MTH 385 2022-04-06 Worksheet

Exercise 1. *Historically, which was considered simpler, differentiation or integration?*

Exercise 2. *What is the earliest example of the limit process*

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}?$$

Exercise 3. *When did this limit process appear for polynomials?*

Exercise 4. *Suppose $\varepsilon^2 = 0$ and we can divide by $\varepsilon \neq 0$. Compute $\dfrac{f(x+\varepsilon) - f(x)}{\varepsilon}$ for the following polynomials.*

(a) $f(x) = 1$

(b) $f(x) = x$

(c) $f(x) = x^2$

(d) $f(x) = x^3$

(e) $f(x) = 2x^2 - 3x + 5$

Exercise 5. *Comment on the results of the previous exercise. What was Fermat's method? And, how is it related to the method of the previous exercise?*

Exercise 6. *Let $p(x, y)$ be a polynomial. And, consider the curve $p(x, y) = 0$. What is the formula of Hudde and Sluse for $\dfrac{dy}{dx}$ when*

$$p(x, y) = \sum a_{ij} x^i y^j?$$

For evidence that tangents to algebraic curves may be found without calculus, it is enough to look more closely at what we called Diophantus's tangent method in Section 3.5. In his *Arithmetica*, Problem 18, Book VI (previously mentioned in Exercise 3.5.1), Diophantus finds the tangent $y = \frac{3x}{2} + 1$ to $y^2 = x^3 - 3x^2 + 3x + 1$ at the point $(0, 1)$, apparently by inspection. Without mentioning its geometric interpretation, he simply substitutes $y = \frac{3x}{2} + 1$ for y in $y^2 = x^3 - 3x^2 + 3x + 1$.

Exercise 7 (8.3.1)**.** *Check that this substitution gives the equation*

$$x^3 - \frac{21}{4}x^2 = 0$$

What is the geometric interpretation of the double root $x = 0$?

Exercise 8 (8.3.2)**.** *What would you substitute for y to find the tangent at $(0, 1)$ to the curve $y^2 = x^3 - 3x^2 + 5x + 1$?*