HW 2

Saturday, January 30, 2021 6:54 PM

470hw2(1)

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MTH 470/570
Winter 2021
Assignment 2.

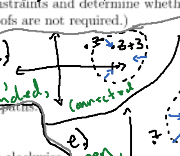
This assignment is due before class on Tuesday, February 2. Please submit your solutions via Blackboard. Solutions are required – answers must be justified.

(Circle, open D, closed D) $\ni C, D, \bar{D}[a, r]$ $\{z \in \mathbb{C} : |z-a| =, <, \leq r\}$

p.16, #1.27. Sketch the sets defined by the following constraints and determine whether they are open, ~~closed~~, or neither; bounded; connected. (Proofs are not required.)

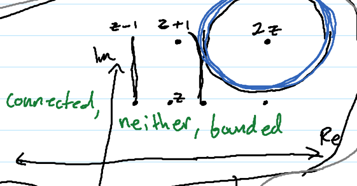
(a) $|z+3| < 2$ (b) $|Im(z)| < 1$
(c) $0 < |z-1| < 2$ (d) $|z-1| + |z+1| = 2$
(e) $|z-1| + |z+1| < 3$ (f) $|z| \geq Re(z) + 1$.

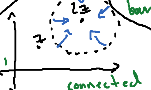
1.27

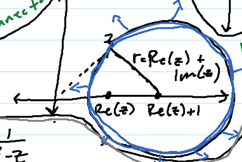
a) 
open, bounded, connected

b) 
open, bounded, connected

c) 
open, bounded, connected

d) 
connected, neither, bounded

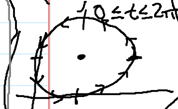
e) 
open, bounded, connected

f.) 
closed, not bounded, connected

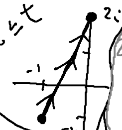
#1.33. Find a parametrization for each of the following paths.
(a) the circle $C[1+i, 1]$, oriented counter-clockwise.
(b) the line segment from $-1-i$ to $2i$.
(d) the rectangle with vertices $\pm 1 \pm 2i$, oriented counter-clockwise.

1.33

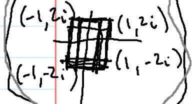
a) $\gamma(t) = (1+i) + e^{it}$, $0 \leq t \leq 2\pi$



b) $\gamma = -1 + t + i(-1+3t)$, $0 \leq t \leq 1$



d. $\gamma = \begin{cases} 1 - r + 2i \\ -1 + i(2-2r) \\ -2i - 1 + r \\ 1 - i(2-2r) \end{cases}$ $0 \leq r \leq 2$



p.31, #2.12. Consider the function $f : \mathbf{C} \setminus \{0\} \to \mathbf{C}$ given by $f(z) = 1/z$. Apply the definition of the derivative to give a direct proof that $f'(z) = -1/z^2$.

2.12

$f = z^n \Rightarrow f_z = (n) z^{(n-1)}$; $f = z^{-1} \Rightarrow f_z = -z^{-2}$; $f_z = \lim_{\zeta \to z} \frac{f(\zeta) - f(z)}{\zeta - z} = \lim \left(\frac{1}{\zeta} - \frac{1}{z}\right) \cdot \frac{1}{\zeta - z}$
$= \lim \frac{z - \zeta}{\zeta z} \cdot \frac{1}{(\zeta - z)} = \lim \frac{-(\zeta - z)}{\zeta z} \cdot \frac{1}{(\zeta - z)} = \lim_{\zeta \to z} \frac{-1}{\zeta z} = \frac{-1}{z^2} = f_z$

2.18. For which values of z are the following functions differentiable? Holomorphic? Determine their derivatives at points where they are differentiable. (Recall that you are required to justify all of your answers.)

a)

(a) $f(z) = e^{-x}e^{-iy}$ (c) $f(z) = x^2 + iy^2$.
(g) $f(z) = |z|^2 = x^2 + y^2$ (j) $f(z) = 4(Re\, z)(Im\, z) - i(\bar{z})^2$

Recall that a function f is *holomorphic* at z if f is differentiable at every point in some disk centered at z. $Re\, z$ and $Im\, z$ are the real and imaginary parts of z, respectively.

2.20. Prove: If f is holomorphic in the region $G \subset C$ and always real valued, then f is constant in G. (Hint: Use the Cauchy-Riemann equations to show that $f' = 0$.)