MTH 385 2022-03-14 Worksheet

Here, we walk through Cardano's method for find a solution to the (general) cubic equation $x^3 + ax^2 + bx + c = 0$.

Exercise 1. *Substitute $x = y - a/3$ into $x^3 + ax^2 + bx + c = 0$. Rewrite the equation in the form*

$$y^3 = py + q.$$

Express p and q in terms of a, b and c.

Exercise 2. *Substitute $y = u + v$ into y^3. By collecting terms, rewrite the expression in the form*

$$y^3 = py + q.$$

Express p and q in terms of u, and v.

Exercise 3. *Use the expression from the previous exercise for p to eliminate v from the expression for q (also from the previous exercise).*

Exercise 4. *Write and solve the quadratic in u^3 obtained from the previous exercise. This gives two expressions for u^3.*

Exercise 5. *Find expressions for v^3 similar to the ones for u^3 found in the previous exercise.*

Exercise 6. *We now have four expressions that might be $u^3 + v^3$. What are they? Use your knowledge of $u^3 + v^3$ from above to choose a valid expression.*

Exercise 7. *Assume our expressions for u^3 and v^3 are real numbers. Find real values for u and v. Then, find y.*

Exercise 8. *Find a solution to $x^3 + ax^2 + bx + c = 0$.*

The two equations $3uv = p$, $u^3 + v^3 = q$ provide another instance of the phenomenon noted in Exercise 5.2.2: when a variable is eliminated between two equations, the degrees of the equations are multiplied.

Exercise 9 (5.5.1)**.** *The equation $3uv = p$ is of degree 2 in u and v, and $u^3 + v^3 = q$ is of degree 3. What about the equation obtained by eliminating v?*