Exercise 5 (8.4.2). *Deduce Viète's product by substituting $x = \pi/2$.*

Solution. Recall that our starting point is

$$\frac{\sin x}{x} = \cos \frac{x}{2} \cos \frac{x}{2^2} \cos \frac{x}{2^3} \cdots .$$

So, if we use the half angle formula $\cos \frac{\theta}{2} = \pm\sqrt{\frac{1}{2}(1 + \cos \theta)}$ (we take the positive root because the angle is in the first quadrant), we have the following.

$$\frac{\sin \frac{\pi}{2}}{\pi/2} = \cos \frac{\pi}{4} \cos \frac{\pi}{8} \cos \frac{\pi}{16} \cdots$$

$$\frac{\pi}{2} = \sqrt{\frac{1}{2}} \cos \frac{\pi}{8} \cos \frac{\pi}{16} \cdots$$

$$= \sqrt{\frac{1}{2}} \sqrt{\frac{1}{2}\left(1 + \sqrt{\frac{1}{2}}\right)} \cos \frac{\pi}{16} \cdots$$

$$= \sqrt{\frac{1}{2}} \sqrt{\frac{1}{2}\left(1 + \sqrt{\frac{1}{2}}\right)} \sqrt{\frac{1}{2}\left(1 + \sqrt{\frac{1}{2}\left(1 + \sqrt{\frac{1}{2}}\right)}\right)} \cdots$$

$\square$