MTH 470/570
Winter 2021
Assignment 6

This assignment is due before class on Tuesday, March 9. Please submit your solutions via Blackboard. Solutions are required – answers must be justified.

Problem Z. Find the value of the constant C that makes the theorem below true, and prove the theorem.

Theorem 5.1. (continued) Suppose f is holomorphic in the region G and γ is a positively oriented, simple, closed, piecewise smooth, G-contractible path. If w is inside γ then $f'''(w)$ exists, and

$$f'''(w) = C \int_\gamma \frac{f(z)}{(z-w)^4} dz.$$

Hint: Follow the argument from class for the cases of f' and f''. (The proof from class is simpler than the proof in the textbook.) The algebraic identity $a^3 - b^3 = (a-b)(a^2+ab+b^2)$ is likely to be helpful.

5.1. Compute the following integrals, where S is the boundary of the square with vertices $-4-4i$, $4-4i$, $4+4i$,$-4+4i$, (oriented counterclockwise).

(a) $\displaystyle\int_S \frac{\exp(z^2)}{z^3}\, dz.$ (b) $\displaystyle\int_S \frac{\exp(3z)}{(z-\pi i)^2}\, dz.$ (e) $\displaystyle\int_S \frac{\sin(z/3)}{(z-\pi)^4}\, dz.$

5.3. Integrate the following functions over the circle C[0, 3]:

(h) $\displaystyle\frac{1}{(z+4)(z^2+1)}.$ (i) $\displaystyle\frac{\exp(2z)}{(z-1)^2(z-2)}$

5.15. Suppose f is entire with bounded real part, i.e., writing $f(z) = u(z) + iv(z)$, there exists $M > 0$ such that $|u(z)| \le M$ for all $z \in \mathbf{C}$. Prove that f is constant. (Hint: Consider the function $\exp(f(z))$.)