MTH 385 2022-02-23 Worksheet

Exercise 1. *Recount the textbook's description of* Gaussian elimination.

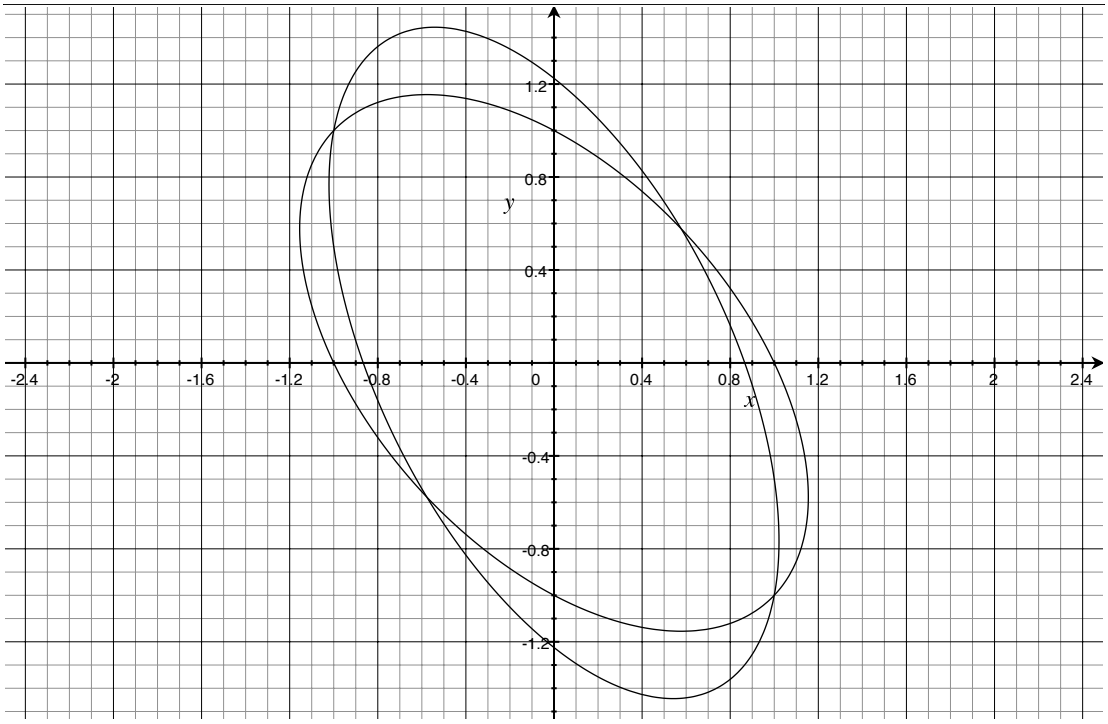
Exercise 2. *Recount the textbook's description of elimination for simultaneous polynomial equations in two (or more) variables.*

Exercise 3. *The textbook mentions Cramer's rule in passing. Describe Cramer's rule.*

Exercise 4 (5.2.1)**.** *Derive an equation that is linear in y from the two equations*

$$x^2 + xy + y^2 = 1,$$
$$4x^2 + 3xy + 2y^2 = 3,$$

and hence show that $y = (1 - 2x^2)/x$.



Exercise 5 (5.2.2)**.** *Deduce that the intersections of the two curves in Exercise 5.2.1 occur where x satisfies* $3x^4 - 4x^2 + 1 = 0$.

This example, where the two equations of degree 2 yield a single equation of degree $4 (= 2 \times 2)$, illustrates a general phenomenon where degrees are multiplied. We will observe other instances, and study it more deeply, as the book progresses.

The present example is not a typical equation of degree 4, since it is quadratic in $x^2 = z$. However, this makes it a lot easier to solve.

Exercise 6 (5.2.3)**.** *Solve* $3z^2 - 4z + 1 = 0$ *for* $z = x^2$ *by factorizing the left-hand side, and hence find four solutions for* x*.*

Give geometric reasons why you would expect two curves of degree 2 *to have up to four intersections. Could they have more than four?*