MTH 385 2022-03-16 Worksheet

Exercise 1. *Check that the equation*

$$x^3 + ax + b = 0$$

can be reduced to the equation

$$4y^3 - 3y = c$$

by setting $x = ky$ and choosing k so that

$$\frac{k^3}{ak} = -\frac{4}{3}.$$

Express c in terms of b, a and k.

Exercise 2. *Recall the formulas for the sine ane cosine of the sum of two angles.*

$$\cos(\alpha + \beta) =$$
$$\sin(\alpha + \beta) =$$

Exercise 3. *Verify.*

$$4\cos^3\theta - 3\cos\theta = \cos 3\theta$$

Exercise 4. *Explain why solving the cubic equation*

$$x^3 + ax + b = 0$$

is equivalent to the problem of trisecting an angle (if $|c| \le 1$).

Exercise 5. *Check that Cardano's solution to the cubic does not envolve complex numbers when $|c| > 1$.*

Exercise 6. *Use induction to prove de Moivre's formula. That is, prove that for all positive integers n and all angles θ,*

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta.$$

Exercise 7 (a variant of 5.6.1)**.** *Revisit Exercise 3 in light of de Moivre's formula.*

Exercise 8. *Recall the Maclaurin series (Taylor series at $z = 0$) for e^z.*

Exercise 9. *Find the real and imaginary parts of the Maclaurin series for e^{ix}.*

Exercise 10. *Comment upon the results of Exercise 9.*