Exercise 8 (8.3.2)**.** *What would you substitute for y to find the tangent at $(0,1)$ to the curve $y^2 = x^3 - 3x^2 + 5x + 1$?*

Solution. The equation of the line through $(0,1)$ with slope m is $y = mx + 1$. We will substitute $y = mx + 1$ and look to see which values of m yield a double root at $x = 0$.

$$\begin{aligned}(mx+1)^2 &= x^3 - 3x^2 + 5x + 1\\ m^2x^x + 2mx + 1 &= x^3 - 3x^2 + 5x + 1\\ 0 &= x^3 - (m^2+3)x^2 + (5-2m)x\end{aligned}$$

Evidently, $y = mx+1$ is tangent at $(0,1)$ to the curve $y^2 = x^3 - 3x^2 + 5x + 1$ exactly when $5 - 2m = 0$. So, the tangent line is $y = \frac{5}{2}x + 1$. □