MTH 470/570
Winter 2021
Assignment 3.

This assignment is due before class on Tuesday, February 9. Please submit your solutions via Blackboard. Solutions are required – answers must be justified.

p.62, #3.1. Show that if $f(z) = \dfrac{az+b}{cz+d}$ is a Mobius transformation, then
$$f^{-1}(z) = \frac{dz-b}{-cz+a}.$$

#3.7. Show that the Mobius transformation $f(z) = \dfrac{1+z}{1-z}$ maps the unit circle (minus the point z = 1) onto the imaginary axis.

#3.14. Find Mobius transformations satisfying each of the following. Write your answers in standard form, as $\dfrac{az+b}{cz+d}$.

(a) $1 \to 0,\ 2 \to 1,\ 3 \to \infty$.

(b) $1 \to 0,\ 1+i \to 1,\ 2 \to \infty$

(c) $0 \to i$, $1 \to 1$, $\infty \to -i$.

3.30. Prove that $\overline{\sin(z)} = \sin(\overline{z})$ and $\overline{\cos(z)} = \cos(\overline{z})$.

3.32. Prove that the zeros of $\sin z$ are all real numbers. Conclude that they are precisely the integer multiples of π.

Problem I. Let C be the circle with center a and radius r. Suppose that 0 does not lie on C. (In other words, 0 could be inside or outside of the circle.) Show that the image of C under the function $f(z) = 1/z$ is the circle with center $\dfrac{\overline{a}}{|a|^2 - |r|^2}$ and radius $\dfrac{r}{|\,|a|^2 - r^2\,|}$.

Hints: This is an exercise in complex algebra. Start by writing down the equations for both circles. Use the fact that, for any complex number w, $w\overline{w} = |w|^2$, to eliminate all of the complex norms in both equations. Starting from the equation of the second circle, clear all of the fractions and factor out a common term to work toward the equation of the first circle.