MTH 385 2022-03-09 Worksheet

An elementary proof that $\sqrt[3]{2}$ is not constructible was found by the number theorist Edmund Landau (1877–1938) when he was still a student. It is broken down to easy steps below. But first we should check that $\sqrt[3]{2}$ is actually irrational.

Exercise 1 (5.4.1)**.** *Show that the assumption $\sqrt[3]{2} = m/n$, where m and n are integers, leads to a contradiction.*

Landau's proof now organizes all numbers involved in a construction into sets $F_0, F_1, \ldots$, according to the depth of nesting of square roots.

Exercise 2 (5.4.2)**.** *Let*

$$F_0 = \{rationals\}, \qquad F_{k+1} = \{a + b\sqrt{c_k} \mid a, b, c_k \in F_k\} \text{ for some } c_k \in F_k.$$

Show that each F_k is a field*, that is, if x, y are in F_k, then so are $x + y$, $x - y$, xy, and x/y (for $y \neq 0$).*

Exercise 3. *Consider the situation of Exercise 5.3.5. Given the equations of two circles in the form*

$$\begin{aligned}(x-a)^2 + (y-b)^2 &= r^2\\ (x-c)^2 + (y-d)^2 &= s^2,\end{aligned}$$

When do we get the the equation of a line by subtracting one of the equations from the other? When we get the equation of a line, how is the line related to the circles?