MTH 385 2022-04-18 Worksheet

Exercise 1. *Why was there a dispute over whether Newton discovered calculus?*

Exercise 2. *What were Leibniz' contributions to calculus? How did he think about calculus?*

Exercise 3. *Who introduced the word "function"?*

Exercise 4. *What is an algebraic function? What is a transcendental function? How do they differ?*

Exercise 5. *What is a "closed-form" expression?*

Exercise 6. *According to the textbook, "the search for closed forms was a wild goose chase." Give an example of an algebraic function that does not have an algebraic anti-derivative.*

Exercise 7. *Sketch a proof of the Fundamental Theorem of Calculus.*

Leibniz (1702) was stymied by the integral $\int \frac{dx}{x^4+1}$, because he did not spot the factorization of x^4+1 into real quadratic factors.

Exercise 8 (8.6.1)**.** *Writing* $x^4+1 = x^4+2x^2+1-2x^2$ *or otherwise, split* x^4+1 *into real quadratic factors.*

Exercise 9 (8.6.2)**.** *Use the factors in Exercise 8.6.1 to express* $\frac{1}{x^4+1}$ *in the partial fraction form*

$$\frac{x+\sqrt{2}}{q_1(x)} + \frac{x-\sqrt{2}}{q_2(x)}$$

where $q_1(x)$ *and* $q_2(x)$ *are real quadratic polynomials.*

Exercise 10 (8.6.3)**.** *Without working out all the details, explain how the partial fractions in Exercise 8.6.2 can be integrated in terms of rational functions and the* $\tan^{-1}$ *function.*