MTH 385 Homework due 2022-02-07

Exercise 1 (2.3.3)**.** *By finding some parallels and similar triangles in Figure 2.5, show that the diagonal x of the regular pentagon of side 1 satisfies $x/1 = 1/(x-1)$.*

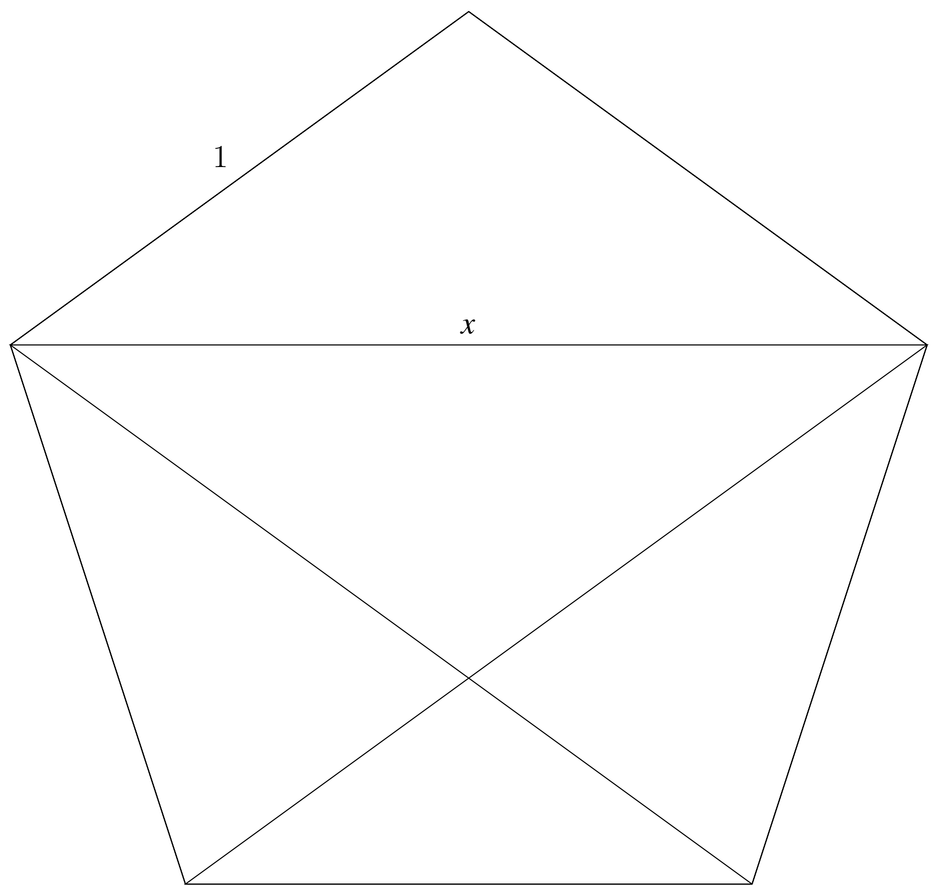

Figure 2.5: The regular pentagon

Exercise 2 (2.3.4)**.** *Deduce from Exercise 2.3.3 that the diagonal of the pentagon is $(1+\sqrt{5})/2$ and hence that the regular pentagon is constructible.*

Exercise 3 (2.4.2)**.** *By introducing suitable coordinate axes, show that a curve with the above "constant sum" property indeed has an equation of the form*

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

(It is a good idea to start with the two square root terms, representing the distances F_1P and F_2P, on opposite sides of the equation.) Show also that any equation of this form is obtainable by suitable choice of F_1, F_2, and $F_1P + F_2P$.

Another interesting property of the lines from the foci to a point P on the ellipse is that they make equal angles with the tangent at P. It follows that a light ray from F_1 to P is reflected through F_2.

A simple proof of this can be based on the shortest-path property of reflection, shown in Figure 2.7 and discovered by the Greek scientist Heron around 100 CE.

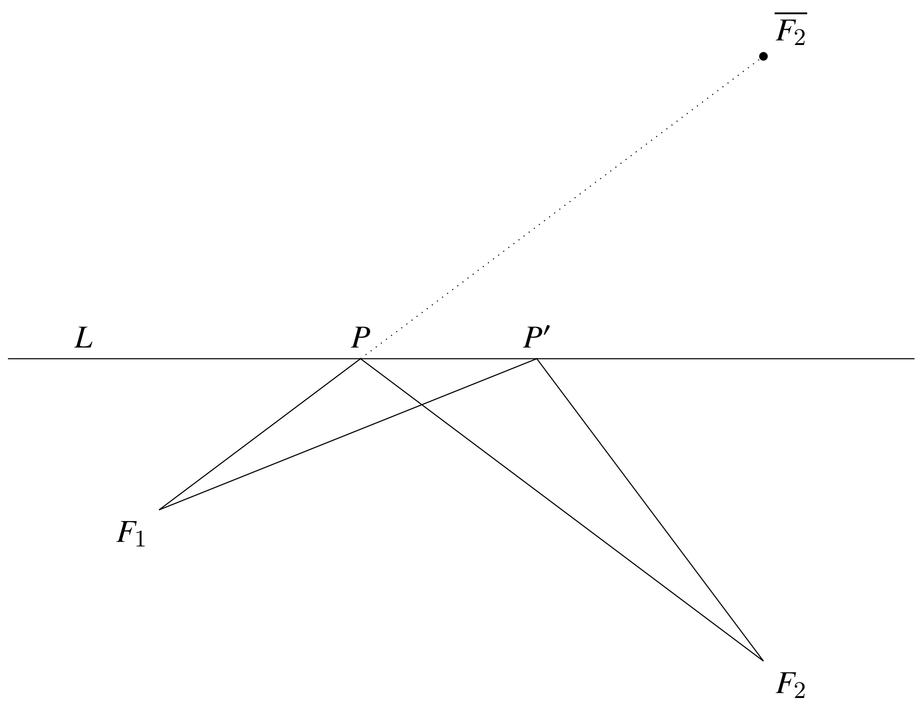

Figure 2.7: The shortest-path property

Shortest-path property. The path F_1PF_2 of reflection in the line L from F_1 to F_2 is shorter than any other path $F_1P'F_2$ from F_1 to L to F_2.

Exercise 4 (2.4.3)**.** *Prove the shortest-path property, by considering the two paths F_1PF_2 and $F_1P'F_2$, where $\overline{F_2}$ is the reflection of the point F_2 in the line L.*

Thus to prove that the lines F_1P and F_2P make equal angles with the tangent, it is enough to show that F_1PF_2 is shorter than $F_1P'F_2$ for any other point P' on the tangent at P.

Exercise 5 (2.4.4)**.** *Prove this, using the fact that F_1PF_2 has the same length for all points P on the ellipse.*