd(a, b) = ma + nb$.

This in turn gives a general way to find integer solutions of linear equations.

Exercise 2 (3.3.3). *Deduce from Exercise 3.3.2 that the equation $ax+by = c$ with integer coefficients a, b, and c has an integer solution x, y if* $\gcd(a, b)$ *divides c.*

Exercise 3 (3.3.5). *(Solution of linear Diophantine equations) Give a test to decide, for any given integers a, b, c, whether there are integers x, y such that*

$$ax + by = c.$$

Exercise 4 (3.4.3). *Show that*

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\ddots}}}}$$

Exercise 3.4.3 implies that $\sqrt{2} + 1$ is the periodic continued fraction

$$2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\ddots}}}}$$

Exercise 5 (3.4.4). *Show that $\sqrt{3} + 1$ also has a periodic continued fraction, and hence derive the continued fraction for $\sqrt{3}$.*