MTH 385 Homework due 2022-01-17

Exercise 1 (1.2.3)**.** *Show that any integer square leaves remainder* 0 *or* 1 *on division by* 4*.*

Solution. $\square$

Exercise 2 (1.2.4)**.** *Deduce from Exercise 1.2.3 that if* (a, b, c) *is a Pythagorean triple then* a *and* b *cannot both be odd.*

Solution. $\square$

The parameter t in the pair $\left(\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2}\right)$ runs through all rational numbers if $t = q/p$ and p, q run through all pairs of integers.

Exercise 3 (1.3.1)**.** *Deduce that if* (a, b, c) *is any Pythagorean triple then*

$$\frac{a}{c} = \frac{p^2 - q^2}{p^2 + q^2}, \qquad \frac{b}{c} = \frac{2pq}{p^2 + q^2}$$

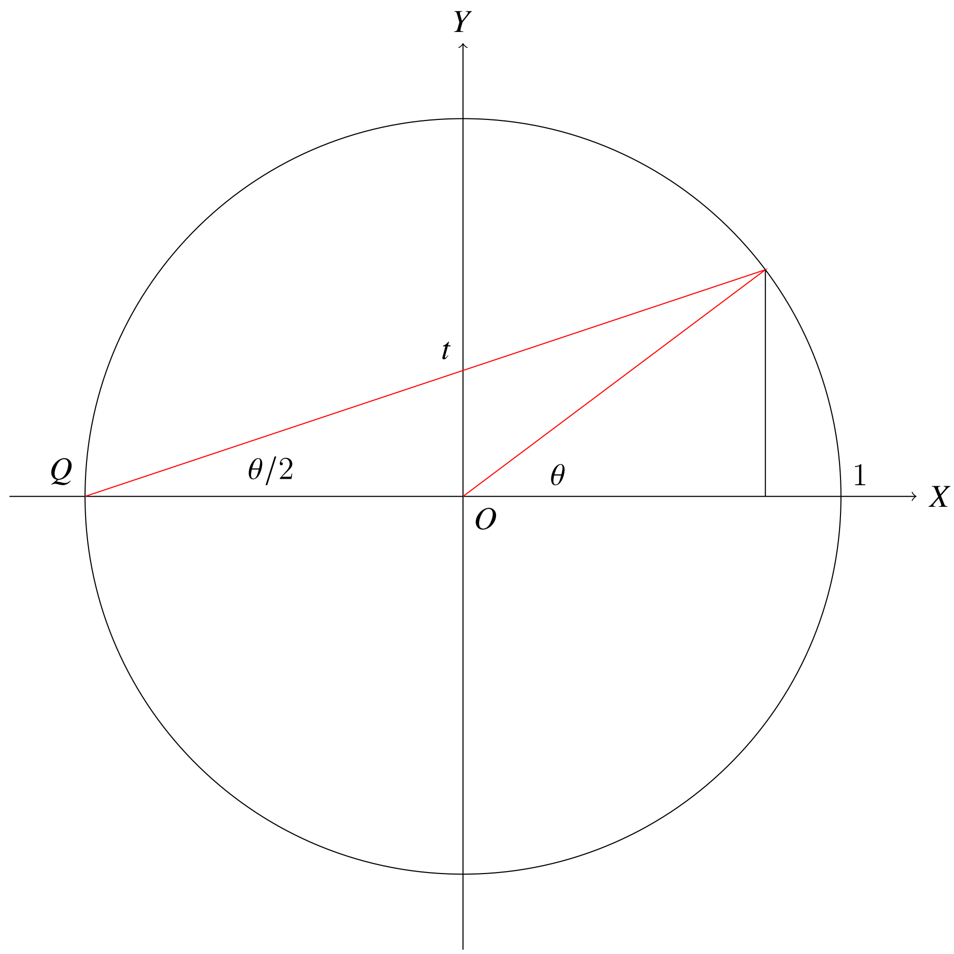
for some integers p *and* q*.*

Solution. $\square$

Exercise 4 (1.3.2)**.** *Use Exercise 1.3.1 to prove Euclid's formula for Pythagorean triples, assuming* b *even. (Remember,* a *and* b *are not both odd.)*

Solution. $\square$

Some important trigonometry may be gleaned from Diophantus's method if we compare the angle at O in Figure 1.4 with the angle at Q in Figure 1.5. The two angles are shown in Figure 1.7, and high school geometry shows that the angle at Q is half the angle at O.



Exercise 5 (1.3.4)**.** *Why does the angle at Q equal $\theta/2$? (Hint: consider angles in the red triangle.)*

Solution. □

Exercise 6 (1.3.5)**.** *Use Figure 1.7 to show that $t = \tan\frac{\theta}{2}$ and*

$$\cos\theta = \frac{1-t^2}{1+t^2}, \qquad \sin\theta = \frac{2t}{1+t^2}.$$

Solution. □