CS 156a Set 2

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The following algorithm was used to answer questions 1-2

```
import random
import numpy

def coinflip():
    return int(numpy.floor(random.randint(0,1)))


v_1 = 0
v_rand = 0
v_min = 0

for x in range(0, 100000):
    if x%1000 == 0:
    freq_1000 = [0] * 1000
    for i in range(0, 1000):
        coin_10 = [0] * 10
        for q in range(0,10):
            coin_10[q] = coinflip()
        heads_frequency = sum(coin_10)/10
        freq_1000[i] = heads_frequency
    v_1 = v_1 + freq_1000[0]/100000
    v_rand = v_rand + random.choice(freq_1000)/100000
    v_min = v_min + min(freq_1000)/100000

print(v_1)
print(v_rand)
print(v_min)
```

problem 1

After running the algorithm above, the average value for $v_m in$ over 100000 iterations was 0.037869. This is closest to 0.01.

The answer is [b]

problem 2

After running the algorithm above, the average values for v_1, v_{rand}, and v_{min} were:

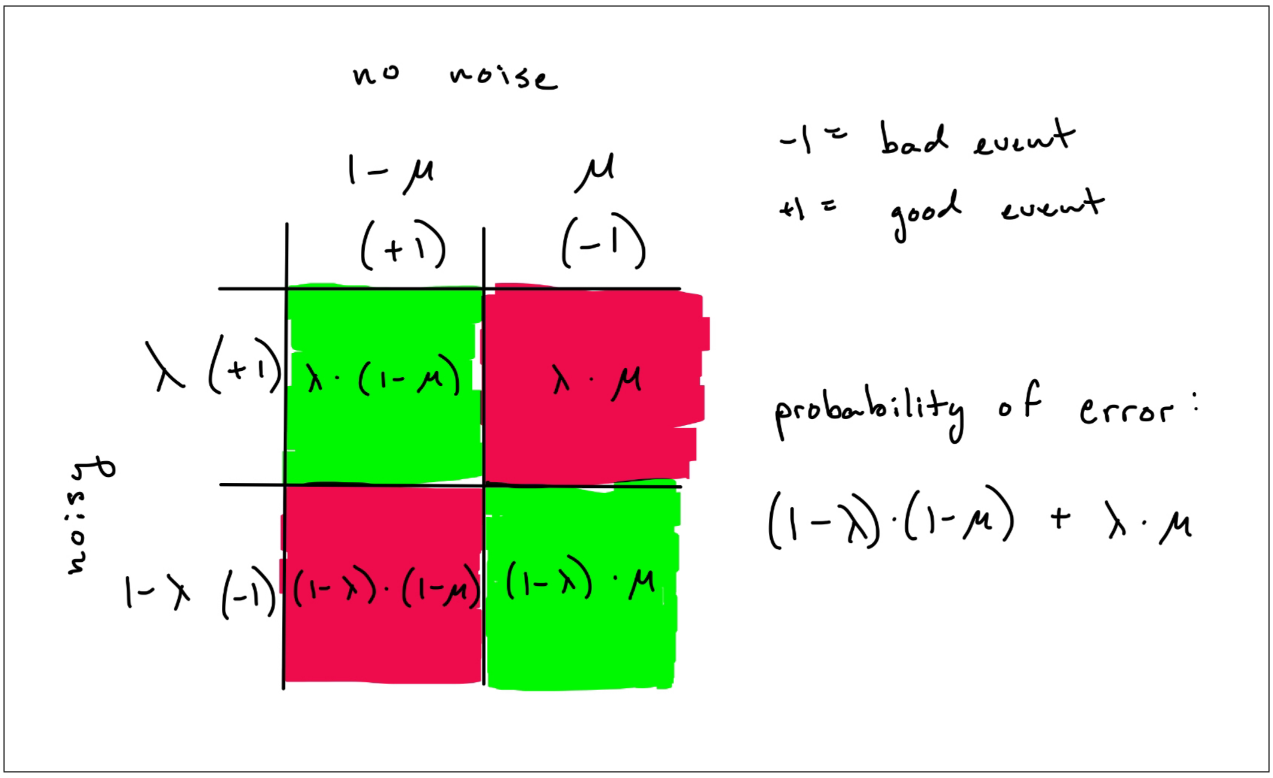
$$v_1 = 0.4998539999997137$$
$$v_{rand} = 0.5009359999997081$$
$$v_{min} = 0.03786900000001721$$

In order for a distribution of v to satisfy the Hoeffding (single-bin) Inequality, v must be close to the true probability of flipping a coin and getting heads. Thus, the only two values that satisfy this are v_1 and v_{rand}.

