MTH 385 2022-02-21 Worksheet

Exercise 1. *What do we mean when we say a point in $\mathbb{R}^2$ is* rational*?*

Exercise 2. *How does one find (algebraically) the intersection(s) of a curve $p(x, y) = 0$ with a line $y = mx + b$?*

Exercise 3. *If $p(x, y)$ is a degree d polynomial, how many intersections do we expect the curve $p(x, y) = 0$ to have with a line $y = mx + b$?*

Exercise 4. *What is the* chord method*? What is the goal of the method?*

Exercise 5. *Suppose $q(x)$ is a polynomial with rational coefficients. Further suppose $q(x) = k(x - r_1)(x - r_2)$ and r_1 is rational. Prove k and r_2 are also rational.*

Exercise 6. *Prove: If we know two rational points on a cubic curve $p(x, y) = 0$, then a third intersection point on the line through them will also be rational.*

Exercise 7. *What is the* tangent method*? What is the goal of the method? When can it be used?*

Exercise 8. *Consider the curve $x^3 - y = 0$ and the line $y = 3x - 2$. Check that they are tangent at $(1, 1)$. Apply the tangent method.*

Exercise 9. *If the curve $p(x, y) = 0$ has degree 3 and the line $y = mx + b$ is tangent to the curve at some point (x_0, y_0), what can we expect from the solutions to $p(x, mx + b) = 0$?*