CS 156a FINAL

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January 18, 2023

problem 1

We will use the formula $d = \frac{Q(Q+3)}{2}$. Since $Q = 10$, we get $d = \frac{130}{2} = 65$. This is not an option.

The answer is [e]

problem 2

Consider the average of two logistic functions. Sometimes the results will simply not be logistic.

The answer is [d]

problem 3

In this instance, d is false. This is because we cannot generalize to all instances. When over fitting occurs, we have a hypothesis that generates a very small E_{in}, but a large E_{out}. Consider we have a small training set, but a very large test set. In this case we can expect our hypothesis function to be very well fitted to the training data, thus E_{in} will be very small. However, due to our small training set, the E_{out} could be very large. In this case, we might be wary that our data is over fitted, but a small training set prevents this determination. Thus, we cannot *always* determine if the data is over fitted through comparing E_{in} and E_{out}.

The answer is [d]

problem 4

Stochastic noise is random noise that is a part of the target function we are trying to evaluate, where as we view deterministic noise as a byproduct of the hypothesis we choose. Thus, stochastic noise is independent of the hypothesis set.

The answer is [d]

problem 5

We know that $w_{reg}^T w_{reg} = C$. So, from the textbook, we know that $w_{reg} = w_{lin}$ to satisfy the equation.

The answer is [a]

problem 6

It is known that augmented error constraints can be used to measure the goodness of a soft-order constraint. Thus, soft-order constraints can be translated into augmented error.

The answer is [b]

problem 7

After running the code attached, the lowest E_{in} was found for the 8-versus-all classifier.

(a) $E_{in} = 0.076$

(b) $E_{in} = 0.091$

(c) $E_{in} = 0.088$

(d) $E_{in} = 0.074$

(e) $E_{in} = 0.088$

The answer is [d]

problem 8

After running the code attached, the lowest E_{out} was found for the 1-versus-all classifier.

(a) $E_{out} = 0.107$

(b) $E_{out} = 0.022$

(c) $E_{out} = 0.099$

(d) $E_{out} = 0.083$

(e) $E_{out} = 0.100$

The answer is [b]

problem 9

Using the 5-versus-all classifier, the following E_{out} was produced:

Non-Transformed: $E_{out} = 0.079\mathbf{72097658196313}$

Transformed: $E_{out} = 0.079\mathbf{22272047832586}$

The transformed improved the 5-versus-all out-of-sample performance by 0.6% Thus, e is true.

The answer is [e]

problem 10

After running the code attached, the following values for E_{in} and E_{out} was produced:

Lambda: 0.01

E_{out} : 0.0283

E_{in} : 0.0045

Lambda: 1

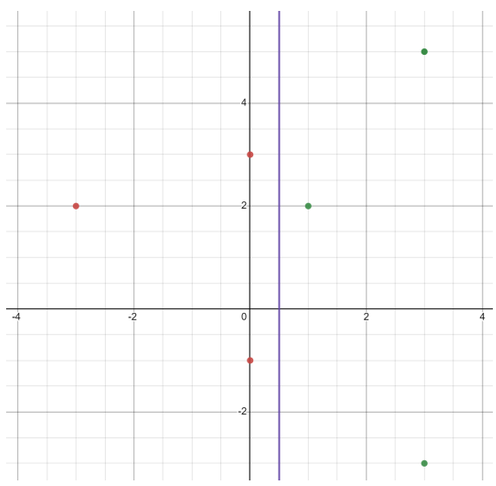
E_{out} : 0.0259

E_{in} : 0.0051

It follows that a is correct. As you can see, when $\lambda = 1$, the E_{in} was higher than when $\lambda = 0.01$, but the E_{out} was lower. This is a sign of over fitting. In this case, we would prefer the classifier for $\lambda = 1$, as this results in a lower E_{out}.

The answer is [a]

problem 11



When the transformed points are graphed, we see that each classifier lies on a different side of the line $x = 0.5$. Thus, only the z_1 value for each point matters. To the right of the line, the points are classified as $+1$, and -1 on the other side, thus, w_{z_1} is 1, and w_{z_2} is 0. The bias is -0.5 as this is the shift of the separating plane. Thus, $w_1 = 1, w_2 = 0, b = -0.5$.

The answer is [c]

problem 12

See code attached. I get 5 support vectors.

The answer is [c]

problem 13

See code attached. For 100 iterations, the data set generated was never not separable by the RBF kernel. (0% of the time)

The answer is [a]

problem 14

See code attached. For $K = 9$, and $\gamma = 1.5$, the kernel form had a lower E_{out} than the regular form 78% of the time.

