MTH 385 2022-02-09 Worksheet

Exercise 1. *Recreate Figure 3.1.*

Exercise 2. *Define the following phrases.*

(a) triangular number

(b) square number

(c) pentagonal number

(d) polygonal number

Exercise 3. *According to the textbook, "From Figure 3.1 it is easy to calculate an expression for the mth n-gonal number as the sum of a certain arithmetic series ..." Do this for:*

(a) the triangular numbers;

(b) the square numbers; and,

(c) the pentagonal numbers.

Exercise 4 (3.2.1)**.** *Show that any square leaves remainder* 0, 1, *or* 4 *on division by* 8.

Exercise 5 (3.2.2)**.** *Deduce that a sum of three squares leaves remainder* 0, 1, 2, 3, 4, 5, *or* 6 *on division by* 8.

Bonus Material

For each nonnegative integer k, let $P_k(t)$ be the polynomial with rational coefficients given by

$$P_0(t) = 1, \text{ and } P_k(t) = \binom{t}{k} = \frac{t(t-1)\cdots(t-k+1)}{k!} \text{ for } k \geq 1.$$

Exercise 6. *Find the following.*

(a) $P_1(t)$

(b) $P_2(t)$

(c) $P_3(t)$

Exercise 7. *Complete the following table.*

t	$P_0(t)$	$P_1(t)$	$P_2(t)$	$P_3(t)$
0				
1				
2				
3				
4				
5				

Given a(n integer valued) polynomial $f(t)$ define a new integer-valued polynomial $(\Delta f)(t)$ by

$$(\Delta f)(t) = f(t+1) - f(t).$$

Exercise 8. *Find the following.*

(a) $(\Delta P_0)(t)$

(b) $(\Delta P_1)(t)$

(c) $(\Delta P_2)(t)$

(d) $(\Delta P_3)(t)$

Think of the operator Δ as a discrete analog of differentiation. We write $\Delta^2 f$ for $\Delta(\Delta f)$.

Exercise 9. *Let a, b, and c be integers. And, let $f(t) = aP_0(t) + bP_1(t) + cP_2(t)$. Complete the following table.*

t	$f(t)$	$(\Delta f)(t)$	$(\Delta^2 f)(t)$
0			
1			
2			
3			
4			
5			

Exercise 10. *Suppose you are given a sequence of integers $a_0, a_1, \ldots$ and you are asked whether that sequence is of the form $a_n = f(n)$ for some quadratic polynomial (with rational coefficients), how might you test that idea? And, how you find f when such a polynomial exists?*