MTH 385 Homework due 2022-03-28

Exercise 1 (5.7.2)**.** *If* $p(x) = a_k x^k + a_{k-1}x^{k-1} + \cdots + a_1 x + a_0$*, use Exercise 5.7.1 to show that* $p(x) - p(a)$ *has a factor* $x - a$*.*

Exercise 2 (5.7.3)**.** *Deduce Descartes's theorem from Exercise 5.7.2.*

Recall

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}.$$

This property gives an easy way to calculate Pascal's triangle to any depth, and hence compute a fair division of stakes in a game that has to be called off with n plays remaining. We suppose that players I and II have an equal chance of winning each play, and that I needs to win k of the remaining n plays to carry off the stakes.

Exercise 3 (5.8.2)**.** *Show that the ratio of I's winning the stakes to that of II's winning is*

$$\binom{n}{n} + \binom{n}{n-1} + \cdots + \binom{n}{k} : \binom{n}{k-1} + \binom{n}{k-2} + \cdots + \binom{n}{0}.$$

The sum property of the binomial coefficients also explains the presence of some interesting numbers in Pascal's triangle.

Exercise 4 (5.8.3)**.** *Explain why the third diagonal from the left in the triangle, namely* $1, 3, 6, 10, 15, 21, \ldots$*, consists of the triangular numbers.*

Exercise 5 (5.8.4)**.** *The numbers on the next diagonal, namely* $1, 4, 10, 20, 35 \ldots$*, can be called* tetrahedral numbers*. Why is this an apt description?*