The answer is [e]

problem 15

See code attached. For $K = 12$, and $\gamma = 1.5$, the kernel form had a lower E_{out} than the regular form 62.6% of the time.

The answer is [d]

problem 16

See code attached. The frequency of each happening (over 1000 iterations) were as follows:

(a) 190

(b) 85

(c) 78

(d) 420

(e) 16

The answer is [d]

problem 17

See code attached. The frequency of each happening (over 1000 iterations) were as follows:

(a) 138

(b) 174

(c) 284

(d) 173

(e) 12

The answer is [c]

problem 18

See attached code. When run over 1000 iterations using $K = 0$, and $\gamma = 1.5$, the regular RBF achieved $E_{in} = 0$, 1.4% of the time.

The answer is [a]

problem 19

Since f is unknown across the interval $[0, 1]$, and since our random selection had a heart attack, we can describe the posterior probability. Lets say h is closer to 0, $I\!P(h = f)$ will be much smaller than when h is near 1. Thus the posterior probability is not uniform. However, as each successive step (regardless of step size) towards 1 from 0 will result in $I\!P(h = f)$ to also increase by the same step size. Thus, the posterior probability ($I\!P(h = f)$) is linear over $f \in [0, 1]$

The answer is [b]

problem 20

Since we are using mean-squared error, consider some point $\mathbf{x}$ that produces E_{g_1} and E_{g_2}. Since g is the aggregate of g_1 and g_2, we know that g($\mathbf{x}$) will lie somewhere between $g_1(\mathbf{x})$ and $g_2(\mathbf{x})$. Thus, $E_g(\mathbf{x})$ will also lie between E_{g_1} and E_{g_2}. Thus, $E_{out}(g)$ cannot be worse than the average of $E_{out}(g_1)$ and $E_{out}(g_2)$.

The answer is [c]

The following code was used to answer questions 7-9:

```
import numpy

def versus_all(n, data_set):
    #splits data and classification info
    data = [[1, data_set[x][1], data_set[x][2], data_set[x][1]*data_set[x][2], data_set[x][1]*data_set[
    classifier = [1 if x[0] == n else -1 for x in data_set]
    return classifier, data

lam = 1

for i in [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]:
    class_i, data_i = versus_all(i, train_data)

    X_points = numpy.array(data_i)
    class_array = numpy.array(class_i)
    first_half = numpy.linalg.pinv(numpy.add(numpy.matmul(X_points.T, X_points), lam*numpy.identity(6))
    w = numpy.matmul(first_half, numpy.matmul(X_points.T, class_array))


    true_class, test_data_i = versus_all(i, test_data)

    test_class_calc = [sign(numpy.dot(test_data_i[x], w)) for x in range(0, len(test_data_i))]


    E_out = (count_diff(true_class, test_class_calc))/len(true_class)
```

The following code was used to answer question 10:

```
import numpy

lam = 1

train_1v5 = []
test_1v5 = []

for x in train_data:
    if (x[0] == 1 or x[0] == 5):
        train_1v5.append(x)

for x in test_data:
    if (x[0] == 1 or x[0] == 5):
        test_1v5.append(x)



for lam in [0.01, 1]:
    class_i, data_i = versus_all(1, train_1v5)

    X_points = numpy.array(data_i)
    class_array = numpy.array(class_i)
    first_half = numpy.linalg.pinv(numpy.add(numpy.matmul(X_points.T, X_points), lam*numpy.identity(6))
    w = numpy.matmul(first_half, numpy.matmul(X_points.T, class_array))

    true_class, test_data_i = versus_all(1, test_1v5)

    test_class_calc = [sign(numpy.dot(test_data_i[x], w)) for x in range(0, len(test_data_i))]

    E_out = (count_diff(true_class, test_class_calc))/len(true_class)
    E_in = count_diff([sign(numpy.dot(data_i[x], w)) for x in range(0, len(test_data_i))], class_i)/len

    print("Lamda: "+str(lam))
    print("E_out: "+str(E_out))
    print("E_in: "+str(E_in))
```

The following code was used to answer question 12:

```
from sklearn import svm
import numpy
from math import inf

data = [[1, 0], [0, 1], [0, -1], [-1, 0], [0, 2], [0, -2], [-2, 0]]
y = [-1, -1, -1, 1, 1, 1, 1]

clf = svm.SVC(kernel="poly", C=inf, gamma=1, coef0=1)
clf.fit(data, y)
print(clf.support_vectors_)
```

