MTH 470/570
Winter 2021
Assignment 4.

This assignment is due before class on Tuesday, February 16. Please submit your solutions via Blackboard. Solutions are required – answers must be justified.

p.52 #3.41. Convert the following expressions to the form $x + iy$. (Reason carefully.)
(a) $e^{i\pi}$.   (c) i^i.   (d) $e^{\sin(i)}$.   (e) $\exp(\text{Log}(3 + 4i))$ (f) $(1 + i)^{1/2}$

3.51. Prove that $\exp(b \log a)$ is single valued if and only if b is an integer. (Note that this means that complex exponentials do not clash with monomials z^n, no matter which branch of the logarithm is used.) What can you say if b is rational?

p.68 #4.3. Integrate the function $f(z) = \bar{z}$ over these three paths from Example 4.1:
(a) The line segment from 0 to $1 + i$.
(b) The arc of the parabola $y = x^2$ from 0 to $1 + i$.
(c) The union of the line segment from 0 to 1 and the line segment from 1 to $1 + i$.

4.8. Compute $\int_\gamma f(z)\, dz$ for the following functions f and paths γ:

Hint: All but one of these can be computed using the complex version of the Fundamental Theorem of Calculus.

(a) $f(z) = z^2$ and $\gamma(t) = t + it^2$, $0 \le t \le 1$.
(b) $f(z) = z$ and γ is the semicircle from 1 through i to -1.
(c) $f(z) = \exp(z)$ and γ is the line segment from 0 to a point z_0. (The answer will be a function of z_0.)
(d) $f(z) = |z|^2$ and γ is the line segment from 2 to 3+i.