MTH 470/570
Winter 2021
Assignment 2.

This assignment is due before class on Tuesday, February 2. Please submit your solutions via Blackboard. Solutions are required – answers must be justified.

p.16, #1.27. Sketch the sets defined by the following constraints and determine whether they are open, closed, or neither; bounded; connected. (Proofs are not required.)

(a) $|z+3| < 2$ (b) $|Im(z)| < 1$
(c) $0 < |z-1| < 2$ (d) $|z-1| + |z+1| = 2$
(e) $|z-1| + |z+1| < 3$ (f) $|z| \geq Re(z) + 1$.

#1.33. Find a parametrization for each of the following paths:
(a) the circle $C[1+i, 1]$, oriented counter-clockwise.
(b) the line segment from $-1-i$ to $2i$.
(d) the rectangle with vertices $\pm 1 \pm 2i$, oriented counter-clockwise.

p.31, #2.12. Consider the function $f : \mathbf{C} \setminus \{0\} \to \mathbf{C}$ given by $f(z) = 1/z$. Apply the definition of the derivative to give a direct proof that $f(z) = -1/z^2$.

2.18. For which values of z are the following functions differentiable? Holomorphic? Determine their derivatives at points where they are differentiable. (Recall that you are required to justify all of your answers.)
(a) $f(z) = e^{-x}e^{-iy}$ (c) $f(z) = x^2 + iy^2$.
(g) $f(z) = |z|^2 = x^2 + y^2$ (j) $f(z) = 4(Re\, z)(Im\, z) - i(\overline{z})^2$
Recall that a function f is *holomorphic* at z if f is differentiable at every point in some disk centered at z. $Re\, z$ and $Im\, z$ are the real and imaginary parts of z, respectively.

2.20. Prove: If f is holomorphic in the region $G \subset C$ and always real valued, then f is constant in G. (Hint: Use the Cauchy-Riemann equations to show that $f' = 0$.)