MTH 385 2022-03-21 Worksheet

Exercise 1. *According to the textbook, the general* quartic *equation*

$$x^4 + ax^3 + bx^2 + cx + d = 0$$

can be reduced to an equation of the form

$$x^4 + px^2 + qx + r = 0$$

using a linear transformation. Carry out this reduction.

Exercise 2. *Show that the equation*

$$x^4 + px^2 + qx + r = 0$$

can be rewritten

$$(x^2 + p)^2 = px^2 - qx + p^2 - r.$$

Exercise 3. *Deduce from the previous exercise that*

$$(x^2 + p + y)^2 = (p + 2y)x^2 - qx + (p^2 - r + 2py + y^2).$$

Exercise 4. *Give a criterion for determining when*

$$Ax^2 + Bx + C$$

is a square.

Exercise 5. *Apply the criterion from the previous exercise to determine when the equation*

$$(x^2 + p + y)^2 = (p + 2y)x^2 - qx + (p^2 - r + 2py + y^2)$$

has a solution.

Exercise 6. *The answer to the previous exercise is a polynomial equation in y. What is its degree?*

Exercise 7. *Outline a procedure for solving a general quartic equation.*

Exercise 8. *According to the textbook, does a general* quintic *equation have a* solution by radicals*?*

Exercise 9. *State Descartes's Theorem.*

Exercise 10 (5.7.1)**.** *Show that $x^n - a^n$ has a factor $x - a$. What is the quotient $(x^n - a^n)/(x - a)$? (And what does this have to do with geometric series?)*