MTH 385 2022-02-14 Worksheet

Exercise 1. *Define* greatest common divisor.

Exercise 2. *Define* relatively prime.

Exercise 3. *Use the Euclidean Algorithm* as presented in the textbook *to compute* $\gcd(1001, 65)$.

Exercise 4. *Based on your experience with Exercise 1, suggest a refinement to the algorithm.*

Exercise 5. *Find integers m and n such that* $\gcd(1001, 65) = 1001m + 65n$.

Exercise 6. *Find a counterexample to the following statement: If n is a number that divides ab, then n divides a or b.*

Exercise 7. *Prove: If n is relatively prime to a and n divides ab, then n divides b.*

Exercise 8. *State and prove the* Fundamental Theorem of Arithmetic.

Exercise 9. *State the* Well-Ordering Principle.

Exercise 10 (3.3.1)**.** *Use the prime divisor property to show that the proper divisors of* $2^{n-1}p$*, for any odd prime* p*, are* $1, 2, 2^2, \ldots, 2^{n-1}$ *and* $p, 2p, 2^2p, \ldots, 2^{n-2}p$.

Exercise 11 (3.3.4)**.** *The equation* $12x + 15y = 1$ *has no integer solution. Why?*