MTH 385 2022-01-19 Worksheet

Exercise 1. *As you may recall, the proof that $\sqrt{2}$ is irrational is often used to teach an important proof technique. What is that proof technique? And, how does it work?*

Exercise 2. *State the Well-Ordering Principle. (And, look for its appearance in the following.)*

Exercise 3. *The proof recounted in our textbook uses the fact that any (positive) rational number can be written as the ratio of two realtively prime (positive) integers. Prove this fact.*

Exercise 4. *The proof recounted in our textbook also uses the fact that, if the square of a positive integer is even, then that positive integer is also even. Prove this fact.*

Exercise 5. *Do Exercise 1.5.1: Writing an arbitrary odd number m in the form $2q + 1$, for some integer q, show that m^2 also has the form $2r + 1$, which shows that m^2 is also odd.*

Exercise 6. *What happens if we replace 2 by 3 in Exercise 1.5.1? Write $m = 3q + 1$, for some integer q, does m^2 also have the form $3r + 1$?*

Exercise 7. *What happens if we replace 2 by 4 in Exercise 1.5.1? Write $m = 4q + 1$, for some integer q, does m^2 also have the form $4r + 1$?*

Exercise 8. *What happens if we replace 2 by 5 in Exercise 1.5.1? Write $m = 5q + 1$, for some integer q, does m^2 also have the form $5r + 1$?*

Exercise 9. *What, if anything, can be said if we replace 2 in Exercise 1.5.1 by an arbitrary (positive) integer?*

Exercise 10. *The proof that $\sqrt{2}$ is irrational is an early instance of the recurring interplay between geometry on the one hand and arithmetic/algebra on the other. Are there other examples of such interplay between streams of mathematical thought in the 100-level and 200-level courses at UM-Flint? If so, list some.*