The answer is [d]

problem 3

Since we are interested in the probability of error that h makes in approximating y given the noisy data, we must consider the event in which we either get a false-positive or false-negative result by the confusion matrix below. By adding the error boxes, we get the probability of error: $(1 - \lambda) * (1 - \mu) + \lambda * \mu$



The answer is [e]

problem 4

To find when h is independent of μ, we must find the value of λ where μ can be eliminated from the equation:

$$\epsilon = (1-\lambda)*(1-\mu)+\lambda*\mu$$
$$\epsilon = 1-\mu-\lambda+\lambda\mu+\lambda\mu$$
$$\epsilon = 1-\mu*(2\lambda-1)$$

To eliminate μ, we can set $2\lambda-1$ equal to 0 and solve for λ. This gives us:

$$\lambda = 0.5$$

The answer is [b]

The following Linear Regression algorithm was used/manipulated to answer questions 5-10. The algorithm was made into one function so that it could be iterated 1000 times with ease later.

```
from random import uniform
import numpy
import random

def Lin_Reg():
    #SETUP
    """x_1 = uniform(-1, 1)
    y_1 = uniform(-1, 1)
    x_2 = uniform(-1, 1)
    y_2 = uniform(-1, 1)

    a, b = numpy.polyfit([x_1, y_1], [x_2, y_2], 1)"""

    def f(x, y): #target function
        return (x*x + y*y - .6)

    def sign(x):
        if x == 0:
            return 0
        elif x > 0:
            return 1
        else:
            return -1

    def count_diff(list_1, list_2):
        count = 0
        for x in range(0, len(list_1)):
            if list_1[x] != list_2[x]:
                count += 1
        return count

    N = 1000
    y_target = [0] * N
    point_list = [] * N
    classification_list = [0] * N
    PLA = [0] * N

    for i in range(0,N):
        temp_x1 = uniform(-1, 1)
        temp_x2 = uniform(-1, 1)
        point_list.append([1, temp_x1, temp_x2, temp_x1*temp_x2, temp_x1*temp_x1, temp_x2*temp_x2])
        y_target[i] = f(point_list[i][1], point_list[i][2])

    y_data = [x[2] for x in point_list]

    for i in range(0,N):
        if sign(y_target[i]) == 1:
            classification_list[i] = 1
        else:
            classification_list[i] = -1

    idx_array = random.sample(range(0, 1000), 100)
    for x in range(0, 100):
        classification_list[int(idx_array[x])] = classification_list[int(idx_array[x])] * -1

    idx_array = 1000 * numpy.random.sample((100,))
    X_points = numpy.array(point_list)
    X_dagger = numpy.matmul(numpy.linalg.pinv(numpy.matmul(X_points.T, X_points)), X_points.T)
    w = numpy.matmul(X_dagger, numpy.array(classification_list))

    E_in = (1/N)*numpy.dot(numpy.subtract(numpy.matmul(X_points, w), classification_list),numpy.subtrac

    """
    itter_count = 0
    while PLA != classification_list:
        itter_count = itter_count + 1
        wrong = []
```

```
        for p in range(0, N):
            if classification_list[p] != PLA[p]:
                wrong.append(p)
        index_point = random.choice(wrong)
        random_point_x = point_list[index_point]
        #grab sign of correct classification
        temp_sign = classification_list[index_point]
        # unpack and repack weight tuple
        w = [w[0] + temp_sign*random_point_x[0], w[1] + temp_sign*random_point_x[1], w[2] + temp_sign*ra
        PLA = [0] *N
        for idx in range(0, len(PLA)):
            if numpy.dot(point_list[idx], w) > 0:
                PLA[idx] = 1
            else:
                PLA[idx] = -1
                """

    y_correct = [0] * 1000
    points = []
    classification = [0] * 1000
    classification_check = [0]*1000

    for i in range(0,1000):
            temp_x1 = uniform(-1, 1)
            temp_x2 = uniform(-1, 1)
            points.append([1, temp_x1, temp_x2, temp_x1*temp_x2, temp_x1*temp_x1, temp_x2*temp_x2])
            y_correct[i] = f(points[i][1], points[i][2])

    y_random = [x[2] for x in points]

    for i in range(0,1000):
        if sign(y_correct[i]) == 1:
            classification[i] = 1
        else:
            classification[i] = -1

    idx_array = random.sample(range(0, 1000), 100)
    for x in range(0, 100):
        classification[int(idx_array[x])] = classification[int(idx_array[x])] * -1

    for idx in range(0, len(classification_check)):
        if numpy.dot(points[idx], w) > 0:
            classification_check[idx] = 1
        else:
            classification_check[idx] = -1

    diff = count_diff(classification, classification_check)
    E_out = diff/1000


    return (E_out, E_in, w)
```

problem 5

After 1000 iterations, my algorithm got an E_{in} of 0.0389, which is closest to 0.01.

The answer is [c]

problem 6

After 1000 iterations, my algorithm got an E_{out} of 0.04852, which is closest to 0.01.

The answer is [c]

problem 7

After processing the PLA using the w value determined by the linear regression algorithm, the PLA converged in an average of 5.9 iterations over 1000 trials. This is closest to 1.

The answer is [a]

problem 8

After carrying out the linear regression on the new target function with the added noise, I got an average value of 0.6974350480139204 for E_{in} over 1000 trials. This is closest to 0.8.

The answer is [e]

problem 9

After transforming the w vector to $\tilde{w}$ and running this algorithm 1000 times, the average $\tilde{w}$ values were:

$$\tilde{w} = (-.9916, -.0009, -.0002, .0054, 1.560, 1.557)$$

These values are closest to the coefficients of the function of [a].

The answer is [a]

problem 10

After running this algorithm on a new set of data outside the original sample, including noise, the average E_{out} over 1000 trials was 0.126 which is closest to 0.1.

The answer is [b]