MTH 470/570
Winter 2021
Assignment 7

This assignment is due before class on Tuesday, April 6.

Please submit your solutions via Blackboard. Solutions are required – answers must be justified.

p.123, #8.5. Find the terms through third order and the radius of convergence of the power series for each of the following functions, centered at z_0 . (Do not find the general form for the coefficients.)

(c) $f(z) = (1+z)^{1/2}$, $z_0 = 0$. (d) $f(z) = \exp(z^2)$, $z_0 = i$.

#8.22.

(a) Find the power series representation for $\exp(az)$ centered at 0, where $a \in \mathbf{C}$ is any constant. (Hint: Substitution!)

(b) Show that $\exp(z)\cos(z) = \frac{1}{2}(\exp((1+i)z) + \exp((1-i)z))$.

(c) Find the power series expansion for $\exp(z)\cos(z)$ centered at 0. (The complete expansion, that is, not just the first few terms.)

#8.32. Find the three Laurent series of $f(z) = \dfrac{3}{(1-z)(z+2)}$, centered at 0, defined on the three regions $|z| < 1$, $1 < |z| < 2$, and $2 < |z|$, respectively. (Hint: Use a partial fraction decomposition.)

Problem H: Find the Laurent series for the function $g(z) = \dfrac{1}{1-\cos z}$ about 0, up through the term $c_6 z^6$. (Hint: This is similar to Example 8.23 in the textbook, which we did by another method in class. Choose your favorite method.)

#8.20. Find the terms $c_n z^n$ in the Laurent series for $\dfrac{1}{\sin^2(z)}$ centered at $z = 0$, for $-4 \le n \le 4$. (Same hint as for the previous problem.)