MTH 385 2022-03-23 Worksheet

Exercise 1. *Compute.*

$$
\begin{aligned}
(a+b)^0 &= \\
(a+b)^1 &= \\
(a+b)^2 &= \\
(a+b)^3 &= \\
(a+b)^4 &= \\
(a+b)^5 &= \\
(a+b)^6 &= \\
(a+b)^7 &=
\end{aligned}
$$

Exercise 2. *Make a table of the coefficients of the polynomials (when written in decreasing a-degree) from the previous exercise.*

Write $\binom{n}{k}$ for the kth element of the nth row (indexing starting with 0). That is, $\binom{n}{k}$ is the coefficient on $a^{n-k}b^k$ in $(a+b)^n$. These numbers are called *binomial coefficients*.

Exercise 3 (5.8.1)**.** *Use the identity*

$$(a+b)^n = (a+b)^{n-1}a + (a+b)^{n-1}b$$

to prove the sum property of binomial coefficients:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}.$$

Exercise 4. *Prove $\binom{n}{k}$ is the number of combinations of n things taken k at a time. That is, prove $\binom{n}{k}$ is the number of k-element subsets of an n-element set.*

Exercise 5. *Prove $n!$ is the number of permutations of an n-element set.*

Exercise 6. *Prove*

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}.$$

Exercise 7. *Prove*

$$\binom{n}{k+1} = \binom{n}{k}\frac{n-k}{k+1}.$$

Exercise 8. *Prove*

$$\binom{r}{n}\binom{n}{k} = \binom{r}{k}\binom{r-k}{n-k}.$$