The following code was used to answer questions 13-15:

```
import numpy
from random import uniform
from sklearn import svm, cluster
import math

N = 100
gam = 1.5
clusters = 12
iterations = 1000

kernel_wins = 0

for i in range(0, iterations):
    data = [[uniform(-1, 1), uniform(-1, 1)] for x in range(0, N)]
    data_out = [[uniform(-1, 1), uniform(-1, 1)] for x in range(0, N)]

    def f(point):
        return sign(point[1] - point[0] + 0.25 * numpy.sin(numpy.pi*point[0]))

    class_data = [f(point) for point in data]
    class_data_out = [f(point) for point in data_out]

    clf = svm.SVC(kernel="rbf", C=math.inf, gamma=gam, coef0 = 1)
    clf.fit(data, class_data)

    Kmeans = cluster.KMeans(n_clusters=clusters, init='random').fit(data)

    for i in range(0,clusters):
        if (i not in Kmeans.labels_):
            iterations += 1
            continue

    phi = numpy.array(build_phi(data, clusters, Kmeans.cluster_centers_))
    class_data = numpy.array(class_data)

    w = numpy.matmul(numpy.matmul(numpy.linalg.pinv(numpy.matmul(phi.T, phi)), phi.T), class_data.T)

    #KMEANS OUT PREDICTION
    phi_out = numpy.array(build_phi(data_out, clusters, Kmeans.cluster_centers_))

    y_out = numpy.matmul(phi_out, w.T)
    y_out = [sign(x) for x in y_out]

    E_out_Kmeans = count_diff(y_out, class_data_out)/len(y_out)
    E_out_RBF = count_diff(clf.predict(data_out), class_data_out)/len(data_out)

    if E_out_RBF < E_out_Kmeans:
        kernel_wins += 1/iterations

def build_phi(points, clusters, centers):
    #BUILD PHI TO GET WEIGHTS
    phi = []
    for m in range(0, len(points)):
        point = points[m]
        row = []
        for n in range(0, clusters):
            center = centers[n]
            temp = numpy.exp(gam*-1 * numpy.linalg.norm(numpy.subtract(point, center))* numpy.linalg.no
            row.append(temp)
        phi.append(row)
    return phi

print(kernel_wins)
```

The following code was used to answer questions 16-18:

```
import numpy
from random import uniform
from sklearn import svm, cluster
import math

N = 100
gam = 1.5
#clusters = 9
iterations = 1000

answer = [0, 0, 0, 0, 0]

E_i = 0
for i in range(0, iterations):
    compare = []
    for clusters in [9]:
        data = [[uniform(-1, 1), uniform(-1, 1)] for x in range(0, N)]
        data_out = [[uniform(-1, 1), uniform(-1, 1)] for x in range(0, N)]

        def f(point):
            return sign(point[1] - point[0] + 0.25 * numpy.sin(numpy.pi*point[0]))

        class_data = [f(point) for point in data]
        class_data_out = [f(point) for point in data_out]

        """clf = svm.SVC(kernel="rbf", C=math.inf, gamma=gam, coef0 = 1)
        clf.fit(data, class_data)  """

        Kmeans = cluster.KMeans(n_clusters=clusters, init='random').fit(data)

        for i in range(0,clusters):
            if (i not in Kmeans.labels_):
                iterations += 1
                continue

        phi = numpy.array(build_phi(data, clusters, Kmeans.cluster_centers_))
        class_data = numpy.array(class_data)

        w = numpy.matmul(numpy.matmul(numpy.linalg.pinv(numpy.matmul(phi.T, phi)), phi.T), class_data.T

        #KMEANS OUT PREDICTION
        phi_out = numpy.array(build_phi(data_out, clusters, Kmeans.cluster_centers_))

        y_out = numpy.matmul(phi_out, w.T)
        y_out = [sign(x) for x in y_out]

        y_in = numpy.matmul(phi, w.T)
        y_in = [sign(x) for x in y_in]

        E_out_Kmeans = count_diff(y_out, class_data_out)/len(y_out)
        E_in_Kmeans = count_diff(y_in, class_data)/len(y_in)
        compare.append(E_out_Kmeans)
        compare.append(E_in_Kmeans)

    if compare[1] == 0:
        E_i += 1/iterations

    E_in_goes_up = compare[1] < compare[3]
    E_in_goes_down = compare[1] > compare[3]
    E_out_goes_up = compare[0] < compare[2]
    E_out_goes_down = compare[0] > compare[2]
    no_change = bool((compare[1] == compare[3]) & (compare[0] == compare[2]))
    if E_in_goes_down & E_out_goes_up:
        answer[0] += 1

    if E_in_goes_up & E_out_goes_down:
        answer[1] += 1

    if E_in_goes_up & E_out_goes_up:
        answer[2] += 1
```

```
    if E_in_goes_down & E_out_goes_down:
        answer[3] += 1

    if no_change:
        answer[4] += 1
print(answer)
```